Fundamentals Of Particle And Fluid Solid Processing
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Lecture – 24
Fluidization (Contd.)

Hello everyone, welcome to another class of Fundamentals of Particle and Fluid Solid Processing. In the last class, we have seen that different characteristics of powders or fine particles and how they are classified based on their Fluidization property at ambient condition that is Geldart's part group A, B, C, D. And we have also seen in details, that what are the examples, what are the behaviors. Now we will see, we will dig into details on two types of fluidization, that is broadly categorized as non bubbling fluidization and bubbling fluidization.

But before going into that, let us have a quick revisit of the single particle terminal velocity in a settling condition; when both the fluids and the particles were settling at that downward velocity.

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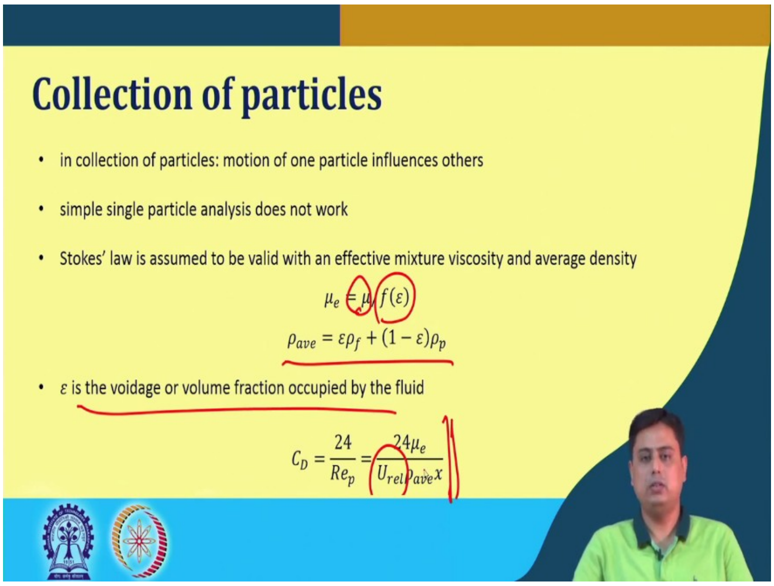

So, in case of collection of particles, because here in fluidization we are talking about several particles together not a single particles anymore; now in collection of particles this motion of one particle influences, the other particle ok.

So, this simple single particle analysis does not work in this case. So, Stokes law assumed to be valid with an effective mixture viscosity and average density; for this whole suspension or the solid particles including the fluid media. So, in this case when there are multiple particles; that means, the solid the fluid density is bound to change or the fluid viscosity, the property overall property will change; but remember at the beginning of the fluidization we mentioned that, we assume that the bed behaves as if a single media with a mixture density ok.

So, similar concept or the analogy if we draw here, that is to we assume in a collection of particle when that is settling, that follows Stokes law in the Stokes law regime; but with the effective mixture viscosity and average density. So, the mixture viscosity is defined by

$$\mu_e = \mu / f(\varepsilon)$$

ok. The volume fractions occupied by the fluid and the solid fractions is $(1-\varepsilon)$of that parameter.

So, the average density is then written in this form,

$$\rho_{ave} = \varepsilon \rho_f + (1-\varepsilon)\rho_p$$

where this ε is the voidage or the volume fraction occupied by the fluid. In Stokes law region we know this relation ok, where this U is the relative viscosity of the particle that is settling, if the fluid is also in motion fine.

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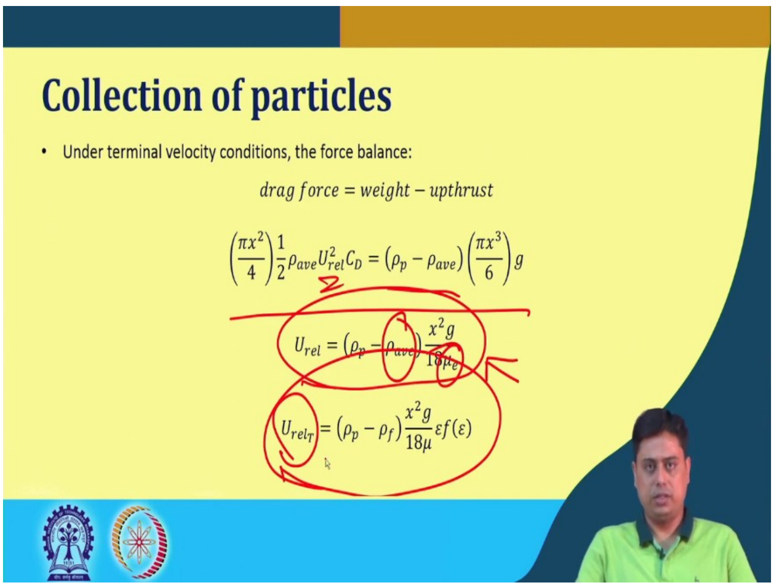

