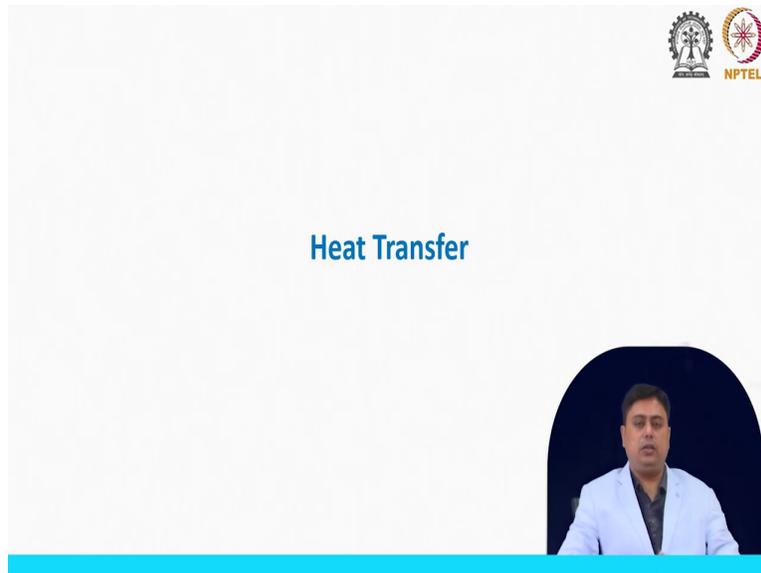**Chemical Engineering Fluid Dynamics and Heat Transfer**
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Lecture - 35
Fundamentals and Mechanism of Heat Transfer (Contd.)

Hello everyone. Welcome back once again in the another class of Chemical Engineering Fluid Dynamics and Heat Transfer.

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In the last class we spoke about the heat equation or the heat diffusion equation. And we have seen in three coordinate system how the form of heat diffusion or the heat equations can be in which for the Cartesian coordinates we have derived those expressions and have written its analogous form for the cylindrical as well as the radial coordinates.

Now, the thing we should remember that the temperature gradient in Fourier's law that has to have an unit of temperature per unit length. So, accordingly when we do this for different coordinate system for the Cartesian coordinates its simple its easier, but for the polar or the cylindrical systems particularly when it is not in the radial direction.

We must take care while having evaluate I mean when we try to evaluate this q or the heat transfer rate in the proper format. Now, considering this case let's apply our heat diffusion equation or let's understand how this heat diffusion equation in Cartesian coordinate helps us to analyze certain system or the thermal systems. Now, say the heat distribution the problem is given that a heat distribution across a wall.

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So, say the wall that we have is a thick wall. Now, say this wall is of 1 m width. Now, the temperature distribution across this 1 m thick wall at a certain or at a given point of time is

$$T(x) = a + bx + cx^2$$

Where, T is given in ºC.

However, and at the same time a is given as 900 ºC, b is having a value which is -300 ºC/m and c is -50 ºC/m^2. These values are given for this temperature profile. So, there is a 1 m thick wall inside which there is a temperature variation that has a form of $(a + bx + cx^2)$ where the coefficient values a, b and c are given.

And in this case that we have the heat transfer that is happening, in the x-direction. The other properties that are necessary to know for this material, say density of this material it is known say 1600 kg/m^3. Thermal conductivity value k is also given 40 W/mK.

C_p the heat capacity it is also mentioned as 4 kJ/kgK. So, these are the things that are given here. What we have to determine is the rate of heat transfer entering the wall at (x=0) and leaving the wall which is at (x = L = 1 m). We have to find out what is the rate of heat transfer at these two positions.

So, the first question is that what is the rate of heat transfer at (x = 0) and (x = 1 m). So, how do we solve such problem? For this the other information that we require is the area of this heat transfer. Because now we see that the direction of heat flux is in the x-direction. So, this is the first assumption that we do is that we have one dimensional heat transfer or heat conduction that is happening here.

So, the area through which this is happening that must be mentioned. Say, for example, here if I consider as 10 meter square. So, now we try to solve this problem. So, this is my q in this is my q out. So, how do we solve this problem? So, the first assumption that we have done here is that 1D heat transfer is happening.

The medium is isotropic and say there is because there is a temperature distribution, or there is a temperature profile and also there is it is mentioned that there the reason is that we have a heat generation term which is $\dot{q}$. Say, the uniform heat generation is happening there. This $\dot{q}$ value is also given here is 1000 W/m^3 , per unit volume it is mentioned.

The wall area is 10 m^2 having all these properties. So, whatever information we need it is given here. So, this is the uniform heat generation, isotropic, 1-D and uniform heat generation. So, these are the things that are happening the assumptions that we have made here.

