Fundamentals Of Particle And Fluid Solid Processing
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Lecture - 39
Centrifugal Separation (Contd.)

Hello everyone and welcome back to the another class of Fundamentals of Particle and Fluid Solid Processing. We were discussing some problems related to the Centrifugal Separation and particularly the efficiency of cyclone separator, its grade efficiency, are the scale of operations and related to the problems that we did on Stokes number and the Euler number.

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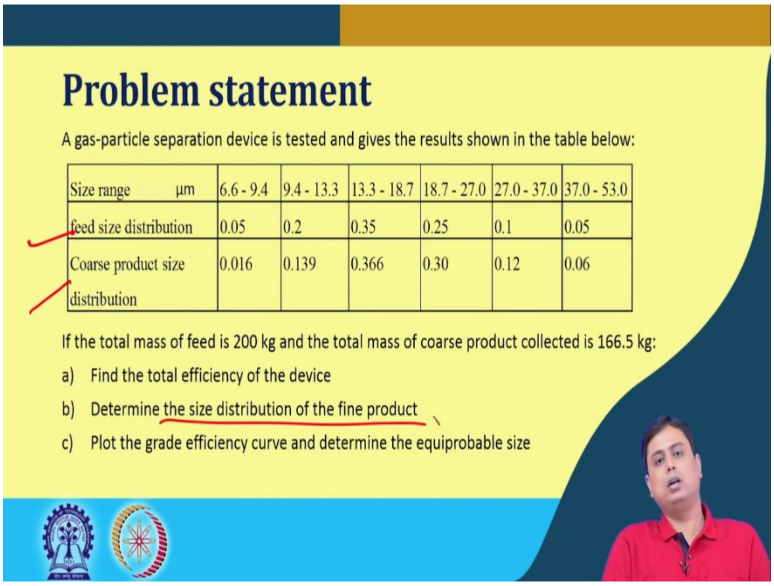

So, now we will see this relatively simple problem, which says that up gas particle separation device is tested and provides the following result; that is given here, that we have the first row says the size range in micron, the second row is the feed size distribution, and the third one is the coarse product size distribution.

Now, if the total mass of feed is 200 kg and total mass of the coarse product that is collected is to be 116.6 kg; then the questions are, find the total efficiency of the device, determine the size distribution of the product, and plot the grade efficiency curve to determine the equiprobable size, that is the x_{50}, the value of x_{50}.

So, how to proceed on this? So, we have a size range, we have feed size distribution, we have coarse product size distribution. The total mass of the feed is given, the total mass of coarse product that is also given. So, we have to find out the total efficiency of the device, this is the first question which is very simple; because the total efficiency is nothing, but the mass fraction of the coarse product that is collected with respect to the feed that is supplied.

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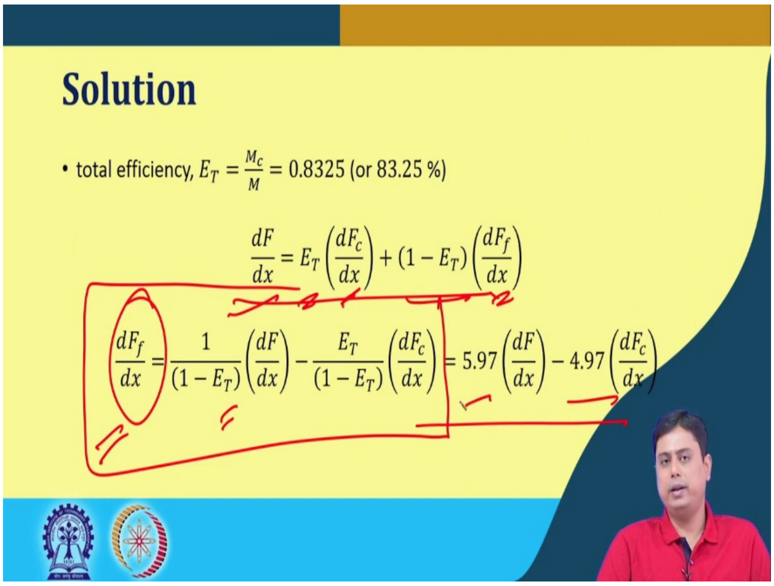

So, in this case

$$E_T = \frac{M_c}{M} = 0.8325 \text{ (or } 83.25 \text{ \%)}$$

So, this is the efficiency, total efficiency of the separator or the cyclone separator. Now, coming to the second part that asks determine the size distribution of the fine product; because we have here feed size distribution, coarse size distribution, ok. So, naturally what is the size distribution of the fine products that are escaping with the cleaner gas?

So, we know this relation now,

$$\frac{dF}{dx} = E_T\left(\frac{dF_c}{dx}\right) + (1 - E_T)\left(\frac{dF_f}{dx}\right)$$

this was the size distribution expression that we derived; that is this is the differential size distribution of the feed on mass basis, this is of the coarse product, this is of the finer product multiplied by the efficiency and the finer product multiplied by 1 minus efficiency, the total efficiency.

