Fundamentals Of Particle And Fluid Solid Processing
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Lecture - 37
Centrifugal Separation (Contd.)

Hello everyone, and welcome back to the another class of Fundamentals of Particle and Fluid Solid Processing.

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We will continue our discussion on the Centrifugal Separation. We started in the last class the discussion the on the, on its working principle on introduction. And we have seen this part that is the efficiency and separation. So, we will quickly go to this part once again in order to maintain the coherency.

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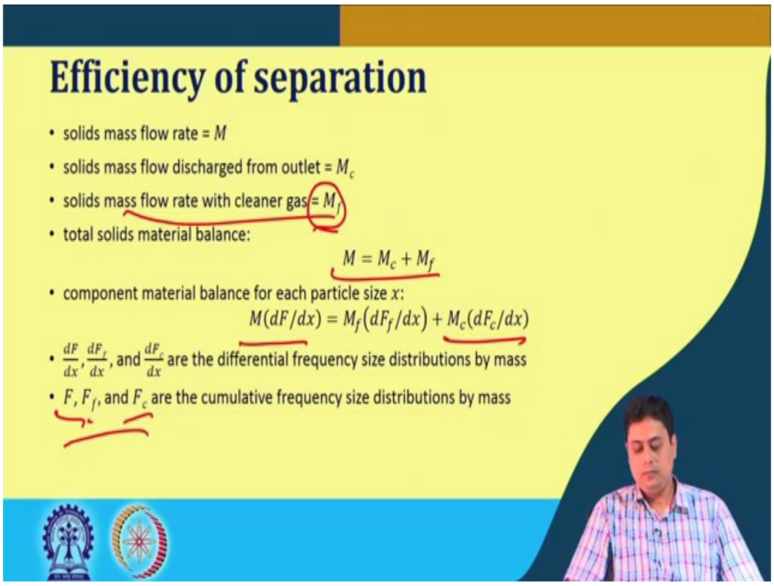

So, say the solid mass flow in the cyclone is M, the solids mass that is discharged from the outlet it is M_c, c stands for the coarse particle. So, that is actually the particles that we are separating, and the other flow rate, that is a solid mass flow rate with the cleaner gas this is basically a finer particles that escapes with a clean gas and since the cyclone efficiency is not 100 %, although theoretically it can be. So, these are the impurities that goes with the clean gas.

Now, if you do the total solids material balance, we can write this expression that

$$M = M_c + M_f$$

M which is the total mass flow rate of the solid is equals to the mass that is being discharged or the solid that is being discharged from the coarse outlet and the final outlet. And if we do the component material balance for each particle size of x, then we write that

$$M(dF/dx) = M_f(dF_f/dx) + M_c(dF_c/dx)$$

M multiplied by the size distribution of the inlet that is this dF/dx is the differential frequency size distributions of the solid particles for the inlet or the flow rate that is coming in the cyclone separator.

Similarly, $\frac{dF}{dx}$, $\frac{dF_f}{dx}$,and $\frac{dF_c}{dx}$ are the differential frequency size distributions by mass. So, this is the differential size distributions of the solids in the finer particle collection or say the clean gas connection outlet. And, accordingly this dF_c/dx is basically the differential size distribution frequency by mass of the coarser particles. Now, we know the relation between the differential size distribution and the cumulative size distribution. So, this F, F_f and F_c, this small f and small c these are the subscripts for the finer and the coarser particles. These are the cumulative frequency size distributions by mass.

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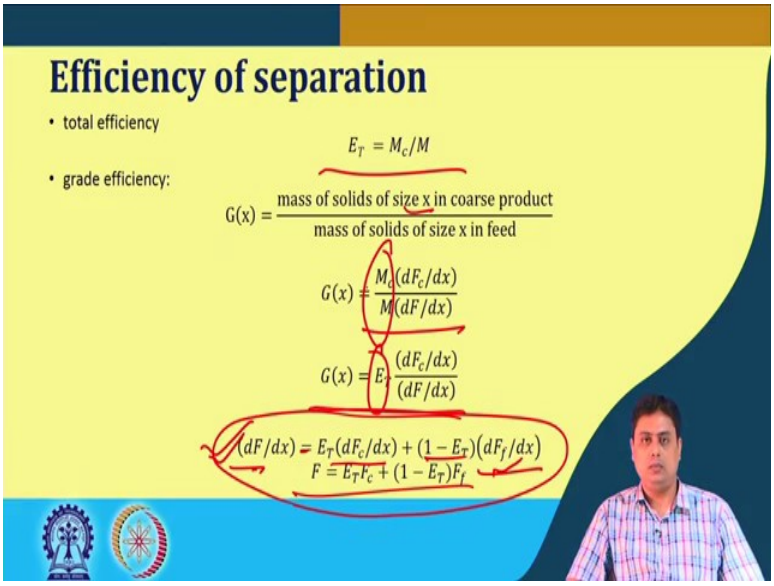

