Fundamentals Of Particle And Fluid Solid Processing
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Lecture – 52
Fluid – solid transport (Contd.)

Hello everyone, and once again when come to the Fundamentals of Particle and Fluid Solid Processing lectures. We were discussing about the transport of fluid solid system and specifically the slurry transport. We have introduced this topic.

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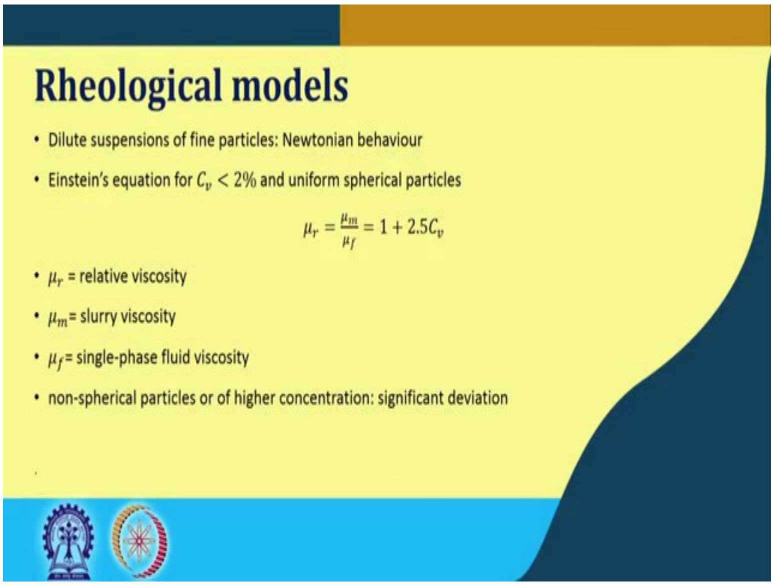

And we have seen that what happens, how to calculate this average density or say the mixture density of the slurry. Now, coming to the viscosity of this mixture we have seen Einstein equation, specifically for the solids concentration when it is very low or say the volume concentration of the solid particles is lesser than 2 %. Then also the slurry consists of uniform spherical particles then this expression is valid.

$$\mu_r = \frac{\mu_m}{\mu_f} = 1 + 2.5\,C_v$$

But for non spherical particles and of higher concentrations, there is significant deviations from the predictions that it provides.

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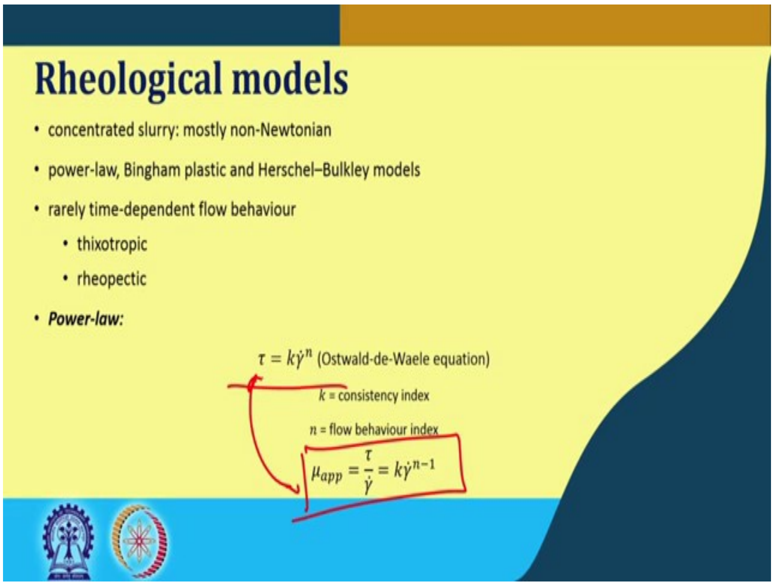

Now, for the concentrated slurry, which mostly behaves as a non-Newtonian liquid, there are several rheological models available, like the power law, Bingham plastic and Herschel Bulkley models. Now, there are also slurries, but rarely in practice which basically exhibit time dependent flow behavior, like the thixotropic or the rheopectic material.

