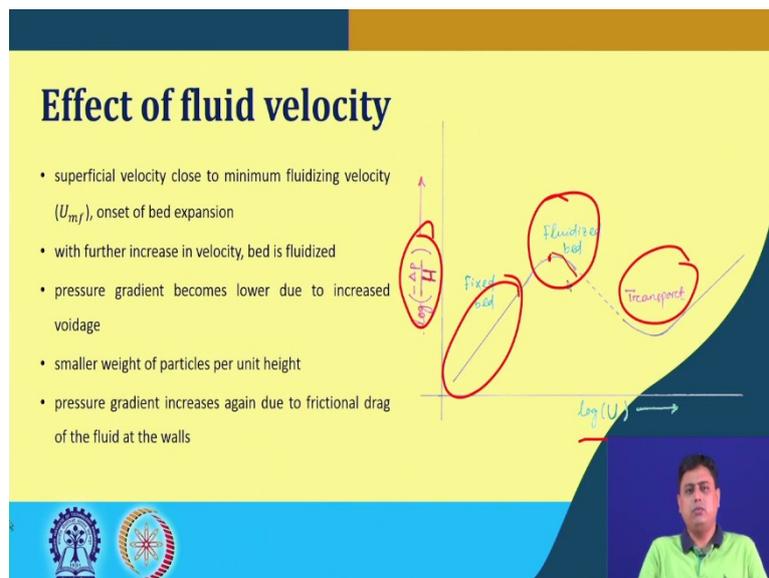


Fundamentals Of Particle And Fluid Solid Processing
Prof. Arnab Atta
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 22
Fluidization (Contd.)

Hello everyone, welcome back to another class of Fundamentals of Particles and Fluid Solid Processing. In the last class, we started with the concept of Fluidization. Today, we will go into the details of estimating minimum fluidization velocity. We have already understood that, what is minimum fluidization velocity, what is bed expansion and how it varies with the or when there is of gas velocity or the fluid velocity, how the pressure inside the bed varies from a fixed bed to the fluidized bed. During this expansion or this transition, how it this parameter varies that we will see today.

(Refer Slide Time: 01:15)



So, to start with if we have sufficient fluid superficial velocity ok, when it goes to the minimum fluidization velocity that is the U_{mf} we designate here mf stands for the minimum fluidization velocity. Then, the onset of bed expansion occurs, which means the bed starts to expand and it goes till a point when there is the maximum voidage that is possible for that kind of packing. So, with further increase in the velocity we have the bed as fluidized.

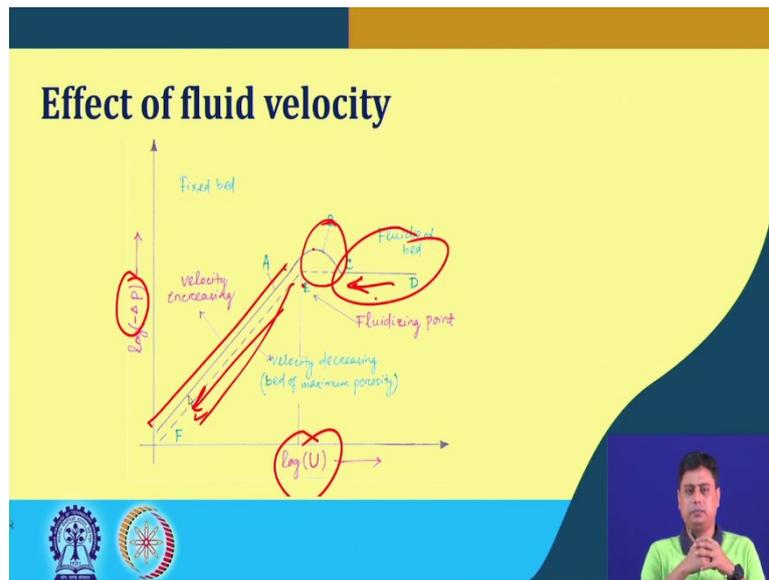
So, now if you consider about the scenario that we have seen for the fixed bed case, that we had velocity superficial velocity we have a packed bed of particles. And then we increase the velocity, we saw that till a point there is this variation this is the linear variation ok. In case of laminar flow, that is the first part of the Ergun equation.

Then, if we start continuing increasing that superficial velocity, what will happen that we have repeatedly have heard and discussed that the bed will now start to expand. The voidage will start to increase ok. And until the point it reaches it is apparent weight that is just balanced by this upward thrust, that till that point the bed expands and the pressure does not change with that much ok, it remains some kind of a constant value.

