Fundamentals Of Particle And Fluid Solid Processing
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Lecture - 26
Sedimentation

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Hello everyone, welcome back to the another class of Fundamentals of Particle and Fluid Solid Processing. Until the last class we have seen the fluidization the fundamentals of fluidization and during that we have also find an analogy of fluidization with a opposite of settling. Now, this settling has immense importance in this particular area because when we try to separate solids from fluids, there are several techniques available and one of the primitive as well as less expensive technique is the settling or sedimentation.

So, here what happens that, in a pool of liquid or some fluid we put those solid particles mixture and we have seen the fundamentals that due to different size density and all these, the particles will have its different terminal velocity while coming downward to that top surface. Now, based on that settling speed settling time you can segregate different size particles. In fact, we solved a problem during the particles motion in a fluid on that section.

We also in the fluidization we introduced the concept of hindered settling because when there are multiple particles, the simple scenario that we have understood for single particle movement in a pool of liquid in this case of collection of particles that physics is not relevant.

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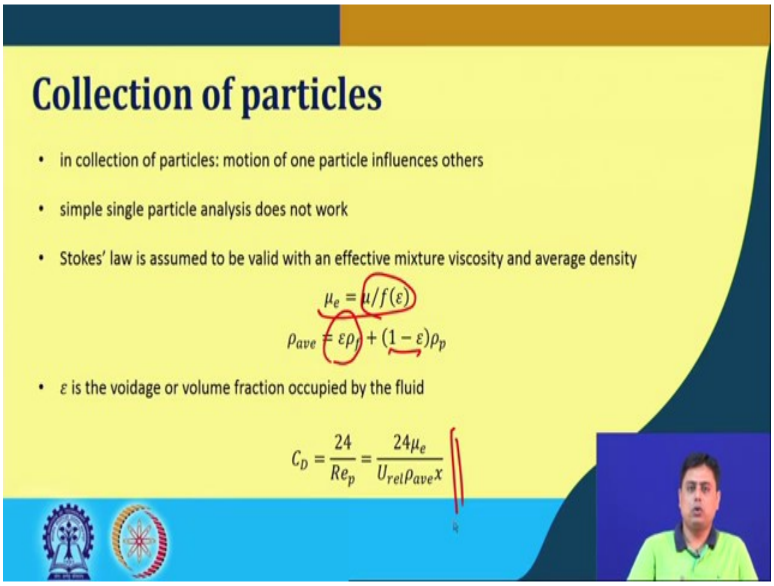

So, this couple of initial slides, I have again kept it for the sake of coherensive so, that we can quickly go through this concept or the fundamentals once again before we start to look at these different settling phenomena. So, the settling operations can have we can operate in a two different way that one is the batch settling that we just put certain particles in the fluid and we wait for that to sediment the settling to have its final stage and the other case there is a continuous operation which is called a continuous settling.

So, we will see these two in details in this section. Now before that as I said just look we will again glance through these equations expression so, that we can find we can remember the relevant terms is that when there are collection of particles this viscosity density of the mixture has been defined in terms of effective viscosity as well as the average density. How we have done that? It is that viscosity becomes a function of the void fraction or the void fraction occupied by the fluid phase

$$\mu_e = \mu/f(\varepsilon)$$

and the average density is basically this volume fraction of the liquid phase multiplied by the respective fluid density and the addition of the solids fraction multiplied by the solid density.

$$\rho_{ave} = \varepsilon\rho_f + (1-\varepsilon)\rho_p$$

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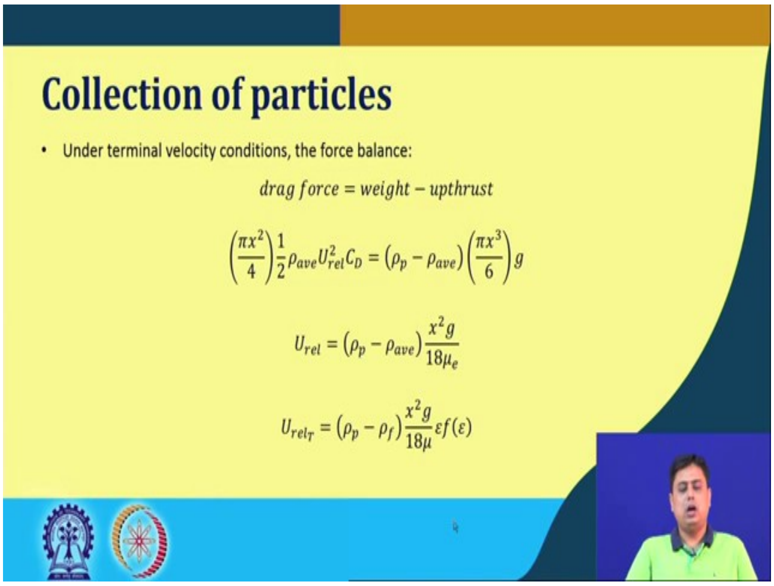