We will not go into this details of these things because I assume that these terminology and these models you know. We will have an overview of the power law the Bingham plastic and related to these models, how we calculate or how we predict pressure drop for the slurries that exhibits this power law and the Bingham plastic behaviour. So, we will mainly focus on these two categories.

So, for the power law, we know its constitutive equation is

$$\tau = k\dot{\gamma}^n$$

which is also popularly known as Ostwald-de-Waele equation, where this k is the consistency index and n is the flow behaviour index. So, the shear stress and the strain rate is related by consistency fact consistency index and the flow behaviour index.

And since in this case, the viscosity is not a constant because the viscosity varies with the shear rate. So; that means, there is this absolute viscosity or viscosity see this terminology becomes insufficient or it is meaningless. So, that is why in non-Newtonian fluids, we call that as apparent viscosity at a certain shear rate. This provides,

$$\mu_{app} = \frac{\tau}{\dot{\gamma}} = k\dot{\gamma}^{n-1}$$

which is the apparent viscosity which we can have from previous relation.

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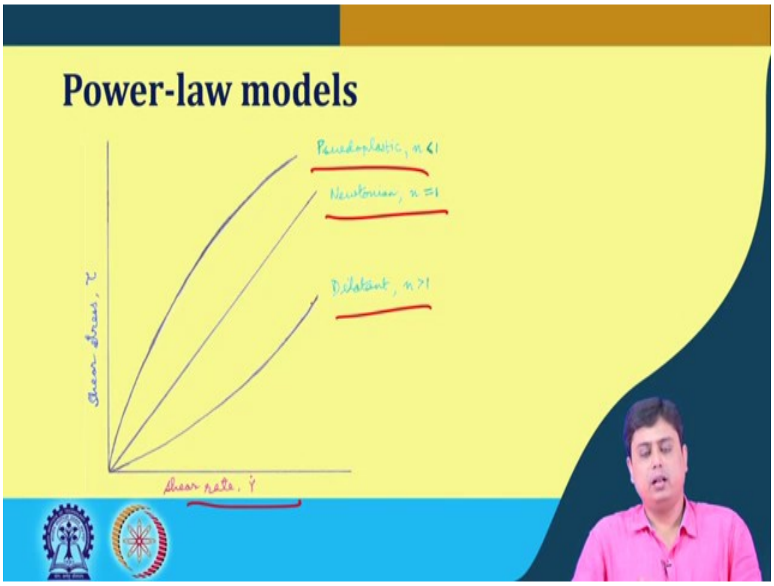

Now, we can see from this schematic how this relation varies. So,

$$\tau = k\dot{\gamma}^{n}$$

this flow behaviour index when it is 1, then this expression basically the k becomes the μ which is for the Newtonian fluid or the Newtonian liquid for $n=1$. For $n>1$, this fluids shows the shear thickening behaviour or we say the Dilatant fluid or the liquids.

And in other case when $n<1$, we say these are the pseudo plastic material or the pseudo plastic liquid, what the shear thinning. Because with shear rate we can see in the Dilatant case the shear stress increases and the viscosity will increase. In the shear thinning cases, we see that the as we increase the shear rate the viscosity goes down. So, these are the basic things we have seen in the fluid mechanics or the fluid dynamics classes.

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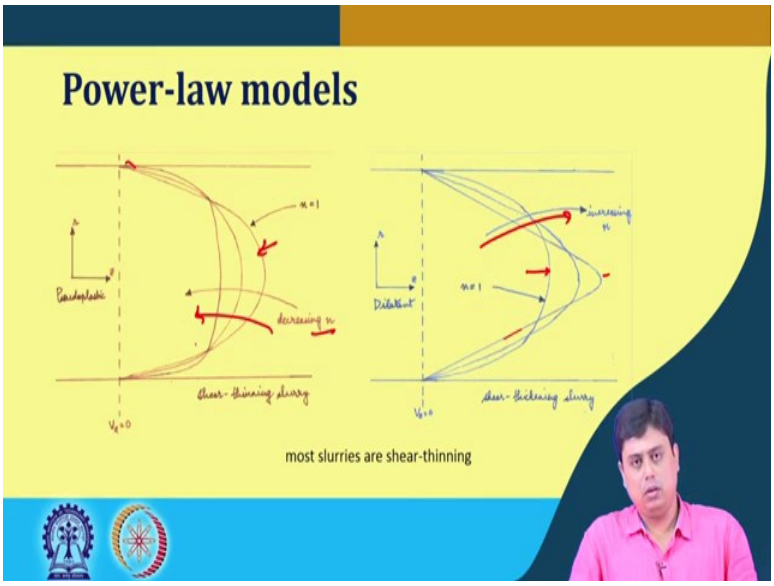