So, here this parameter is the pressure gradient that is mentioned here against U . So, it reaches a maximum pressure value ok. Then actually if we look into details, then we will see that it drops, because of this increase in porosity or voidage in the bed. It slightly drops and then again it increases ok, because this drag force now becomes dominant after a certain period or period of this superficial flow rate ok. So, these pressure gradient increases due to the frictional drag of the fluid on the walls, when that is transporting this solid particles ok. So, we have a movement ok, were the whole bed is fluidized and then we have the transport region were now the particles are being carried with the fluid or the fluidizing fluid.

In between that some period of time the pressure gradient basically decreases, the pressure drop per unit length that decreases and that decrease happen as I said due to this increase in the voidage. The particles or the bed it have now a smaller weight of the particles per unit height of the bed. This is the balance between that we have discussed balance between the apparent weight and the pressure gradient. So, this is what basically happens during the fluidization. What happens with the pressure drop instead of pressure gradient which is the pressure drop per unit length.

(Refer Slide Time: 05:27)



If we look into that we see such profile that we have already discussed during a problem. So, what happens, that if you now simply plot ΔP versus U , you get a linear profile you get a point ok. Till this end of this point A, line A you have the minimum fluidization velocity that is the onset of fluidization. After that for a certain period the bed expands ok. And as the voidage increases ok, pressure drop slightly decreases and then it remains unaltered.

For a period until the transport happens ok. So, then if you come back on the same line like we went from this point to the end of point A by increasing velocity, further increment in velocity leads to point B, further increases in velocity we have a point C, after that the bed expansion is completed ok. And, the pressure drop remains unaltered, till the point let us say D. Now, from this stage ok. If, you now decrease the superficial velocity or the fluidizing fluid velocity, it would follow this path D to C ok, then to E instead of going to through this B path. And, it follows this E F path.

So, when we have the highest voidage possible from there if we decrease the superficial velocity it would follow a same linear path, but not identical path while increasing the velocity. There are several studies, which relates or finds expression between the start up and this shut down procedure. If I say in a start-up operation and it is closing down or the deaeration process.

So, this is the characteristics of fluidization or the fluidized bed. So, we have the pressure drop and velocity and in the previous slide, I showed you the pressure gradient versus the

velocity. The reason I hope is now clear to you that from a fixed bed as we increases the superficial velocity of the fluidizing fluid, what happens the bed starts to disengage the expansion happens, reorientation of the particles happens.

It finds it is maximum porosity with those particles by reorienting themselves. So, that the flow resistance inside them becomes lesser and lesser and the minimum value ok. And then the transport region occurs after a certain period, until and unless the it cannot withstand that much pressure gradient the whole apparent weight of the bed cannot withstand further, the pressure gradient of the bed. The transport happens an as the transport happens we all know that as a suspension flows or such kind of flow happens with increasing velocity again the pressure gradient increases. Due to the drag exerted on the wall fine.

(Refer Slide Time: 09:49)

The slide is titled "Effect of fluid velocity" and contains the following bullet points:

- U_{mf} can be determined experimentally by increasing and decreasing velocities
- plotting pressure drop vs. velocity results as shown
- intersection of two straight lines as the minimum fluidizing velocity
- linear plots are popular than logarithmic plots
- logarithmic plots for nonlinear pressure gradient against velocity in the fixed bed

The slide also features a video inset of a person in the bottom right corner and logos of institutions in the bottom left corner.

So, this U_{mf} or the minimum fluidization velocity can be determined experimentally by increasing and decreasing velocities these two lines and plotting this pressure drop versus velocity results as we have seen ok. And practically when we get the intersection of these two straight lines or the best fit between this experimental points, while increasing and while decreasing, the superficial velocity we get this two pressure drop lines.

And when we plot those or find the best fit linear fit and we have the intersection between the 2 lines that is detected as the minimum fluidizing velocity. So, typically linear plots are popular than the logarithmic plots, but logarithmic plots are important or becomes unavoidable, when there is a non-linear pressure gradient against velocity in the fixed bed.

And when that happens you know that, when that flow region shifts from laminar to the transition or the turbulent region.

Then, the pressure drop does not vary linearly with the superficial velocity, then to have this straight line intersection you must plot those on the logarithmic plots. So, this is the effect or the influence of fluid velocity on the packed bed to the fluidization or during fluidization the influence of fluid velocity.

