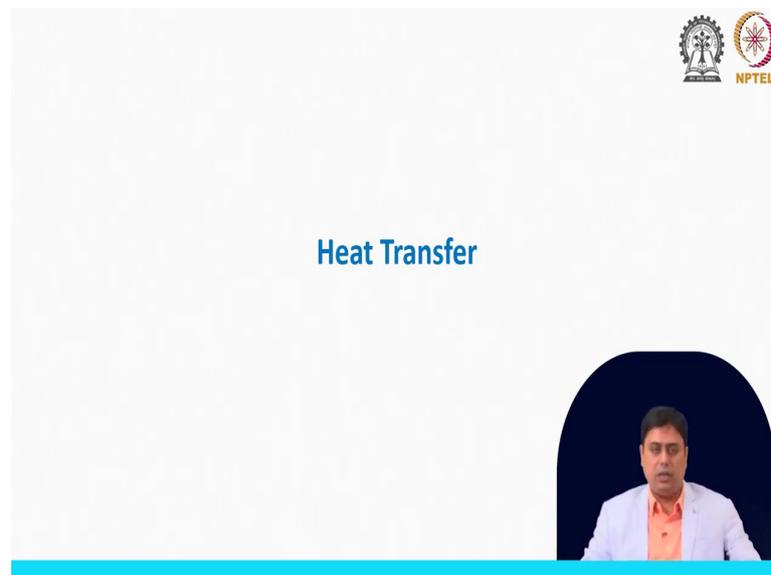


Chemical Engineering Fluid Dynamics and Heat Transfer
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Lecture - 50
Forced Convection (Contd.)

Hello and welcome back once again with the another lecture on Forced Convection in Chemical Engineering Fluid Dynamics and Heat Transfer. We were discussing several relations or correlations that are available in order to estimate Nusselt number in case of external flow.

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That is the flow that is happening on for example, on a flat plate or we will see when it flows across a cylinder or a sphere. What are the correlations that are available to estimate Nusselt number that will help us to calculate the convection heat transfer coefficient, a convection coefficient particularly h . Now, we have seen the relations that for uniform heat flux in the last class that for uniform heat flux. What we have seen?

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Handwritten notes on a whiteboard:

- Uniform heat flux
- $Nu_{lam} = 0.453 Re^{0.5} Pr^{1/3}$ → Laminar
- $Nu_{turb} = 0.0308 Re^{0.8} Pr^{1/3}$ → Turbulent
- 36% | Isothermal plate
- 4% | $Nu_{lam} =$
- $Re & Pr$ | $Nu_{turb} =$
- h_s
- $q_s = h_s \Delta T_s$
- $q_s = h_s [T_s(x) - T_\infty]$
- $T_s(x) = T_\infty + \frac{q_s}{h_s}$

$$Nu_{laminar} = 0.453 Re^{0.5} Pr^{\frac{1}{3}}$$

$$Nu_{turbulent} = 0.0308 Re^{0.8} Pr^{\frac{1}{3}}$$

So, these are the relations we have seen. Now, from this we can I mean once for a particular Reynolds number we calculate for a particular value if we calculate for a particular value of Prandtl number. If we calculate this Nusselt numbers or the heat transfer coefficient, what we see that these conditions I mean compared to this for the laminar flow and this is for the turbulent flow. In case of isothermal plate, the relations that we have seen, what we see that for fixed or a constant Reynolds number and Prandtl number or for identical fluid and flow condition, in this case we have uniform heat flux condition.

Previously we say we have seen isothermal plate condition. For there we have seen the Nu_{ux} for both the cases, laminar there are relations we have seen. If you compare those two equations from the last class last couple of classes, you would see that this H_x that we will calculate is nearly 36 % higher in case of laminar flow and 4 % higher when it comes to the turbulent flow compared to this isothermal plate.

So, these are of 36 % and 4 % greater than this isothermal condition. Nearly for the same Reynolds number and Prandtl number for the two conditions or in a more simplified way say for example, of Reynolds number 1000, there is a situation which means its a laminar

condition. The Prandtl number say for example, 1 a gaseous flow. These two value once you put in the uniform surface heat flux condition, you get a certain value of h_x .