Which in this case; if it is rearranged, because the question is the finer product distribution which is if $\frac{dF_f}{dx}$; where again the small f represents the finer product, c stands for the coarse product, and simple F stands for the feed. So, if we rearrange this expression, we can have this expression we got on the left hand side, if we put the unknown value, unknown factor.

$$\frac{dF_f}{dx} = \frac{1}{(1-E_T)}\left(\frac{dF}{dx}\right) - \frac{E_T}{(1-E_T)}\left(\frac{dF_c}{dx}\right)$$

Now, here E_T is known that we have calculated. So, the expression becomes

$$\frac{dF_f}{dx} = \frac{1}{(1-E_T)}\left(\frac{dF}{dx}\right) - \frac{E_T}{(1-E_T)}\left(\frac{dF_c}{dx}\right) = 5.97\left(\frac{dF}{dx}\right) - 4.97\left(\frac{dF_c}{dx}\right)$$

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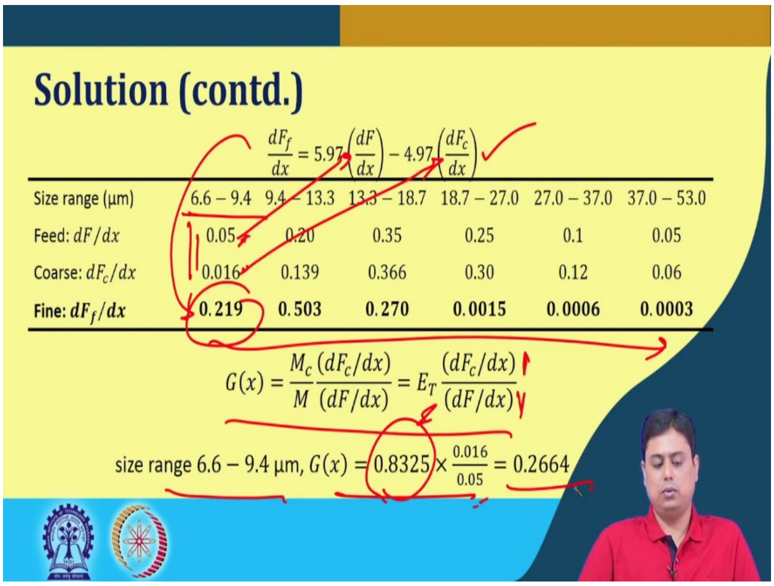

If we have this expression, now we can find what is the fine particle distribution from the table itself? Because now, for each and every size fraction, size range we have the feed $\frac{dF_f}{dx}$, we have the coarse one, so we find what is the $\frac{dF}{dx}$ from this expression. Similarly, for all the values we can find this value; this is the finer product size distribution of the finer particle size distribution.

Now, the third question is that the grade efficiency curve. If we go back to the problem it says, the plot the grade efficiency curve and determine the equiprobable size. So, the grade efficiency again by definition is

$$G(x) = \frac{M_c}{M} \frac{(dF_c/dx)}{(dF/dx)} = E_T \frac{(dF_c/dx)}{(dF/dx)}$$

Now, again here we can use this simple table once again; because here $\frac{dF_c}{dx}$ and $\frac{dF}{dx}$ both these values are known, E_T is known. So, for each and every size range we can calculate the $G(x)$, say for this size range 6.6 to 9.41 one sample calculation is given here;

$$G(x) = 0.8325 \times \frac{0.016}{0.05} = 0.2664$$

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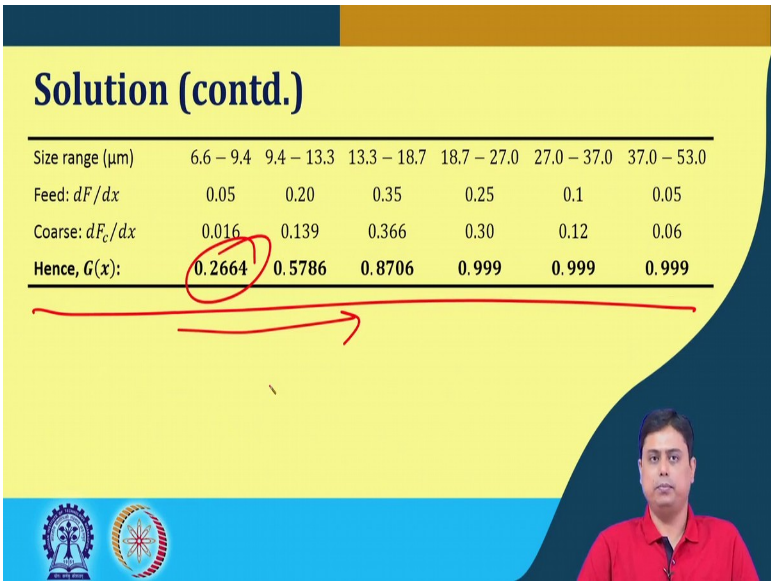

So, similarly we can form such table and calculate the $G(x)$ values from the previous expression; this would give us the grade efficiency plot as well and which would look like something like this.