Now, what it leads to this shear thinning or the shear thickening behaviour by with changing the value of n. So, say the velocity profile of a Newtonian fluid in a pipe or a liquid in a pipe, it would typically give us a parabolic profile which is represented by this line.

Now, in case of pseudo plastic fluid that is $n<1$, it shows a blunt profile, ok. As we decrease the value of n; the profile becomes more flatter and flatter or the flow profile becomes more plug likes scenario; plug flow like behaviour. But in case of Dilatant fluid as we increase the value of n; because this is for $n=1$ that is the Newtonian fluid parabolic profile. As we increase the value of n, we see this becomes more and more sharp at the front end.

For very high value of n, that is greater than 1 much greater than 1. This velocity is seen to be linearly varying in the directions that we can see here that it is almost a linear variations till its center. So, this becomes the flow behaviour or the flow pattern of the slurry according to its rheological characteristics.

So, if it behaves as a shear thinning liquid it falls in this category, if it is a shear thickening fluid it goes in this category. So, most slurries exhibits shear thinning behavior; because the hypothesis is that this slurries contains the fine particles which creates aggregate with the increasing shear. And these aggregates basically have a lower resistance to the flow. So, it becomes more flat like profile in this case, the gradient is lesser in the middle.

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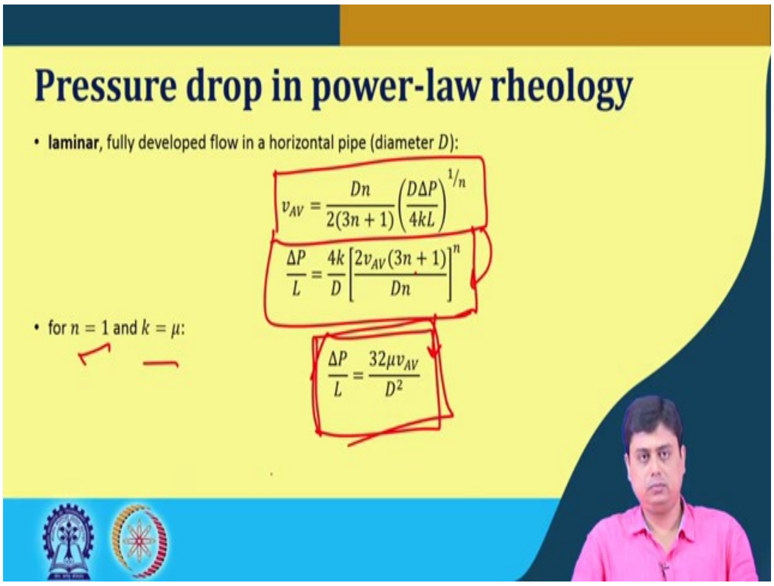

Now, the most important factor in the slurry transfer is to predict or calculate the pressure drop that is required, because this in other way dictates the mechanical energy that is required or the pumping cost that is required. So, now, if we do the momentum balance, like we have done in fluid dynamics for a Newtonian fluid.

In order to have a velocity profile we see that for laminar fully developed flow in a horizontal pipe of diameter D, we can have

$$v_{AV} = \frac{Dn}{2(3n+1)}\left(\frac{D\Delta P}{4kL}\right)^{\frac{1}{n}}$$

as a value of the average velocity of the slurry that exhibits power law behaviour. So, this profile basically depends on the value of k which is the consistency index as well as the flow behaviour index along with the diameter of the pipe.