Now, we know that in stokes region the drag coefficient is:

$$C_D = \frac{24}{\Re_p} = \frac{24\mu_e}{U_{rel}\rho_{ave}x}$$

which now under terminal velocity condition we can write this simple balance,

$$drag\,force = weight - upthrust$$

that you have already seen and we find a relation between this terminal velocity based on the relative velocity concept and a single particle terminal velocity.

$$\left(\frac{\pi x^2}{4}\right)\frac{1}{2}\rho_{ave}U_{rel}^2 C_D = \left(\rho_p - \rho_{ave}\right)\left(\frac{\pi x^3}{6}\right)g$$

$$U_{rel} = \left(\rho_p - \rho_{ave}\right)\frac{x^2 g}{18\mu_e}$$

$$U_{rel_T} = \left(\rho_p - \rho_f\right)\frac{x^2 g}{18\mu}\varepsilon f(\varepsilon)$$

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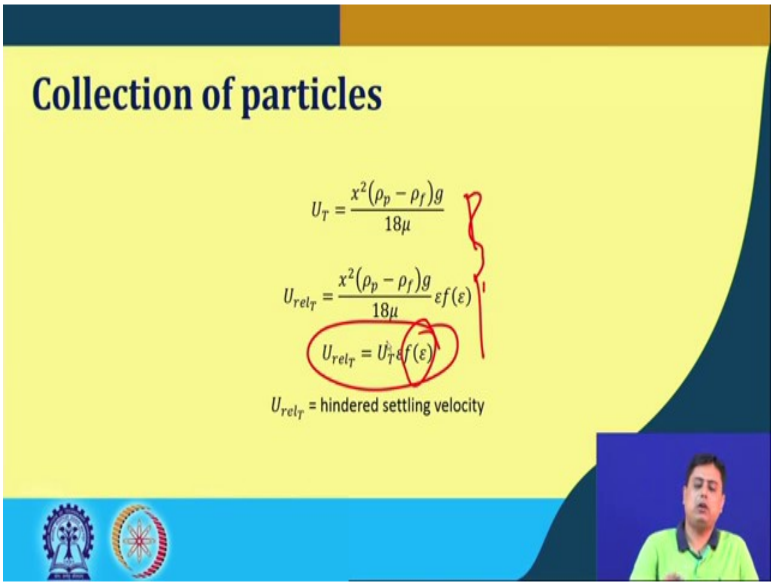

By comparing,

$$U_T = \frac{x^2(\rho_p - \rho_f)g}{18\mu}$$

and

$$U_{rel_T} = \frac{x^2(\rho_p - \rho_f)g}{18\mu}\varepsilon f(\varepsilon)$$

we find this relation that, the hindered settling velocity is basically the single particle terminal velocity multiplied by the void fraction multiplied by a function dependent on the void fraction or the volume fraction of the fluid.

$$U_{rel_T} = U_T \varepsilon f(\varepsilon)$$

Now, this expression we have also seen that how it depends or what are its expression in different regions ok.

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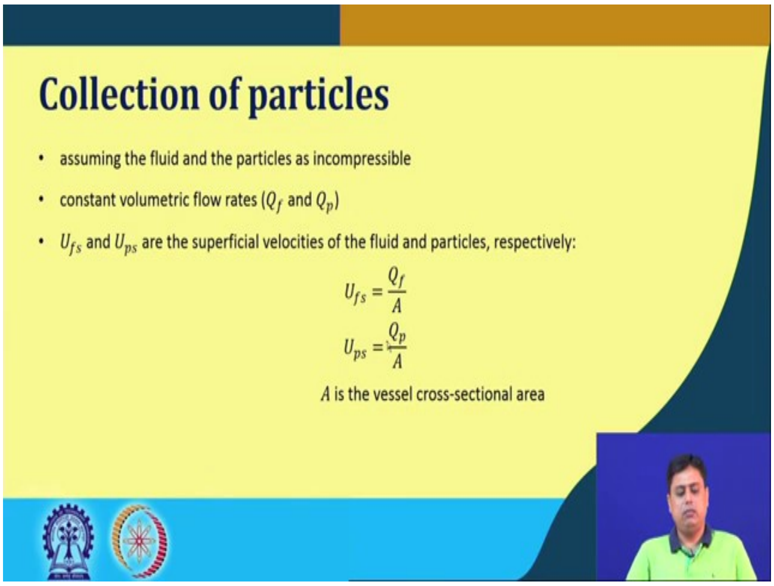

