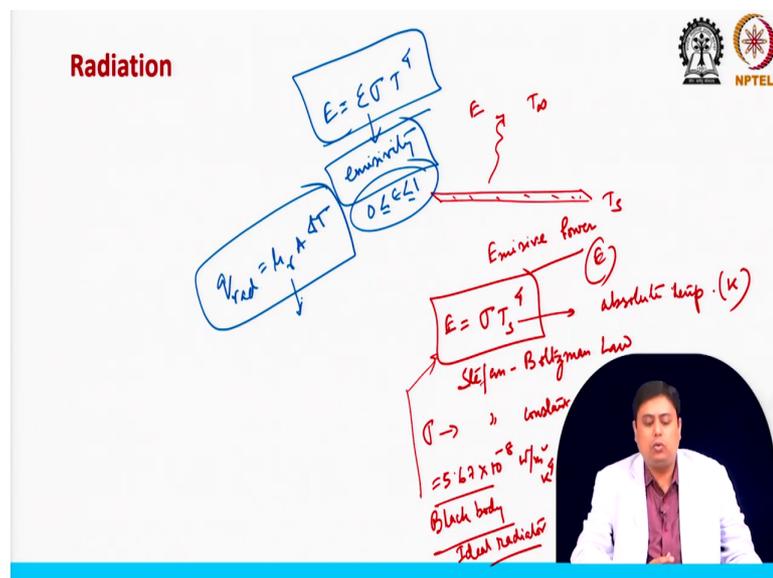


**Chemical Engineering Fluid Dynamics and Heat Transfer**  
**Prof. Arnab Atta**  
**Department of Chemical Engineering**  
**Indian Institute of Technology, Kharagpur**

**Lecture - 33**  
**Fundamentals and Mechanism of Heat Transfer (Contd.)**

Hello everyone. Welcome back once again with a new class on this Chemical Engineering Fluid Dynamics and Heat Transfer.

(Refer Slide Time: 00:39)



We are discussing about the different mechanisms of heat transfer and its fundamentals. We have covered conduction and convection very basic principles. Now, regarding the radiations, we have briefly spoke about it. Now, the thermal radiation that we typically consider that it is the energy that is emitted by a body of non-zero temperature.

So, any body that is of non-zero temperature emits energy. Now, mostly in fact in this course we will consider that this body as solid, but that does not mean that the gas or the liquid does not emit energies, it also have radiations already if those are at non-zero temperature. So, when we speak about radiations in this topic, in this course we mostly talk about the solid bodies.

Now, this transfer of energy by conduction and convection we know that it requires a medium a presence of a material, but radiation does not. This radiation heat transfer occurs

based in vacuum. Now, if we consider the radiation transfer or the radiation heat transfer, we can as I mentioned that we will talk about only solid bodies and the radiation from it. So, if the solid body is at a temperature say  $T_s$  and the ambient temperature is  $T_\infty$  or a vicinity of the solid surface having a temperature  $T_\infty$ .

Now, there is radiation heat transfer and it's measure it measured by its emissive power; by its emissive power. Now this body is when it emits there is a maximum limit or there is a upper limit to this emissive power and that is prescribed by a law called Stefan - Boltzmann law. The maximum emissive power of a body can be estimated from this law.

So, where  $T_s$  is the absolute temperature in K and  $\sigma$ , we call that as the Stefan Boltzmann constant that has a value of  $5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$ . Now, if we come across such a body that emits this much power exactly that body is called the black body or an ideal radiator.

It radiates to its maximum possible amount, but the heat flux emitted by real body is not of this much magnitude and that is given by where this  $\epsilon$ , we call this as the is a radiative property of the surface and we call the emissivity. A factor that varies between 0 to 1.

So, this is a material property or a radiative property of a material that takes a value when this is not a black body or ideal radiator for the real case it would be lesser than the black body radiation. So, this value as I mentioned depends on the material property as well as now since it is radiating its finish the surface finish the surface type the surface category.

Now, we will talk on this in detail when we talk about the radiation, but eventually also in radiation when we try to estimate the amount of radiative energy that is being transferred. We similar to the previous discussions like the convection and conduction we try to find out that the amount of radiation the  $q_{\text{rad}}$  that can happen through a simple and generic expression we try to find out by this way, where this  $h_r$  is the radiation heat transfer coefficient which again, we need to estimate.

A is the area through which that is happening and the  $\Delta T$  is the body temperature and the surrounding temperature the difference between these two. So, eventually what it boils down to that in all the modes of heat transfer we will our objective are essentially is to find out how much heat transfer is happening what is the rate of that heat transfer and in order to do so, in convection and in radiation what are the values of convection heat transfer coefficient and radiation heat transfer coefficients.