So, the pressure drop this if it is rearranged the pressure drop per unit length can be expressed in terms of this average velocity and k as well as and the n values.

$$\frac{\Delta P}{L} = \frac{4k}{D}\left[\frac{2v_{AV}(3n+1)}{Dn}\right]^n$$

So, this gives us the pressure drop estimation, laminar condition in a horizontal pipe for power law slurries.

Now interestingly, if we look at this expression we see that for $n=11$, we have seen that k becomes μ. And in that case this expression reduced to the Hagen Poiseuille equation

$$\frac{\Delta P}{L}=\frac{32\mu v_{AV}}{D^2}$$

which should be in this case; because if it is 1 it becomes a Newtonian fluid. And we know for the laminar fully developed flow of a Newtonian liquid in a horizontal pipe the pressure drop is related by this famous expression.

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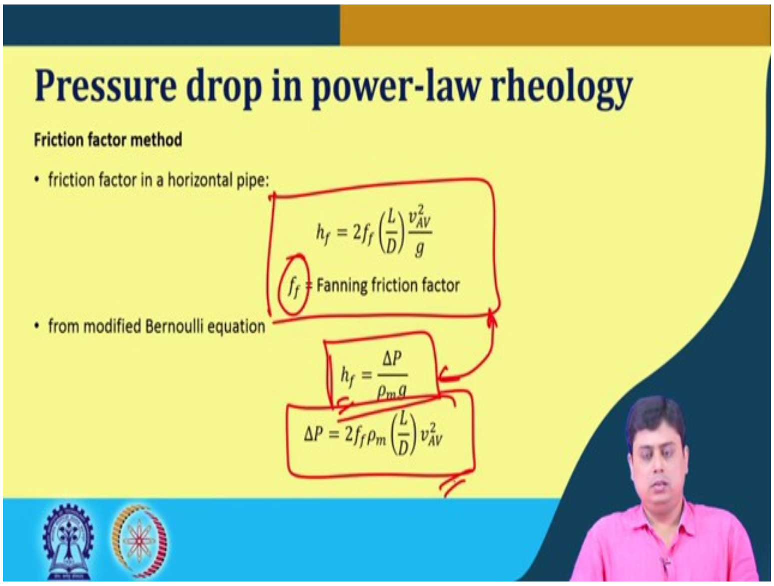

The other way to estimate the pressure drop is by friction factor method. Now, the friction factor in a horizontal pipe, we have also seen, this expression in fluid dynamics,

$$h_f=2f_f\left(\frac{L}{D}\right)\frac{v_{AV}^2}{g}$$

where h_f is the head loss the frictional head loss, this is the fanning friction factor the other parameters are well known to us.

Now also from modified Bernoulli's equation in case of this horizontal flow, we know the frictional head loss and the pressure drop how it is related.

$$h_f=\frac{\Delta P}{\rho_m g}$$

This head loss is basically having the unit of length ρ_m is the mixture density. So, which means from these two expressions, we have ΔP and the fanning friction factor relation which is also well known to us.

$$\Delta P = 2 f_f \rho_m \left(\frac{L}{D}\right) v_{AV}^2$$

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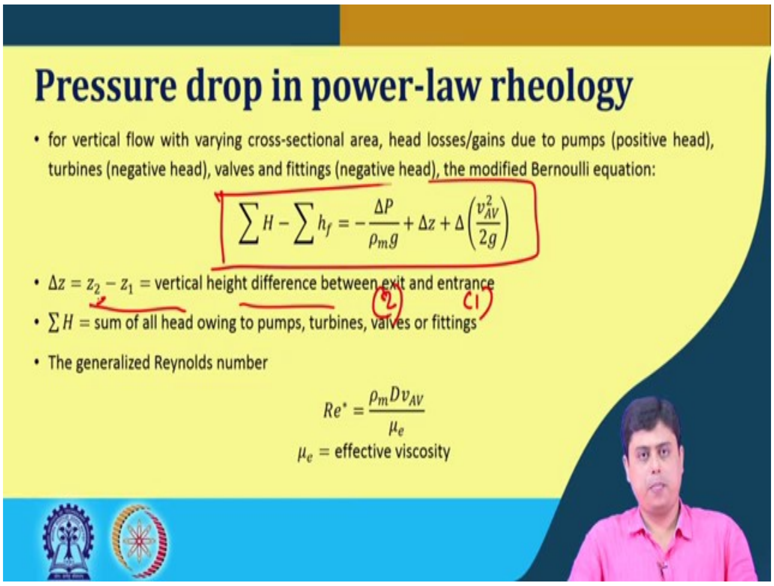