(Refer Slide Time: 11:47)

Minimum fluidization velocity

- Pressure drop = $\frac{\text{weight of particles} - \text{upthrust on particle}}{\text{bed cross-sectional area}}$
- $\Delta P = \frac{HA(1-\varepsilon)(\rho_p - \rho_f)g}{A}$
- $\Delta P = H(1-\varepsilon)(\rho_p - \rho_f)g$
- $\left(\frac{-\Delta P}{H}\right) = 150 \frac{(1-\varepsilon)^2 \mu U}{\varepsilon^3 x_{sv}^2} + 1.75 \frac{(1-\varepsilon) \rho_f U^2}{\varepsilon^3 x_{sv}}$
- $(1-\varepsilon)(\rho_p - \rho_f)g = 150 \frac{(1-\varepsilon)^2 \mu U_{mf}}{\varepsilon^3 x_{sv}^2} + 1.75 \frac{(1-\varepsilon) \rho_f U_{mf}^2}{\varepsilon^3 x_{sv}}$

So, how do we estimate in other way this minimum fluidization velocity? Typically, in fluidization when we say the pressure drop, it is basically the weight of particles minus the up thrust on the particle ok, by the bed cross sectional area ok. This is the balance we have already done. If you remember this balance we have already done, that this is a pressure drop of the bed or the pressure gradient that happens in the bed balanced by the apparent weight of the bed.

And, this is the apparent weight, solid fractions relative density multiplied by the g fine. Now, if you remember again or recall the Ergun equation ok, we can write this expression, because if you think of it is kind of a similar manner that the flow is happening upward in a packed bed until the minimum fluidization velocity.

$$(1-\varepsilon)(\rho_p - \rho_f)g = 150 \frac{(1-\varepsilon)^2}{\varepsilon^3} \left(\frac{\mu^2}{\rho_f x_{sv}^3} \right) \left(\frac{U_{mf} x_{sv} \rho_f}{\mu} \right) + 1.75 \frac{(1-\varepsilon)}{\varepsilon^3} \left(\frac{\mu^2}{\rho_f x_{sv}^3} \right) \left(\frac{U_{mf}^2 x_{sv}^2 \rho_f^2}{\mu^2} \right)$$

This relation should give you an estimate if not the accurate answer for the pressure gradient because until that minimum fluidization velocity the bed is not fluidized.

So, you can think of that is we are having a fixed bed structure, but with the maximum porosity. So, the point is that if we use Ergun equation until the point of this minimum fluidization velocity, we can replace then this ΔP expression in fluidized bed at that fluidizing state here in place of this ΔP ok. So, we have this relation we further simplify this relation ok, because there are so, many terms or so, many parameters involved. So, it is wiser to convert it to some dimensionless number, which will essential involve this parameters, but in a unified way ok.

(Refer Slide Time: 14:31)

Minimum fluidization velocity

- $(1 - \varepsilon)(\rho_p - \rho_f)g = 150 \frac{(1-\varepsilon)^2}{\varepsilon^3} \left(\frac{\mu^2}{\rho_f x_{sv}^3} \right) \left(\frac{U_{mf} x_{sv} \rho_f}{\mu} \right) + 1.75 \frac{(1-\varepsilon)}{\varepsilon^3} \left(\frac{\mu^2}{\rho_f x_{sv}^3} \right) \left(\frac{U_{mf}^2 x_{sv}^2 \rho_f^2}{\mu^2} \right)$
- $(1 - \varepsilon)(\rho_p - \rho_f)g \left(\frac{\rho_f x_{sv}^3}{\mu^2} \right) = 150 \frac{(1-\varepsilon)^2}{\varepsilon^3} Re_{mf} + 1.75 \frac{(1-\varepsilon)}{\varepsilon^3} Re_{mf}^2$
- $Ar = 150 \frac{(1-\varepsilon)^2}{\varepsilon^3} Re_{mf} + 1.75 \frac{1}{\varepsilon^3} Re_{mf}^2$
- $Ar = \frac{\rho_f (\rho_p - \rho_f) g x_{sv}^3}{\mu^2}$

So, this is further rearranged by this manner, that we deliberately multiply or try to find out the very common flow dimensionless number is the Reynolds number. So, we try to write those expression in this form in each of the term the first part and the second part. So, for that whatever the parameters we have multiplied here we divide that ok. So, to have this mu at the denominator we have multiplied that with μ at the numerator as well. So, basically we again rearrange this expression to find out or to express the relation in non-dimensional form or non-dimensional number ok.

