CFD APPLICATIONS IN CHEMICAL PROCESSES

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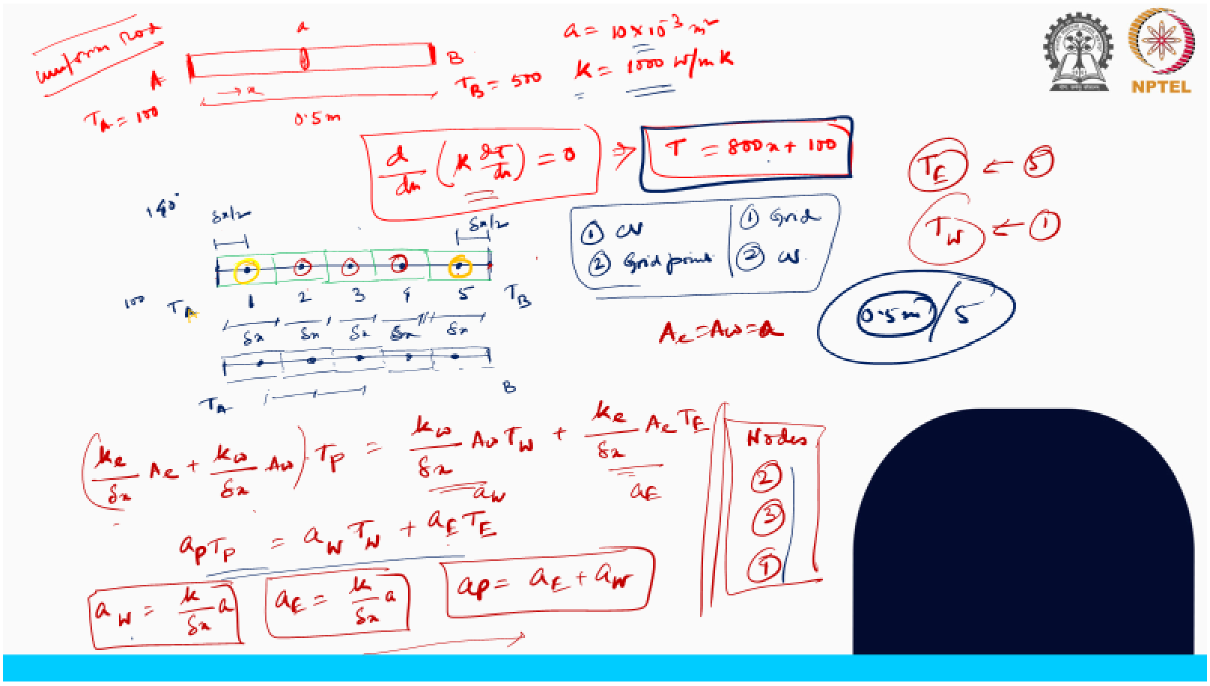
Week-04

Lecture 18: Finite Volume Method

Hello everyone, welcome back with another lecture on CFD applications in chemical processes. We were discussing finite volume methods in the last two lectures, the details of it. In the last lecture, we started a problem where we had one dimensional steady state conduction with all values known that were required including the two boundary conditions, the thermal conductivity we know its actual analytical solution. We will verify that with our, we will, once we develop our model finite volume method, then we will cross-check our model predictions against this actual result, that how much accurate is our model.



So, while doing so, we have discussed that we have taken a domain, this domain of 0.5 meter length, and we have divided that in five equal spaced, five equal width control volume set. And accordingly, we have developed a generic expression for nodal points 2, 3, 4, 2, 3, 4, For point one and five, we discussed that we require some special attention or we have to pay some special attention at these two nodal points because these two are nearer to the boundary And it does not actually have the point 5 does not have the point east that is the TE and this point 1 does not have the west side point that in the generic expression we have considered.

So for nodal point 1 taking this specific consideration that at the east west side we have boundary condition TA that comes into play here now. And how that creates a pseudo generation term or the source term. Generation or the source term in this expression. Although in the problem there was no heat generation or heat sink term. but that we have simplified or we have tried to write that in a form which is further again generic for that specific purpose or in the similar form that APTP is equals to AWTW plus AETE plus the source term here that we have distinguished.