Now, for vertical flow with varying cross sectional area including head losses due to say positive head or negative head that comes from pump valve fittings in a pipeline network. We can apply Bernoulli's equation in a modified form which would look like this expression.

$$\sum H - \sum h_f = \frac{-\Delta P}{\rho_m g} + \Delta z + \Delta\left(\frac{v_{AV}^2}{2g}\right)$$

Where $\sum H$ is the sum of all the head owing to this pumps turbines say valves fittings bends all this kind of a scenarios.

The Δz is the vertical height difference between exit say this exit, we designate at 2 and the entrance is 1; so z_2 minus z_1. So, this can be the generic expression for a pipeline network depending on the orientation beta vertical flow or they say the horizontal flow. Now, the thing becomes important in friction factor related pressure drop calculation is the regime calculation or the flow regime. Because is it in laminar or in turbulent depending on that we can consult the friction factor chart or we can consult the appropriate correlations to find out what is the friction factor value.

Now, this regime we identify with the help of Reynolds number in Newtonian fluid. Similar to that here also we apply the concept of Reynolds number definition but in a modified form. Here the modification is done through the effective viscosity; because this is we are talking about non-Newtonian fluids or non-Newtonian liquids. So, there is no absolute viscosity we have effective viscosity μ_e. So, this how this effective viscosity is now defined. If we because we have seen the definition of ρ_m which is the mixture density v_{AV} we know.

We have seen the expression previously for the power law fluid. So, if we know now this effective viscosity expression, then we can calculate Reynolds number for that particular power law exhibiting slurry.

$$\Re = \frac{\rho_m D v_{AV}}{\mu_e}$$

And we can understand whether that is in the laminar or the turbulent regime. So, one part is that to calculate the Reynolds number of the existing slurry that exhibits power law behaviour.

The other part is that what is the transition Reynolds number in such slurry? Because for the Newtonian case we know that margin is to be 2000 or to the 21000. Below this it is laminar and on higher than that its transitional and then we have the turbulent regime. So, here what is that critical Reynolds number or the transition Reynolds number? So, we have two fold issues here.

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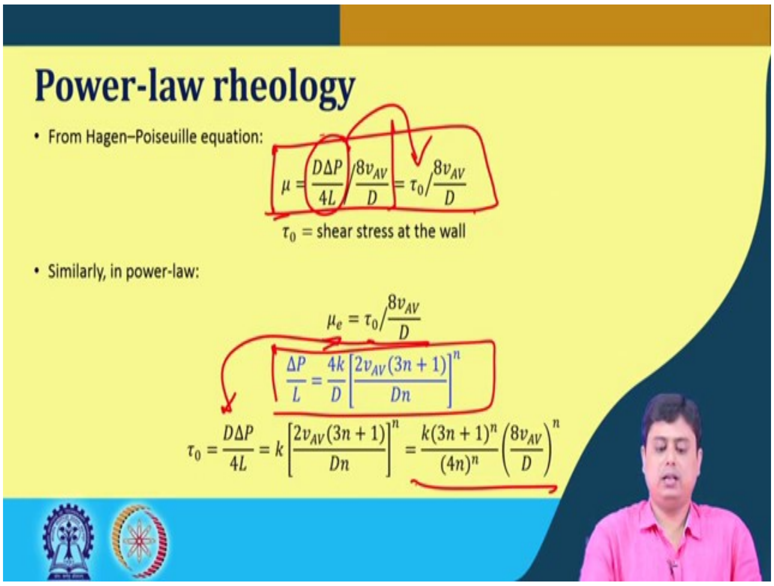

So, let us try to see one by one. So, now, from Hagen Poiseuille equation, we can write this in a slightly oriented form reoriented form.

$$\mu = \frac{D \Delta P}{4L} / \frac{8 v_{AV}}{D} = \tau_0 / \frac{8 v_{AV}}{D}$$

So, this is the expression of the Hagen Poiseuille, we have just rearranged that in order to write this term has τ_0; which is shear stress at the wall. So, for Newtonian liquid, we can write the μ in this form by rearranging the Hagen Poiseuille equation.