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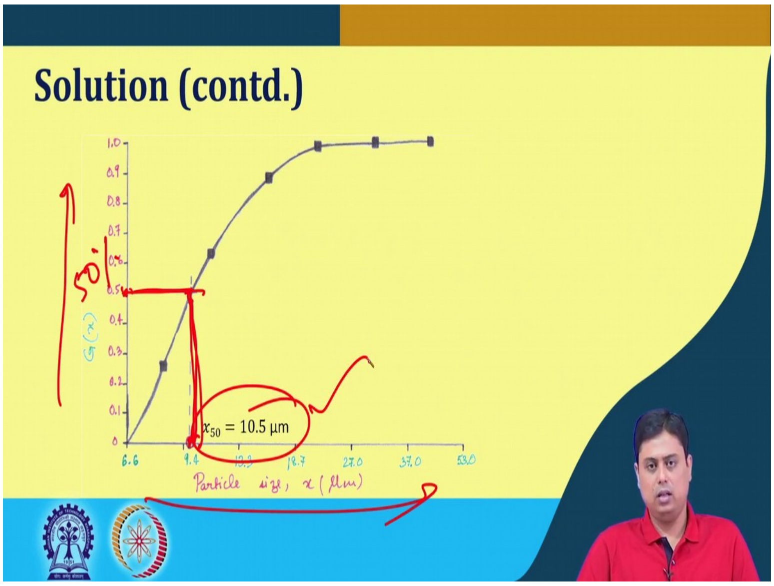

And from this how to find the equiprobable size; because here we have the table, so this is the size range on the x axis and on the y axis you plot the grade efficiency. Again this is not to the scale, this is a schematic and how do you find the equiprobable size by definition; it is the cut the particle size at 50 % of the grade efficiency which is at 0.5 what is the value of x, which comes out to be 10.5 µm.

So, we draw a horizontal line at 50 % efficiency, grade efficiency and we find the x coordinate of the bisection between this grade curve and the 50 % efficiency straight line; and we get the x_{50} as 10.5 µm. So, this is our equiprobable cut size. I hope this is clear to you; that if we have the size distribution known of at least two inlet or outlets, then we can calculate the third one. We can also calculate the grade efficiency from the size range data, from that grade efficiency we can find out the equiprobable size.

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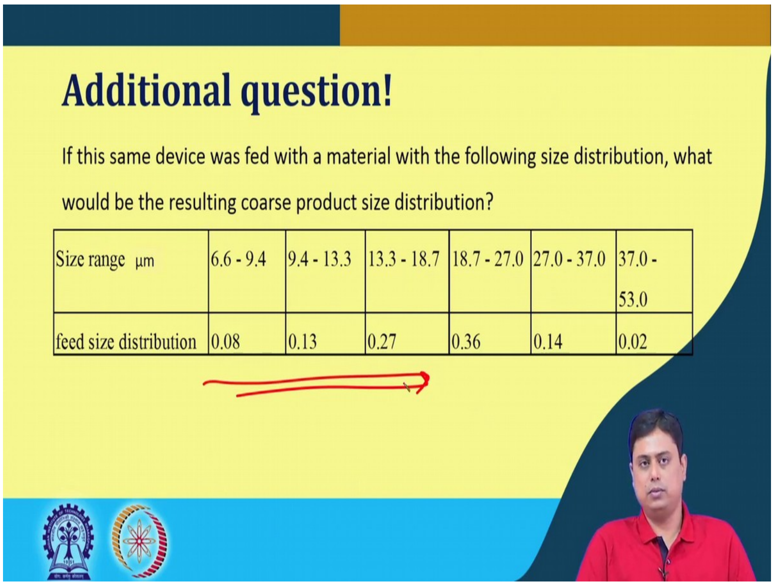

The additional question is that, if same device was fed with the material of the following size distribution; that means, that is given in the table, what would be the resulting coarse size, coarse product size distribution.

So, cyclone separator remains same, it is characteristics remains same; only there is the variation in feed, the size distribution is different. So, the question is what would be the coarse product of the coarse product size distribution.

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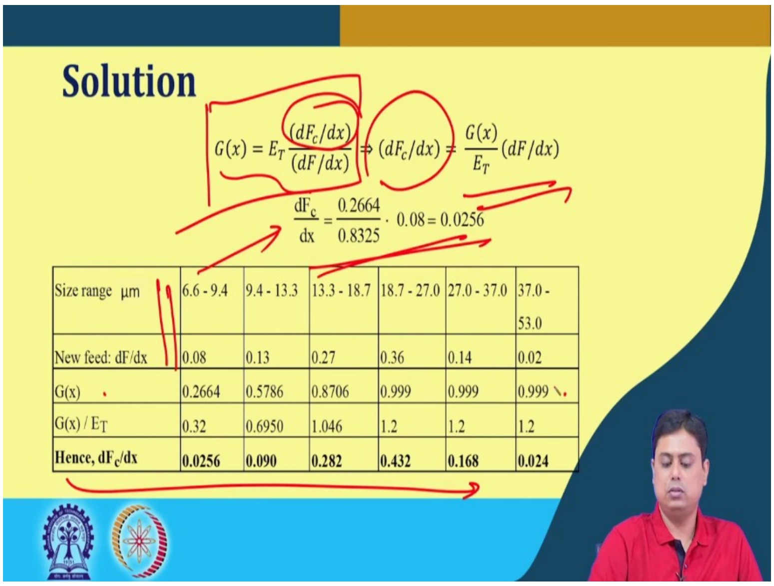

