**Chemical Engineering Fluid Dynamics and Heat Transfer**
Prof. Arnab Atta
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 48
Forced Convection (Contd.)

Hello and welcome back to the another lecture on Heat Transfer in Chemical Engineering Fluid Dynamics and Heat Transfer. We were discussing Forced Convection or more or less on the fundamentals of convection related to forced convection.

(Refer Slide Time: 00:49)
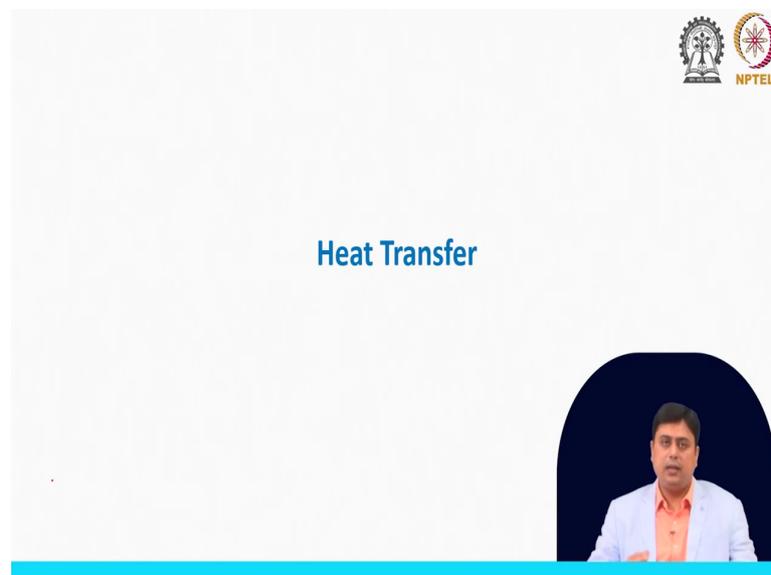
So, what we have gone through is that the concept of thermal boundary layer, the mechanism of its development. We have also seen the governing equations, mass, momentum and energy. Although, we have not gone with the detailed derivations or the solution of those equations for a particular case that is say flow over a flat plate.

But we have seen the form of thermal velocity boundary layer thickness, how it varies and now we will discuss what will happen with the thermal boundary layer because once we have solved the velocity boundary layer, now we can proceed to the energy equations. And, there for this particular case that is the flow over a flat plate, we can solve this this energy equation with the help of the information's that is the velocity components.

And, what we see again skipping the derivations part, what we see that the Nusselt number, I mean eventually our target is to find out the heat transfer, convection heat transfer coefficient value or how it is varying with the flow parameter.

(Refer Slide Time: 02:31)
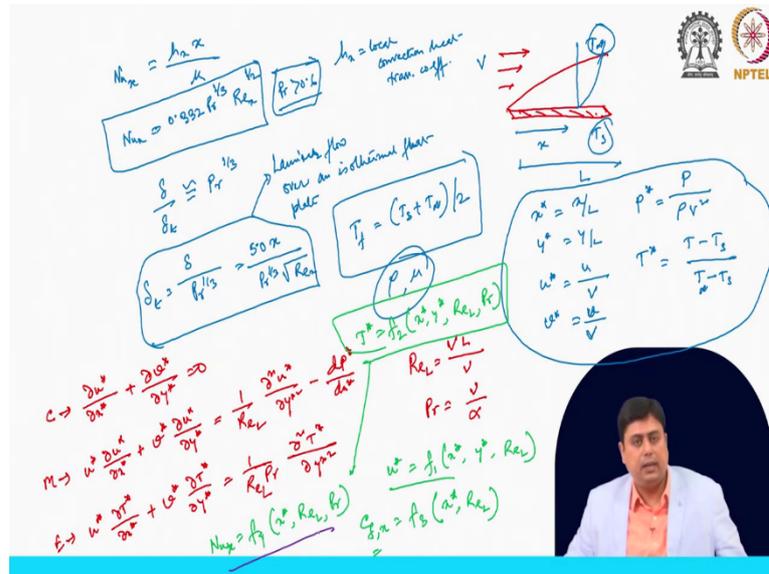
So, in that case what we see is that Nusselt number or the local Nusselt number specifically, the reason is that it is varying along with x that is from the leading edge for a flow over a flat plate what we see is that the variation of Nusselt number x stands for the local Nusselt number is essentially:

$$Nu_x = \frac{h_x x}{k} = 0.332 Pr^{\frac{1}{3}} Re_x^{\frac{1}{2}}$$

for (Prandtl number, Pr > 0.6) scenarios. So, Nusselt local Nusselt number, now these relations you have to remember because those are important while solving the problem. Now, further what is seen, that this relation:

$$\frac{\delta}{\delta_t} \cong Pr^{\frac{1}{3}}$$

This relation is only valid for laminar flow over an isothermal flat plate. So, this relation that what it becomes eventually which can be rearranged as follows:

$$\delta_t = \frac{\delta}{Pr^{\frac{1}{3}}} = \frac{5.0x}{Pr^{\frac{1}{3}}\sqrt{Re_x}}$$

because we have seen this δ hydrodynamic boundary layer expression in the last class.

So, that means, we can estimate the thermal boundary layer thickness for laminar flow over an isothermal flat plate. Now, the there are say influence of temperature gradient on the fluid properties that sometimes we cannot neglect. So, the effect of variable properties that usually we take into consideration in such problems and we take gross value of various properties because say the temperature of the surface is T_s and this is at T_∞.

Now, the fluid temperature is changing across along its normal. Now, the point is at which temperature we will consider the fluid properties in order to estimate the Prandtl number or whether Reynolds number, the density, viscosity, etcetera. In those cases what we consider is the film temperature which is defined as:

$$T_f = \frac{T_s + T_\infty}{2}$$

If the properties are significantly varying for a fluid in between these two temperature range or in this temperature range, then we consider film temperature which is defined as the average of these two temperatures and then accordingly we find the value of all variables or the varying properties, not variables the varying properties specifically. And, then we insert it here and find out the numerical values.

So, this is again for a specific flow scenario that is flow over a flat plate which we are discussing depending on the flow scenarios or the flow conditions, these formulation changes. Now, for a given geometry what we have now realized is that u or say in most of the cases in chemical engineering problems whenever we find opportunity, we try to non-dimensionalize the problem.

That means this x the distance from the leading edge and if the length is L of this flat plate, what we do? We define a non-dimensional parameter:

$$x^* = \frac{x}{L}$$

$$y^* = \frac{y}{L}$$

$$u^* = \frac{u}{V}$$

$$v^* = \frac{v}{V}$$

$$p^* = \frac{P}{\rho v^2}$$

$$T^* = \frac{T - T_s}{T_\infty - T_s}$$

So, once we introduce these values or these non-dimensional parameters, we eventually can non-dimensionalize the continuity equation, momentum equation and energy equation and it will have its relevant form. So, let me write those forms for the sake of continuity here. So, you would understand these introduction of non-dimensional terms actually leads to, if I write the continuity equation, the continuity equation would now look like:

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$$

Momentum equation would look like:

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{1}{Re_L} \frac{\partial^2 u^2}{\partial y^2} - \frac{dp^2}{dy^2}$$

we are writing it for only one x-momentum equation and energy equation would look like in this form:

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{Re_L Pr} - \frac{dT^2}{dy^2}$$

Where,

$$Re_L = \frac{VL}{\vartheta}$$

$$Pr = \frac{\vartheta}{\alpha}$$

So, what it shows eventually the reason of writing this is that to show you eventually what happens that:

$$u^* = f(x^*, y^*, Re_L)$$

By this understanding we can also find out the dependencies of shear stress and the friction factor coefficient. Those are eventually function of so, local friction factor coefficient, similarly will be a function a different function that is why I am writing as a different function which is:

$$C_{f,x} = f_3(x^*, Re_L)$$

which means local friction factor coefficient is eventually a function of the distance from the leading edge and the Reynolds number.

So, now these are the local values and similarly, what will happen? The Nusselt number local Nusselt number will be again a function of:

$$Nu_x = f_4(x^*, Re_L, Pr)$$

The reason is that temperature is say function:

$$T^* = f_2(x^*, y^*, Re_L, Pr)$$

Temperature would be dependent on the x, y the domain definition, the domain size, the Reynolds number as well as the Prandtl number and if it is dependent then accordingly the Nusselt number from the definition of Nusselt number you would see that it also depends on the value of a or the x^*, Reynolds number and Prandtl number. But, these are all the local values.