Similarly, for this nodes, this nodal 0.5, this 0.5, what we can write is that now here we do not have for the east side nodal 0.5, point where my which is essentially coinciding with the temperature of the rod at the end B, this is my fifth nodal point. So, here what we have? T b minus T p divided by delta x by 2 because we discussed that if this is my one control volume of having delta x or delta x width,

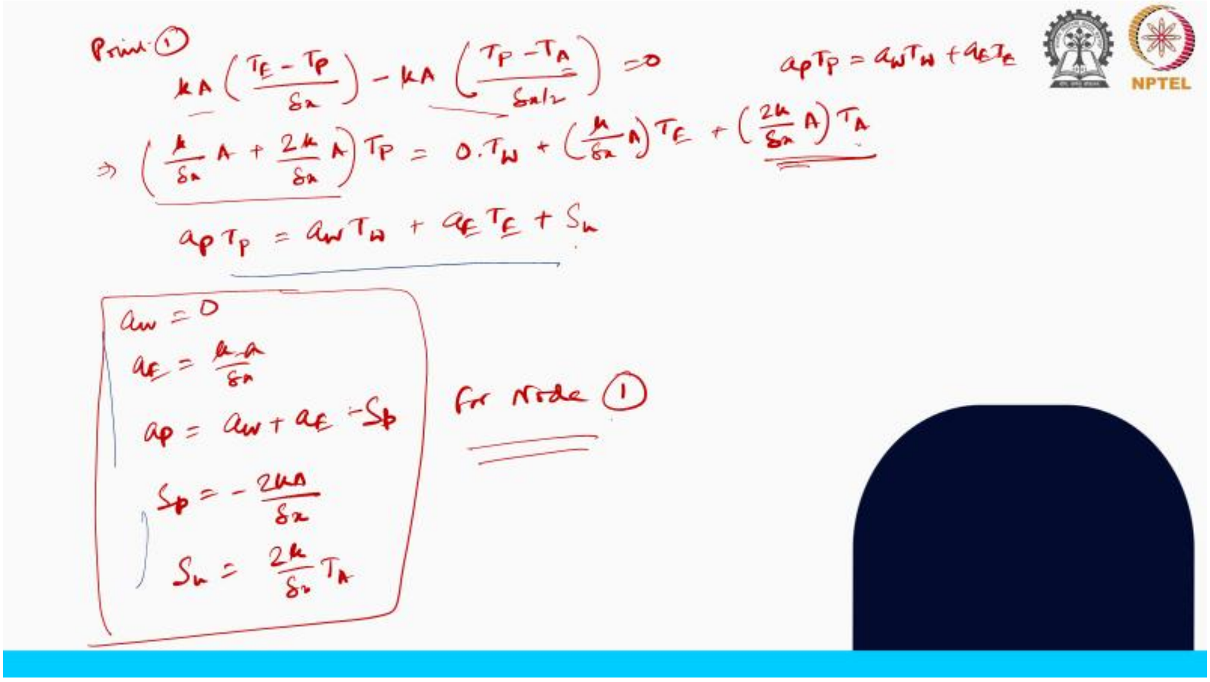
then naturally the distance between the point 5 and the boundary point is delta x by 2. minus K a where a again this a is the same a as small a that we have considered T p minus T w divided by delta x is equals to 0. So, further if we try to find out what is T p because my point p is now here for fifth nodal point. the thing that we segregate is that delta X A plus 2 k delta X A multiplied by T p is equals to k by delta X A T w plus there will be no coefficient for the Te because there is no Te point plus rest of the term after segregation is similar to the nodal point A.

multiplied by T b. So, similar to nodal point 1, we can write A p T p is equals to A w T w plus A e which is 0 here, this is 0 here plus 2 times k delta x A Or say here, this term we consider similar to that is SU, where AW is equals to KA by delta X, AE is equals to 0, AP is equals to AW plus EE. minus sp where sp is equals to minus 2 ka divided by delta x and su is equals to 2 ka delta x Tb. for node 5. So, for 2, 3, 4, we have this expression AE, AW, AP.

