Chemical Engineering Fluid Dynamics and Heat Transfer

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Lecture - 42
One-dimensional Heat Conduction (Contd.)

Hello and welcome back once again. In another lecture of heat transfer in the online NPTEL course that is Chemical Engineering Fluid Dynamics and Heat Transfer, we were discussing in the last class a problem that is related to heat generation 1 dimensional heat conduction system.

So, there is a thermal energy generation 1-dimensional heat conduction problem where we are trying to find out what would be a certain temperature of this problem.

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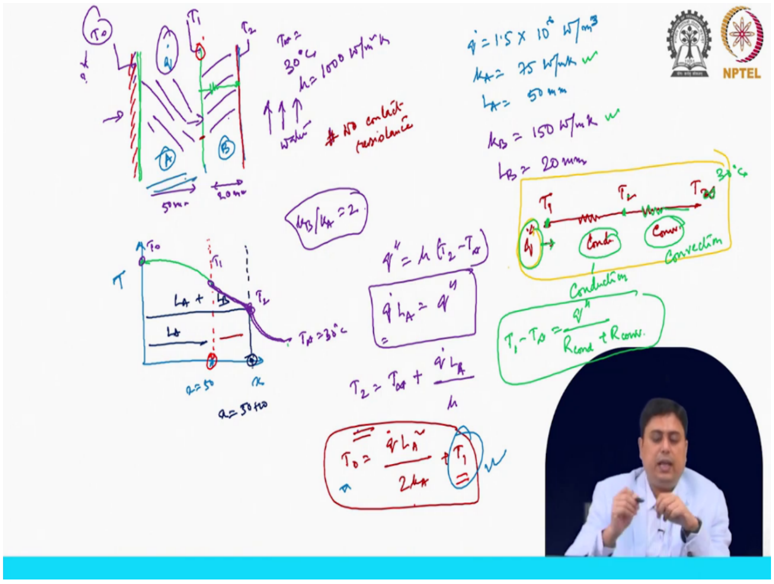

Where the problem was that we had a composite wall of material A material B. The inner surface of material A is the this surface of material A is insulated, material A has a certain thickness, material B has a different thickness the temperature outside of material B is say cooled by a water by water that is having a temperature 30 °C.

So, our task the first task was to find out what would be the temperature profile. Not exactly to the scale, but we had to draw what could be the schematic of the temperature profile and which we have done already, because of the heat generation in material A there will be a parabolic profile from material A interface that is interfacing with material B that from junction till the end of the domain that is the outer surface of material B. We have linear profile, because there is no heat generation.

The slope of that linear profile also we understood that $(k_B/k_A) = 2$. And from the outer surface of material B till the flowing fluid that is there at the outside there would be a temperature drop again due to convection. So, now based on that another data that are provided here. What we had first did we equated the convection heat the heat gain by the convection and the heat lost by material B part B.

Now, in material B in this part the amount of heat that is coming in since it is a 1 dimensional problem we have and assumed. This heat is essentially what is there in material A which is generating, because this side it is insulated. So, there is no chance of any heat input or heat loss from this side.

So, whatever energy thermal energy being generated is being transferred to material B or being dissipated by convection through material B. So, the amount of heat that comes in the heat flux is essentially is equals to heat generation multiplied by the volume of material A. Now, if we consider unit cross section that is why it is written $(\dot{q}L_A)$ is equals to heat flux i.e $(\dot{q}L_A = q^{"})$.

Now, if we just equate these two expressions, the amount of heat flux:

$$\dot{q}L_A = q^{"}$$

$$T_2 = T_\infty + \frac{\dot{q}L_A}{h}$$

Where we know T_∞, we know $\dot{q}$, the generation term is given L_A the thickness of material A is given and h is also given in the problem statement.

So, we can find out the value of T_2. We are here not doing this calculation you can do it this trivial calculation. So, the value of T_2. Now for T_0 that is the second part that we have to find out what is the value of T_0? Now, for this if you consider only this part the material A part there you will see you can consider this system like previously, we have done that one sided insulated system where we know the temperature profile and that looks like in this format.