So, how do we do that? We just rearrange the grade efficiency expression, this was the grade efficiency expression.

$$G(x) = E_T \frac{(dF_c / dx)}{(dF/dx)}$$

Now, there it was related with the coarse product size distribution and the feed size distribution. Now, here all the parameters are known except dF_c/dx. So, dF_c/dx we can write in this manner;

$$(dF_c/dx) = \frac{G(x)}{E_T}(dF/dx)$$

and again one of such calculation is given here for 6.6 to 9.4 size range and we can have this table completed.

$$\frac{dF_c}{dx} = \frac{0.2664}{0.8325} \cdot 0.08 = 0.0256$$

This is the size range that is given, this is the $G(x)$ values that we have got from here, for this size range; because remember the cyclone separator did not change, it remained the same identical separator. So, the grade efficiency would be same, only the feed size has changed. So, based on that $G(x)$, we find this dF_c/dx value and this is the result that is required.

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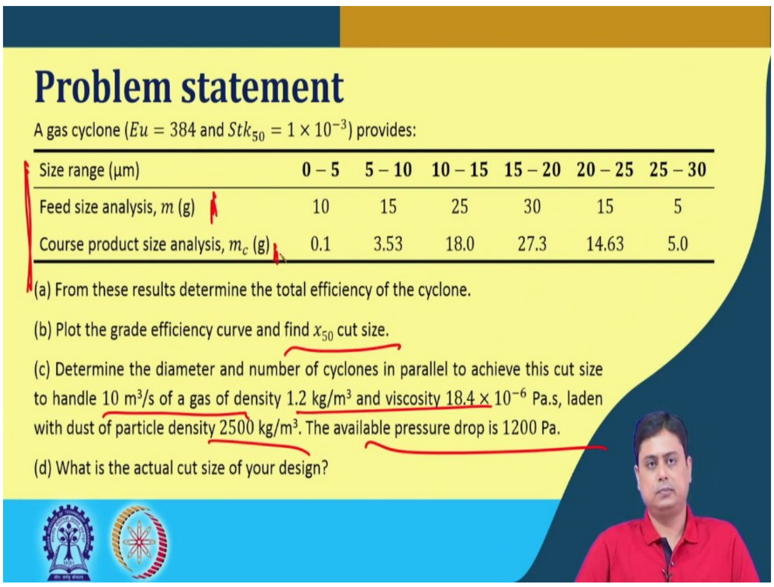

Now, again if we quickly go through another similar problem, but there is a small twist I would say; that again a gas cyclone of known Euler number and Stokes number gives this kind of separation, the size range, mass of the feed, mass of the product size is given. So, what is the efficiency; plot the grade efficiency, find out the x_{50}.

Now, determine the diameter and number of cyclones in parallel to achieve this cut size; that is this equiprobable size to handle a certain flow rate of known density, known viscosity, and known particle density, the allowable pressure drop is also mentioned.

So, it is a combination of the last two problems that we did. So, let us quickly see, go through this example and see whether you can do it yourself. So, the mass here the values are given in mass, the cumulative fraction, the cumulative distribution.

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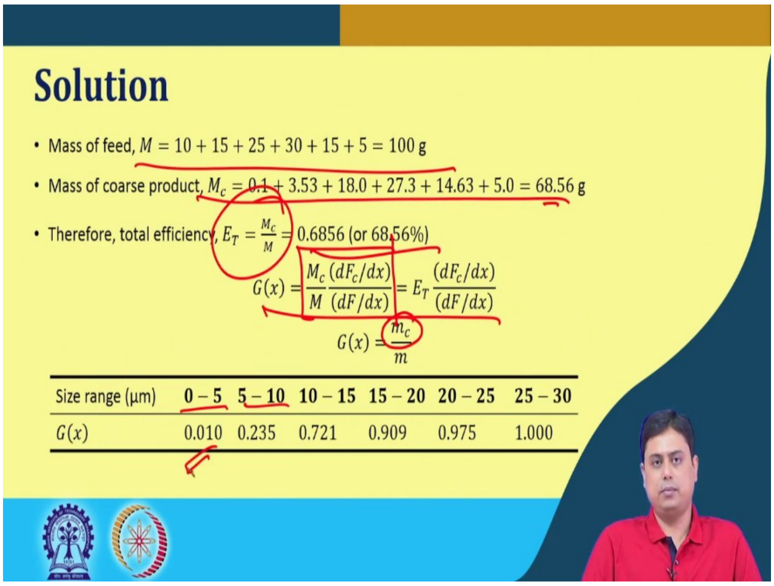