For node 1, we have again The expressions of those plus additional SP and SU. For node 5 also we have those expressions. Now if we write for each and every node these equations separately. So we took this generic expression. For 2, 3, 4, so for 0.2, I can have this expression AP 5P, AP TP here is equals to AW TW plus AET. The SIMP, I can write for the three times.

For 0.1, I have to write this expression with the numerical values that are already given. We have provided here, which is k value is given a is given delta x is here as 0.1 because we have a 0.5 meter length we divided that by 5 equal control volume so delta x is 0.1 this numerical values when we replace it what we find a set of algebraic equation that would be 300 300 T 1 is equals to 100 T 2 plus 200 T a for the node point 1 T 2.

for 2 it would be 3 200 T2 because 2, 3, 4 are same the generic expressions were same 200 T4 is 100 T3 plus 100 T5 and 300 T5 is equals to 100 T4 plus 200 TB. So we get this set of equation. We have to solve by any means that the processes I have told you. we consider that these are trivial for the time being, that such set of equations you can solve by any means, any CFD code, any your programming or by MATLAB.

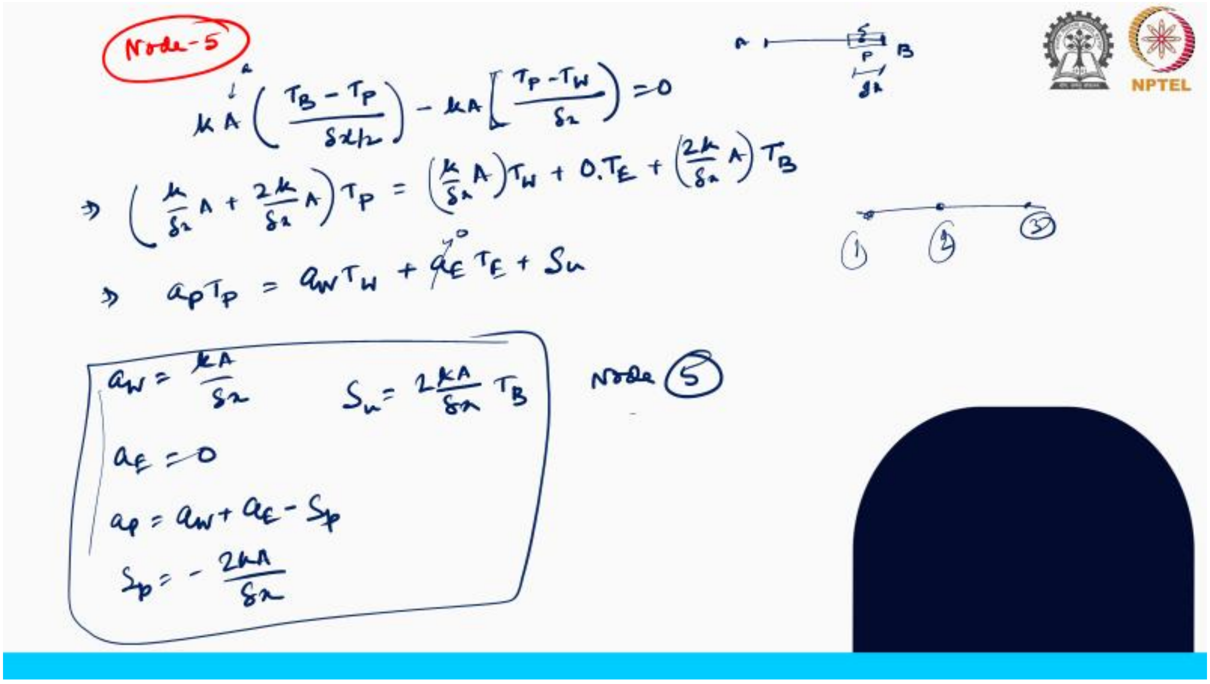


And what would happen is that it would give a solution So which means once you have this, you have this matrix to solve. 300. minus 100, 0, minus 100, 200, 00, 200, 100, minus 100, 200 minus 100, 000 minus 100, 300 you have this AX is equals to B kind of a

to solve. So, you have this kind of AT is equals to B kind of a formulation that you have to solve. You solve this and find that The solution would be 140, 220, 300, 380 and 460. Which means in this problem if you realize that you had this is the point at delta x by 2 which is 0.05.