These are the primary objectives when we will discuss we will see that those are mainly being addressed in the relevant sections. So, with this preamble and this understanding on the radiation convection and conduction. So, this emission before I go to the next topic. So, this emission this emission energy that is happening, we can consider this as the electromagnetic waves by this it is being emitted.

So, again as I mentioned that all these things we will discuss in detail in the relevant section.

(Refer Slide Time: 09:01)

**Conduction rate equation**

Fourier's law is phenomenological

Handwritten notes and diagrams on the slide:

- Left side:  $q_n = -k \frac{dT}{dx}$  and  $= -k \left( \frac{dT}{dx} \right)$
- Middle:  $q_n \propto A \frac{dT}{dx}$  and  $q_n = kA \frac{dT}{dx}$
- Bottom middle:  $\frac{q_n}{A} = \frac{q_n}{A} = -k \frac{dT}{dx} = -k \frac{dT}{dx}$
- Right side: A diagram of a cylindrical rod with temperatures  $T_1$  and  $T_2$  at its ends and heat transfer rate  $q_n$ . Below it,  $\Delta T (T_1, T_2)$  and a box labeled "insulators".
- Center right:  $q_n \propto \frac{1}{dx}$ ,  $q_n \propto \Delta T$ , and  $q_n \propto A$

Now, what we do we move on to the conduction in details and the first point in conduction in details that we talk about is the Fourier's law because we have seen its formed that the conduction rate equation is essentially the Fourier's law that we have seen.

Now, we have to see its origin how it came. It is a phenomenological model that means this Fourier's law came after doing several experiments or observing the phenomena, what are those kinds? Say for example, I have a slab and here I have a temperature  $T_1$  and here I have a temperature  $T_2$  on this side.

Now, if there is a  $\Delta T$  that means the temperature varies between  $T_1$  and  $T_2$  that the  $T_1$  is say the hotter side and  $T_2$  is my cooler side then what has been seen that if we maintain this  $\Delta T$  somehow. If we maintain this  $\Delta T$  somehow the heat transfer rate is essentially

proportional to  $\frac{1}{\Delta x}$ , if this is the  $\Delta x$ , inversely proportional to the  $\Delta x$ , that as the  $\Delta x$  increases the rate of heat transfer decreases.

Similarly, for a fixed  $\Delta x$ , if we keep now  $\Delta x$  constant and we study this transfer  $q_x$  from hotter to cooler side what we see that it is proportional to  $\Delta T$  and at the same time  $q_x$  has been seen to be directly proportional to the area through which this heat transfer is happening. So, now combination of all these leads to a form that is:

$$q_x \propto A \frac{\Delta T}{\Delta x}$$

This is a collective form for this process to understand this conduction.

Now, if this is the case then in the next stage what we can do is that we eliminate this proportionality and we introduce the proportionality constant what happens with that we come across this format where again  $K$  is thermal conductivity that we have seen earlier  $W/mK$  and this is the most important parameter in conduction for the material.

Now, when this expression in the limit of  $\Delta x$  tends to 0 what we can write the about this expression is that

$$q_x = -kA \frac{dT}{dx}$$

$$\text{Or, } \frac{q_x}{A} = q_x'' = -k \frac{dT}{dx}$$

this is what we have seen earlier and you may remember that this negative sign is introduced or is there is necessary because heat always transfer from a higher temperature to a lower temperature or a decreasing temperature with a negative slope.

The slope has a negative value and in order to be this heat flux to be a positive quantity this negative sign is introduced. So, this heat flux is normal to the cross sectional area of  $A$  wherever that we consider. So, it is in general the direction of heat flow happens always normal to a surface of uniform temperature. So, if this is the surface that I have of uniform temperature  $T_1$  the  $q_x''$  happens normal to this uniform surface temperature. So, this uniform surface temperatures are usually called as isotherms.

So, heat flux is a vector quantity and if we try to write this in more generic form then what we can write about this is that is

$$q_x'' = -k \nabla T$$

$$= -k \left( i \frac{dT}{dx} + j \frac{dT}{dy} + k \frac{dT}{dz} \right)$$

(Refer Slide Time: 15:39)

The slide titled "Conduction rate equation" contains the following content:

- Title:** Conduction rate equation
- Text:** Fourier's law is phenomenological
- Text:** isotropic
- Equation:**  $q'' = -k \nabla T$
- Equation:**  $q'' = -k \left( i \frac{\partial T}{\partial x} + j \frac{\partial T}{\partial y} + k \frac{\partial T}{\partial z} \right)$
- Equation:**  $q'' = -k \frac{\partial T}{\partial n}$
- Diagram:** A cylinder with heat flux  $q''$  and temperature  $T$ .
- Diagram:** A rectangular block with heat flux  $q''$  and temperature  $T(x, y, z)$ .
- Equation:**  $q'' = -k \frac{\partial T}{\partial x}$
- Equation:**  $q'' = -k \frac{\partial T}{\partial y}$
- Equation:**  $q'' = -k \frac{\partial T}{\partial z}$
- Equation:**  $q'' = i q''_x + j q''_y + k q''_z$
- Equation:**  $q'' \propto \frac{1}{\Delta x}$
- Equation:**  $q'' \propto \Delta T$
- Equation:**  $q'' \propto A$
- Equation:**  $q'' \propto k \frac{\Delta T}{\Delta x}$
- Equation:**  $q'' = kA \frac{\Delta T}{\Delta x}$
- Equation:**  $q'' = -kA \frac{\Delta T}{\Delta x}$
- Equation:**  $q'' = -k \frac{\Delta T}{\Delta x}$
- Equation:**  $q'' = -k \frac{\partial T}{\partial n}$
- Diagram:** A person in a white lab coat speaking.
- Logo:** NPTEL

Well, this is a  $\nabla$  operator and in three dimensional the  $\nabla$  operator is having a this form.  $T$  is a scalar temperature field. So, what we can see from this expression as well or in a more compact way or in a different way as well alternate if way to write the Fourier's equation that we can do is when we call this as the

$$q_n'' = -k \frac{\partial T}{\partial n}$$

where  $q_n''$  is the heat flux in a direction  $n$  that is normal to the isotherms.

So, this quantity further what we can do heat flux we can also write in this form in all the direction :

$$q'' = i q_x'' + j q_y'' + k q_z''$$

where our  $q''$  it will be:

$$q_x'' = -k \frac{\partial T}{\partial x}$$

$$q_y'' = -k \frac{\partial T}{\partial y}$$

$$q_z'' = -k \frac{\partial T}{\partial z}$$

Now, each of this expression shows that the heat flux across a surface to the temperature gradient in a direction that is perpendicular to the surface it is happening. So, in particular when we talk about this expression, we have considered here that  $k$  is not varying inside the medium or the medium is isotropic that the thermal conductivity value does not change spatially.

So, what we have seen here? The origin of Fourier's law, its vector form, the heat flux vector form, the components of the heat flux in 3 dimension and the understanding of isotropic material that means, where the thermal conductivity value does not change.

(Refer Slide Time: 19:22)

**A few relevant thermophysical properties**

- Transport
  - thermal conductivity  $(W/mK)$
  - kinematic viscosity  $(m^2/s)$
- Thermodynamic
  - Density  $\rho$
  - specific heat  $C_p$
  - Volumetric heat capacity
    - ability of a material to store thermal energy

Handwritten notes on the slide:

- $\alpha = \text{Thermal diffusivity} = \frac{k}{\rho C_p}$
- $\rho C_p \rightarrow \text{volumetric heat capacity } (J/m^3K)$

And the other properties that are important for conduction analysis those are mainly called the thermo physical properties which has mainly two component one is the transport component the other one are the thermodynamic components. So, in the transport component the points that we have or the properties that we usually consider are the thermal conductivity of the material and kinematic viscosity of the material.

In thermodynamic properties the things that are important in conduction analysis are the density and specific heat because this density,  $\rho$  and specific heat,  $C_p$  ( $\rho c_p$ ) this quantity together we know these are called the volumetric heat capacity ( $J/m^3K$ ).

So, volumetric heat capacity what it does? It gives us a measure of how much thermal energy can be stored in a material ( $\rho c_p$ ) value that we know already and here the thermal conductivity value small  $k$  and the kinematic viscosity which is  $\nu$  that we typically designate. This requires as I mentioned that thermal conductivity is purely related to the conduction or the diffusion.

Now, in convection analysis we require the transport property which comes from this kinematic viscosity that is the transport property it is for the momentum transfer. Now, this volumetric capacity volumetric heat capacity that is ( $\rho c_p$ ), its a ability of a material to store thermal energy and the other factor that now relates in combination of this property is the  $\alpha$  value that we call the thermal diffusivity which is :

$$\frac{k}{\rho c_p}$$

what it means?

It means, that the material how much it is able to conduct with respect to how much it can store the thermal energy. So, its a relative measure the value thermal diffusivity gives us an idea about a material that how much it can conduct with respect to how much it can store, the larger the value of  $\alpha$  which means, it would quickly respond to a temperature change from one end to other end the conductivity is higher.