So, then under this terminal velocity condition on the force balance; the drag force is

$$drag\,force = weight - upthrust$$

this we have seen. So, here if we write the detailed expression, that the drag force on a single particle and it is apparent weight, we equate that with the relative velocity.

$$\left(\frac{\pi x^2}{4}\right)\frac{1}{2}\rho_{ave}U_{rel}^2 C_D = \left(\rho_p - \rho_{ave}\right)\left(\frac{\pi x^3}{6}\right)g$$

We find an expression for this relative velocity,

$$U_{rel} = \left(\rho_p - \rho_{ave}\right)\frac{x^2 g}{18\mu_e}$$

in terms of this ρ_p, the ρ_{ave} where ρ_{ave}e is the average density of the medium or of the whole system and there is another term which is the effective viscosity(μ_e).

Now these two expressions we have seen here. Now if we replace these two expressions here in this equation, we find a simplified relation in terms of the particle dimension and the voidage as well as the relative density of the fluid and the particle; where this is at the relative velocity at the terminal velocity condition.

$$U_{rel_T} = \left(\rho_p - \rho_f\right)\frac{x^2 g}{18\mu}\varepsilon f(\varepsilon)$$

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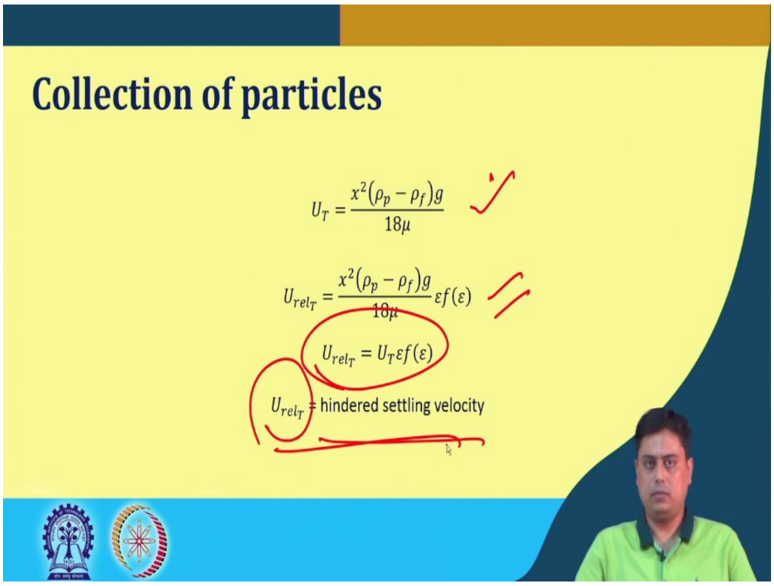

So, now if we look at this expression and compare this with for a single particle system, because this is the thing that we did for the multiple particles or the collection of particles; now these two expressions, this is for the single particle, this is for the multiple particles.

$$U_T = \frac{x^2(\rho_p - \rho_f)g}{18\mu}$$

$$U_{rel_T} = \frac{x^2(\rho_p - \rho_f)g}{18\mu}\varepsilon f(\varepsilon)$$

If we look at these two expressions and compare them, we find that this relative terminal velocity is basically a single particle terminal velocity multiplied by the void fraction ok, the fraction of liquid, multiplied by a function that is dependent on this voidage; where this is called the relative velocity at the terminal condition or specifically we can say this is the hindered settling velocity, because there are multiple particles in the medium.

$$U_{rel_T} = U_T \varepsilon f(\varepsilon)$$

$$U_{rel_T} = \text{hindered settling velocity}$$

So, considering their interaction, the inter particle interactions and their crossing of flow path and all the things. So, basically the presence of one particle will hinder the movement of the other particle and that is why this relative velocity is then called as the hindered settling velocity ok.

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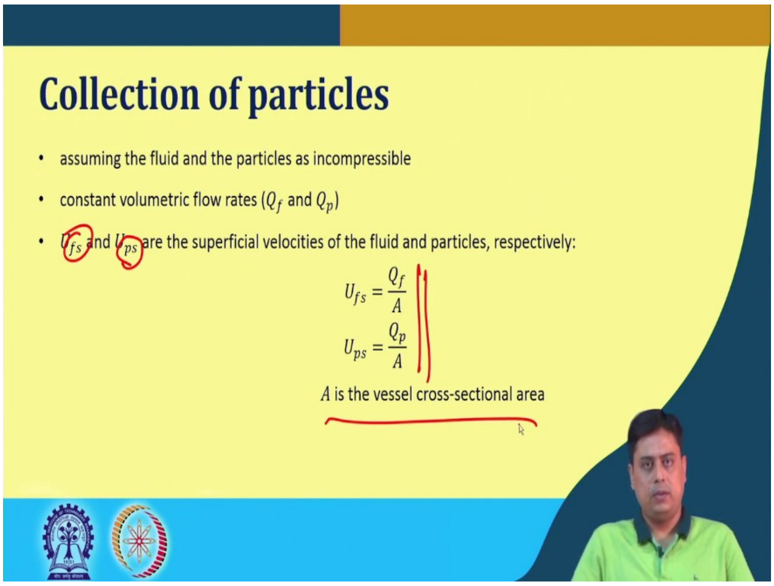