Now, once the temperature distribution is known in the medium the rate of heat transfer we can easily calculate from Fourier's law. So, heat equation gives us the temperature profile. Once the temperature profile is known, we can apply Fourier's law to estimate heat flux or heat transfer rate. These are the two key steps you need to remember while analyzing a problem.

So, for any given problem either you have to find out at first the temperature profile or the temperature variation in the domain. Once it is known be it a steady state or unsteady state you can apply Fourier's law to estimate your heat transfer rate or heat flux, if you know the area through which this is happening.

So, here since we need to have this heat transfer rate say q_x, now that we are calculating at ($x = 0$) or in other form that we can write, $q_x(x = 0)$ and it is in the x-direction. What we write

$$= -kA \frac{\partial T}{\partial x}\Big|_{x=0}$$

which means this is the variations that we have,

$$T(x) = a + bx + cx^2$$

We take a derivative with respect to x and replace it here what we get is:

$$q_x = -kA(b + 2cx)|_{x=0}$$

So,

$$q_x|_{x=0} = q_{in} = -kbA$$

Now, we replace all the numerics here and find out the value whatever it comes. Similarly, here now we know this is the k value the, b is given and area is also mentioned. So, it is essentially the amount of heat transfer that is happening the b is given 300. Now, the negative sign cancels out k is 40, A is 10. And the unit is Watt, which is coming out to be 120 kW. Similarly, you can find out for $q_x(x = L)$ or q_out where L is equals to 1 m.

$$q_x|_{x=L=1m} = q_{out} = -kA \frac{\partial T}{\partial x}\Big|_{x=L} = -kA(b + 2cx)|_{x=L=1m}$$

Again, we replace all now the known values that are given and we find out a numeric value accordingly. So, these calculations are trivial and I am leaving this to you. So, which means once the temperature field is known we apply Fourier's law in conduction to get the heat transfer rate or the heat transfer flux by conduction that is happening.

So, now the next question if it happens to be that what is the rate change of storage heat storage by the wall? Because in is known, out is known, generation is known. So, all of its values once it is known, if we go back to this energy conservation equation that is $q_{in} + q_{gen} - q_{out} = E_{storage}$. Then what we get the rate change of energy storage in the wall.

So, here how it is happening how what is the value that would come? So, in this case again if I write about this.

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So, that I mention that:

$$E_{in} - E_{out} + E_{gen} = E_{st}$$

So, that means, what it implies $E_{storage}$ is equals to here that the value we have in this case we have found 120kW i.e

$$E_{st} = 120 - X + 1000 \times 10 \times 1$$

Where X is the value of q_{out}.

q_g or E_{gen} is given as the value of per unit volume. It is multiplied by the area and the width to get the complete volume of the domain and then it is multiplied with the per unit volume generation term to reach at the complete E_g value that is the energy generation. This gives us the energy storage value in terms of again kW because here we have use the unit as kW.

Now, if it is further required that what determine the time rate of change of temperature at (x=0) and at (x=0.5), in the middle of it. At, (x = 0.5 m). In such, case what we should do? So, now the

question is time rate of change of temperature at any point if we can find out that expression then whatever the value that would be ask that is either (x = 0) or (x = 0.5) or (x = 0.75), whatever value they would ask you can immediately calculate.

So, for this what we have to understand or what we have to now check that the time rate of change in temperature.

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So, the temperature distribution that is happening that we know from this expression is that, this is again we are writing the heat equation or the heat diffusion equation for this specific problem having several simplifications. Because, now the this is one dimensional heat transfer that is happening the medium is isotropic that is considered. And for such scenario what we can write.

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{\rho c_p}$$

So, from this heat diffusion or the heat equation which is now in one dimensional form and the isotropic media that equation that we have seen earlier can be simplified and written in this form. Now, from here our objective is to find the rate of change of temperature at a certain point. So, what we can write or what we can see from here is that which we can write:

$$T(x) = a + bx + cx^2$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right)$$

$$= \frac{\partial}{\partial x}(b + 2cx)$$

$$= 2c = 2 \times -50 \, °C/m^2$$

$$= -100 \, °C/m^2$$

So, this is this derivative that we can see is independent of the position in the medium, there is no x term on the right hand side.

So, the time rate of change of temperature in the medium would also be independent of the position. So, that if we see here, this value when it goes or replaced here, we would see that the time rate of change of temperature in the domain is now also irrespective of the value of x. So, be it either at 0, be it in the middle of the domain or quarter whatever the positions inside the media that would be a constant value.

So, here you have already got the value of this parameter all these other parameters are already given here, all the numerical values are mentioned here which you can easily replace in this expression and find out the value of the time rate of change of temperature inside the medium. And if you do the calculations it would you see that the value would come numerically close to :

$$\frac{\partial T}{\partial t} = -4.7 \times 10^{-4} \, °C/s$$

Such values you may arrive at this if you do the calculations properly. Because here why I am living with this trivial calculations is that all the parameters are known to you the (ρc_p) values k values $\dot{q}$ which is already known here, we just simply replace this numerical values and calculate the time rate of change. So, what is the bottom line from this problem? The bottom line is that we know the temperature profile, we apply Fourier's law, we get heat transfer rate or heat flux values.