Because if we assume that the fluid and particles are incompressible and there is constant volume flow rates of fluid and the particle. So, we can write this we can find out what are the superficial velocity for the solids and the particle.

$$U_{fs} = \frac{Q_f}{A}$$

$$U_{ps} = \frac{Q_p}{A}$$

A is the vessel cross-sectional area

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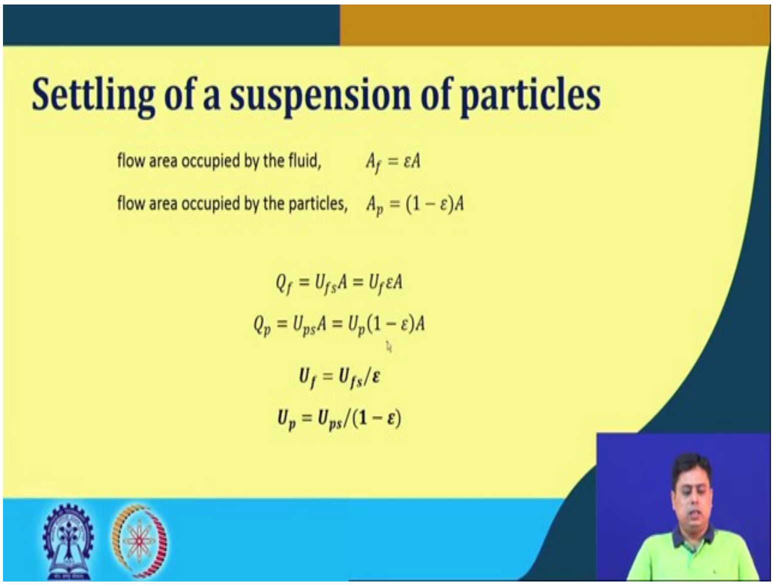

And then the flow area occupied by the fluid is basically its fraction multiplied by the total available area. Similarly the area occupied by the solids is its solids fraction multiplied by the total available area.

flow area occupied by the fluid, $A_f=\varepsilon A$

flow area occupied by the particles, $A_p=(1-\varepsilon)A$

We do this conservation because the mass is conserved

$$Q_f=U_{fs}A=U_f\varepsilon A$$

$$Q_p=U_{ps}A=U_p(1-\varepsilon)A$$

and then we find that this actual velocity is this superficial velocity divided by the respective volume fraction.

$$U_f=U_{fs}/\varepsilon$$

$$U_p=U_{ps}/(1-\varepsilon)$$

So, superficial velocity of each phase be the fluid phase or the particles, it is their corresponding superficial velocity divided by respective volume fraction. These things we have already seen, but just to recall, I am showing you these expressions once again.

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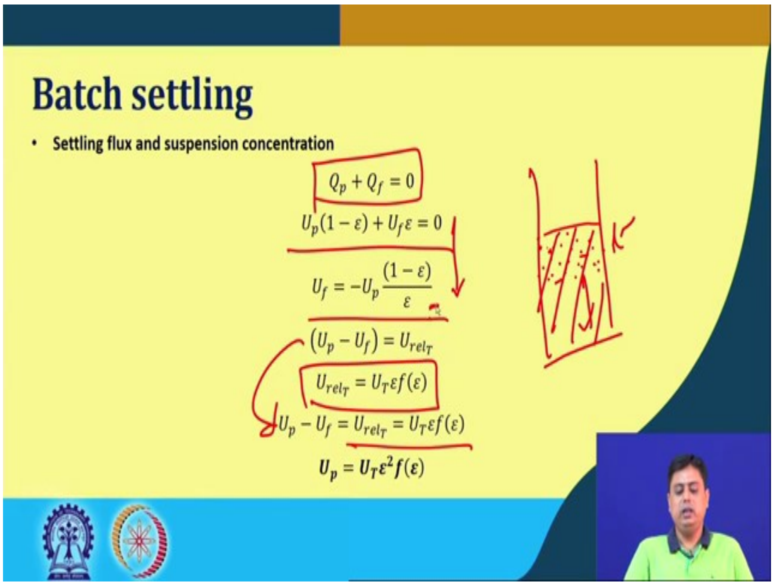

Now, in case of batch settling; that means, say you have a measuring cylinder and then there is a pool of liquid is there, some solid particles are now settling. This is at let us say certain time t this particles will sediment or will settle at the bottom of this measuring cylinder at a certain time. Now, in batch settling there is no inflow or outflow of this particle or say the fluid. So, for the whole system

$$Q_p + Q_f = 0$$

This is the batch system which we can write because it has a uniform cross sectional area.

$$U_p(1-\varepsilon) + U_f\varepsilon = 0$$

So, we write that in terms of its actual velocity.