And then the total efficiency by definition becomes the mass fraction of the coarser particle or the coarse particles that is collected from this operation which is

$$ET = M_c/M$$

and the grade efficiency is the mass of solid of size x in the coarse product divided by the mass of solids of size x in the feed.

$$G(x)=\frac{mass\ of\ solids\ of\ ¿¿\ coarse\ product}{mass\ of\ solids\ of\ ¿¿\ feed}$$

So, grade efficiency as I told last time it is of a certain particle size.

The example that I gave say you want the suspension or say the particle (Refer Time: 04:18) gas to be free from say certain particle size that is your x in this expression. Say you want that particle (Refer Time: 04:33) gas to be free from size of particles of more than 15 µm. So, this is the size that is x is 15 µm in this case; so, 15 µm of particle in the coarse product divided by the 15 µm of the particle in the inlet or in the feed. This ratio is the grade efficiency of that cyclone separator.

So, by this definition grade efficiency can also be written in this form.

$$G(x)=\frac{M_c(dF_c/dx)}{M(dF/dx)}$$

This is the mass of solids multiplied by the size distribution of size in the coarse product divided by the mass of the solids in the feed multiplied by its size distribution. And again we know this M_c/M is basically the overall efficiency or the total efficiency of the cyclone separator, so which means we can write the grade efficiency in terms of total efficiency as well.

$$G(x)=E_T\frac{(dF_c/dx)}{(dF/dx)}$$

Now, this three expressions that is this overall efficiency, total expression and this component material balance, this yields to this expression

$$(dF/dx)=E_T(dF_c/dx)+(1-E_T)(dF_f/dx)$$

which is the size distribution of the particles, in the field, in the coarse product and in the cleaner gas with the fine particles related with the overall efficiency. If we integrate this we get this cumulative mass fraction or size distribution based on the mass. So, this was the point we ended or I ended the last class.

$$F=E_TF_c+(1-E_T)F_f$$

So, basically this expression or these two above mentioned expressions of $G(x)$ and $F$are of important because these two expressions give us the product size distributions or say the particle size distributions from the feed in the coarse and in the finer particle in the clean gas outlet. So, if we know these two, we can calculate the third one with the help of its total efficiency. We will see example related to that that how these expressions are utilized in some problem.

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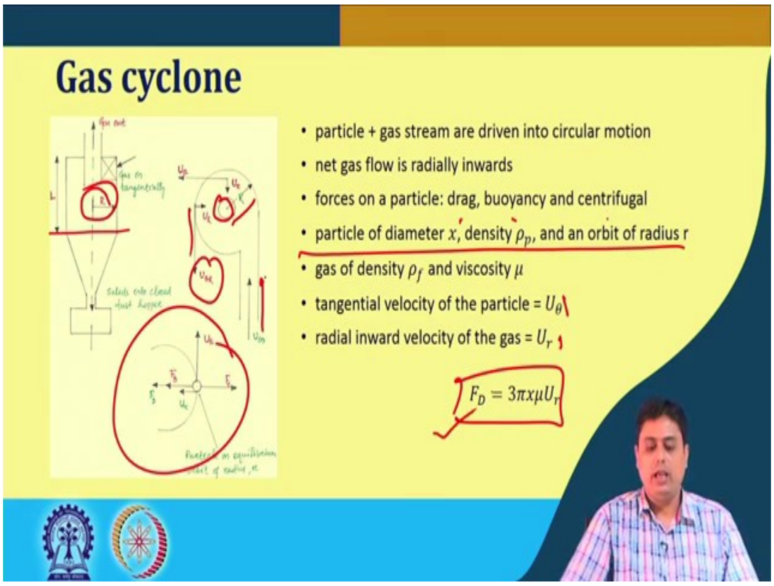

Before that let us look into the flow pattern inside this gas cycle. Now, as I mentioned last time, as its working principle this particle and the gas stream or the particle (Refer Time: 07:29) gas stream are forced into circular motion inside the cyclone by introducing it tangentially with a very high velocity. So, it creates vortex inside that. And once it comes down in the conical section, it changes its direction and the clean gas goes at the top because the particles this free vortex move the particles towards the radial directions and it hits the wall and due to the gravity it moves down and is collected at the bottom of the cyclone.

