CFD APPLICATIONS IN CHEMICAL PROCESSES

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Week-05
Lecture 22: Finite Volume Method

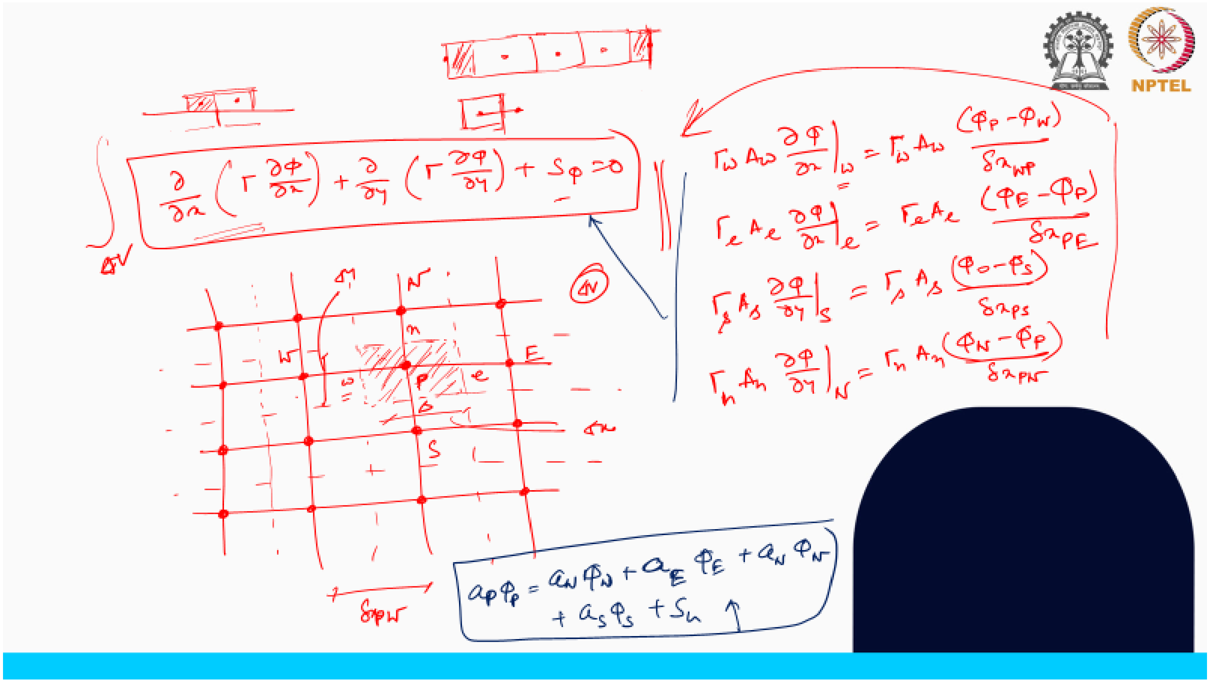
Hello everyone, welcome back with another lecture on finite volume method in the course CFD applications in chemical process. So, in the last lecture I discussed about two different strategies on creating the meshes in finite volume method and how it is applied based on its pros and cons. Regarding the strategy in one case I mentioned that if you have two different boundary condition at a particular boundary. In that case if you require the control that I will have my control volume like this. So, that I need not bother about one control volume sharing my two different boundary condition.

and accordingly the propagation of the boundary condition value into the problem. And that we overcome by creating the control volume first and then placing the grid points at the center of it. But by doing Again, the drawback, let me repeat it, the drawback is the faces may not be exactly at the center, at the midway between the two grid points. So, the flask approximation or the discretization have to be in a proper way or more appropriate way.

The other thing is that if we draw the grid points at first, So, that we have seen earlier that leads to half of the control volume at this boundary conditions at the near it to the boundary conditions. So, this is the half of it. So, after this control volume we can have say this is my the other point. So, we have seen this during the previous examples when we did that that the solution of this half of the control volumes would require your special attention.

So, that means drawing the grid points first can also lead to creation of the half of the control volume. That also can be eliminated by placing the control volumes at first coinciding with the boundary values and then you have one control volume full in throughout the domain. but the major disadvantage of this process is that the approximation of the flask at the faces are not appropriate by the central differencing scheme because the faces are not at the center of the two nodal points. So, this is the part of understanding 2 different strategies that we discussed in the last class.