This is the kind of expression that we derived earlier for that part. But in this expression if we try to look at it. So, here this is the generation term the dot was missing earlier. So, here. So, $\dot{q}$ is known, L_A is known, K_A is known, but T_A this interface temperature which is in between A and B material A and B this is unknown. So, we have to find it out. Before we apply this expression that what would be and for that we have drawn a thermal circuit like this.

This is the amount of heat flux that you see coming out from material A or the domain A to material B. So, this is the interface between A and B where the temperature is T_1. It goes through material B; that means, it goes through a resistance related to conduction. This is inside material B it goes to the outer surface of material B here after this resistance it goes to T_2.

The temperature drops to T2 from T2 it further drops to $T_\infty = 30\ ^\circ\mathrm{C}$. So here the resistance it goes through is because of convection and this is conduction. Now, to find out T_1 what we can write from this expression is that $(T_1 - T_\infty)$ which is the driving force of the flow of heat here or the thermal energy.

$$(T_1 - T_\infty) = \frac{q''}{R_{cond} + R_{conv}}$$

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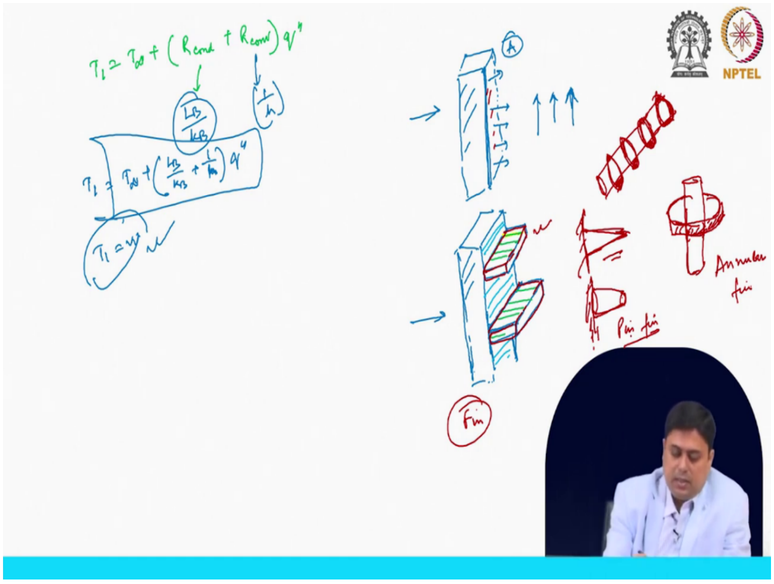

So, in other words what we can write is that:

$$T_1 = T_\infty + (R_{cond} + R_{conv})q''$$

Where the resistance are given by:

$$R_{cond} = \frac{L_B}{K_B}$$

$$R_{conv} = \frac{1}{h}$$

So, now it becomes trivial simple thing because here now we have to calculate T_1,

$$T_1 = T_\infty + \left(\frac{L_B}{K_B} + \frac{1}{h}\right)q''$$

where now everything on the right hand side are known to us and we replace it with the numerical values and find out the value of T_1. So, T_1 is now known to us. Once we know T_1 we go back to this expression. Now, here also everything else was known except the T_1. Now, we know T_1 we can calculate the value of T_0.

So, I hope this concept is now clear to you. That when we had composite wall with heat generation in one part the inner side was insulated. So, what we did to calculate the heat flux we multiplied by the heat generation term with the unit volume with the volume that is the heat flux that is coming out from that system or that domain the material A, because it cannot go on the other side it is completely insulated on the other side.

And then sequentially the steps that I have mentioned here. And again, here we have plugged in the concept of thermal circuit in order to find out a certain unknown, which is the interface of material A and B temperature. And we have written in this way to find out that interface temperature.

So, now we will move on to another concept which is called the extended surface. Extended surface means that we are now realizing that heat transfer the amount of heat being transferred is essentially proportional to the area to which it is exposed or on which it is exposed we have a surface like this. So, from this surface the amount of heat being transferred is essentially proportional to the area which is exposed to the flowing fluid this area A.

There are situations where we want to enhance the rate of heating or the rate of cooling or we may want the may want to enhance the rate of heat transfer that is happening between the flowing fluid and this surface that is exposed to the flowing fluid. Now, in those cases what is what is usually done is that we augment the surface with some additional areas.