So; that means, if you add those you get the total mass of the feed;

$$M = 10+15+25+30+15+5 = 100 \text{ g}$$

if you add the coarse product you get the total coarse product.

$$M_c = 0.1+3.53+18.0+27.3+14.63+5.0 = 68.56 \text{ g}$$

So, that efficiency, the total efficiency is the ratio of these two which is 68.56 %.

$$E_T = \frac{M_c}{M} = 0.6856 \text{ (or } 68.56\,\%)$$

The grade efficiency expression we have seen this as such value.

$$G(x) = \frac{M_c}{M} \frac{(dF_c/dx)}{(dF/dx)} = E_T \frac{(dF_c/dx)}{(dF/dx)}$$

And in this case, if you look at only this term this M c multiplied by d F c by d x is nothing, but the mass fraction, the cumulative mass of the coarse product, on that particular size range.

$$G(x) = \frac{m_c}{m}$$

So, the size range for 0 to 5, 5 to 10 that are given there, we can find out what is the $G(x)$ or the grade efficiency. So, this is 0.1 divided by 10 the first value, the second value is 3.53 divided by 15 is the grade efficiency in this range; similarly the rest of the table can be completed. Once we complete; that means we have the size range versus grade efficiency data.

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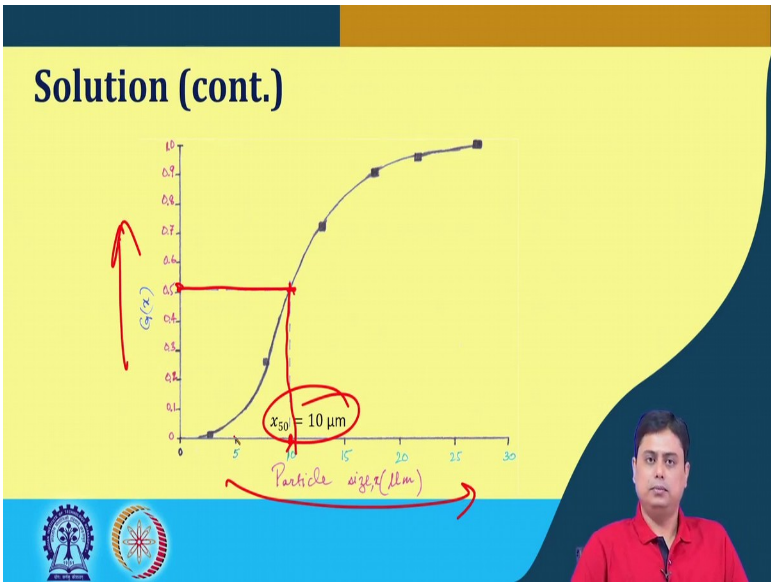

Which can again be plotted on the particle size on the x axis, $G(x)$ on the y axis; once we plot we find that 50 % of the grade efficiency intersect with this grade curve for x as 10 µm. So, this is the x_{50}. So, x_{50} is the 10 µm; this answers the second part, find the x_{50} cut size.

Now, for this cut size with all the other information given; we have to determine the diameter of the cyclone and the number of cyclones that is required, to treat 10 m^3/s of gas, of known density, known viscosity, and known particle diameter with the available pressure drop of 1200 Pa.

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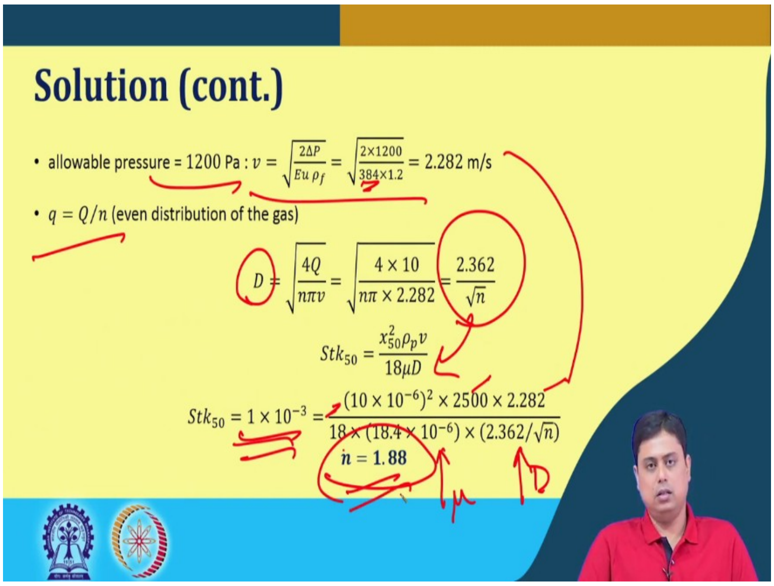

So, similarly like we did in the last problem, we see that the allowable pressure is 1200 Pa; the Euler number was given which is 384. So, from the Euler number expression, we can find what is the characteristic velocity.

$$v = \sqrt{\frac{2\Delta P}{Eu\,\rho_f}} = \sqrt{\frac{2\times1200}{384\times1.2}} = 2.282\,m/s$$

Again assuming that if n numbers of cyclones are required that connected in parallel and the flow gets evenly distributed; then the flow rate becomes overall flow rate divided by n. So, from v and this q we can find out what is the diameter. We find the diameter having an expression of n the number of the cyclones.