So, now the point is that, if we assume that the fluid and the particles are incompressible; which is logical as well in most of the cases, except for the gas phase as the fluid media. And if we assume that there is constant volumetric flow rate that is the flow rate of the fluid, f stands for the fluid, p stands for the particle ok.

And these f s, $U_{fs}$and U_{ps} these are the superficial fluid velocity and superficial particle velocity respectively. Then this U_{fs} and U_{ps}can simply be written in this form, this we know now,

$$U_{fs} = \frac{Q_f}{A}$$

$$U_{ps} = \frac{Q_p}{A}$$

where A is the cross sectional area of the vessel or of the container where this phenomena is happening.

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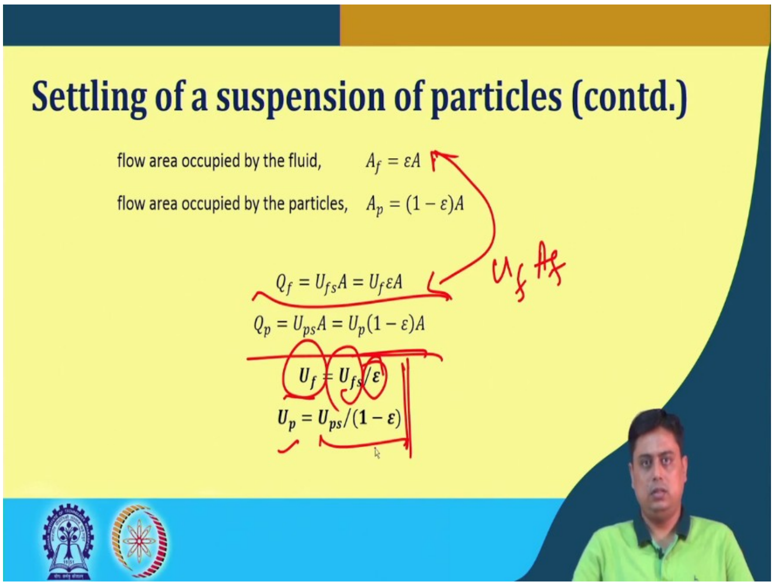

Then this flow area occupied by the fluid is basically A f which is

$$A_f = \varepsilon A$$

And flow area occupied by the particle is

$$A_p = (1-\varepsilon)A$$

And since the flow rate is same, by and assuming as I mentioned it is the incompressible fluid and the particle system we can write such continuity expressions that the superficial velocity, because the flow rate would be same.

Superficial velocity multiplied by the cross sectional area is equals to the actual velocity of the fluid which are

$$Q_f = U_{fs}A = U_f\varepsilon A$$

Basically, this is $U_f$multiplied by A_f; where A_f is εA. Similarly for the particle phase or the particle we can write

$$Q_p = U_{ps}A = U_p(1-\varepsilon)A$$

which means again we came back to these expressions which we already know, but still to recall these expressions that the actual velocity is basically the superficial velocity divided by the respective volume fraction.

$$U_f = U_{fs} / \varepsilon$$

$U_p = U_{ps} / (1 - \varepsilon)$

So, similarly the actual particle velocity is basically the superficial particle velocity, divided by it is fraction. So, what we have realized here is that, this relation that this terminal velocity and this factor how it appeared, ok.

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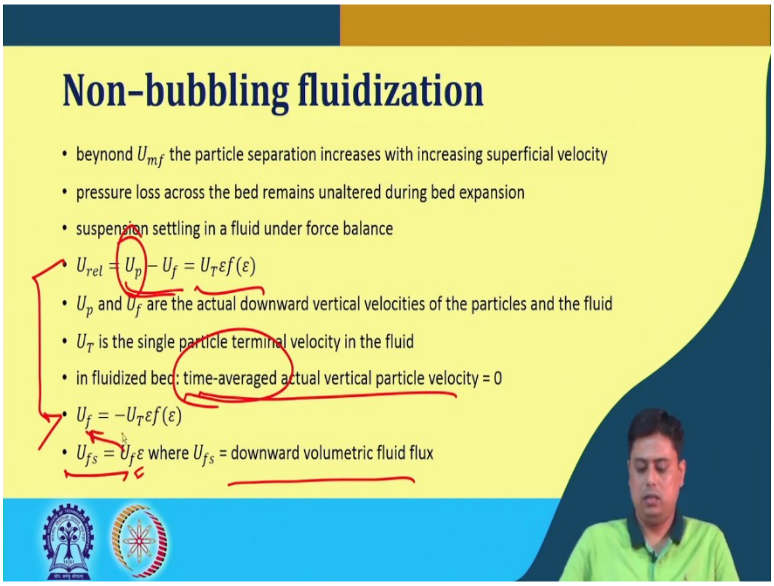