So, what you see here is that say this surface that we had you first consider this surface which was exposed to a flowing fluid. The second one the similar surface or say the identical surface now it is having some extended areas that has been attached to it. And those areas are marked by this say red lines that I am drawing now.

So, it is easier for you to distinguish. Once you add these new surfaces or attach this to the base if I consider this now as the base. This becomes an extended surface area. Now, as we have increase the surface area the rate of heat transfer increases. And that sometimes is necessary, because in some case we are having natural convection; that means in our mobile chip or computer chip or CPU etcetera.

There we have a limitation of space we are trying to make it as much compact or smaller as we can for the sake of portability. But there is some heat generation. Now, apart from the fan. There in the CPU there are fans but in certain mobiles or certain laptops there is no such fans are there.

So, how that heat what whatever the chip heat that is being generated by the chip microchips how those are being dissipated. Now, for those conditions it becomes extremely important to come up with the innovative design. Where we can think of or this kind of designs are being implemented where extended surfaces are being created. So, that the natural convection or the natural circulation of air or the fluid improves the rate of heat transfer.

So, in these cases it becomes important to understand that how many number of such plates or different other designs can be incorporated. Also, it is not like for only for the Cartesian coordinate there can be also for a flowing fluid there can be some concentric plates that are attached to it in order to enhance its surface area artificially.

So, either this can be integral while designing it designing such equipment or it can be externally attached by pressing molding etcetera. So, these are the points where we need to understand what would happen if there are extended surfaces to my considered domain.

In those case how the temperature profile would vary or what would be the heat transfer rate or how much it is being enhanced. So, we will discuss those in this section or this part, but before that understand that like this one of the domain I have or geometry of the shape I have drawn here.

So, this rectangular plate that I have drawn here. It can also be in triangular shape. If this is my surface it can be also in triangular shape. As I mentioned for a cylindrical part it can be a circular shape of a certain width or even this can also be a different shape like this.

So, all these have different names. Particularly these all the combinations are called fin. So, this is a straight fin of uniform cross section. This is a straight fin of non-uniform cross section. This can be annular fin and this particularly is called the pin fin it looks like a pin. So, several types of fins can be seen in the real world as examples.

Now, if we try to have a generic understanding of how this works or what would be the governing equation.

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Let us consider a simple scenario where we have a non-uniform case and we consider this differential element where it is attached to the base that mean this is the base surface it is attached to original surface and this is the surface that is exposed to the flowing fluid.

So, for a differential element that looks like this. So, say this kind of a thing where this is (x+dx), this is the dx, this is q_x, this is the surface area if I say that this is dA_S and the heat that is lost from the surface area is say $dq_{convection}$ because if this is the dx part. So this is the area that is A(x) varying with x.

So, for this differential analysis what we can write is that:

$$q_x = q_{x+dx} + dq_{convection}$$

So, q_x amount of heat is coming, going or leaving the domain q_{x+dx}.

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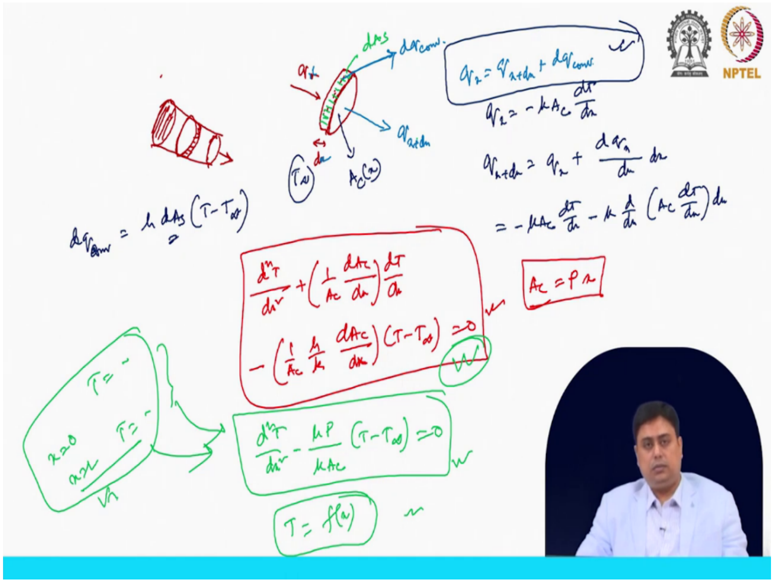