So, for average values, average friction factor or average heat transfer coefficient we have to integrate it over the entire surface.

And, if we look at the dependencies in those cases what we will see that friction factor coefficient is a function of Re$_L$, $C_f = f(Re_L)$. This is the average friction factor coefficient value and Nusselt number is say function of: $Nu = g(Re_L, Pr)$ .Now, when it is measured experimentally what has been seen that in most cases Nusselt number simply follows this simple power law relation:

$$Nu = CRe^m Pr^n$$

m and n are the constant properties or the constant exponents that varies between 0 and 1 and the constant C it depends on the geometry.

Various experiments have shown that the Nusselt number variations for any kind of geometry usually follow this relation the simple power law relation. C a constant parameter that is geometry dependent, m and n this usually varies between 0 and 1 depending on how the flow is happening or what condition of the flow. Now, we will come to the values of m and n later for whenever we talk about any specific flow condition or specific flow problem.

Now, the point is that there exist analogies between this momentum and heat transfer. What we have seen is that from the momentum and energy what we see is that:

$$\frac{C_{f,x}}{2} Re_L = Nu_x$$

From this previous equations the non-dimensional equations and the understanding what we see that this analogy or this equality exist which is called the Reynolds analogy. Because, what it helps it is important because it helps in determining heat transfer coefficient for the fluids say with it is specifically for Prandtl number 1.

For such flow problem where Prandtl number is 1, this Reynolds analogy provides us information on the convection heat transfer coefficient from the knowledge of friction coefficient. In this respect let me introduce another variable this:

$$\frac{C_{f,x}}{2} = St_x \; [Pr = 1]$$

St is called the Stanton number another non-dimensional number which is the combination of Nusselt number and Prandtl number.

$$Stanton\ No., St = \frac{Nu}{Re_L Pr}$$

It is also a non-dimensional or dimensionless heat transfer coefficient value. But this Reynolds analogy is of limited use because the restriction of Prandtl number is equals to 1 and also the other criteria which is to be mentioned here that this is taken into consideration here while deriving this analogy. But what we should do or we should try to have a analogy which is applicable for a wide spectrum of the problems.

In order to do so, there is Prandtl number correction. If we introduce that correction then this relation or this analogy becomes more widely acceptable. So, what happens in this cases that this friction factor coefficient and Nusselt number for a flat plate problem it becomes:

$$C_{f,x} = 0.664 Re_x^{\frac{1}{2}}$$

$$Nu_x = 0.332 Pr^{\frac{1}{3}} Re_x^{\frac{1}{2}}$$

So, these are the things that we had now considering this Prandtl number correction. What we do here is that this part we already had and now, we introduce Prandtl number correction which is:

$$C_{f,x} \frac{Re_L}{2} = Nu_x Pr^{-\frac{1}{3}}$$

$$\frac{C_{f,x}}{2} = \frac{\mu}{\rho c_p V} Pr^{\frac{2}{3}} = \dot{J}_H$$

$\dot{J}_H$ term as the Colburn j-factor. And, this is valid for a much wider range.

And, this analogy we called the modified Reynolds analogy or Chilton Colburn analogy. This is the Chilton Colburn analogy where $\dot{J}_H$ is the Colburn j-factor Prandtl number to the power.

(Refer Slide Time: 24:22)

So, I mean this relation was developed using the relations for laminar flow over a flat plate for which this relation was valid for:

$$\frac{\partial P^*}{\partial x^*} = 0$$

But it is experimentally I mean it has been shown that this relation is valid for a wide spectrum even in presence of pressure gradient. Now, the analogy is actually is not applicable until and unless we have:

$$\frac{\partial P^*}{\partial x^*} = 0$$

And, therefore, it we cannot apply for this analogy in case of laminar flow even in pipe because there this gradient exist i.e.

$$\frac{\partial P^*}{\partial x^*} = 0$$

But, this analogies of the friction factor and the Reynolds number in later it has been developed more accurately and there are several relations available, but we are leaving those not and or not discussing in this context.