So, basically what happens? This net gas flow is basically radially inward and the forces that are acting on the particle are the drag buoyancy and the centrifugal force. Now, the direction of drag force if you realize here the particles are being pushed towards the wall, the gas is inward, ok. So, the relative velocity based on the particle is basically outwards because it has to be captured, so the back velocity is higher and that means, the drag force is basically inwards because it is opposite to the direction of relative velocity. So, the drag force F_D is inward.

So, here this picture shows that based on the balance of these forces on a particle if it comes to a balanced expression or the forces are balanced then it assumes a certain orbit of say (Refer Time: 09:30) radius small r. So, particle size x of density ρ_p has attained orbit of radius r and it is moving in that radius of orbit. Also assume the gas density as ρ_f and viscosity is μ. Say, the tangential velocity of the particle is U_θ, this is the U_θ and the radial velocity is U_r. This is a view from the top and this is the particle motion, the particle trajectory.

So, say the radius of this cylindrical portion is r, the length till this position is L, ok. And this is the velocity inlet, if you look from the top this would be the velocity inlet, this is the gas outlet, this is the radius of the cylindrical section. U_θ is the tangential velocity, so $U_{\theta R}$ is a tangential velocity at the wall and U_R is the radial velocity of the particle say on the wall of this vessel. So, if we assume say for the time being it is not touching the wall, but having a trajectory or orbit of radius r, that is the diameter x of the particle density is rho p and the orbit of radius r, tangential velocity U_θ and U_r.

So, the drag force and if we assume say all this operation is happening in the stokes correlation then the drag force acting on the particle is this expression that we know already which is

$$F_D = 3\pi x \mu U_r$$

So, this scenario the particle on equilibrium orbit of radius r. When this forces are in equilibrium that is the drag force buoyancy force and centrifugal force. So, out of which

$$F_D = 3\pi x \mu U_r$$

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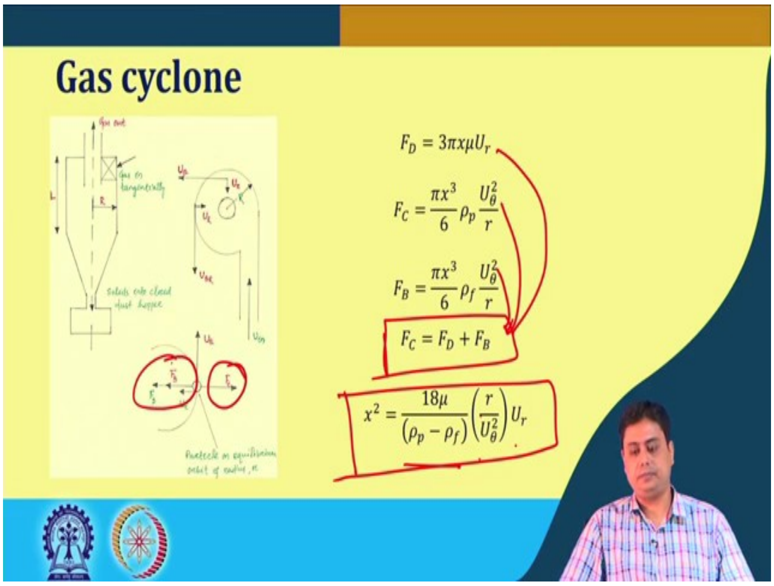

Then, this centrifugal force which is F_C ok. We can write

$$F_C = \frac{\pi x^3}{6} \rho_p \frac{U_\theta^2}{r}$$

where U_θ is the tangential velocity of the particle that is having trajectory that is moving in a orbit of radius r on equilibrium. Similarly, the buoyancy force we can write

$$F_B = \frac{\pi x^3}{6} \rho_f \frac{U_\theta^2}{r}$$

and at equilibrium based on the direction of the force as I mentioned that F_D and F_B is inward and centrifugal force is acting out word, which means F_C is equals to F_D plus F_B, C stands for the centrifugal force, D stands for the drag force, B stands for the buoyancy.