Now, if we carry on with this concept and we see for the fluidization, that in non bubbling fluidization beyond this minimum fluidizing velocity, the particle separation increases with increasing superficial velocity; but the pressure loss across the bed remains unaltered during this bed expansion till the bed expansion happens.

If we now try to relate this concept ok, with the previous one that I have discussed that is the particles are settling in a fluid media ok, that both are flowing downward. So, this relative velocity is basically,

$$U_{rel} = U_p - U_f = U_T \varepsilon f(\varepsilon)$$

fine. Now, in fluidized bed, the time averaged actual vertical particle velocity is 0; if we take the time average term, then this U_p is basically 0. And if we apply that concept, this expression is simplified to

$$U_f = -U_T \varepsilon f(\varepsilon)$$

Now again, we know that the superficial velocity or we can say that the downward volumetric fluid flux is basically actual velocity multiplied by the void fraction or the voidage

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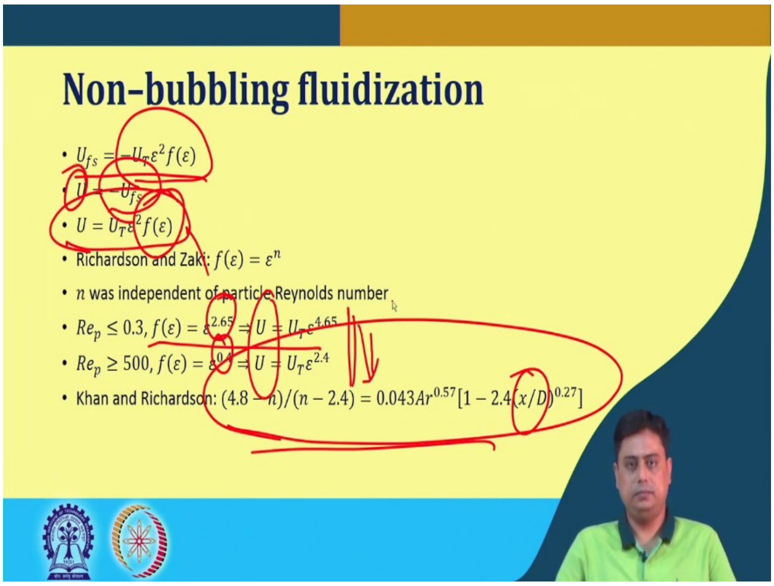

So, if we apply it here, this expression becomes superficial velocity or the volumetric downward liquid; this fluid flux becomes

$$U_{rel} = U_p - U_f = U_T \varepsilon f(\varepsilon)$$

$$U_{fs} = -U_T \varepsilon^2 f(\varepsilon)$$

Now this is the case for the downward movement or considering we are finding the analogy while settling the multiple particles in a fluidic media.

Now, this downward motion or negative of downward is basically the upward motion in fluidized case. So, this superficial velocity in fluidized bed is basically the negative of this downward motion, this downward volumetric fluid flux; in fluidized bed that is the opposite. So, which means in case of fluidized bed, the superficial velocity is

$$U = U_T \varepsilon^2 f(\varepsilon)$$

Now, we are repeatedly telling this term; Richardson and Zaki proposed a relation to find out this value, what is that factor that is applied on both the cases that is the hindered settling and the fluidization.

$$f(\varepsilon) = \varepsilon^n$$

And they found that this n was independent of particle Reynolds number, be it a low or the higher case; it was the value around 2.5. But for better accuracy, for the complete range of particle Reynolds number these relations; that is the when particle Reynolds number beyond 0.3 that is in the Stokes region this n value is 2.65.

$$\Re_p \leq 0.3,\ f(\varepsilon) = \varepsilon^{2.65} \Rightarrow U = U_T \varepsilon^{4.65}$$

And this for larger than 500 it is 0.4;

$$\Re_p \geq 500,\ f(\varepsilon) = \varepsilon^{0.4} \Rightarrow U = U_T \varepsilon^{2.4}$$

which means when we apply this relation here, we get the expression $U = U_T \varepsilon^{2.4}$ in case of particle Reynolds number is greater than 500. Khan and Richardson the other scientist, they proposed this relation for a wide range of Reynolds number, particle Reynolds number which can be applicable; that brings the introduction of the vessel diameter and the particle diameter.