Now similarly in power law liquid also we can write this effective viscosity as this expression.

$$\mu_e = \tau_0 / \frac{8 v_{AV}}{D}$$

Now, also we have seen what is the del P by L previously.

$$\frac{\Delta P}{L} = \frac{4k}{D}\left[\frac{2 v_{AV}(3n+1)}{Dn}\right]^n$$

So, if we now try to write this τ_0; in terms of ΔP and substituting this ΔP in this expression we find an expression something like this.

$$\tau_0 = \frac{D \Delta P}{4L} = k\left[\frac{2v_{AV}(3n+1)}{Dn}\right]^n = \frac{k(3n+1)^n}{(4n)^n}\left(\frac{8v_{AV}}{D}\right)^n$$

Where we can see, the shear stress at the wall is basically depending on the consistency index, flow behaviour index, average velocity and the diameter of the pipe.

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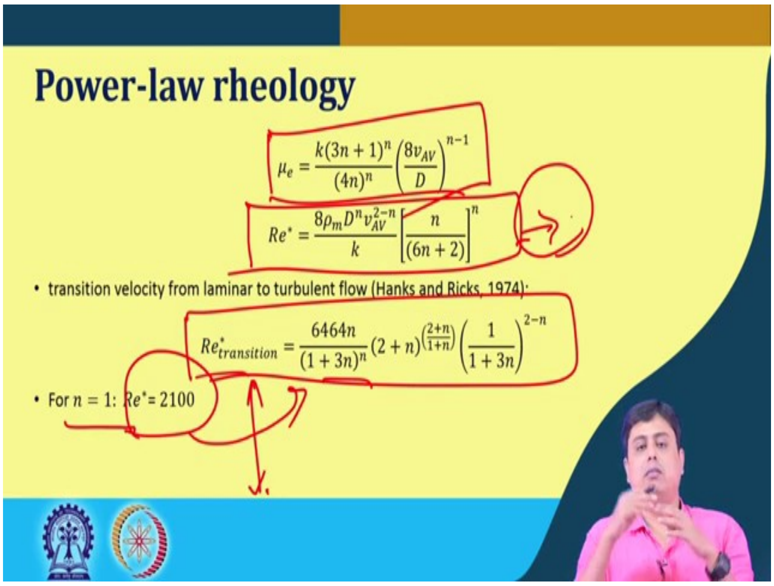

So, now; that means, this effective viscosity which is

$$\mu_e = \tau_0 / \frac{8v_{AV}}{D}$$

if we now, substitute

$$\tau_0 = \frac{k(3n+1)^n}{(4n)^n}\left(\frac{8v_{AV}}{D}\right)^n$$

here to find the effective viscosity, we see we can derive this expression for effective viscosity.

$$\mu_e = \frac{k(3n+1)^n}{(4n)^n}\left(\frac{8v_{AV}}{D}\right)^{n-1}$$

So, which means the effective viscosity is a function of the consistency index as well as the power law index or the flow behaviour index.

Now, the Reynolds number consequently, because we started with

$$\Re^{¿} = \frac{\rho_m D\, v_{AV}}{\mu_e}$$

We wanted to find what is effective viscosity what is the expression of that? Now, we have now got that expression. So, we substitute that effective viscosity in this above expression and we get this expression has the modified Reynolds number for power law liquids or power law fluids.

$$\Re^{¿} = \frac{8\rho_m D^n v_{AV}^{2-n}}{k}\left[\frac{n}{(6n+2)}\right]^n$$

So, by this expression we should be able to calculate what is the Reynolds number that flow which exhibiting power law behaviour so, from this expression we can find what is the Reynolds number? Now the second issue was that what is the critical or the transition Reynolds number based on which we can understand that this flow is either laminar or turbulent.