The lower the value of thermal diffusivity which means the denominator is now dominant it means, it would store the most of the energy that is being transferred to it and its response to this temperature change would be lesser. And that is why considering different material different type of material iron, steel, wood the temperature the response to a change in temperature at one end of a bar be it a iron or say wood.

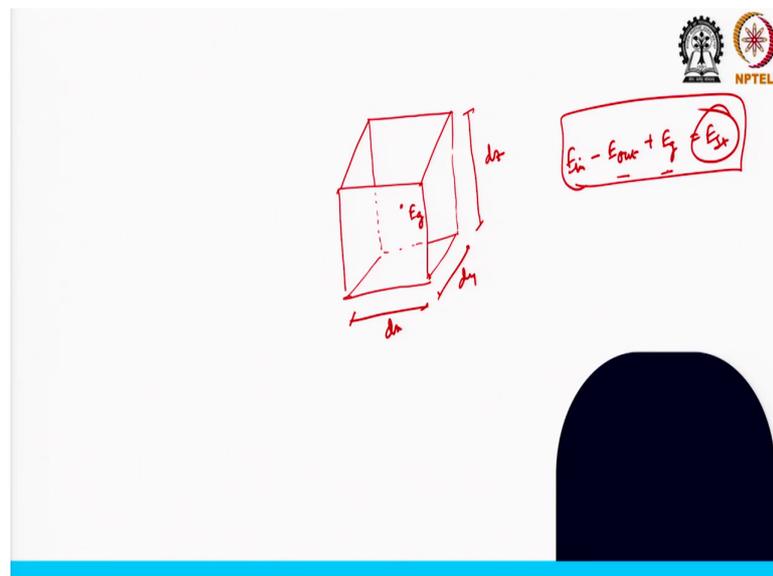
The response to the temperature change is much higher in case of the iron or the steel bar because of this higher thermal conductivity value in the other cases that is not the case the

case is the value of case much lower that is that is why the thermal diffusivity value is lower in this cases.

So, these properties are essential and in most of the cases these properties are already estimated and are usually given in order to solve a problem be it a design problem, be it a conceptual problem and those values are usually mentioned in the appendix of a textbook or in various reference books the text book that I mentioned in the first class of my part there in the appendix you may find several this for the values of several such materials thermal conductivity, diffusivity, the  $(\rho c_p)$  etcetera explicitly those values are mentioned.

So, these are the fundamentals that we need to know and with this we then move on to the understanding of the heat diffusion equation in particular that is contextual to the heat conduction equation.

(Refer Slide Time: 25:29)



So, if we consider a homogeneous medium say we have a homogeneous medium and we know at the same time the control volume analysis. So, if we consider a homogeneous media in which there is no bulk motion; that means, advection is not there the temperature distribution if we try to understand the this and the Cartesian coordinate, we apply the conservation of energy concept.

So, say for example, we have this differential element where these are the dimensions and there is say energy generation that is happening that is now this energy generation can

happen by several ways either chemical or thermal ways. Chemical means say for example, inside this domain some chemical reaction is happening which is exothermic in nature.

So, that chemical that energy that comes out from the chemical reaction eventually is converted to thermal energy and get dissipated in the domain inside the domain and that is how a energy generation can happen inside a domain or in other way say there is a electrical wire that is wrapped around it or has gone through this material.

Now, when it passes through the electric current passes through that that electrical wire due to resistance, it also generates heat that can be a continuous source of heat externally. So, when we talk about the energy balance we consider energy inflow, energy outflow the energy generation inside it and eventually all these sums up to be the energy storage by the domain.

So, the amount of energy that it can store is eventually the

$$E_{in} - E_{out} + E_g$$

is eventually the energy that is being stored. So, in this differential balance for the Cartesian coordinate we will apply this simple concept in order to understand what would be the temperature distribution inside the domain in all three direction.

So, we will take this derivations in the next class, but the point that you must go through to do this is the basics of the fluid mechanics where the differential analysis part has been done. Again, refresh your memory because that would help in to understand this derivation in a more better way because we will not go into the every details of it how it is done.

We just sum up the energy in, energy out and energy generation part in order to derive an expression for the temperature profile inside the domain. And now this is how and this is why you can see that these two subjects have a common thread that in order to solve this heat transfer problems we need needed to know the fluid mechanics the fundamentals at the very beginning.

And this is how this course has been structured so that we have a common thread and we can find the analogies and we can use the concept that we have learned continuously in the heat transfer part. So, with this preamble we stop here because we take up this derivations

in the next class we solve it and we also solve a work out problem in order to understand how it works.

With this I thank you for your attention and we will see you in the next class.