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So, the influence of the equipment size now comes into play in the whole range, while calculating the value of n. So, the point is again similar to the other expressions that I mentioned, that there are expressions available or correlations available to find out these required parameters. So, once we have it then we can understand that with all these relations, we can find out what is the relation between epsilon and the fluidizing velocity in case of non bubbling fluidization; that is the relation between U and epsilon.

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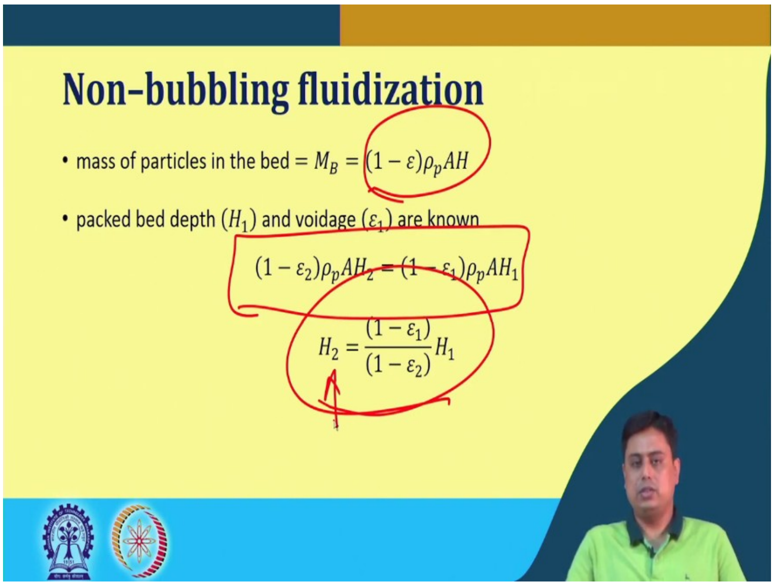

Once we have it, once we know that the voidage, then it is required to know that; what is the bed expansion height. It was initially of some epsilon, when the bed expansion starts ok. We know that pressure loss remains unaltered during this bed expansion period. And then what happens at this height, how much is the bed height becomes from a fixed state. So, if we say that the mass of particles in the bed is M_B that we can write by this expression

$$M_B = (1-\varepsilon)\rho_p AH$$

And if say at any time or even it is say at initial stage before the minimum fluidization velocity, we knew what is the packed bed height or the depth; that time if we have the information of ε_1 or the epsilon bed voidage value; then at any stage of let us say the stage 2, that height will be H_2 and the voidage will be ε_2. Now for that superficial velocity we already have that relation, that for that superficial velocity and epsilon we just saw in the last slide what is the relation, in case of non bubbling fluidization.

$$(1-\varepsilon_2)\rho_p A H_2 = (1-\varepsilon_1)\rho_p A H_1$$

So, from there we can find out what would be the ε_2, which means ε_1 and H_1 which is of the packed bed was known with a certain superficial velocity ε_2 is known, so what would be our H_2? Considering this mass remains same, for the bed weight remains same and no particles are escaping the equipment or the fluidized bed, by alliteration or some other means. If it is contained inside the equipment then; that means, the mass remains same, whatever it was during the packed bed stage of the particles; by then we can have the height H_2 at the required superficial velocity in case of non-bubbling fluidization.

$$H_2 = \frac{(1-\varepsilon_1)}{(1-\varepsilon_2)} H_1$$

This is how we can calculate the bed expansion height in case of non bubbling fluidization by these relations and for that again these relations are important. And similar to these expressions there are other relations available, several researchers have proposed these relations in a different way these are the correlations.

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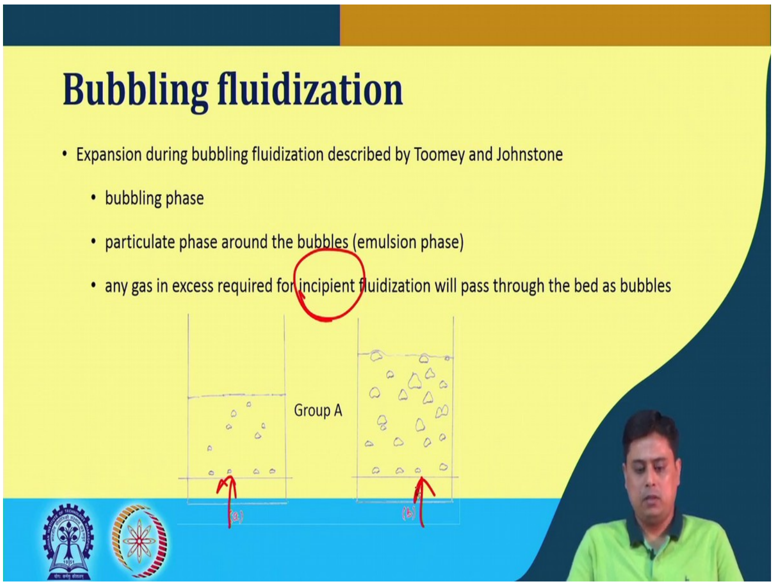