With the same set of value, you get another h_x , but for a different condition which is for isothermal plate condition and it has been observed that this value h_x that you get here considering uniform surface heat flux condition is nearly 36 % higher in the laminar case. So, it means that we have to be very careful when we apply these correlations. We have to look at the suitable correlations of the appropriate correlations depending on the problem that we have in hand.

Now, when the heat flux that is the \dot{q}_s is known the heat the rate of heat transfer from or to the plate and that is surface temperature that is at a distance x from the leading edge. The way that we can calculate is that:

$$\dot{q}_s A_s = Q_s$$

$$\dot{q}_s = h_x [T_s(x) - T_\infty]$$

above surface temperature changes with the direction x ; when we have uniform heat flux condition.

So, which means the surface temperature in this case we can calculate by having this relation:

$$T_s(x) = T_\infty + \frac{\dot{q}_s}{h_x}$$

or particularly here it is h_x because this is the local one, we are calculating. So, when we know the amount of heat flux, we can calculate the surface temperature by this way when we have uniform heat flux condition. So, now let us see this for a problem what we have is that an engine oil.

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The diagram shows a horizontal flat plate of length $L = 5\text{ m}$ exposed to a fluid flow. The fluid has a temperature of 60°C and a velocity of $v = 2\text{ m/s}$. The plate is maintained at a temperature of 20°C . The flow direction is indicated by three arrows pointing right. The plate is labeled with 'A' at its leading edge. The NPTEL logo is in the top right corner. Handwritten notes in blue ink list the objectives: 'Total drag force' and 'Rate of heat transfer per unit width'.

Engine oil is flowing over a surface flat plate, say consider this scenario there is a flat plate oil is flowing over it which is at 60°C velocity is 2 m/s . This plate has a length of 5 meter this is kept at 20°C . The surface area across which the heat transfer is happening or on the heat transfer on the surface which is heat transfer is happening at A.

So, we have to determine total drag force. The question is how much is the total drag force and rate of heat transfer per unit width of the plate? So, the width is not mentioned. So, we consider it has a unit width.

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This diagram is identical to the one in the previous slide, but includes the handwritten note 'Steady flow' in the top left corner. The NPTEL logo and the objectives 'Total drag force' and 'Rate of heat transfer per unit width' are also present.

So, engine oil flows over it. We have to determine total drag force and the rate of heat transfer per unit width. So, what we consider or what we assume at the first place that this flow is steady. So, we consider that the flow is steady that is the steady flow is happening.

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Steady flow
Incompressible
 $Re_{cr} = 5 \times 10^5$

60°C
20°C
 $L = 5m$

o Total drag force -
o Rate of heat transfer per unit width -

$T_f = \frac{60 + 20}{2} = 40^\circ C$
 ρ, μ, Pr, k

$\rho = 876 \text{ kg/m}^3$
 $Pr = 2870$
 $k = 0.144 \text{ W/m}\cdot\text{K}$
 $\nu = 242 \times 10^{-6} \text{ m}^2/\text{s}$

$Re = \frac{VL}{\nu} = \frac{2 \times 5}{0.242 \times 10^{-3}}$

We also consider that it is incompressible because in absence of any other information we consider that this is incompressible. Flow over a flat plate we know the critical Reynolds number is, $Re_{cr} = 5 \times 10^5$.

We need to know the properties of the oil at a temperature which is we have discussed this there is a significant variation of the temperature 60 °C and 20 °C. So, we calculate or estimate the film temperature as 40 °C and look out for the values of the fluid properties, Prandtl number, thermal conductivity etcetera at this temperature that is the film temperature.

Usually, this fluid properties are mentioned whenever a problem statement is given if not we have to look it in the reference book or we have to search for it. Usually, these properties are listed as the appendix of any textbook or the reference books. These physiochemical properties are usually mentioned or given.