$q = Q/n$ (even distribution of the gas)

$$D = \sqrt{\frac{4Q}{n\pi v}} = \sqrt{\frac{4\times10}{n\pi\times2.282}} = \frac{2.362}{\sqrt{n}}$$

We put this in the Stokes number that is mentioned,

$$Stk_{50} = 1\times10^{-3} = \frac{(10\times10^{-6})^2\times2500\times2.282}{18\times(18.4\times10^{-6})\times(2.362/\sqrt{n})}$$

that is fixed for the problem to find what is the value of n, and we see that the value of n is 1.88; that means, we need at least two cyclones of this Stokes number that is 1×10^{-3}, and 384 Euler number with the pressure drop of 1200 Pa to have a cut size of 10 µm.

Because here the cut size we have calculated from the equiprobable line here all, is 10 µm. So, x is 10 µm, this is 2500 the particle density, 2.282 m/s is the characteristic velocity that we have calculated, 18µ value is given and this is the diameter in terms of n. So, we find n to be 1.88 and since it cannot be a fraction; it has to be a whole number.

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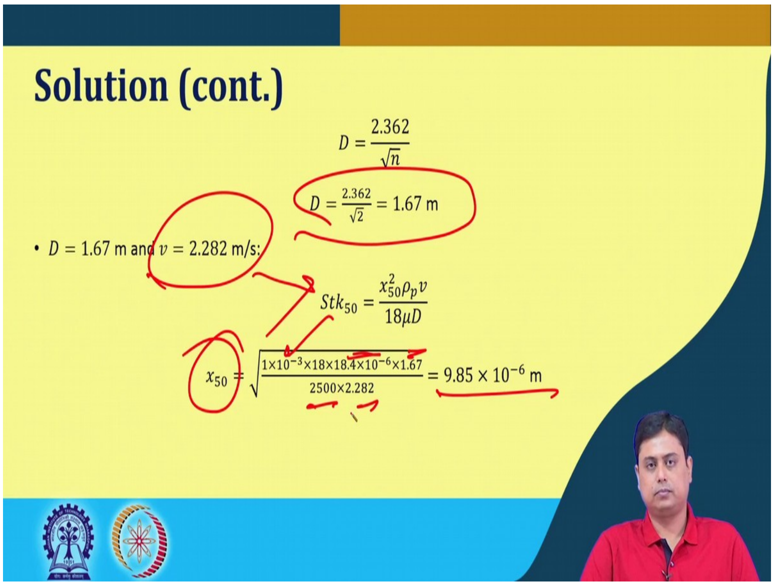

So, we need at least two cyclones to affect this cut size; that is the minimum number and since two is more than 1.88, we find a new D the diameter 1.67 m. So, with this new diameter and the same characteristic velocity because the Euler number remains same, this is a design parameter.

We put it in the Stokes number to find the new value of cut size, which is 9.85 µm.

$$x_{50} = \sqrt{\frac{1\times10^{-3}\times18\times18.4\times10^{-6}\times1.67}{2500\times2.282}} = 9.85\times10^{-6}\,m$$

So, that answers our last part as well that what is the actual cut size of your design.

So, I hope this problem is also clear to you and you have now understood that if this size distribution data is given; Stokes number, Euler number, physical properties are given; we can easily find that what would be the x_{50} value. And intermediately to have this x_{50} we have to get the grade efficiency, from grade efficiency we calculated x_{50}, with the help of x_{50} we calculated the number of cyclones that are required.

And since the number was not of a whole number, it was a fraction we have to take the next whole number to be the number of cyclones. And since the value has now increased, so we have to recalculate all the diameter and the cut size; because this would be the new or the realistic values and that gives us the result that we need at least two cyclone separators of this family. Which means, the family has Euler number 384 and Stokes number of 1×10^{-3} with a pressure drop of 1200 Pa, will give us a cut size of 9.85 μm for the treatment of 10 m^3/s of gas, having this much of density, this much of viscosity and this much of particle density.