Now, in case of Bubbling fluidization, we see that the expansion during bubbling fluidization has been best described by some simple theory and the researchers they proposed this are Toomey and Johnston. They proposed, that basically in bubbling fluidization we have two phases; one is the bubbling phase, the other is the particulate phase. Bubbling phase or in other words it is the gaseous phase mostly and the particulate phase around that bubbles which is also called the emulsion phase.

Their simple theory is that any gas that is provided in excess which is actually required for the incipient fluidization will pass through as bubbles through the bed. What is incipient fluidization? It is the beginning of fluidization or the minimum fluidization; that is also called in other way is the incipient fluidization, just the onset of fluidization. So, anything that you supply in excess of what is required for this fluidization to happen, that passes through the bed as bubbles.

Now, this is one of the simple illustration schematic illustration, for group Geldart's group A powders or the particles when air is flown through this stack of particles; this is what happens exactly at just above the minimum fluidization velocity. And this case when it is sufficiently higher than that, more bubbles are passing through as more gas phase actually passing through the bed as bubbles.

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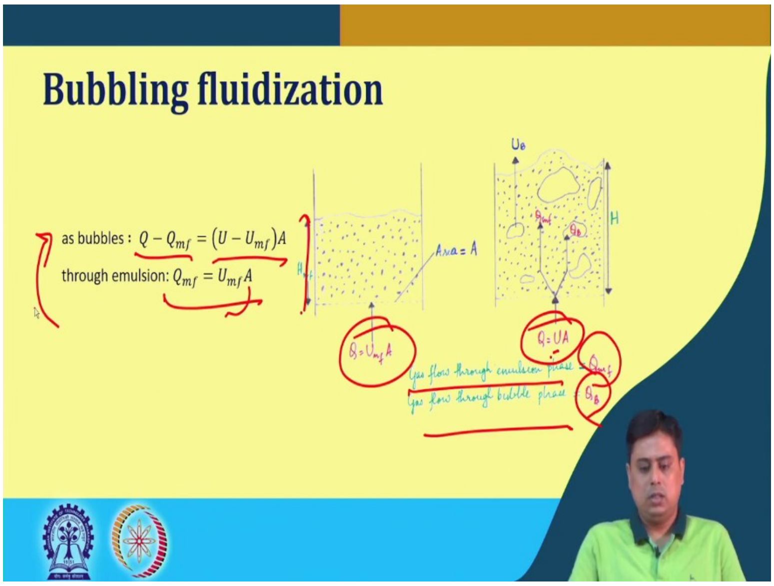

Now, considering this scenario, this is what let us say was required this Q flow rate or let us say the mf is the minimum fluidization velocity we know, U_{mf}. Now, if this is the minimum fluidization velocity that is required for this bed of H_{mf} of cross sectional area A and we actually supply something else; that is here as U multiplied by A, which is actual velocity or actually given flow superficial velocity ok. So, the gas flow through the emulsion phase ok, the gas flow through the emulsion phase is basically this $Q_{mf}$and the gas flow through the bubble or as bubble is Q_B. Which means, as bubbles the gas flow is

$$Q-Q_{mf}=\left(U-U_{mf}\right)A$$

Q_{mf} is what actually going through the emulsion phase and Q is what actually we are supplying for this fluidization, which is higher than the minimum fluidization flow rate, ok.

Then we can write this expression, where Q_{mf}that is passing through the emulsion is

$$Q_{mf}=U_{mf}A$$

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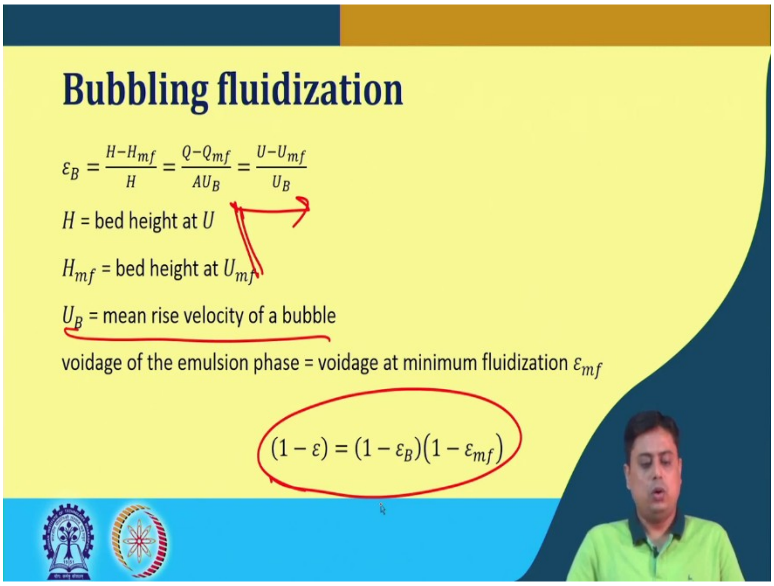

