CFD APPLICATIONS IN CHEMICAL PROCESSES

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Week-04

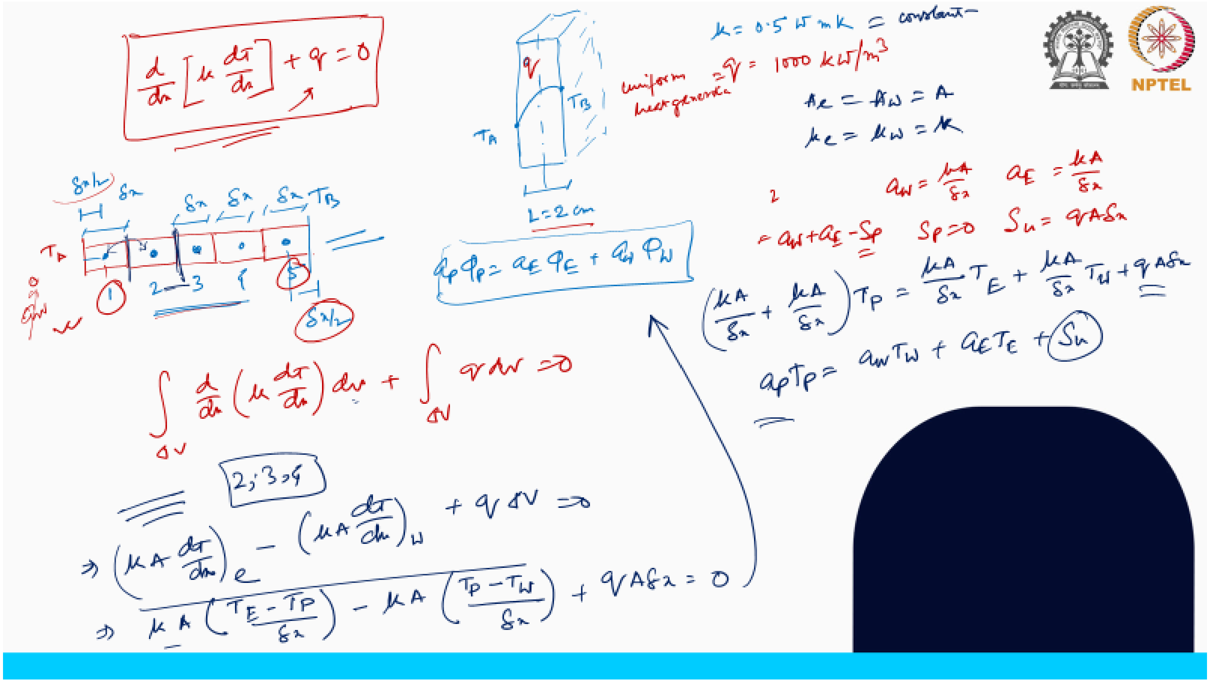
Lecture 20: Finite Volume Method

Hello everyone welcome back with another lecture on safety applications in chemical processes and specifically we are discussing in this week finite volume method. So, in the last couple of classes we have seen the details of finite volume method including in the last class that we started an example And, in fact we have solved it that steady state heat conduction equation with generation. So, we have seen because for the simple case the first part was discretized, but in the second example we took the generation term. So, that it became a bit more complex, but we have seen that how accurate the finite volume method with even only 5 nodal points although you have to remember that these are the very simple problem.

one dimensional problem and apparently the easiest or the easier problem to solve because we had its analytical solution also here. So, the reason for writing this analytical solution is to validate our model that whatever we are predicting from the finite volume formulation we were checking that whether that predictions are close enough with the exact values or not. And in the previous case we saw that it has it is giving maximum of 3 percent error when we chose 5 nodal points. Again, you can give it a try with 7 or 10 nodal points and see whether you get further accurate solution or not.