From the above expression we can have what is the U_f?

$$U_f = -U_p\frac{(1-\varepsilon)}{\varepsilon}$$

Now,

$$(U_p - U_f) = U_{rel_T}$$

ok. Now here this relative terminal velocity we have also seen its expression in the previous slide that this is nothing, but the single particle terminal velocity multiplied by this factor ok.

$$U_{rel_T} = U_T\varepsilon f(\varepsilon)$$

So, this expression can be written in this form that now we have this one which if we now replace it here we get that what is U_p in terms of U_T the actual velocity of the particle in terms of the single particle terminal velocity what is the relation.

$$U_p - U_f = U_{rel_T} = U_T \varepsilon f(\varepsilon)$$

So, from here what we get? We get what is U_f we replace this here and we get this final form of U_p which is the actual particle velocity in case of batch settling ok.

$$U_p = U_T \varepsilon^2 f(\varepsilon)$$

So, that means, this dimensionless velocity or now the point is that we have seen before I go into that. The point I have seen is that this function what its expression because then you can calculate what is U_p.

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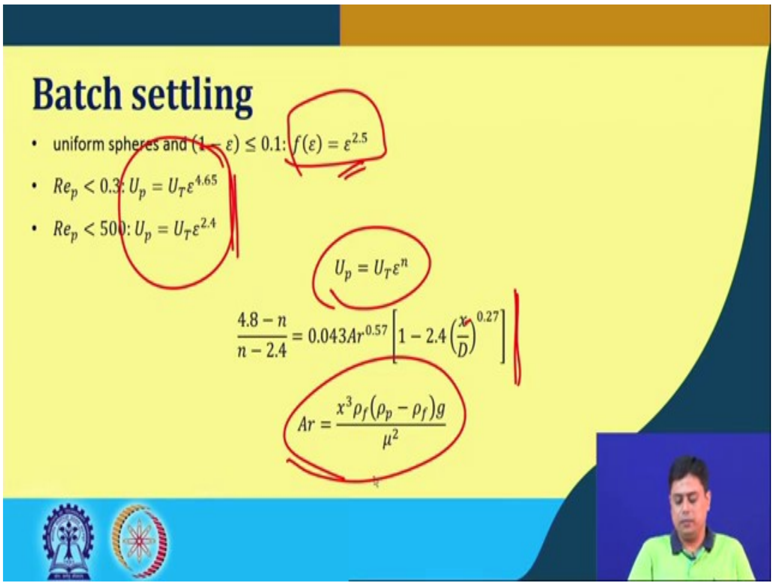

Now, this expression theoretically has been shown that for uniform spheres and solids volume fraction or suspension of less than 0.1.

$$f(\varepsilon) = \varepsilon^{2.5}$$

This $f(\varepsilon)$ the function of a ε is basically $\varepsilon^{2.5}$ok. We have also seen these two criteria that

When

$$\Re_p<0.3: U_p = U_T \varepsilon^{4.65}$$

$$\Re_p>500: U_p = U_T \varepsilon^{2.4} \text{ (Newton's law regime)}$$

So, in Stokes' law region ε the function the viscosity will be influenced by ε to the power 2.5, 2.65 and in case of Newton's law region when this drag force is independent of the viscosity there we have ε to the power 0.4. In general U_p; that means, is basically U_T multiplied by ε to the power n because if you replace this expression in here you basically get this U_p relations.

$$U_p = U_T \varepsilon^n$$

So, U_p is basically U_T multiplied by ε to the power n. We have also seen or also have covered this expression by Khan and Richardson that covers the entire range of n entire range of Reynolds number and predicts the value of n where x and D are the particle and the vessel diameter or the measuring cylinder diameter in this case.

$$\frac{4.8-n}{n-2.4} = 0.043\, Ar^{0.57}\left[1-2.4\left(\frac{x}{D}\right)^{0.27}\right]$$

And Archimedes number we already know this expression

$$Ar = \frac{x^3 \rho_f \left(\rho_p - \rho_f\right) g}{\mu^2}$$

and in this case again x is logically the mean surface volume diameter in case of irregular or non-spherical particles. So, this we have seen.