$$F_C = F_D + F_B$$

If we equate or if we now replace these expressions of F_C, F_D and F_B, you would achieve an expression of particle diameter with this parameter that is

$$x^2 = \frac{18\mu}{(\rho_p - \rho_f)} \left(\frac{r}{U_\theta^2}\right) U_r$$

So, 18 μ divided by the density difference of the particle and the fluid, r is the radius of the orbit that a particle of dimension or diameter x has taken at equilibrium, U_θ is the tangential velocity U_r is the radial velocity of the particle. So, we find an expression of the particle diameter and its equilibrium radius, radius of the orbit.

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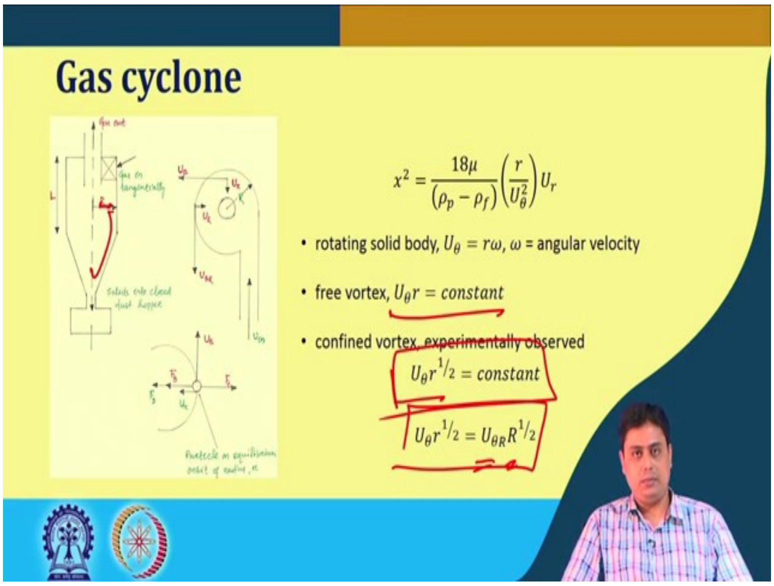

So, now, the point is that before we proceed any further we need a relation of this U_θ or expression for U_θ, because it is the U_θ at an arbitrary position having orbit of radius small r which hardly can be measured. So, U_θ now for a rotating solid body, U_θ the tangential velocity is the orbit radius multiplied by the angular velocity,

$$U_\theta = r\omega$$

this we know. And, for free vortex

$$U_\theta r = constant$$

and when this vortex is confined it has been seen experimentally and has been observed that

$$U_\theta r^{\frac{1}{2}} = constant$$

This is experimentally determined or experimentally observed for confined vortex, that the tangential velocity and the radius of orbit to the power half or 0.5 this is constant. So, which means this

$$U_\theta r^{\frac{1}{2}} = U_{\theta R} R^{\frac{1}{2}}$$

the solid say touching the wall multiplied by the radius of the cylindrical section because the particles basically touches this wall and then falls. When it impacts on the solid wall then it is being collected by gravity it rolls down at the bottom.

So, now that means, we have an expression for U_θ in some known quantity, because $U_\theta r$ is basically a design parameter. It comes from the design of the cylindrical section and the operating condition, as well as R is known from the design parameter the R.

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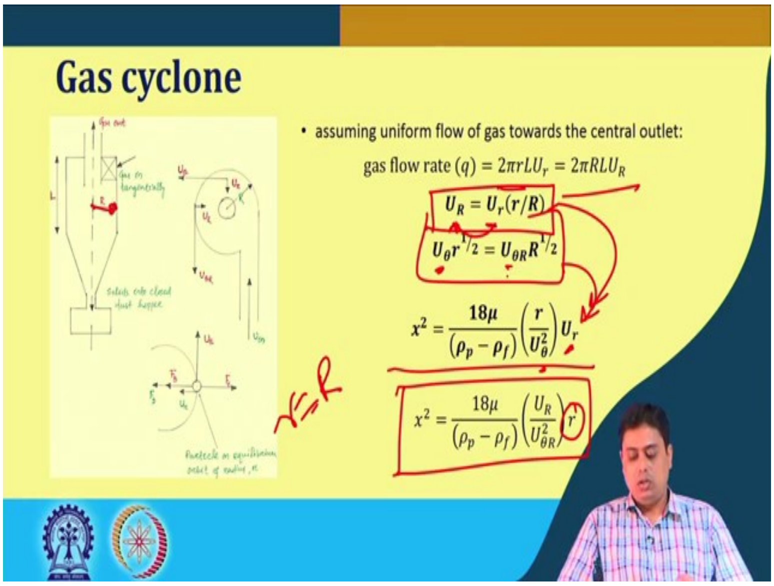