So, if you do this ok. And, this is nothing, but the particle Reynolds number at the minimum fluidization velocity or minimum fluidizing velocity; this is the Reynolds numbers square ok. So, this parameter is here 1 by this parameter. So, the right side is basically simplified in this

way that this is 150 again at this stage this 1 minus this thing goes out $(1-\epsilon)$. So, you basically have

$$150 \frac{(1-\epsilon)}{\epsilon^3} \mathfrak{R}_{mf} + 1.75 \frac{1}{\epsilon^3} \mathfrak{R}_{mf}^2$$

And this parameter we have already seen earlier, it is another dimensional less number called the Archimedes number ok, or if we clearly write that this has this expression.

So, which means within the Archimedes number and the Reynolds number we have a relation that involves the minimum fluidization velocity. The Reynolds number expression is having the minimum fluidization velocity into it and the Archimedes number deals with the fluid properties. So, this from this relation

$$Ar = 150 \frac{(1-\epsilon)}{\epsilon^3} \mathfrak{R}_{mf} + 1.75 \frac{1}{\epsilon^3} \mathfrak{R}_{mf}^2$$

we have on one hand this is the relation between the Archimedes number and Reynolds number at minimum fluidization velocity.

(Refer Slide Time: 17:57)

Minimum fluidization velocity

- $\epsilon_{mf} = 0.4$
- $Ar = 1406 Re_{mf} + 27.3 Re_{mf}^2$
- $Ar = 1652 Re_{mf} + 24.51 Re_{mf}^2$ (Wen and Yu)
- $Re_{mf} = 33.7[(1 + 3.59 \times 10^{-5} Ar)^{0.5} - 1]$
- spheres and $0.01 < Re_{mf} < 1000$
- for particles larger than $100 \mu\text{m}$

This, now here at this case at this minimum fluidization velocity, typically the maximum voidage in such case by for the case of spherical particle ok, this maximum porosity that in the minimum fluidization velocity has been seen to be of 0.4. So, we if we replace this value

in this expression, so then we can find all this expression are simplified and we get Archimedes number and Reynolds number at this minimum fluidization velocity with this coefficients.

$$Ar = 1406 \Re_{mf} + 27.3 \Re_{mf}^2$$

For spherical particle and wen and yu has had proposed similar correlations that feeds a wide range of particle ok, or wide range of sizes it covers spheres and this much of Reynolds number or the particle Reynolds number for sizes larger than $100 \mu m$.

$$Ar = 1652 \Re_{mf} + 24.51 \Re_{mf}^2 \quad (\text{Wen and Yu})$$

$$\Re_{mf} = 33.7 \left[\left(1 + 3.59 \times 10^{-5} Ar \right)^{0.5} - 1 \right]$$

So, which means once we have found out the relation between this Archimedes number and the Reynolds number at the minimum fluidization velocity. Based on this all other parameter here, because here usually all this parameters will be known to you, because you are handling the fluid you have chosen that fluid the particles are also known the size mean size ok. And here these parameters are also know except the U_{mf} .

And this minimum U that would be required to have this balance between the Archimedes number and Reynolds number will give you the minimum fluidization velocity. And there are several such relations in the literature ok, which helps for a range of particles or a for a range of different sized particles to have or to calculate the minimum fluidization velocity. And, this relation as it is shown here had some restriction to you or in other way it can be used safely, if these criteria's are made. So, I hope the estimation of minimum fluidization velocity is clear to you.

(Refer Slide Time: 21:05)

Particle and powder properties

- for particles less than 100 μm : $U_{mf} = \frac{(\rho_p - \rho_f)^{0.934} g^{0.934} \mu_p^{1.8}}{1110 \mu^{0.87} \rho_f^{0.066}}$
- particle density (ρ_p) = $\frac{\text{mass of a particle}}{\text{hydrodynamic volume of particle}}$
- absolute density = $\frac{\text{mass of a particle}}{\text{volume of solids material making up the particle}}$
- bed density (ρ_B) = $\frac{\text{mass of particles in bed}}{\text{volume occupied by particles and voids between them}}$
- $\rho_B = (1 - \epsilon)\rho_p$
- bulk density for powders




The other relevant property, that we look or that we consider during the fluidization ok, for particles less than 100 μm , like in the previous slide we have seen the particles of more than 100 μm , we had relations for that. Then other researchers have also come up with the U_{mf} expression for particles less than 100 μm . So, which means we have expressions, because this fluidization bed has been a area of interest or research for a long time for several decades ok. So, there are sufficient amount of information of this minimum fluidization velocity for any kind or any shape, any size, of particles. Whenever that is required we should choose the appropriate 1 for our design purpose. Now, as I said the other parameters that are relevant or are typically we must know or we should be dealing with during the fluidization are the particle density.