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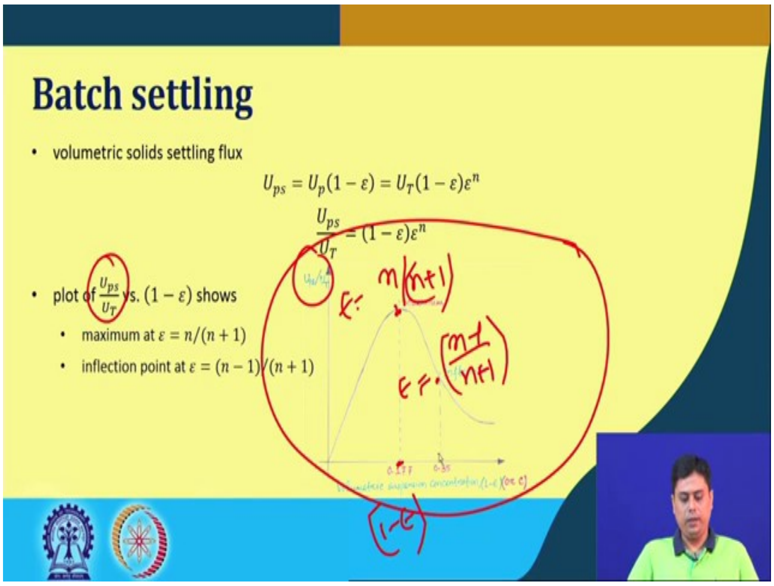

Now, the point is that, this if now we know that what is the actual particle velocity based on the Reynolds numbers region you can find out what is the Richardson Zaki expression this is this is the famous Richardson Zaki expression

$$U_p = U_T \varepsilon^n$$

and then you can find out what is U_p that is the actual particle velocity. In terms of volumetric solid settling or solid settling flux, we can write U_{ps} is the actual velocity multiplied by its volume fraction which again we write in terms of the terminal velocity we replace the expression of U_p by $U_T \varepsilon^n$.

$$U_{ps} = U_p(1-\varepsilon) = U_T(1-\varepsilon)\varepsilon^n$$

So, which means we have got the volumetric solids settling flux in terms of the single particle terminal velocity and the volume fraction of the solids and the liquids. In terms of a dimensionless quantity, if we divide this by U_T we get this expression.

$$\frac{U_{ps}}{U_T} = (1-\varepsilon)\varepsilon^n$$

Now, if we plot this; that means, the plot of this parameter on the left hand side $\frac{U_{ps}}{U_T}$ and $(1-\varepsilon)$ which is the solids fraction, we find there is a maxima because a simple thing of making doing the derivatives first and second derivatives will give you this ideas that this curve or this plot will have maxima at $\varepsilon = n/(n+1)$ and inflection point at $\varepsilon = (n-1)/(n+1)$.

So, this plot would look like something like this that is shown here this graph. Where x axis is the volumetric suspension concentration that is $(1-\varepsilon)$ and y axis is this non dimensional quantity, non-dimensional settling velocities. So, it will have a maxima at $\varepsilon = n/(n+1)$ and inflection point at $\varepsilon = (n-1)/(n+1)$. In case of Stokes' law region we know what is that factor that n what is n ok. In case of Stokes' law region n is 4.65. So, for Stokes' law region this value remember this is $(1-\varepsilon)$ becomes 1.77 and here we get at 0.35 3 something.

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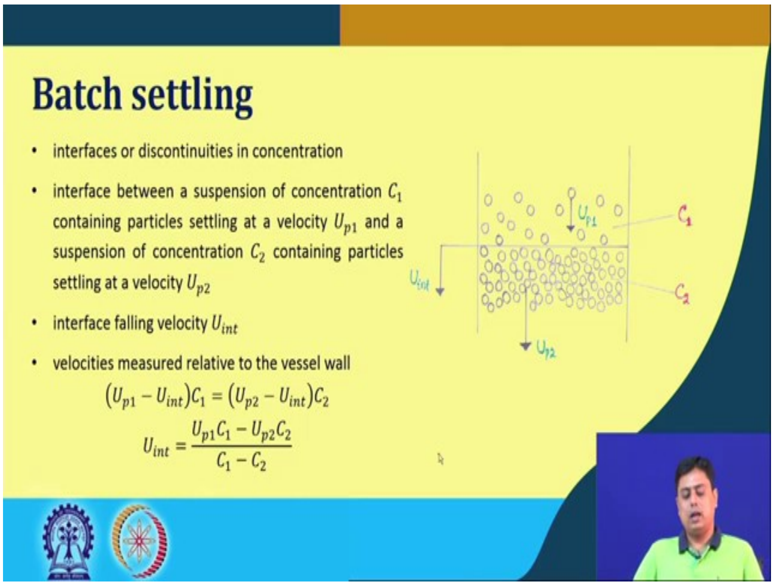