So, now looking at this all these relations that why we need and if we try to summarize this whole thing what I would like to convey here that what we started with a simple understanding that in convection overall heat transfer value we can calculate from the Newton's law of cooling which is $(hA\Delta T)$.

(Refer Slide Time: 26:43)
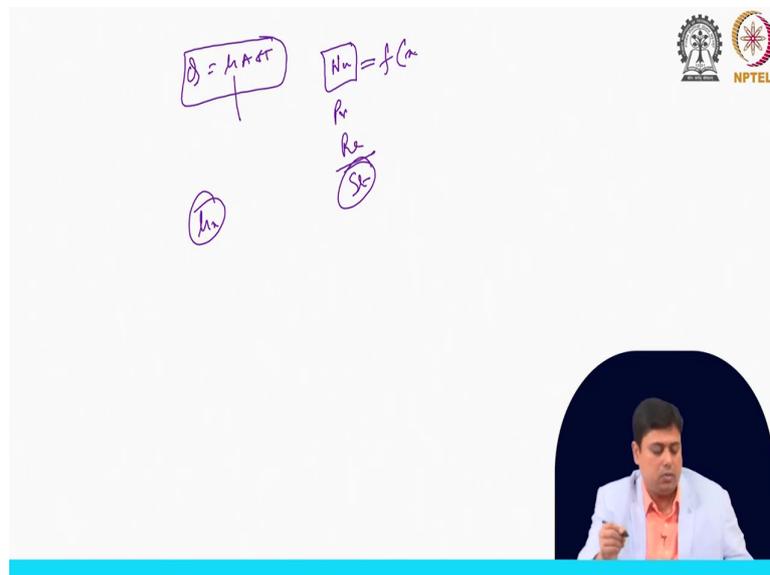
we have seen that h how we can estimate adjacent to the surface for flow over a flat plate and from there we introduced a dimensionless number which is Nusselt number. We have also seen the definition of Prandtl number. We already knew what was Reynolds number. We have seen the governing equations that is continuity momentum and energy equations.

We have understood very briefly what is and what and when viscous dissipation is important. We have seen the analogies between the friction factor and the Nusselt number and why it is important because again let me reiterate in convection bulk movement of the flow is important or essential the movement of the fluid is related with the friction that it come across. This friction factor is that is why important in heat transfer studies.

So, if we can find out the convection heat transfer coefficient from the friction factor value. For a specific case of Prandtl number 1, we saw the Reynolds analogy. Since Reynolds analogy is very specific that is laminar flow over a flat plate and where there is no gradient of pressure along the x-direction or in the flow direction that analogy was valid.

But, since in most of the cases or most of the fluid flow cases that much restrictions cannot be assumed. It was modified by the Chilton-Colburn analogy. From there for a Prandtl number range of $(0.6 < Pr < 60)$ we had a relation between the local Nusselt number and the friction factor coefficient.

So, all these things eventually contributes to the calculation of h_x or the local heat transfer coefficient convection heat transfer coefficient. Once we have it we can integrate it over the entire domain or like we have seen in the cases where we can calculate the Nusselt number over the entire domain. Eventually Nusselt number and another dimensionless number we have introduced is the Stanton number which is also a dimensionless heat transfer coefficient value.

So, once we find out the average Nusselt number we can always find out what is the average heat transfer coefficient. We have also seen how Nusselt number can vary. It is eventually a function of x for the local it is the thing that we have seen that the Nusselt now local Nusselt number eventually varies with the x Reynolds number and Prandtl number and the average reynolds Nusselt number varies with Reynolds number and Prandtl number.

In most of the flow cases, we have seen or we have understood that the Nusselt number varies by this relation: $(Nu = CRe^mPr^n$) where C is the constant introduced due to various geometries or it is a constant that is dependent on the geometry specification. It has different value for different geometries. The value of m and n we will see in the next class. It varies between 0 to 1 this m and n values this indices.

And, how it varies we will see that in the next class when we talk about several flow problems related to forced convection. Forced convection as we have discussed in the flow classification can happen for internal flow for external flow. Now, depending on the flow condition or the flow situation this values would be different. How those are different, what are the magnitudes – those we will look into in to the next class.

Till then go through these materials, consult the textbook reference book that we have mentioned and we will see in the next class.

Thank you for your attention.