that means whether this percentage error goes lower than 3 percent maybe even 1 percent 1.5 percent if it is if it goes and if you think that that much accuracy you need for this problem then you must find this mesh or the grid that the number of meshes has to be taken higher. But if you think that 3 percent mesh is acceptable and when you took after 5 say 10 number or 7 number of meshes and the maximum deviation from the exact solution improves to 2.5 percent error. Then again the point that we mentioned earlier that decision is called the grid sensitivity analysis or the grid sensitivity study that you have taken 5, 7, 10 and you have seen that at 5 you have the acceptable accuracy which is maximum of 3 percent if you go

to 7 nodes possibly you can check although it is just to say this example you may have 2.5 percent accuracy and with 10 you can have possibly say 1.5 percent accuracy. Now again if 3% accuracy or 3% maximum deviation is acceptable to you for this problem you consider that you reach grid independent study or grid independent solution with 5 number of nodes. So this study is very essential to check whether you are capturing the right physics or not. Because, in this problem what happens is that your solution has is not further a straight line or a linear variation of the temperature from 100 to 200 that usually happens if you did not have the volumetric generation in that problem. But, since you have it what we will have is that the maximum temperature somewhere between the two nodal two extreme boundaries or the two nodal points. So, which will be higher than 200 because you have a volumetric heat generation term. So, here what you will see that once you plot this CFD prediction or the finite volume prediction will be on this exact solution curve mostly because this 3 percent deviation would be hardly noticeable when you plot it on a graph that has this temperature versus the distance. but now this the similar development that we had d by dx k a d t by dx.



Now, say we consider there is some convective motion of the fluid that exists

And by looking at this form you can relate again with the temperature since I have mentioned it here d t by d x the example of a fin say the extended surface. If you have a surface and if you have a fin extend on this. So, what you have here is that this extended surface of which here say we consider a boundary condition which is insulated. That means, 0 heat flux across this boundary of this extended surface you have 0 heat say this temperature is the base temperature we call this as T b that is of this surface on which this extended surface is embedded.

So, kind of a this kind of a scenario that you have a flat surface on which of there is a extended surface. So, here Now there is ambient air is flowing or air at a different temperature is flowing across this extended surface. You have a constant heat flux or the constant temperature at this end. So, the point is what will be your temperature profile along the length of this extended surface. So, it is a pin fin kind of an example.

where you have here T A which is the T ambient or say T infinity that we call. So, that is why it is the T infinity the ambient temperature. If we know all the dimensions say the length of this fin say length is 1 meter, the base temperature this T B is 100 degree centigrade T ambient is 20 degree centigrade, then what we have here and also say the values that are known is the convective coefficient, heat transfer coefficient H and perimeter P of this fin extended surface which is say uniform in this case. In this problem, you have a uniform cross section of the fin.

So, this is the convective part and this is that you have the diffusive part and it must be balanced. The heat loss by the conduction is the heat gain by convection of this ambient air. So, this form or this equation also can be solved analytically, but which is not trivial. It has a form of exact solution something similar to or in the line of this. this is the cos hyperbolic N into L minus X. I am coming to this all the values divided by cos H in L.

where N square is equals to HP by KA, H is the convective heat transfer coefficient, H is the perimeter of this surface extended surface, K is the thermal conductivity, A is the cross sectional area. So, if you know this convective coefficient perimeter K and the cross sectional area you would be knowing the value of N square or the N values here. say this n square value is also given for this problem because all the physical properties are known for us. So, what we have this as the exact solution of this equation. Now, this problem we will solve by finite volume method and we will see what is really the temperature profile here or along the x.

we have this as the 1 meter distance. So, similar to our previous understanding the same formulation goes that say we divide this domain in 5 control volume 5. So, what we have now this phase is insulated this phase is insulated. So, here we have Q is equals to 0. Now, here we have T v is equals to 100 degree centigrade.