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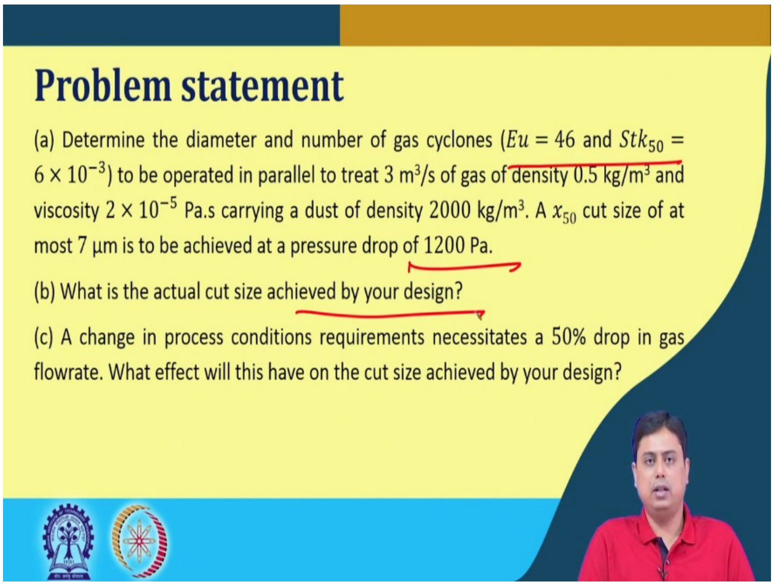

So, let us see the final problem to sum up all the ideas now; that we determine a diameter and number of gas cyclones of this family which has Euler number 46, Stk_{50}as 6×10^{-3} to treat 3 m^3/s of gas having density known, viscosity known, particle density is also known; a cut size at most of 7 μm is mentioned the pressure drop is given.

Simple problem like we did the last time; this part is also exactly similar that what would be the actual cut size achieved by your design. We will come to the third question, before that let us see whether you can quickly go through with these values, whether you can also get the similar values through your calculations.

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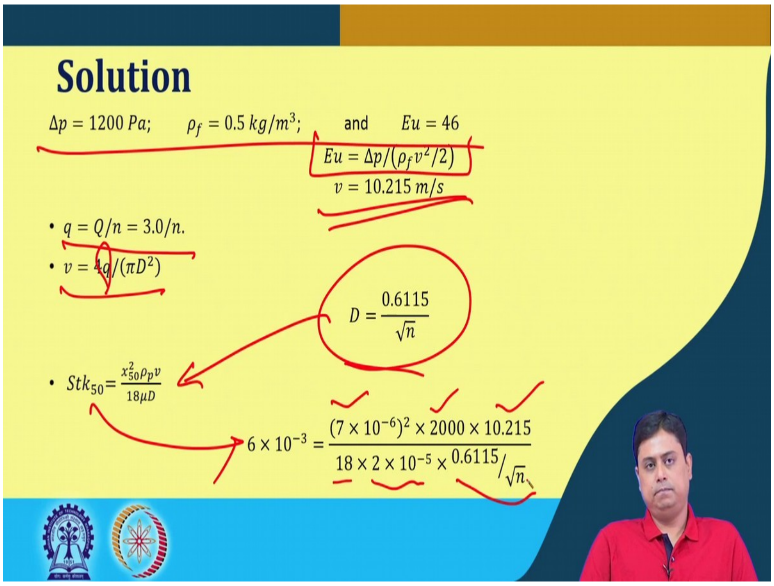

So, these values are mentioned

$\Delta p = 1200\,Pa;$ $\rho_f = 0.5\,kg/m^3;$ and $Eu = 46$

So, by the definition of Euler number we find the characteristic velocity;

$Eu = \Delta p / (\rho_f v^2 / 2)$

$v = 10.215\,m/s$

say number n is required number of cyclone. So, the flow rate to each cyclone is $3/n$.

From v and q, we can calculate what is the required diameter in terms of n.

$q = Q/n = 3.0/n$

$v = 4q/(\pi D^2)$

$$D = \frac{0.6115}{\sqrt{n}}$$

We put it in the Stokes number to find the value of n, because Stokes number is known.

$$Stk_{50} = \frac{x_{50}^2 \rho_p v}{18 \mu D}$$

$$6 \times 10^{-3} = \frac{\left(7 \times 10^{-6}\right)^2 \times 2000 \times 10.215}{18 \times 2 \times 10^{-5} \times \frac{0.6115}{\sqrt{n}}}$$

So, here this is the cut size, particle density, characteristic velocity, 18 μD.

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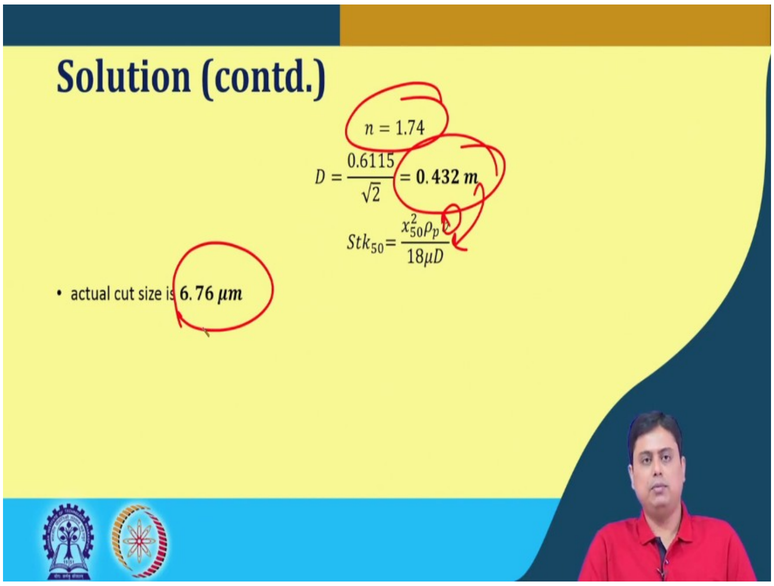