So, at this temperature we have noted that these are the values.

$$\rho = 876 \frac{\text{kg}}{\text{m}^3}; Pr = 2870; k = 0.144 \frac{\text{W}}{\text{m}} \cdot \text{K}; \nu = 242 \times \frac{10^{-6} \text{m}^2}{\text{s}}; L = 5m$$

Before we apply, so we can always apply a correlation to find out what is the drag coefficient or the drag friction factor. At the same time, we can apply any Nusselt-number correlations that is suitable for this problem to find out what is the value of h and accordingly we can find out what is the value of Q .

In order to choose a appropriate relation we have to at first look at this criteria. We have to at first determine whether the boundary layer here is laminar or turbulent and accordingly we choose our appropriate correlation or appropriate relation for Nusselt number and the other parameters.

So, here what we see that Reynolds number is:

$$Re = \frac{VL}{\mu} = \frac{2 \times 5}{0.242 \times 10^{-5}}$$

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Steady flow
Incompressible
 $Re_{cr} = 5 \times 10^5$

$C_f = 1.328 Re^{-0.5}$
 $= 0.0207$

$h_0 = \frac{C_f \rho V^2}{2}$

$Nu = \frac{hL}{k} = 0.242 Re^{0.5} Pr^{0.4}$

$h = \frac{k}{L} (0.242 Re^{0.5} Pr^{0.4})$
 $Q = hA(T_w - T_\infty)$

Diagram: Flow over a plate of length $L = 5m$. Inlet velocity $V = 2m/s$. Wall temperature $T_w = 60^\circ C$. Free stream temperature $T_\infty = 20^\circ C$. Fluid properties: $\rho = 870 kg/m^3$, $\mu = 0.242 \times 10^{-5} Pa \cdot s$, $k = 0.149 W/m \cdot K$.
 $Re = \frac{VL}{\mu} = \frac{2 \times 5}{0.242 \times 10^{-5}} = 41322 < Re_{cr}$
 $T_f = \frac{60 + 20}{2} = 40^\circ C$
 $Pr = 2870$
 $h = 8.1 W/m^2 \cdot K$
 $Q = 4050 W$

So, from this what we will see? We see that this Reynolds number is in fact, is lesser than this critical number once we calculate this value. So, whatever the numerical values will be here you replace it to calculate the Reynolds number of the problem.

We have to check whether it is lesser than the critical value or higher than the critical value if it is lesser than the critical value then if the flow is in laminar zone. So; that means, we have laminar flow over the entire plate because we have used ($L = 5$ m).

So, considering from ($x = 0$) from the leading edge still the end, the Reynolds number at max whatever the value would be it is not going beyond the critical value. So; that means, there is no chance of having a hybrid situation. Since the L value we have used it completely here.

So; that means, there is a complete laminar flow over the entire plate. So, accordingly we use the C_f relation the friction factor relation which:

$$C_f = 1.328Re_L^{-0.5} = 0.0207$$

Now, here we have to understand that the drag force the F_D which we have the component the skin drag as well as the form drag. In this case the pressure drag here is 0 for such flow condition. So, the drag force that we will have here is:

$$F_D = C_f A_s \frac{\rho V^2}{2}$$

Again, we apply all the numerical values from this problem on this relation, on this equation and we can find out what is the total drag force that is acting per unit width because the width we have considered here. As the unit width if the width is known we can multiply it by the width and we can have the complete drag force over the entire plate.

So, once it is known this is the first part. Now, coming to the rate of heat transfer per unit width for that we need to know what is the Nusselt number again the flow condition is known its laminar flow. So, we use this relation that we have seen earlier and we calculate all the parameter.

$$Nu_{laminar} = \frac{hL}{k} = 0.664Re^{0.5}Pr^{\frac{1}{3}}$$

The Prandtl number is given Reynolds number we have already calculated here we use this and find out the value of h from this expressions because L is known and k is also known from this expression.

Once this value is known to us W/m^2 because of this value what we can calculate is the amount of heat transfer:

$$Q = hA(T_\infty - T_s)$$

Again, A here considering the unit width or unit the surface containing unit width. So, here ($A = 1 \text{ m}^2$) we use this value x is known ΔT is known and we find the amount of heat transfer in Watt.