And heat loss by the surface is $dq_{convection}$. Now, from Fourier's law we know that:

$$q_x = -kA_c \frac{dT}{dx}$$

the A_c the cross sectional area, because this is the periphery the peripheral surface area and, but q_x would be the A_c would be the normal to the surface across which the heat transfer is happening by convection.

So, if I consider that as A_c it becomes the cross sectional area and that cross sectional area is in this case if I mention as this is dA_s already we have mentioned this is leveling would be here it is A_c which is varying with x. So, if that is the case then similarly what we can write it write for this is essentially:

$$q_{x+dx} = q_x + \frac{\partial q_x}{\partial x} dx$$

$$= -kA_c \frac{dT}{dx} - k\frac{d}{dx}\left(A_c \frac{dT}{dx}\right) dx$$

And this convection part that we can write is essentially:

$$dq_{conv} = hdA_s(T - T_\infty)$$

So, this dA_s is the surface area of the differential element now once we substitute this in the energy equation what we actually obtain is an expression that would look like:

$$\frac{d^2T}{dx^2} + \left(\frac{1}{A_c}\frac{dA_c}{dx}\right)\frac{dT}{dx} - \left(\frac{1}{A_c}\frac{h}{k}\frac{dA_c}{dx}\right)(T - T_\infty) = 0$$

So, the point is there this is the generic development for any kind of fin that you can do by following such steps. That is the complete energy balance the overall energy balance and then finding out each and every term of the energy balance. Now, this provides the general form of energy equation for an extended surface and solution of this equation would require appropriate boundary condition with respect to x we would need 2 boundary condition and then we can find out the solution of this.

Now, this expression can be simplified if the area or the cross-sectional area is not varying. So, in case of uniform fins like this ok. So, wherever we will have uniform surface uniform cross sectional fin then A becomes constant the A_c the cross sectional area becomes constant which is essentially the perimeter multiplied by the x where the area that we see is the fin perimeter which is attached to the base and in the x directions wherever it is going.

So, and then if we substitute this in this expression if this expression would eventually reduces to:

$$\frac{d^2T}{dx^2} - \frac{hP}{kA_c}(T - T_\infty) = 0$$

Much simpler form, because here we have assume the fin cross sectional area is uniform. So, this expression can be analytically solved again with appropriate boundary condition with respect to x.

Once we solve it like the previous cases that we have seen. Once we know for (x=0) T is something at (x = L) T is other temperature with this kind of boundary conditions, we can solve this equation analytically. Once you solve it you can find out the variation of T with respect to x.

Now, as I mentioned when such kind of boundary conditions are there you remember it is called the Dirichlet boundary condition where the temperatures are specified. The situation can be a mixed boundary condition that at (x = 0) or (x = L) one of the temperature is known, but the other one is the flux that is mentioned which is the Neumann boundary condition. Depending on these boundary conditions this analytical solution would be more convoluted or complicated.

There are several expressions I would suggest that you go through the textbook or the references book that I have mentioned those expressions for different types of boundary conditions how the solutions looks like, because that eventually gives the compact solution or the final solution of this case.

And depending on as I mentioned if the fin is infinite fin which means $(L \rightarrow \infty)$. So, in that case this solution will have a very specific form and sometimes that specific forms is needed, because then for such related problem you need not derive all these expressions.

If you remember that expression it can be directly applied, because derivations of these analytical solutions are not trivial. These are complex, but this can be solved analytically. So, for the extended surface part I will stop here and I will suggest that you go through those expressions as per a various boundary conditions provided our this equation is understandable to you.

This simple derivation of energy balance of variable cross-sectional area once you understand this this expression like the generic heat diffusion equation can be simplified for different scenarios wherever it is needed. So, I will stop here today and in the next class of this week or the next three classes of this week I will take up the transient heat conduction part. Till then thank you for attention and practice this part.