This gives us the value in as 1.74;

$n = 1.74$

which means, we need two cyclones; that means, that D would be 0.432 m. We put it in again in this Stokes number with calculated v, the characteristic velocity we just calculated earlier; and we find the actual cut size

$$D = \frac{0.6115}{\sqrt{2}} = 0.432\, m$$

$$Stk_{50} = \frac{x_{50}^2 \rho_p v}{18 \mu D}$$

actual cut size is 6.76 μm

which is lower than the required cut size; which means, the design is safe or the required number is good enough.

So, this answers the first two parts. We have done three four problems similar to this. Now coming to the last part of this problem; that is, a change in process conditions requirements necessitates a 50 % pressure drop in gas flowrate. So, let us say for some reason, some malfunctioning, or some issues there is a 50 % drop in gas flow rate. So, now what would be the cut size of your design?

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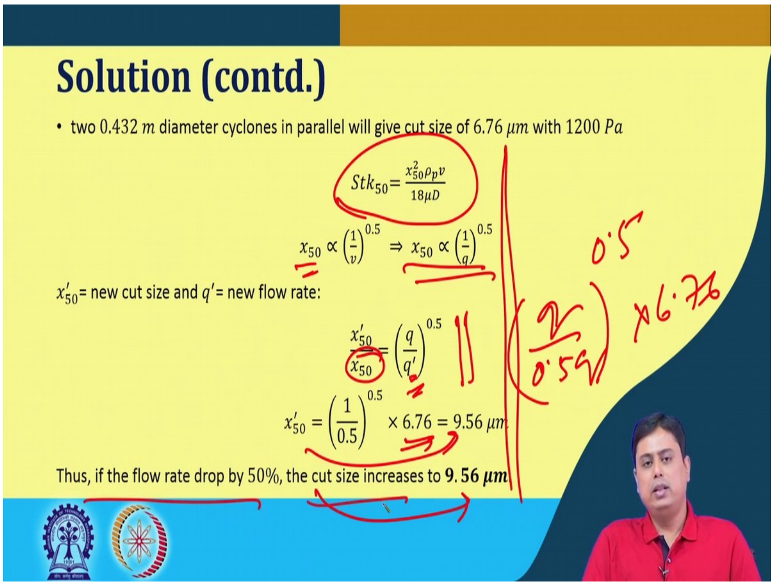

So, for this part, if you think or if you look at the Stokes number, the expression we now know very well;

$$Stk_{50} = \frac{x_{50}^2 \rho_p v}{18 \mu D}$$

that is

$$x_{50} \propto \left(\frac{1}{v}\right)^{0.5} \Rightarrow x_{50} \propto \left(\frac{1}{q}\right)^{0.5}$$

This velocity and flow rate is proportional. So, which means the cut size is proportional to square root of q, which is the flow rate in each cyclone; which means now, if x'_{50}= new cut size and q'= new flow rate:

$$\frac{x_{50}^{'}}{x_{50}} = \left(\frac{q}{q^{'}}\right)^{0.5}$$

And here x_{50} is known value which is 6.76 and $\frac{q}{q^{'}}$ is 1/0.5, because $q^{'}$ is the value that is half of q.

So,

$$x_{50}^{'} = \left(\frac{1}{0.5}\right)^{0.5} \times 6.76 = 9.56\,\mu m$$

gives us the new cut size of 9.56 μm. That means, if the flow rate for that identical scenario of having two cyclones of same family that is means the Euler number and the Stokes numbers were same, identical; and suddenly if the pressure drop goes down to 50 %, the cut size is increases to 9.56 μm from 6.76 μm.

So, I hope this part is also clear to you, that without doing the whole problem once again from this Stokes numbers or and how it is important; its importance, its utility or its application in these problems I hope this becomes clearer to you.

That from this Stokes number expressions; how do we get the cut size when all the operating conditions remain same except the flow rate goes down to 50 %. So, say similar to this if the problem is rephrased in the way of having pressure drop loss. So, pressure loss suddenly goes down to a different level, then you should be able to connect this problem to the Euler number having constant value or the relation between the flow rate and the pressure drop.

That should give you the measurement of pressure drop or measurement of flow rate, if any one of those changes. So, such analytical process you have to think, because we have now solved several problems. And in the next class we will see about the sedimentation in a centrifugal field and then we will conclude this section.

So, before I conclude todays talk, let me reiterate that all these designs, all these design consideration, all these theory remains same for the hydrocyclone operation as well; that means, when there is water or liquid phase instead of gas phase. When the solid particles are suspended in the liquid phase; the design of hydro cyclone or the cyclone handling the liquid phase would be the similar. So, with this note I would like to thank you for your attention and we will see you in the next step.