So, now if we assume that the uniform flow of gas towards the central outlet is there an ideal scenario then the gas flow rate at every plane remains same because it is the net flow rate that is happening at every plane. So, which means if the gas flow rate q is basically known then

$$q = 2\pi rLU_r = 2\pi RLU_R$$

U_R is the radial velocity at the wall, the particle that is touching the wall at that position.

So, which means we find a relation of U_r with U_R. Now, here U_R again is a design parameter because this tangential velocity and radial velocity these are the design parameters of this gas cyclone at the inlet near the wall. So, whatever happening at an arbitrary small r position, we have now found the expressions relating to that position with some known quantity. We did that with U_θ, with $U_{\theta R}$, θR means near the wall or at the wall and U_r in terms of U_R.

So, these two expressions here

$U_R = U_r (r/R)$

$U_\theta r^{\frac{1}{2}} = U_{\theta R} R^{\frac{1}{2}}$

because in this form these were the two unknowns basically U_θ and U_r that we derived by the equilibrium force balance. Now, we replace these two that U_r in terms of U_R and U_θ in terms of $U_{\theta R}$ in terms of r in this expression. We get x and the relation of r in terms of all known quantity.

$$x^2 = \frac{18\mu}{(\rho_p - \rho_f)} \left(\frac{r}{U_\theta^2} \right) U_r$$

Now, the point is that this is this r is still the radius of orbit where a particle of diameter x has attend at equilibrium condition, equilibrium between the 4 forces of drag buoyancy and the centrifugal force. Now, in order to be this size of x be captured or to have in the coarse product outlet which particle has to collide with the or it has to come to the surface of this vessel; that means, the this orbit of its movement this small r has to be the capital R and then it can basically touches the wall and can be captured. So, that is the critical x.

$$x^2 = \frac{18\mu}{(\rho_p - \rho_f)} \left(\frac{U_R}{U_{\theta R}^2} \right) r$$

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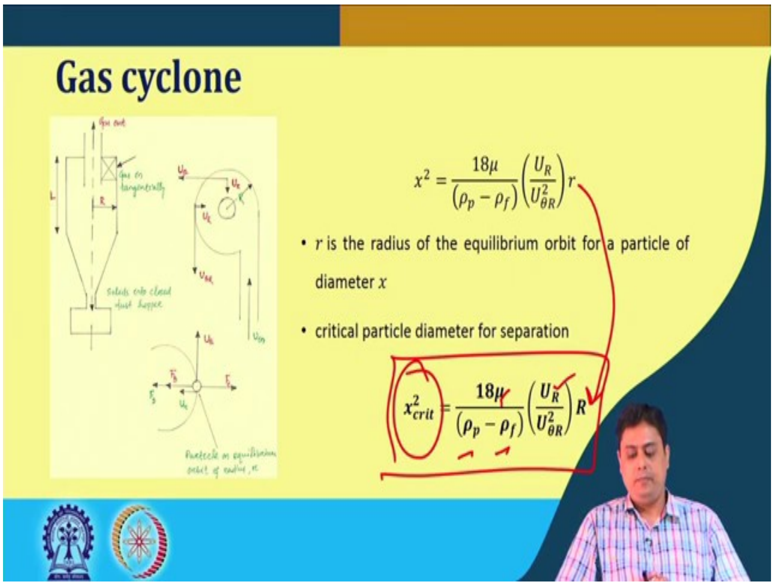

The radius of this equilibrium orbit when it becomes the radius of the vessel; that means, it hits that wall or it touches the wall and then once it touches; that means, that particle can be collected. So, replacing that r with R, we can find the critical diameter of the particle that can be captured and that comes in terms of particle density, fluid density, fluid viscosity, the particle radial velocity and tangential velocity near the surface or say near the wall.

$$x_{crit}^2 = \frac{18\mu}{\left(\rho_p - \rho_f\right)} \left(\frac{U_R}{U_{\theta R}^2}\right) R$$

So, this critical size is sometimes a single quantity that is used to quantify a gas cyclone. The characteristics of a cyclone separator is sometimes quantified by this critical particle size that can be captured for this operating condition. So, I hope this expression is cleared to you that how this critical particle diameter can be calculated provided all other design parameters are given.