So, I hope the strategy of solving such question is clear to you the strategy is that whenever a problem is given, we have to identify whether it is laminar or turbulent or it is hybrid that is some portion of our significant portion or significant length from the leading edge of this flat plate are there is laminar flow occurring and after that there is turbulent flow. In that case we use the hybrid or the piecewise integration equation that we have seen earlier a different correlations or different relation equation is available.

In in other cases if there is purely turbulent flow or purely laminar flow, we use appropriate equation to find out the value of h or to find out the value of Nusselt number at the first place. Once it is known Nusselt number is known we can easily calculate the value of h , we have to remember or we have to have a clear mind on the two things N_{ux} and Nu ; one is the local Nusselt number the other one is the average Nusselt number entire the over the entire plate.

So, whenever we calculate the total heat transfer rate or the total amount of heat has been transfer or being transferred it is considering the average h value or the average friction convection coefficient value and in order to calculate average convection coefficient we have to use the correlation for the average Nusselt number. The average Nusselt number comes from the integration of the local Nusselt number over the entire domain or the length in this case.

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Handwritten notes on a whiteboard showing calculations for Reynolds number, Prandtl number, and Nusselt number for a flat plate. The notes include a diagram of a plate with dimensions 6m and 1.5m, and a small video inset of a man speaking.

$T_f = \frac{20 + 40}{2} = 30^\circ\text{C}$ at 1 atm
 $\mu = \frac{\nu \rho}{P} = \frac{2.1 \times 10^{-5} \text{ m}^2/\text{s}}{0.123} = 1.69 \times 10^{-4} \text{ m}^2/\text{s}$
 $Re_L = \frac{V L}{\nu} = \frac{8 \times 6}{1.69 \times 10^{-4}} = 2.84 \times 10^5$
 $Pr = \frac{c_p \mu}{k} = \frac{1013 \times 1.69 \times 10^{-4}}{0.025} = 6.7$
 $Nu = \frac{hL}{k} = 0.665 Re^{0.5} Pr^{1/3}$
 $h = \frac{k}{L} Nu = \frac{0.025}{6} \times 0.665 \times (2.84 \times 10^5)^{0.5} \times (6.7)^{1/3} = 9.2 \text{ W/m}^2\text{K}$
 $Q = h A \Delta T = 9.2 \times (6 \times 1.5) \times (140 - 20) = 101.7 \text{ W}$

So, say the other problem we have a problem that there is an atmospheric pressure at somewhere in the world or somewhere that we have carried out this study. There atmospheric pressure is say 83.4 kPa and there is air that is flowing over it at 20 °C with 8 m/s. We have a plate which is 6 m in this length and say this is 1.5 m width.

So, (1.5 x 6) m flat plate which temperature say is 140 °C is cooled in an atmosphere where the atmospheric pressure is this atmospheric temperature is this 20 °C and here we have to calculate the rate of heat transfer from the plate if the air flows parallel to the plate in two different condition.

One is the first is it is parallel to the 6 m direction. It is parallel to the 6 m side long the other flow can be in this direction which is parallel to the 1.5 meter side. We will see the difference in orientation of the geometry the influence. These are the two scenario that can happen.

I mean there are many scenarios can happen, but we are looking into to these two aspects ideal aspects that air is in one case is purely flowing in one direction that is parallel to 6 m length and in other case it is flowing only parallel to the 1.5 m long side. So, what are the amount of heat transfer in both the cases whether those are identical or different?

Now, again we consider this is the steady case and there is no radiation the critical Reynolds number in this case is (5×10^5). The values somehow are known to us all the

material properties. Now, here again the material properties we have to look into the reference or the textbook for a temperature which is the film temperature:

$$T_f = \frac{20 + 140}{2} = 80 \text{ }^\circ\text{C}$$

What we see in the chart or in the reference book that usually at 80 °C and one atmosphere pressure the material properties would be given, but here we have a different atmospheric pressure. So, as this is the different atmospheric pressure and this is nearly atmosphere which is slightly different from the atmospheric normal atmospheric pressure that we are the standard atmospheric pressure for which the material properties are known.

$$\frac{83.4}{101.325} = 0.823 \text{ atm}$$

So, among all the parameters specially the kinematic viscosity changes for air with pressure and that we estimate for atmospheric pressure by this P the value that we have calculated. So, if for atmospheric pressure and 80 °C; air has kinematic viscosity of $(2.1 \times 10^{-5}) \text{ m}^2/\text{s}$, it would be divided by this atmospheric value in order to estimate the new value in this case some value which is m/s.