So, on this simple theory we can find out that what is the bubble fraction in the bed which is ε_B, in terms of this ratio of height differences. Initial height was let us say the bed height at velocity is U and H_{mf} is bed height at the minimum fluidization velocity.

$$\varepsilon_B = \frac{H-H_{mf}}{H} = \frac{Q-Q_{mf}}{AU_B} = \frac{U-U_{mf}}{U_B}$$

And considering the fact that the voidage of the emulsion phase is actually the voidage of minimum fluidization; then we can write

$$(1-\varepsilon) = (1-\varepsilon_B)(1-\varepsilon_{mf})$$

which is of solid concentrate or the solid fractions, ok.

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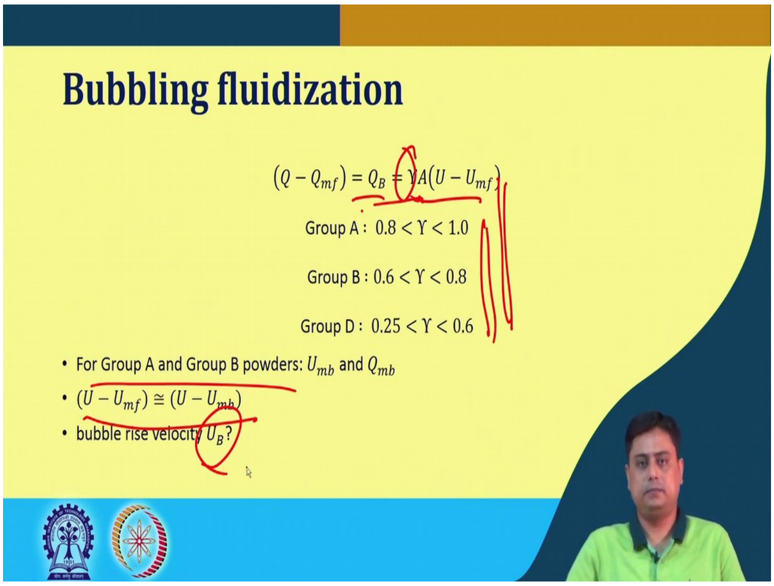

So, then this

$$(Q - Q_{mf}) = Q_B = Y A (U - U_{mf})$$

where this factor depending on the type of the particle has different values.

So, by doing this what we can have, is that this Q_B and this void fraction; but still we require the knowledge of U_B here, to have this volume fraction of the bubbles or the fraction of the bubbles. So, in this expressions, this is the generic form because for A and B this mf or the minimum fluidization is basically the minimum bubbling velocity. So, for particles type A and B this U_{mf} are typically or has to be replaced by U_{mb} which is the minimum fluidization velocity and the Q is the flow rate at that minimum bubbling state.

But it hardly matters, because most of the time this minimum bubbling velocity and minimum fluidization velocities are basically very low than this actual velocity or the actual operating velocity. So, which brings to the such kind of a relation, that this difference is basically or typically numerically equal.

$$(U - U_{mf}) \cong (U - U_{mb})$$

But then still, we need the information of this U_B the bubble rise velocity; how that is this is determined?

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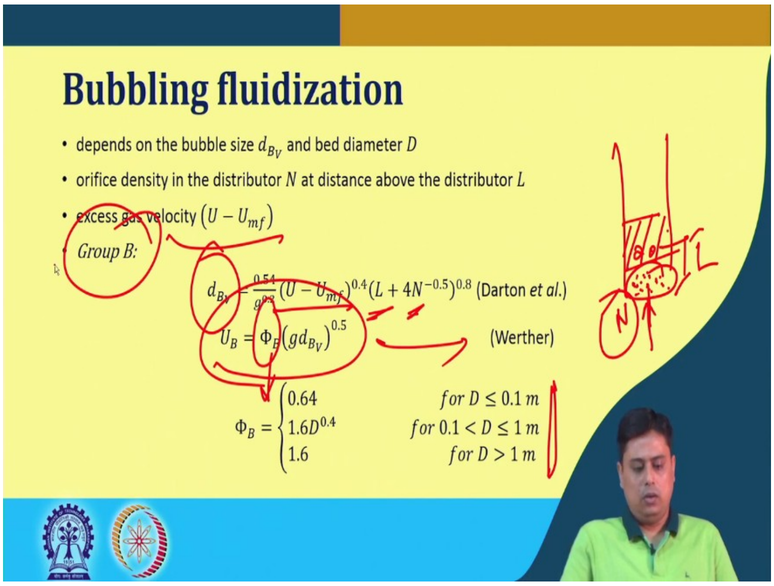