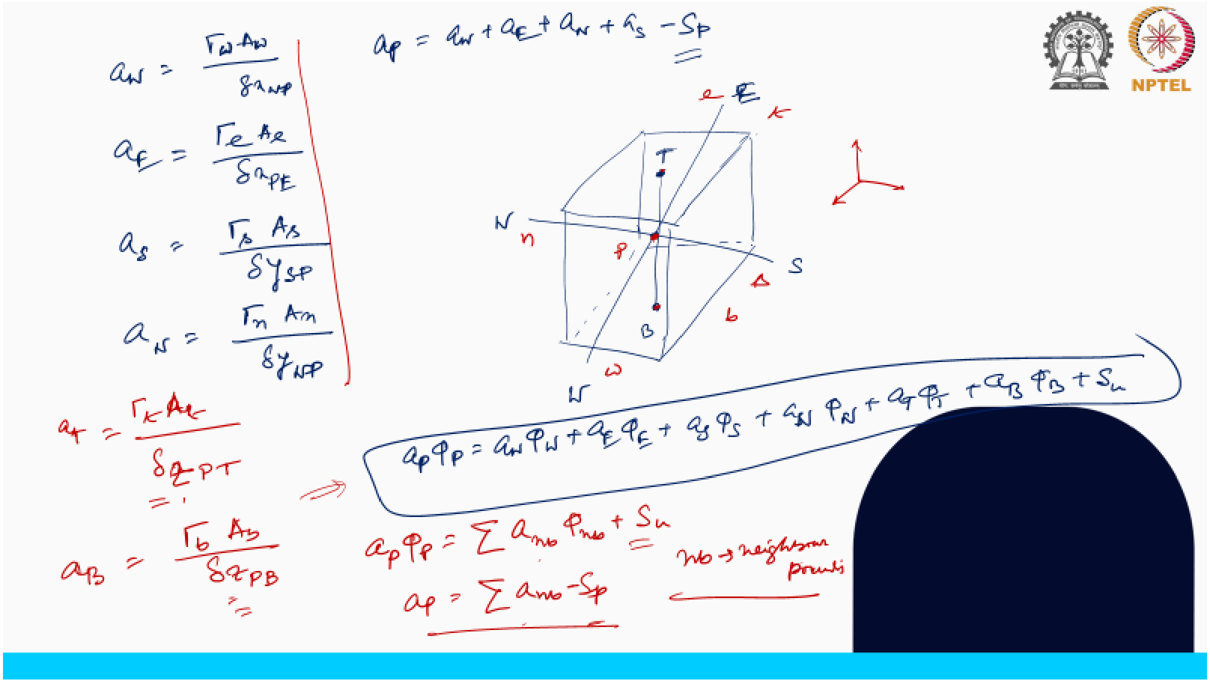
Now, as I told you that we are always looking at one dimensional problem or formulations, but what will happen its implementation the finite volume method implementation in the case of 2D ok. So, in case of 2D say I have again the steady state equation. So, this is a two-dimensional governing equation of variable phi, gamma is the diffusivity value. So, in case of heat or the thermal temperature when phi is temperature this becomes a two dimensional steady state conduction equation with any source term if that appears is phi is not equals to 0 and gamma is the thermal diffusion thermal conductivity in this case.



So, in this case again like we have The grid points are for the sake of say simplicity we consider the grid points are equispaced. So, this solid lines that I have drawn and its junctions are essentially the nodal points or the grid points the nodes. So, and then exactly at the mid faces say we draw the control volumes. So, for point P say consider this is my point P I have this as the control volume.

where this is my east, this is my west nodal points, this is east face, this is west face and this is now the south face or the south point and this is the north point of this control volume. This is the control volume where this is point P. Now in this case so this is this distance say is Δy and this distance is Δx. So, say consider if Δx is equal to Δy for the sake of simplicity then the problem becomes easier in the formulation. But in general what will happen is that this is my north face and this is the small s which is representing the south face of this control volume.

So, in this case the similar to the one dimensional case for this control volume of delta V we integrate this governing equation to have its discretized form and converting it to the area integral.



So, again that means we integrate it over the control volume which is say delta V and then what happens is that in just like the previous case at the waste phase this is the approximation that we have done is a w phi p minus phi w divided by delta x w p if you remember. So, on the waste phase approximation of this becomes this del phi by del x at the waste phase is that

I have phi p minus phi w divided by the distance between the 2 delta x p w. So, similarly what I have here for the these are the 1 dimensional cases we have seen in these 2 part or these 2 expression. phi e minus phi p divided by delta x p capital E. So, in the first 1D we have seen this part, but now since we have 2D the north and south are also required to be discretized. Now, here this del phi by del y for the south is similar to what? So, this is I say phi p minus phi s divided by the delta x p s and

minus phi p divided by x p n just the similar way we did it for the one dimensional case and then we replace all these in this equation to find the form that would look like what we have done previously also Is that now here I have AP phi P is equals to AW phi W plus this is capital W plus AE phi capital E plus AN phi N plus AS phi S plus SU. So, after replacing all these terms here in the governing equation we will find a form the generic form for all the nodal points inside the domain in this form just like the one dimensional case.