Now, there are several expressions, there are several correlations in order to find that one of such is given here.

$$\Re^{¿}_{transition} = \frac{6464\,n}{(1+3n)^n}(2+n)^{\left(\frac{2+n}{1+n}\right)}\left(\frac{1}{1+3n}\right)^{2-n}$$

So, transition Reynolds number is completely dependent on the consistency the power law index basically here not the consistency index. So, we see that for a set of power law fluids set of power law fluids means when n is something a particular value. Then for that set irrespective of the consistency index, the transition Reynolds number has a particular value or a fixed value.

So, which means if $n=1$ in this expression; n is 1 means it falls back to the Newtonian liquid the part of behavior. Reduced to the Newtonian liquid in that case, this expression results in this number which again it should be.

$$\text{For } n=1 : \Re^{¿} = 2100$$

So, similarly for any value of n, we have a fixed value of transition Reynolds number; based on which we will judge whether the existing flow is turbulent or laminar.

So, this is the calculation procedure you should typically follow, when the calculation of pressure drop is involved through friction factor. Because that calculation involves multiple steps one is that you have to find suitable expressions for the friction factor we have to get a chart. If that is available there is two regimes one is the laminar and the turbulent regime.

So, your current operating condition is it laminar or turbulent. To understand that you have to find these two Reynolds number; one is the existing current Reynolds number of the slurry that you are handling, and for that category of power law liquid what is the transition Reynolds number, then we can decide whether the flow is laminar or turbulent.

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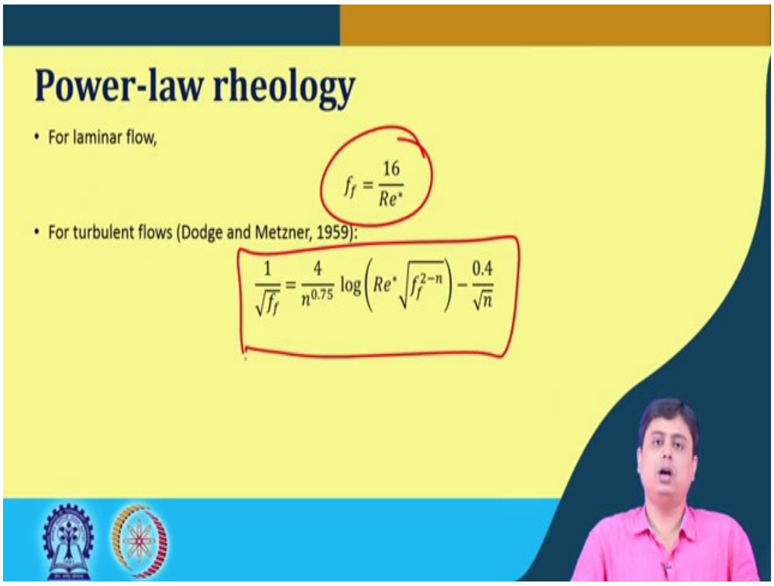

Now, for laminar flow the expression for the friction factor

$$f_f = \frac{16}{\Re^{¿}}$$

where $\Re$ again is the modified one $\Re^{¿}$, modified by the effective viscosity for the power law liquids. And for turbulent flows as I mentioned there are several correlations available and one of such is given here.

$$\frac{1}{\sqrt{f_f}}=\frac{4}{n^{0.75}}\log\left(\Re^{¿}\sqrt{f_f^{2-n}}\right)-\frac{0.4}{\sqrt{n}}$$

Once you find this friction factor you place it in the ΔP expression because we knew now that what is the pressure drop and the friction factor expression.

So, pressure drop versus this friction factor relation we know that. So, once we know this one, we can calculate or estimate the appropriate friction factor we replace that in the pressure drop expression to find out what is the pressure drop in such slurries. So, I hope these steps are clear, it involves multiple steps, but if we follow a new procedure then it becomes very clear.

So, we started with the expression of pressure drop and the friction factor, then we have seen that how to calculate Reynolds number of that particular slurry that involved effective viscosity calculation. And this effective after calculating this effective viscosity, we have find that our found that, what is the Reynolds number of that slurry?

There are relations available or correlations available to estimate what is the transition Reynolds number or the critical Reynolds number for that category of slurry, or that category of slurry, that exhibit particularly power law liquid. Then we calculate the friction factor depending on either the flow is laminar or turbulent. Once the friction factor is known we go back to the pressure drop versus friction factor relation and we get the pressure drop value.