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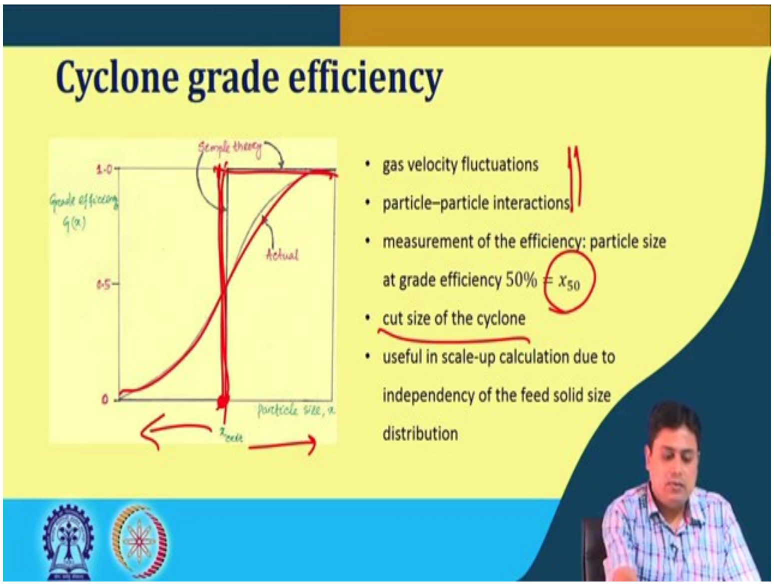

Now, which means if this is the case the grade efficiency for say this x size or x_{crit} would vary very sharply ideally that there will be a sharp line that beyond this line everything will be captured or beyond this size everything will be captured and this particles will escape with the finer rate.

But that does not happen. This simple theory that we have shown this should predict that this is your x_{crit} and this is the critical size that can be captured or higher than that larger sizes than that and any smaller size than critical size will basically still be in rotation and the chances are that it will not be captured because it is not touching the wall. So, ideally this is the case. But the point is that in actual practice it has been seen that even though there is a x_{crit} few particles smaller than that gets away or collected or say higher than that is escaping. And this happens due to gas velocity fluctuations, particle-particle interactions.

These two are the reason the point why we do not get this kind of ideal or 100 % separation with critical size. So, this measurement of efficiency the particle size at grade efficiency 50 % or say x_{50} is the number sometimes these are used for characterizing a gas cyclone operator or its efficiency. So, why this 50 %? x_{50} the particle size at grade efficiency of 50 % because this is a equi-probable number that 50 % chances are there the particles of this size will be collected, of this x_{50} size, the critical size. That this mean that for larger quantities of particle 50 % of the particle that is higher than this size will be collected as the coarse product.

So, this x_{50} is also called as the cut size of the cyclone. Now, this parameter is very useful as a single number in scale up operations or scale up calculations from a laboratory stage to the commercial scale because this cut size is independent of feed size distribution. So, that is the reason that this cut size is one of the vital parameter that is used for the calculation of scale up designs. So, what does that mean? That whatever we have covered now is that we have seen the flow pattern or we have seen that how a critical size of a particle that can be captured for a given operating condition can be calculated or determined.

We have also understood that that ideal scenario is not practical, is not feasible and the grade efficiency of a cyclone typically it takes a shape of this kind of a S shape of curve like the other separator that handles this (Refer Time: 28:08), this centrifugal force and the drag force balance. There is single number we have tried to identify which is called the x_{50} that is the grade efficiency at 50 %. So, this value we say is the cut size of the cyclone because this gives possibility of 50 % chances that the size of this will be collected in the coarse product and this x_{50} is basically independent of feed size distributions and that is why it is useful in scale up calculation.

So, with this I will stop here today. And in the next class, we will be dealing with couple of problems related to this grade efficiency, overall efficiency, how the size distributions can be calculated if these two values if the two outlet values are given, and we will see the range of operation of this cyclone separators.

Till then, I thank you for your attention.