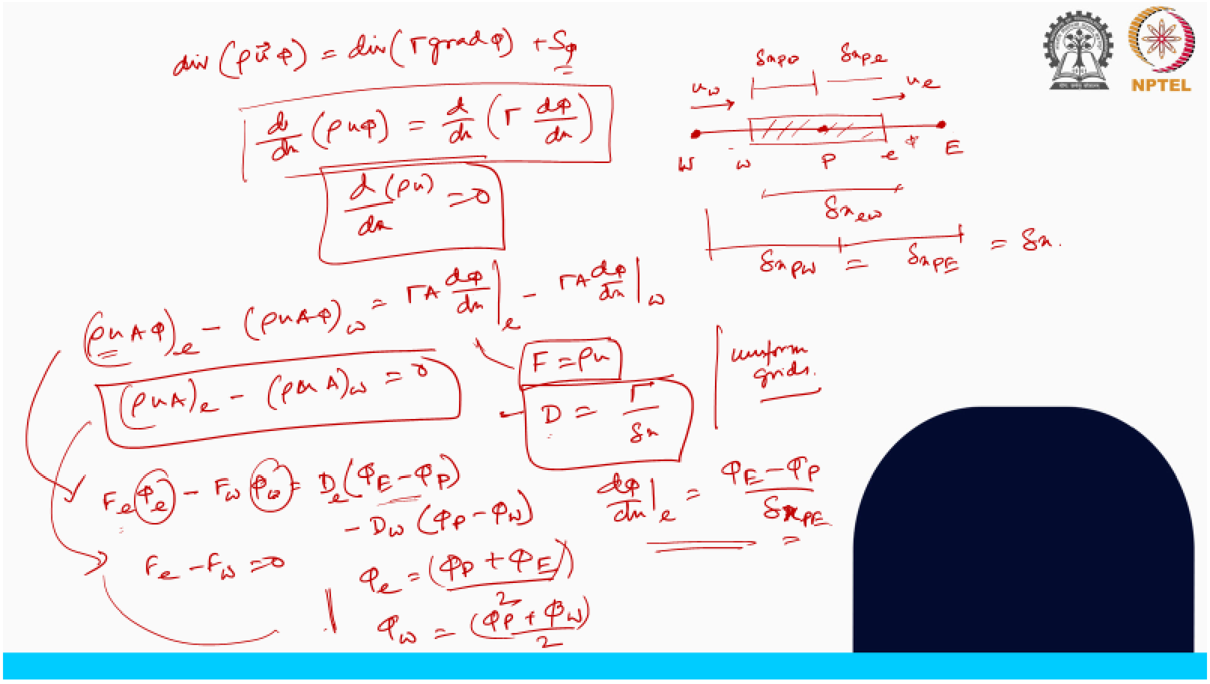
do this and you will see these expressions or you will find this expression which also readily you can find out. So, this divided by the delta x W p. So, these are the small w and this is the capital W p. A e is delta x p e. A S now these are written remember in terms of. So, this is capital S S P which is the same as P S these are written in generic form. Considering that this areas are also different in different directions that means in x direction or the y direction, but most of the problem or the problems that we see or take for solution has uniform surfaces areas. that on the east side west side we consider this as the uniform size or uniform cross section control volumes. In that case AE, AW, AN, AS all are equals to A. Why this is NP and eventually AP would become This AW plus AE plus AN plus AS minus SP where again you have to segregate this term SP from the problem that you have in hand. So, essentially it becomes a similar way of developing, but now for one nodal point instead of two neighbor points, you have four neighbor points.

So, each expression would contain those four neighbor points. And in case of 3D, the situation is further Because in that case what you have that if your point is here say for example P you definitely have say north south east west and then along with that you have top and bottom. So, south and north this is your east or say the capital E and this is the west face or the west nodal points outside it and you have another two surfaces or the two faces top and bottom. So, in this case the formation would be A p phi p is equals to A w phi w

plus A e phi e plus A s phi s A n phi n plus A top plus bottom plus S u. So, all these coefficients you will have for a 3D control volume. the strategy remains same the strategy remains same just like the one dimensional which is further complicated by the presence of two dimension and in this case it will be a three dimension. So, for three dimensional cases again if you try to segregate this variable or try to find out this coefficients you have similar to this that we have done in addition you will have and AB where AT will be capital A this is the top this face is then the top face which we designate small t and this is say small b just like we had say south, north, west and east faces. So in this case, this will be phi small b AB.