Now, the point is that, you can think of this scenario like here that you have this measuring cylinder where you have put it is filled with liquid or say water and then from the top you have put some sand or that kind of fine particle ok. So, there will be in batch set settling there will be a clear interface or discontinuity of concentration; that means, there will be some portions at a certain time where the densities of the particle the concentrations basically the concentrations will be higher and at some portion that would be lower. So, this region you can clearly identify if the measuring vessel is transparent.

Now this interface between the suspension of say concentration C_1 will have say a particle terminal velocity of U_{p1}. And the region where the concentration is C_2 assume that there the particles are settling with the superficial with the settling velocity of p_2 the actual particle velocity ok. And the interface this kind of a interface because here let us say the dilute suspension this is kind of a dense suspension. So, you can identify its interface.

Now, this interface say is falling at a velocity $U_{\int ¿¿}$. Now, this all velocities are basically measured with respect to the vessel walls. Then what happens? That you get this U_{p1} minus U interface multiplied by the concentration in that region or from interface to inside that region wherever the particles are having the settling velocity of U_{p1} multiplied by its concentration is equals to this downward settling flux multiplied by their concentration in the denser region.

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If we assume the concept of incompressibility both particles and the liquids or the fluids, then we can find the rate or the velocity of this interface which we can write in terms of U_{p1} multiplied by C_1 minus U_{p2} multiplied by C_2 divided by the concentration difference between this two layers ok.

$$U_{\int ¿=\frac{U_{p1}C_1-U_{p2}C_2}{C_1-C_2}¿}$$

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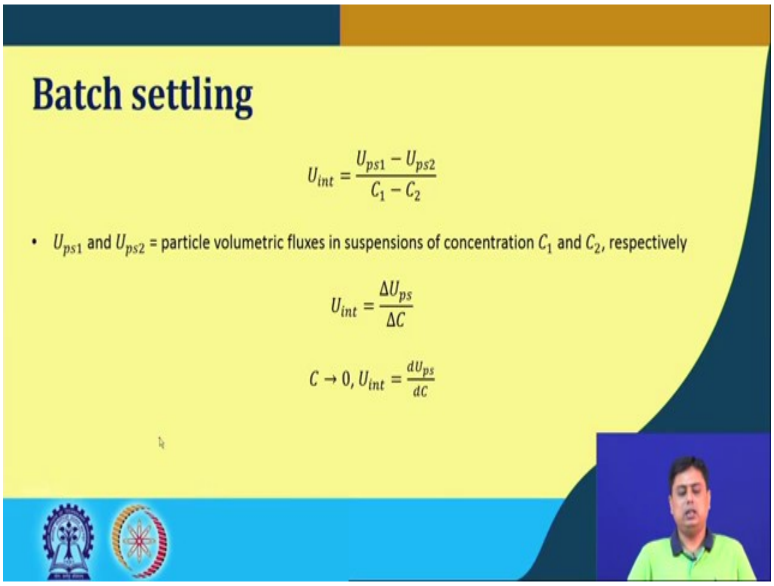

But these terms ($U_{p1}C_1$ and $U_{p2}C_2$)are already these are the downward velocity flux ok or the volumetric flux particle volumetric flux because you remember this expression here

$$U_{ps} = U_p (1-\varepsilon) = U_T (1-\varepsilon) \varepsilon^n$$

when we introduced that term.

Actual velocity multiplied by its concentration here, the point is $(1-\varepsilon)$ is the solids volume fraction. So, now, in settling sedimentation this volume fractions we typically say its concentration here or C. So, $(1-\varepsilon)$ is basically say this concentration of the suspension ok. So, $(1-\varepsilon)$ is the concentration of the solids here. So, that is why that is what it is designated by C.