We apply this energy conservation equation whenever we try to find out the storage, the thermal storage of the medium if it is transient. And also, for the transient case how the temperature is varying when we apply the heat equation that also we can find out provided we knew this spatial

variation of temperature. So, all the aspects of heat diffusion equation or the heat equations we have covered it with this example.

Now, there can be several scenarios of different boundary condition, the heat equation because it is that we can see it is a double derivative with respect to spatial coordinate and it has first order derivative with respect to the time. So, we need three boundary conditions to solve such problem analytically if we try to solve; that means, two with respect to the spatial coordinate and one with respect to the time, if we have to solve this with on a transient case.

For a steady state case as this term would not be there. The transient term vanishes, we would need only two boundary conditions to solve this governing equations or the to solve this heat equation. So, spatially when it is there are several conditions that you may see or you will encounter. Say, for example, there is a surface, and say this is a medium that is there.

The surface temperature say is at temperature which is T_s and anywhere in the medium the temperature that we usually mention is if it is one dimensional is varying with space and time. So, if this the problem says that we have a constant surface temperature which means:

$$T_s = T(0, t)$$

This is called the constant surface temperature boundary condition.

Now, there can be the other such scenario a constant heat flux surface heat flux condition. That means, so for this example is that somehow, we are maintaining the surface temperature at a it's particular constant value. Say for example, a fluid is flowing over a flat plate the plate temperature and the fluid temperature are different.

But somehow by constant heating or constant cooling of the surface externally we are maintaining the surface temperature at a constant value. That means we are externally providing heat to the surface. So, that it can maintain a constant surface temperature. The other scenario can be the constant surface heat flux.

So, this is constant surface temperature some I mean conventionally it is called the Dirichlet boundary condition. The other case is the constant surface heat flux that its temperature may vary,

but your given amount of heat to the surface considering the same example that a fluid is flowing over a flat plate and the temperature of the fluid and the flat plate temperature is different.

Now, the flat plate temperature is varying although there is a constant heat flux or constant amount of heat that you are giving to the surface because the fluid is taking most of that energy and constantly changing the surface temperature that is in contact with it. So, constant surface heat flux in that condition the boundary condition may be written as that is the constant source that we are providing.

$$-k \frac{\partial T}{\partial x}\bigg|_{x=0} = q_s''$$

So, in that case what happens? Instead of a constant surface temperature the thing we have is the constant amount of heat that is given into the system.

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Now, one of its extreme can be or one of its variation can be that this at (x = 0). And this condition is called the adiabatic or insulated boundary condition or insulated surface.

$$\frac{\partial T}{\partial x}\bigg|_{x=0} = 0$$

And, this generic form is called conventionally Neumann boundary condition. So, when we have constant surface temperature, we say this is the Dirichlet boundary condition by the name of the scientist Dirichlet another scientist named Neumann this is the boundary condition that we call is the constant surface heat flux boundary condition is conventionally referred as Neumann boundary condition.

Sometimes you will hear the boundary condition of first kind, boundary condition of second kind. First kind is the same as the Dirichlet boundary condition, second kind is the same as the Neumann boundary condition. And the third kind or the mixed boundary condition can be that along with this scenario when you have a constant flow of a certain fluid.

That means the amount of heat transfer is balanced by the conduction inside the domain as well as on the surface it is by the convection by the other fluid. So, it is then called the boundary condition of third kind or convection surface condition which is:

$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = h(T_\infty - T(0,t))$$

So, then in such case the temperature here at (x = 0) and at a temperature at a time t, at the same time there is a fluid that is flowing over it with a temperature T_∞. At any point T(T, x), this is the

boundary condition of the third kind or mixed boundary condition. So, these boundary conditions are necessary when we solve this heat diffusion equation, this differential equation.

Because we needed here the two-boundary condition with respect to space or with respect to coordinate and if it is transient then we require another boundary condition or usually it is the initial conditions we got a (t = 0) what was the temperature? That is the information that also we need because it then becomes a differential equation of first order with respect to the time.

So, we will see again the examples, the applications when we further discuss the steady state heat conduction from the next class, its applications and the other relevant details. So, I hope you have understood the fundamental mechanism of three modes of heat transfer, where we have particularly focused on the conductions because it would be dealt in the subsequent immediately next lecture in detail.

What we have understood the utility of various thermo physical properties, what is the isotropic medium, what is heat equation, heat diffusion equation in all the three directions, unsteady state scenario, steady state scenario. So, all the basic details you have hopefully learnt in this first week of this course.

With this, I stop here and will be back within in the next class with the one dimensional or steady state conduction in details.

Thank you for your attention.