Now, in fluidization particle density is basically not the absolute density. There are difference between the particle density and the absolute density of the particle. The particle the absolute density basically deals with the density of the solid material by which that particle is made of, but in fluidization particle density is

$$\text{particle density}(\rho_p) = \frac{\text{mass of a particle}}{\text{hydrodynamic volume of particle}}$$

Now, what is an hydrodynamic volume? Hydrodynamic volume is the volume that is seen by the fluid or inside the fluid ok, including it is pore or I mean with pore voidage everything. So, if this is the solid particles having pores ok. And this is the only solid material.

So, this dotted surface the whole dotted surface is basically the hydrodynamic volume including its pores, but the absolute density is

$$\text{absolute density} = \frac{\text{mass of a particle}}{\text{volume of solids material making up the particle}}$$

We have other term which is called the bed density. The bed density is defined as

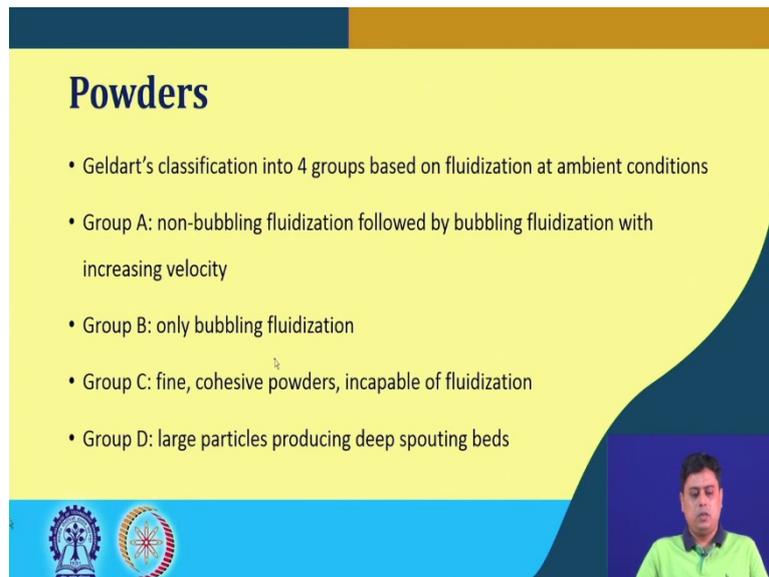
$$\text{bed density } (\rho_B) = \frac{\text{mass of particles} \in \text{bed}}{\text{volume occupied by particles} \wedge \text{voids between them}}$$

Here, we have mass of particles in the bed divided by the volume occupied by the particles and voids in between them, which means we have a relation between the particle density and the bed density, which is that ρ_B the bed density is equal to solid fractions multiplied by the particle density.

This bed density for these particles or the particulate solids also similarly refers to as the bulk density for powder, for powdery materials, for very fine material. So, we will be frequently dealing or calling these names from now onward, that in fluidization, what is the particle density, what is the bed density, which is the equivalent for the powders as bulk density?

So, these are the relevant parameters ok, that we should remember and these are easy to remember. And what are the relation between the particle density and the bed density, if you remember these expressions you can always derive that or if you remember these expressions, you can remember what are the expressions for that.

(Refer Slide Time: 25:51)



Powders

- Geldart's classification into 4 groups based on fluidization at ambient conditions
- Group A: non-bubbling fluidization followed by bubbling fluidization with increasing velocity
- Group B: only bubbling fluidization
- Group C: fine, cohesive powders, incapable of fluidization
- Group D: large particles producing deep spouting beds

Now, as I said it is similar to the powder or the bulk density for powders or for the fine material. Because, the point is that you should understand or you should remember that the powders will have will be very compact when it is packed, it would not be of similar behaviour when there are granular material or let us say the material of considerably bigger size. For example, the behaviour of flowers or weeds is different than the behaviour of gravels or even this the building sands ok.

So, in that next class we will be seeing this characteristics of powder in fluidization or how the powdery material, how the powdery state, actually influence the fluidization characteristic or how this characteristic when there are powder inside a bed and the how the fluidization is influenced? So, these things we will be elaborating in the next class in details.

Till then thank you for your attention.