Chemical Engineering Fluid Dynamics and Heat Transfer
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Lecture - 24
Turbulence - 02

So, welcome back.

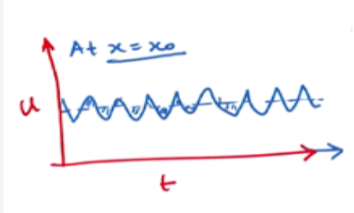
Reynolds decomposition of turbulence,

$$u(y,t) = \bar{u}(y) + u'(\Gamma, t)$$

according to Reynolds Decomposition the description of turbulent flow consists of a superimposed streaming (mean) and fluctuating (eddy) motion. There are two different way the mean can be calculated in turbulent flow.

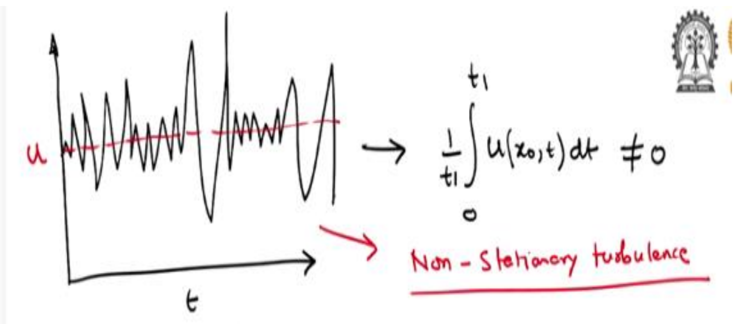
1.Time average of a flow at a particular location.:

$$\bar{u}^t(x_0) = \lim_{t_1 \to \infty} \frac{1}{t_1} \int_0^{t_1} u'(x_0, t)dt$$



Note: If the value of $\bar{u}^t(x_0) = 0$,It means that the fluctuation spectrum remains consistent with time. And so therefore, the fluctuation at a particular point x_0 or at a particular location within the flow field remain consistent with time and such turbulence is called the stationary turbulence.

So, if at a particular location with time this integral is not equal to 0, $\bar{u}^t(x_0) \neq 0$,and such kind of turbulence is called non-stationary turbulence. Because, as you can see from the graph, the areas above and below the graph are not going to be equal.

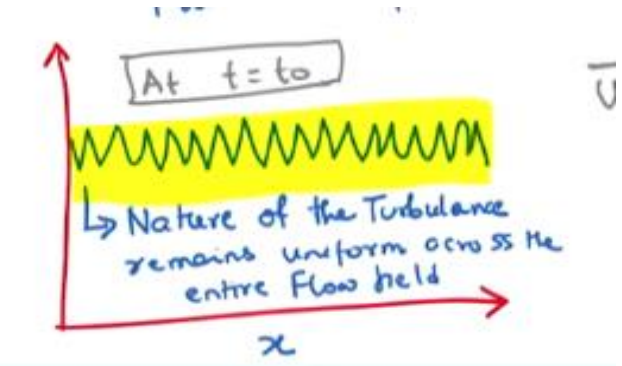


2.Space average of a flow at a particular time:

Here we are considering the fluctuation spectrum across the entire flow field at a particular time.

$$\bar{u}^s(t_0) = \lim_{x\to\infty} \frac{1}{x}\int_0^x u'(x_0, t)dx$$

Note: If the value of $\bar{u}^x(s_0) = 0$,it means that the nature of the turbulence remains uniform across the entire flow field at specific instance of time. So, then that turbulence is called the homogeneous turbulence. That means, that the nature of the fluctuation at any given instance of time is uniform over the entire flow field. If the turbulence is homogenous, then the correlation length is considered to be infinity. If the correlation length is finite, then you will have non-homogenous turbulence.



Is it possible to have a steady state turbulence?

And we know that steady state means the velocity component should be independent of time. If you take Reynolds decomposed form of turbulence, then we can find that the fluctuation spectrum is always a function of time. But probably the nearest analogue of steady state turbulent flow is steady uniform flow. So, from location to location the velocity profile does not change and with time at a particular location the velocity profile also does not change which makes it steady and which makes it uniform. So, the analogue to steady uniform flow will be stationary homogeneous turbulence and also the mean velocity also remains constant.

1.All means $\Longrightarrow$ time average only

2.We are taking the assumption that turbulence as stationary turbulence.

$$\bar{u}^t(x_0) = \lim_{t_1\to\infty} \frac{1}{t}\int_0^{t_1} u'(x_0, t)dt = 0$$

If we are looking into the entity mass flows between the time t_1 and t_2

$m = \int_{t_1}^{t_2}(\rho u A)dt$ ρ is the density, A is the cross-sectional area.

$$m = \int_{t_1}^{t_2}(\rho u A)dt = \rho A \int_{t_1}^{t_2} u' dt = 0$$

There is no flow of mass due to the fluctuations. In order to sustain turbulence, you need to spend higher energy or higher inertial energy that is why the Reynolds number is high. Therefore, in a turbulent flow if you are spending more energy, you have fluctuations, but that is not helping in the mass flow rate. And we also have,

$$\int_0^t u'v' dt \neq 0$$

that means, due to the interaction between the fluctuation spectrum or components, there is macroscopic momentum transport. there is non-zero macroscopic momentum transport due to the fluctuations or the interaction between fluctuation in different directions.If time average we can write,

$$\bar{u} = \frac{1}{t}\int_0^t u dt$$

So, we can write $\int_0^t u'v' dt = \overline{u'v'} \neq 0$

Intensity of turbulence:

which is represented as the measure of the RMS of the fluctuating component

$$I = \sqrt{\frac{\frac{1}{3}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right)}{u_\infty}}$$

If fluctuations are same along all the directions, that type of turbulence is called Isotropic turbulence $\left(\overline{u'^2} = \overline{v'^2} = \overline{w'^2}\right)$,intensity of turbulence, $I = \sqrt{\frac{\overline{u'^2}}{u_\infty}}$

Thank you