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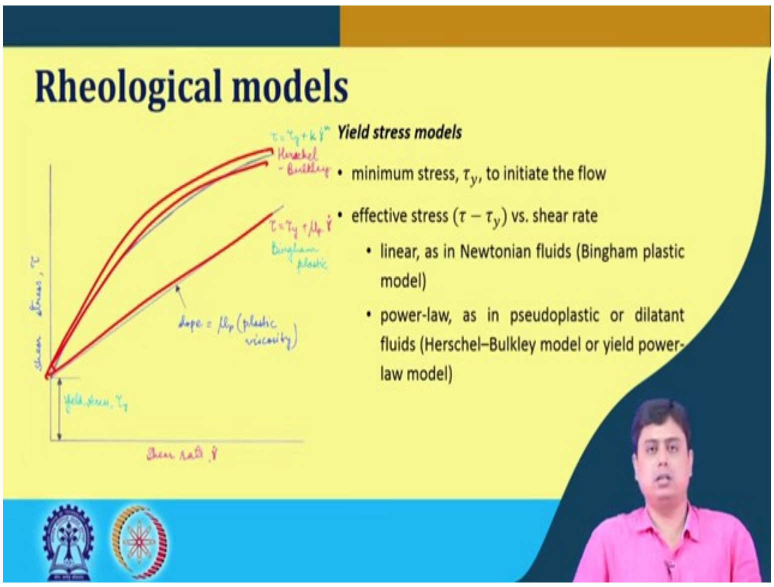

Apart from this power law, there are other rheological models also available for the non-Newtonian fluids, and one of such set of model are the yield stress models. Now here in such slurries or such liquids you need to put a minimum straight, stress in order to initiate the flow or to start the flow it does not flow as it is. It does not flow if you just put on it on a on a surface. For example, say the concrete mixture, until and unless a sufficient angle is provided to the horizontal plane it does not flow itself, you get.

So, we have to have some initial stress on that or say the paint coating or inkjet ink and all this stuff, which requires a minimum or a critical thickness to start the flow by gravity. So, the point is that we have to have some initial stress that we have to apply in order to start the flow. So, in these cases the effective stress versus shear rate can have different profile. Effective shear rate the shear stress is basically the applied shear stress minus this yield stress, this is the effective stress and the strain rate or the shear rate that is being applied. This behaviour either can be, it can be land linear or it can behave as power law.

If it is laminar; it is linear as in the Newtonian fluids, then this particular model is called the Bingham plastic model. If that is not linear and behaves in power law fluid manner then that we call the pseudo plastic or the Dilatant alike, if it follows this kind of behaviour like the pseudo plastic or the Dilatant fluid. Then we have the Herschel Bulkley model or the yield power law model. So, this particular model is called the Herschel Bulkley model. So, it is the pseudo plastic with yield stress, when the yield stress is 0, this Herschel Bulkley basically reduced to shear thinning model.

So, we will continue discussing these models or particularly this Bingham plastic, how do we estimate the pressure drop in such cases. And we will have a look in the other kind of slurry, because this we are talking mostly about the homogeneous slurry. What happens in the heterogeneous slurry? We will continue our discussion on this in the next class.

But, before I conclude my talk today, let me just again summarize that we started discussion on the power law liquid velocity profile, how to estimate pressure drop in such case. The pressure drop calculations we can directly achieve from the theoretical expression or by the friction factor correlations.

To understand, what would be the friction factor we have to understand what is the flow regime? In order to detect the flow regime or to determine the flow regime we know we must know what is the Reynolds number of that flow? Which we have seen to be dependent on the consistency index? And the flow behaviour index and for that set of power law liquid what is the transition Reynolds number that we have also seen as in terms of correlations that is provided by the researchers in the literature.

From that we can detect whether the flow regime is laminar or turbulent and depending on that we calculate the friction factor. Once it is known we go back to the pressure drop expression replace that friction factor and get the pressure drop. And then we started the discussion on the yield stress model which we will continue in the next class till then.

Thank you for your attention.