Again this bubble rise velocity depends on the bubble size and the bed diameter. It also depends on the gas distributor orifice density, that is the N and at distance the L where this value is measured. That means there is a Sparger, we say this is the distributor through which the fluid comes; if this is the stack of liquid solid particles, through this distributor the fluid phase is being distributed. So, it creates the bubble, now this bubble at which distance we are measuring if that is L and this density number density of the orifice if that is N, this information are required for calculating the bubble size and as well as the excess gas velocity, how much excess in excess we are flowing.

$$d_{B_v} = \frac{0.54}{g^{0.2}}(U - U_{mf})^{0.4}\left(L + 4N^{-0.5}\right)^{0.8} \text{ (Darton } et\ al.\text{)}$$

And then for different group of particle there are several correlations available or expressions available to find out the diameter of that bubble. If you look at it you see that depends on this excess velocity, it depends on the length and the orifice density. And then once we get that, we can calculate the bubble rise velocity by another expressions proposed by Werther.

$$U_B = \Phi_B \left(g\, d_{B_v}\right)^{0.5}$$

Where this Φ_B constant is having this value depending on the diameter of the bed or the vessel where the fluidization happen, this is for particularly group B type of particles.

$$\Phi_B = \begin{cases} 0.64 \; for \; D \le 0.1\, m \\ 1.6\, D^{0.4} \; for \; 0.1 < D \le 1\, m \\ 1.6 \; for \; D > 1\, m \end{cases}$$

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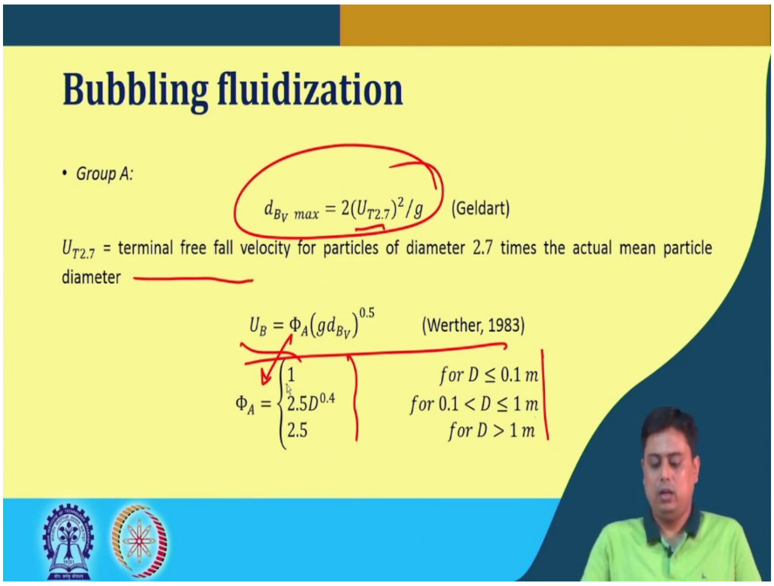

For group A type of particles this is proposed by Geldart, where we can find because this is the case where we find actually the maximum diameter of the bubble that is stable. This we discussed in the earlier class, that group A particle we have a window of non bubbling regime as well as the bubbling regime and there you can have the maximum stable size of the bubble.

$$d_{B_v max} = 2(U_{T2.7})^2/g \qquad \text{(Geldart)}$$

So, here this $U_{T2.7}$ is the terminal free fall velocity of a particle that has a diameter 2.7 times of the actual mean particle diameter. Once we have that again we use this expression, but here the factor changes with the diameters.

$$U_B = \Phi_A \left(g\, d_{B_v}\right)^{0.5} \qquad \text{(Werther, 1983)s}$$

$$\Phi_A = \begin{cases} 1 \; for \; D \le 0.1\, m \\ 2.5\, D^{0.4} \; for \; 0.1 < D \le 1\, m \\ 2.5 \; for \; D > 1\, m \end{cases}$$

So, which means we can calculate this expressions of having the bed expansion height. And in case of bubbling fluidization we can have this different fluidization up rise velocity of the bubbles and from that we can find out what is our these parameters and the bed expansion, the bubbling fraction or the bubble fraction of the bed. So, with these concepts, I will stop here up in this class and in the next class we will see couple of workout problems to clear these concepts in a more better way. And we will cover a certain theoretical topic on gas solid and the gas liquid fluidization in a detailed way; till then thank you for your kind attention.