So, this $U_p C$ terms are nothing, but the volumetric fluxes this one and this one which is U_{ps1} and U_{ps2}.

$$U_{\int} ¿ = \frac{U_{ps1} - U_{ps2}}{C_1 - C_2} ¿$$

These are the particle volumetric fluxes in suspension of concentration C_1 and C_2 respectively which means this interface velocity is

$$U_{\int} ¿ = \frac{\Delta U_{ps}}{\Delta C} ¿$$

in the limit of C tends to 0 or the concentration tends to 0; that means, the clear liquid or the clear fluid this we can write in its derivative term that

$$C \rightarrow 0, \; U_{\int} ¿ = \frac{dU_{ps}}{dC} ¿$$

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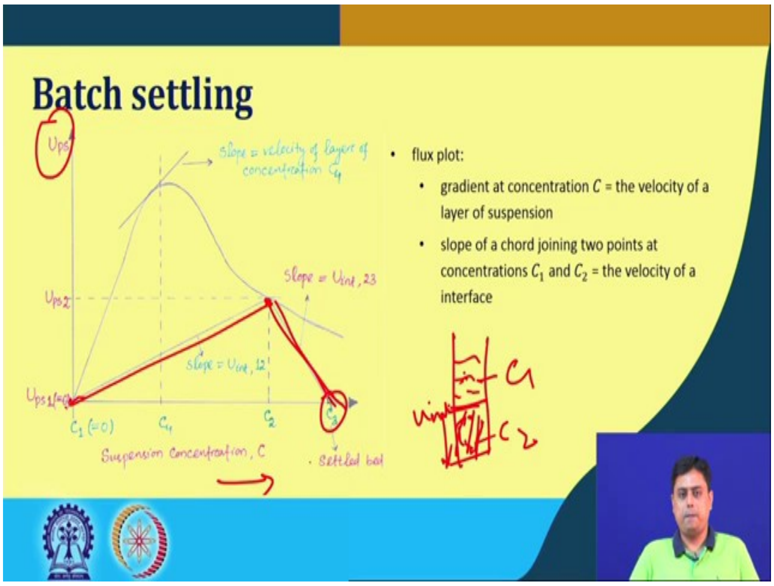

That means the plot of this U_{ps} and the concentration ok. Its gradient at concentration C will give you the velocity for that layer of suspension containing the concentration C is not it because here you get this expression this is the interface velocity ok. And this is the derivative of this U_{ps} with respect to C the concentration. So, U_{ps} versus C at any point the slope of the graph of this curve will give you the velocity of the layer corresponding to that concentration and interestingly the connection or the chord that will join two concentrations that slope of that line will give you the velocity of the interface.

Which means here for example, C_1 if it is 0; that means, it is the clear suspension now; that means, the clear liquid there is no suspension concentration of the solid such 0; that means, the pure fluid ok. As you increase the concentration here fine. So, say at C_3 all particles has settled or the sedimentation is complete ok. So, its a settled bed it will have solid particles filled here and there is a clear liquid at the top.

So, in this case the connection between C_1 and say C_2 for one concentration the slope of this chord that connects C_1 and C_2 in this graph will give you the velocity of the interface having the concentration C_1 and C_2. So, if this is the concentration C_1 and this is concentration C_2 this slope of this line will give you the velocity of this interface.

Similarly, this C_2, C_3 the slope of this line will give you the velocity of the interface between concentration C_2 and C_3 ok. So, if you can come up with this suspension concentration plot with the volumetric flux, then this rate of falling of this interface will be easier to calculate. As well as you can calculate the velocity of the layer that has a certain concentration that at what would be that value.

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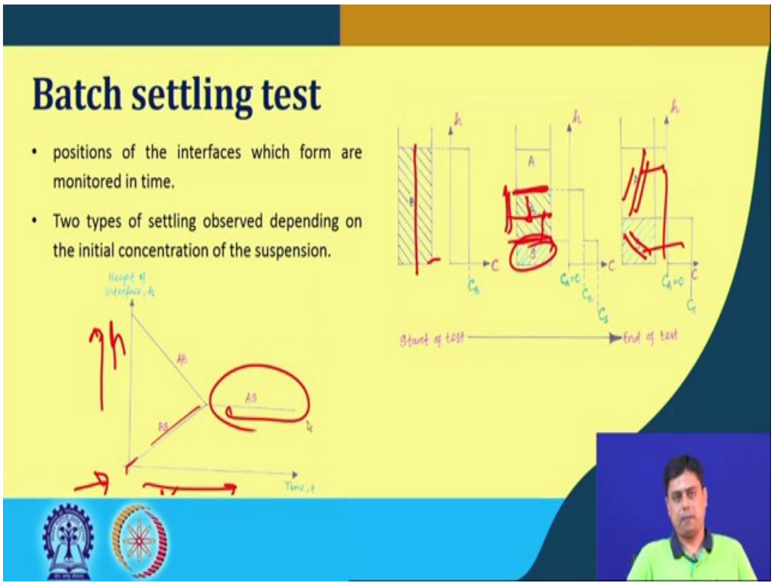