So, this value we have to utilize because the pressure changes this kinematic viscosity in this case. So, when now first condition when air flows in parallel to the 6 m side the Reynolds number calculation involves L which is 6 m in this case. So, $\frac{8 \times 6}{\nu}$ that we have calculated here.

And it would have a value which is in the order of (10^6) or (2×10^6) numerically close to now this is greater than (5×10^5) which means for the entire length there will be fully developed turbulent flow or full turbulent flow over the entire plate when the flow is happening along the side or in parallel to the side that is 6 m long.

So, that means, here what we will do now if we if we consider because now this is happening for the entire length if we do not consider this as 6 say for example, if I consider just 1 here this would be divided by 6 and then the value would come in the laminar range. So; that means, there is chances or there is laminar boundary layer as well or; that means, there is a mixed condition of flow over the entire plate.

And for more accurate estimation in this case then we apply appropriate relation instead of a pure turbulent condition considering the influence of the laminar part as well that we consider here this relation that we have seen earlier kind of hybrid situation in order to find out the Nusselt number.

$$Nu = \frac{hL}{k} = (0.037Re^{0.8} - 871)Pr^{\frac{1}{3}}$$

The reason is simple again let me reiterate because here we have considered till the end of the leading end of the plate from the leading edge if the flow is happening in this direction. Now, somewhere if it is not 6 since it is very close to this value somewhere in between or below say 0.5 or 0.1 m length or similar to that range this value would be in the range of laminar flow the Reynolds number value and in that case there is eventually laminar condition and then the turbulent condition.

So, in order to take into account the effect of both the things what we have done here is we have used a relation that takes into account of this mixed condition. Once it is done we can always find out the value of h from all the parameters that are given here.

We have to understand A_s is eventually the $(1.5 \times 6)m^2$. So, the amount of heat transfer that is happening the h that we have calculated from here multiplied by A_s multiplied by the ΔT we have an answer this is the first point.

In the second case similarly again, we calculate the Reynolds number, but here instead of 6 we use 1.5 because that is the at max plate length that it can have when the flow is in that direction or parallel to that direction. There we will see when we put 1.6 this critical Reynolds number we will see is much lesser than the critical Reynolds number. So, which means when the flow is in this direction or parallel to 1.5 m side long the flow is laminar completely over the entire plate.

So, in that case we use the appropriate relation for Nusselt number in order to estimate:

$$Nu = \frac{hL}{k} = 0.664Re^{0.5}Pr^{\frac{1}{3}}$$

From here we calculate the value of h and we calculate eventually the value of $(hA\Delta T)$ with this value of h that we calculate here and we will see these two values are different.

If you calculate this eventually you would see that this value is much lower if I say that this is YY numerical value and from here if you get XX numerical value you would see that $(XX > YY)$. It is considerably higher in this case and it is considerably lower in the second case.

So, which means the orientation of air flow is also important in convection. If we want this object to be cold faster the positioning of forced convection or the movement of air has to be accordingly oriented. Otherwise, the rate of heat transfer would be much slower if the flow direction is in a different way.

So, I hope these parts are clear to you this is all about the external flow the convections and the external flow. When we talk about flow over cylinder or spheres several like this. Relations are available it is just the different equations we have to choose and, in those cases, remember the Reynolds number critical value also changes for flat plate it is (5×10^5) .

In those cases, the critical Reynolds number value changes. So, the strategy remains same in those problems as well. We at first identify what is their current Reynolds number and whether that Reynolds number is below or above the critical Reynolds number.

Once it is identified we choose the appropriate Nusselt number relation that is with Reynolds number and Prandtl number. We use it we find the value of Nusselt number from there we derive the value of h and we calculate the amount of heat transfer heat flux or the heat transfer rate.

With this I will stop here in the next class I will come back again with the forced convections, but with internal flow that is the pipe flow that is also another interesting part. With this thank you for your attention and we will see you in the next class.