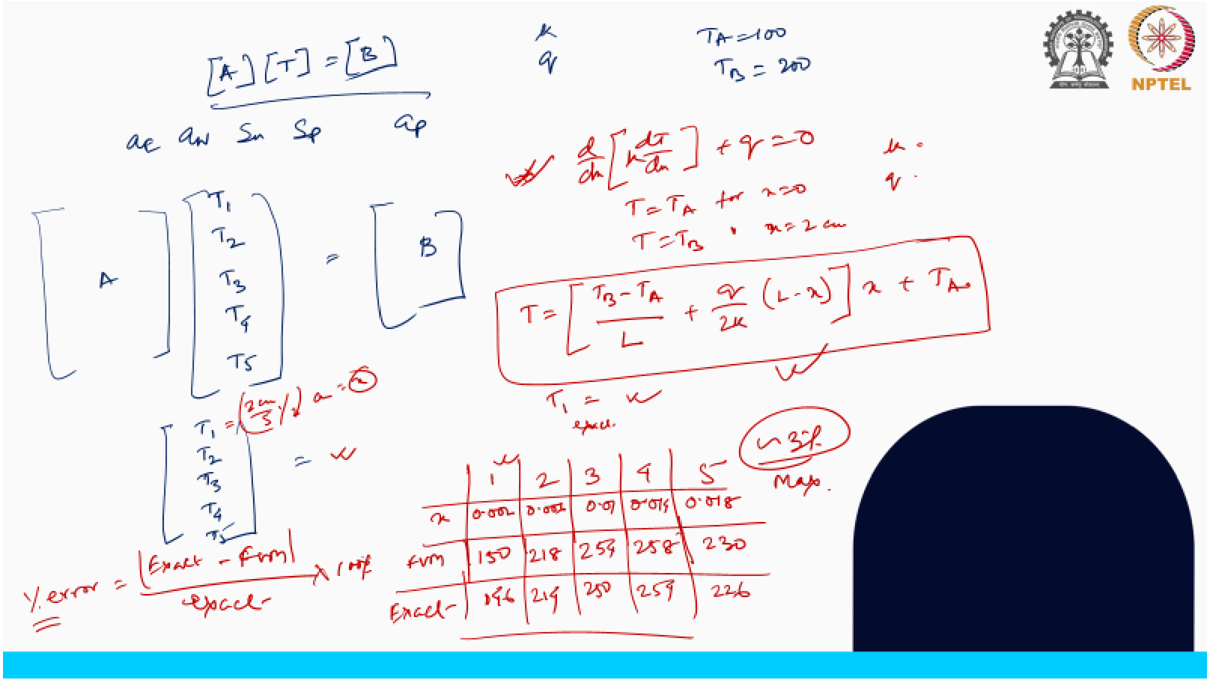
So, these are my nodal points and again this is my delta x by 2 because this is delta x and all these are equispaced or equal size. So, the distance between 2 and 3 is also say if I say 1 this is 2, 3, 4, 5 the distance between 2 and 3 is also delta x. exactly similar to what we have done. Here I have my governing equation as t minus t infinity is equals to 0, where n square is equals to h p by k a because the n square value is given for us. So, what we do here again we integrate it over the control volume which is delta V is equals to 0.

and here what we have is a dt by dx after converting it to surface integral and discretizing So, this is at east face minus A d t dx at the west face minus n square at the point this is tp because that is the point of concern multiplied by A delta x is equals to 0. So, from here again So, if we try to if we write this d t by d x at the east face and all discretized. So, it would be T e minus T p divided by delta x minus T p minus T w divided by delta x.

minus N square T p minus T infinity into delta x is equals to 0. So, this term. So, A is taken out of this formation because we have A everywhere on each term. So, again what we do? We will write that in the form of A p T p is equals to A w T w plus A e T e plus A c u and we try to find out the coefficient.

So, what we will find here the similar to the previous development is this thing that 1 by delta x A w plus 1 by delta x A e plus S u. which means A w is 1 by delta x A e is 1 by delta x A p is basically A w the similar form that we wrote we will try to find the similar form. So, that our S p becomes here minus n square delta x. and s u becomes n square delta x t infinity. So, this is the expression or this is the coefficients for all nodal points 2, 3 and 4 exactly similar to the way that we did earlier I am not repeating it further again for point 1 the formulation would be or would look like T e minus T p divided by delta x minus T p minus T b because of the waste phase divided by delta x by 2 minus n square T p minus T infinity delta x is equals to 0. again, if you separate out these things you would find out that this is delta x A p is A e plus A w plus minus of S p here. where S p again depending on segregation of this parameters is essentially minus 2 by delta x and S u will be in the form of T infinity plus 2 by delta x T b. for 0.5 that is the last point. So, there the fact is that remember here now the boundary condition how it affects the formulation.

Here we have the flask heat flask at this point or this phase is 0 which is the insulated condition which means we will have essentially this whole term would be 0 for that case which means for 0.5 the formulation would look like 0 minus T p minus T w divided by delta x minus n square T p minus T infinity delta x is equals to 0. This whole gradient term would be 0 because that surface is insulated. So, kind of Dirichlet boundary condition and the Neumann boundary condition.