Sorry, the denominator in this case is delta x p t. And in this case, delta z because we have now three dimensions x, y and z. So, this delta x p t similarly a b would be lambda b a b divided by again this is delta z delta z p t and this is delta z p and you have the b distance between p and b nodal points in the z direction. So, top and bottom are in the z direction. So, delta z p t the distance between the point p and t and in this case distance between the point p and the bottom point. So, eventually or compactly what we can write A p phi p is essentially for all the dimensions that we see is in this form.

where AP is nothing but summation of ANB minus SP which is NB stands for here the neighbor points. So, A p phi p is the summation of all neighbor points A phi. A n b phi n b summation of all these if it is in one dimension then you have east west if it is in two dimension along with east west you have north and south and in case of three dimension along with this four you have additional two which is top and bottom. plus the segregated term that is not dependent on these things are the SU, the source term.



Again this may not be actual source term in the problem, but after segregation this acts as if this is a pseudo source term. And in this case the coefficient AP is essentially the summation of all neighbor points coefficients minus the SP again depending on whether this SP is the dependent on the phi or not that specific term that we segregate that we have seen earlier. So, I hope it is now clear to you. that how the finite volume is implemented for a three-dimensional cases.

Now, you understand that this is for only one cell or the mesh or the control volume that results from developing cells or the meshes. you have such multiple grid points or in actual cases when you will have millions of points or millions of cells or in term control volume. In those cases you will have such kind of generic expressions those many algebraic equations that you have to solve either simultaneously or in a iterative way depending on the formulation that you choose. So, I hope the complexity is now clear to you that why such big problems are then are difficult just to solve by a simple computer. Because for the complex geometry you will have possibly millions of points, thousands of points and those have to be accurately calculated depending on the strategy you choose.

Depending on the strategy, we will discuss those later. We already discussed the explicit and implicit formulation, but we will also see how discretization schemes affect or influences the accuracy of the result. But, before that let me introduce this convection diffusion problem that will solve or we will see that how now finite volume is applied in such cases. So, for convection advection the diffusion basically the diffusive term we have seen in terms of convective flow over a pin fin, but say you have divergence of grad phi is equals to S phi. So say I have a form the governing equation that has a form of this everything has the standard nomenclature y is any scalar variable u we have a velocity vector of the velocity field

we have gamma grad phi and the source term. Let us say specific thing so that you can quickly understand that let us drop for the time being this S phi. So, if we have a one dimensional problem for convection of or the advection of phi here specifically advection of phi. So, it is one dimensional in x direction whatever is advected is balanced by the diffusion here. So, in this case this equation is not say a standalone manner it can be solved along with that we have to solve its continuity equation that is

is equals to 0 whatever comes in that goes out we have other continuity expression. So, these two are to be solved together for a domain that how phi is transported in the domain by advection and diffusion. and this property is conserved the flow is conserved. So, in this case again I draw that one dimensional case or the nodal point designation. So, we have a control volume like this

This is point P. This is my west face. This is the east face. So now what I have that U is now being transported. Goes through the west side comes out of the east side or the east face. again this distance is delta x p small w this is delta x p small e this is your delta x small e this distance is delta x p w and this is delta x p capital So, what we will do?

We will integrate this governing equations over this control volume like we have done. So, for the first case what we have rho u phi again the volume integral to area integral and then we are approximating rho u A phi at the west face is equals to lambda A d phi by dx at east face lambda A d phi by dx at the west face. And the second equation what we have? this is indeed the continuity form ok.

So, now, here for the sake of brevity consider that I have rho u I write rho u as the term ok and the diffusive term I write gamma by delta x and I replace this 2 in this 2 equations. So, what I have been this is the f and I consider I have a uniform cross section or the uniform grid. So, all a are dropped from both the hand both the sides. So, what do I have?

f e phi e minus f w phi w is equals to this is the term that I have here as d. So, d and then I discretize this part d phi by dx is I consider that d e minus phi e minus phi p minus. So, this is the E component and this is the waste phase phi P minus phi W capital W. If I consider D as gamma by delta x that delta x is absorbed from this expression because I have e essentially phi e minus phi p divided by the delta x p e and I am considering all these distances are equals to delta x. So, this is the first equation. The second equation simply becomes f e is equals to f w.

Now the point is remember we know the values at these capital letter points or which are essentially the grid points not on the faces. And that is why on the faces whatever the flask and everything we had discretized in terms of nodal points. And then we have we are left with the terms which are this f e d e ok. So, the point is in the case so, here f e and f small w phi e and phi small w these two values are also not known to us because those are at the faces.

This part we have discretized These parts are not known to us yet and that we approximate as the simplest approximation that we can do is the arithmetic average. So, this is the E. for the case of f e small e I can take that whatever the scalar properties that we are estimating for phi here on the phase it is the arithmetic average of these two neighbor points. So, similarly this is the average values.

and then we again incorporate these values in the equation to find a compact form like we did in the previous cases. So, we will continue with this development this formulation and we will see with a numerical value for this particular problem how much accurate would be our finite volume prediction. so, we'll continue this in the next class until then thanks for your attention.