Now, in a batch settling test if you do that yourself or in a laboratory, that initially say you have a suspension of concentration C_B and the interface height or the height of this whole suspension is h from the bottom of the vessel. Say somehow you have mixed it properly the concentration of the solids in the that vessel or the cylinder cylindrical tube is this a uniform, you have made it somehow by vigorous mixing by shaking. And then you just keep it having it upward so, that the particles can settle down at that bottom of the tank or vessel or that measuring cylinder.

And if you can monitor the position of the interfaces in time you would see that there would be two types of settling phenomena. In one type what will happen that this kind of clear liquid zone that is zone A would be there will be a clear sedimented portion; that means, the particles that have settled their and there will be some intermediate portion that is of the concentration C_B that you initially started with. As you leave it with time ultimately there will be a clear liquid zone or the clear fluid zone and the sedimented zone where the concentration of the sedimentation is C_s and the concentration of particles in the A zone is 0.

Now, if you draw if you because you have monitored this interface height in time or the position of interfaces with time. If you plot that the plot would look like that is presented here that is if this is the height of interface and this is the time this is the height of interface h and this is time then AB basically stands for this interface height the weight is decreasing and this is the interface between B and S the way it would increase.

And eventually it would come to a position where we find the clear interface of the sedimented part and the clear part which is A S it would merge at a certain point of time and after that it would be as it is because that is the time when the clear sedimentation has happened or complete sedimentation has been taken place.

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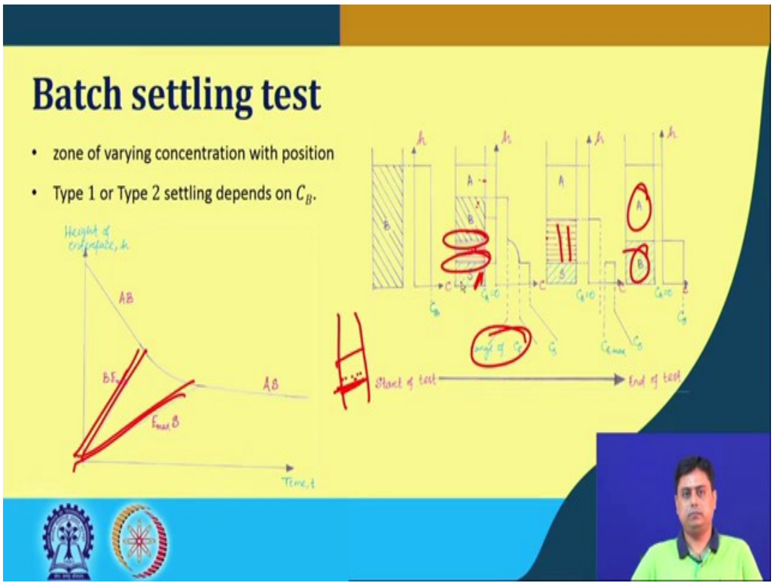

The other type is that say there will be the formation of a zone of varying concentration with position. So, which means this zone will be introduced or formed in the other type that you have a clear section, the section with the initial concentration that you started with a clear sedimented section in between there will be a portion of solids where the concentration will vary between the two extreme that is the concentration C_s and the concentration C_B and this will vary in position.

It will have a range of concentration in that range unlike in the previous case where this was of uniform cross section the whole section. After a certain time again this section B or the zone B would vanish and again it will be a section E where this concentration will vary with position and at the end of the test like the previous one there will be a section A or zone A and zone B where we started with that solution of that suspension. So, this two types of settling are typically observed and this phenomena actually depends on the initial concentration that you started with or the C_B. And in this case if you start with this height of this graph or these interfaces, you would see the similar kind of the pattern, but there is this B E mean.

That means the interface between the B this position and the maximum that could have been just at the end of this E section and at the beginning of S ok. So, this would typically happen if this rise is much faster; that means, this interface height rises much faster than the decreasing height of AB.

So, how it happens again? It depends on the initial concentration that you start with. Either you can land up in type two settling if we say this is the type two or the previous one. We will see this in details we will continue on this ideas in the next class and till then.

Thank you for your attention.