So, we have Dirichlet boundary condition on the east face where T b is given and the Neumann boundary condition where we have the heat flux condition and that heat flux is 0. So, again if you find out this coefficient what you will have of the same form is 1 by A e is essentially 0, A p is A e plus A w minus S p and S p here would be minus n square delta x and S u will be n square delta x T infinity. So, for 0.5 and this is for 0.1. So, you develop the matrix now.

after replacing all these A p, A e, A w all the parameters with the numerical value because all the numerical values are given T b, T infinity, A l, N square all the values are given which essentially are there in A p, A w and A e, S p, S u these terms. So, once you do that and solve this set of matrix after writing for 0.1. So, what you do again for 0.1 you write this A p phi p relation A p T p is equals to A e T e plus A w T w minus S p sorry S u So, for point 1 you find out this values from here for point 2 again you write the similar way for point 3. So, basically you get 5 algebraic equation you have to solve those 5 algebraic equations together to find out the value of t 1, t 2, t 3, t 4 and t 5 of this matrix.

And, the result in this case if you solve it will be in the value range of this 26.50, 22.560 and 21.30. So, this is your finite volume prediction or CFD prediction. Now, the job is to cross check the result against this analytical solution. When you do that I suggest that you calculate this value for each and every point that comes from this delta x calculation and compare the result with this CFD prediction. what you will have is that for node 1 just like the previous case that depending on the x if I just for the sake of 4 5 if I write the first is the finite volume solution and then the exact

the finite volume solution from this previous matrix that you can see I wrote around this values 22.60 and 21.30, but the exact solution the analytical solution would be 68.52 37.86, 26.61, 22.53 and 21.21. Again you calculate your percentage error. if you calculate the percentage error you will find out that the maximum error happens here 2.51, 6.27, 2.51, 0.41 say minus 0.31 again if you take the mod this would not appear and again 0.42 percentage error. The interesting point is that the profile

for this case with distance say x if I have and this is if I have t the profile would look like that from 100 it would go like this. So, for 1 meter this is the total length here it would reach So, if this is the 20 and this is 100 degree centigrade it would reach very close to 20, but it not be exactly 20. And these points actually will be say the first point we have a maximum deviation. So, and then those points essentially will be very close enough to this values exact solution.

but here the maximum deviation is in the range of 6 percent. Now, if you consider that this is quite high for this problem, then you must refine your grid ok. You must take more number of nodal points or the grid points or more number of control again the same procedure like I told for the previous two problems instead of 5 nodal points now you take 10 nodal points. If you take nodal points which in the term of CFD we say that this is the coarser grid when I had 5 nodal points I will go for fine grid.

that means, from 5 to 10. You repeat this process the formulation remains same the numerical part solution that is upper trivial you go for that with any code MATLAB or whatsoever you solve that 10 set of algebraic equations because then you have 10 number of nodal points you set solve 10 nodal points. And, you will see the results are indeed much closer and the maximum deviation in that case would be in the range of 2 percent. That means, it would eventually coincide with these analytical solution on the graph.

And, that is great from 6 percent you went to 2 percent which is quite substantial. which means finer grid in this case is definitely desirable and say again from 10 you then go for 20 and you see that hardly it has possibly gone with the maximum deviation of 1.5 percent. That means, but 20 equation solving would take longer time. So, that is why in one of the last classes I told that then you have a tradeoff. between this time that you spend to get improvement in accuracy of 0.5 percent whether that is worth or not.

If not, you can say that at 10 number of grid points I have reached my grid independent solution. I hope this is clear to you now. So, I will stop here today and in the next class. I will go into the details of the non-uniform meshes and again we will come back to this finite volume. Essentially it is the finite volume method continuation, but we will discuss in the next week regarding the pressure velocity coupling because all these derivations or all the formulations that we have done here considering that the velocity field is known to us or will be known to us when we take up the next problem with the velocity that actual advection.

So, still here we have the convection we started with the conduction diffusion pure diffusion and then we have the convection which is with the help of this example and then we will have the advection in the problem. So, it will be a bit more complex and we will see that how finite volume method can take care of that formulation. with that I stop here today and we will see you in the next week with the next new set of lectures. Thank you.