At 0.05 you have a temperature of 140 degree centigrade. starting here with 100 degree. And then at 1.5, you have 220. So these are the nodal points where you have calculated these values. You plot it now to have an understanding or clear idea that if this is my exact solution for this problem

And as per X, say at 0.05, if you put it here, what is the temperature? What is the exact temperature that you can find out? And corresponding to that 140, how much far is 140 from that value? So that you can easily calculate and plot it in the X versus temperature plot. you would see when x is 0.5 from zero.



So at 0.05, you have calculated at 0.15 and so on at five, one, two, three. So in these locations, when you calculate the exact solution, the exact solution starting from if this is my 100 degree centigrade to 500 degree centigrade, I will have, say, this is my exact solution. And this value, when it is plotted at those locations, you would see that those are mostly matching with the exact solution. Because this is a simple problem, okay, so this is a simple problem, and we see that our CFD predictions by finite volume method is almost equal or same as that of the exact solution.

Now, had you done the same problem instead of five node grid points, if you had done it for three nodal points, I would ask you to do that and see the similar pattern remains, the same formulation remains. Now, instead of when you put it three nodes instead of five, what you will have, you will have one equation in the same form that you have developed for 2, 3, 4. And the other two is the same form that of 0.1 and 0.5.

So, if you have three points, 1, 2, 3, So for one, the same formulation of node one remains. For point three, the node formulation of point five remains. And for two, that you have developed is that we have developed for two, three, four. You solve it and you see you will have T1, T2, T3 in that case.

but it will have at a different location. You plot that and you see that the results would not be that accurate that comes for this fifth point here, that number of points five that are taken here. So what does this indicate is that the necessity of grid independency that we discussed in the previous lectures also. So the number of grids, so if you increase it to say 10 number of points, okay? So if you increase this number of points to 10 instead of five, what will happen is that this accuracy possibly would not increase further.

That also you can try, that instead of five points, if I do for seven points, and then you tally your value that T by FVM, and T exact at those locations. You check those values. For three points, for five points we have already solved, for seven or 10 points. You would realize that possibly after five, the percentage deviation

from the exact solution by the finite volume predictions are not that high or it is say within your acceptance limit. That indicates that by taking five number of nodal points, you possibly have reached a grid independent solution for this problem. But if you go for three, the solution is possibly not accurate. So which means if you consider three number of grids, the solution is not yet grid dependent.

You increase it to five, you check for seven and 10, you see that for 10, you have to spend more time in solving this matrix or by any method that you, because there you will have 10 number of equations for 10 nodes. For seven, you will have seven number of nodes. seven number of equations. So as the number of equations increases, simulation time naturally increases.

So whether that time and this accuracy, the accurate result you are going closer and closer to the exact value, that matters to you. If you think that no, at five I have acceptable accuracy, you can stay with that grid points. And that you call as the grid independent solution. I hope till this part it is clear to you.

So, in the next lecture, we will complicate this problem further and we will have a source term or say the unsteady state part later. And we'll see how finite volume is further added to that problem or can be implemented for such problem. But before that, I want you to have a clear idea of the simple problem. You do it once again on your own by considering such deviation in nodes, that number of nodes you take three, number of nodes you take seven and solve this problem and check what is the accuracy deviation.

And whether 5 is truly the grid independent solution that gives you in this particular case. So with this I will stop here today. Because in the next class we will continue this with another example. But with a source term. With this heat conduction problem. Till then I want you to practice this. Because this is the foundation of finite volume method. I want you to get acquainted with the nomenclature that we have discussed.

This is possibly easy to understand. If it is difficult for you, again we will discuss in the next class how we did that. But again, you go through this once again. So, with this, I thank you for your attention. And we will see you with the next lecture.