Flow through Porous Media
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Lecture - 58
Heat Transfer with Fluid Flow (Contd.)

I welcome you to this lecture of Flow Through Porous Media, what do you what we are discussing is Heat Transfer during Fluid Flow.

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We showed that there is a progression of heat transfer front along with the flow front of obviously, the heat transfer front moves at a much lower rate than the actual fluid flow front. And, there could a major assumption there could be a thermal equilibrium which is very effective and simplify the problem a lot. But, if we do not want to consider thermal equilibrium then probably the next step would be volume averaging that we are discussing in the last lecture and we will continue; we will continue to build on that.

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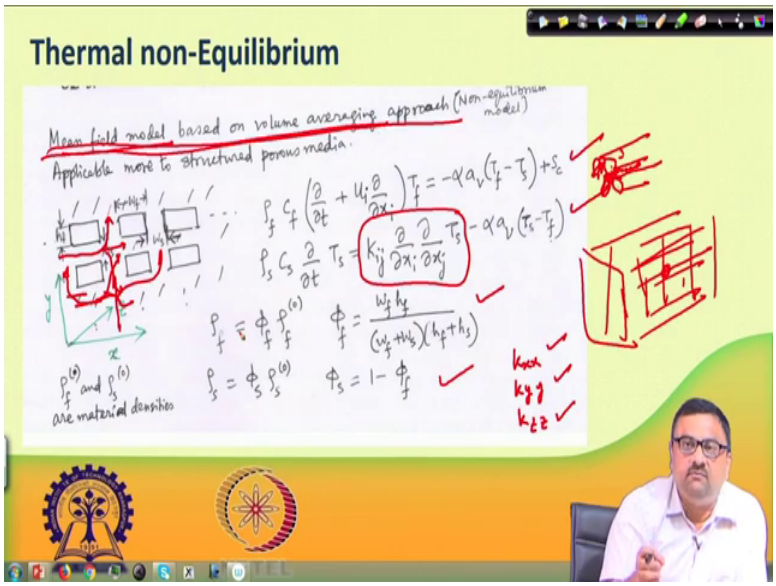

What we discussed in the last lecture was something called a mean field model on, mean field model based on volume averaging approach which is a non-equilibrium model. Mean field model why you are calling it mean field model? Because see we are talking about two temperatures T f and T s, but still we are not considering the temperature it is not a rigorous treatment by that what I mean is let us say I have pores, I have a capillary here, I have another capillary there in between I have some solid phase existing and then another capillary here, another capillary there and in between solid phase.

So, now, if we are supposed to make obtain the temperature profile we expect there is a temperature of gradient within this capillary itself within the fluid phase itself. And, then outside this capillary if we go into the solid phase within that also there would be a gradient in a temperature. We are not taking that level of details in the model instead, we are again using that same brute force treatment we are treating these; let us say this is the porous medium and if we break it into thousand such grids.

So, then these on these grids every grid what we are doing is on every grid we are assigning one T f and one T s two temperatures.

Earlier we had only one temperature instead at every we are still treating this porous medium as homogeneous, still treating this that is as if there is a continuum existing only we are talking about two temperature fields. One is fluid temperature field another is solid temperature field, but they are again treated as within the capillary would be

changing and then as you go into solid phase there also temperature continue to change again you go to another capillary there also temperature is going to change.

So, these discrete changes this is not we are not considering, we are treating two homogeneous temperature fields namely temperature for the fluid and temperature to solid and then within that there are exchanges. So, that is why it is called mean field model. So, mean field model based on volume averaging.

So, now this is the already we said that these are these are the two equations we have, one for the fluid phase another is for the solid phase and we have defined porosity for a structured system here and based on these. So, you can even see here for example, the solid part we can recognize this is arising from thermal conductivity, we have talked about already about this tensor right.

So, here this thermal conductivity is assumed to be, the thermal conductivity is a tensor and then if we are talking about an one dimensional; talking about one dimensional heat flow then it would be simply k xx del square t del x square del square T del square T s del x square, it is not a single temperature field. Earlier we had only temperature field. So, that is why we had k we started with you remember solid was represent represented by rock.

So, we had k r del square t del x square here it will be k xx the thermal conductivity in x direction del square T del x square; similarly thermal conductivity in y direction when we are talking about when it is xi xj right. So, we would be we would be breaking it for simplification we would be talking about when we have a porous medium structure in this way we will be talking about k xx, k yy and k z z.

So, we will have thermal conductivity in three directions. Now, how would you place thermal conductivity in three directions? Earlier we had talked about thermal conductivity, we said depends on porosity. It could be porosity multiplied by the thermal conductivity of fluid plus solid fraction multiplied by the thermal conductivity of solid assuming parallel model, we have talked about this in earlier lecture.

So, there we have put everything on the porosity, here we are going a little bit deeper in the sense that if we have a structure something like this how would you find out k. Because, it is there we are assuming that as if we have an isotropic k right in the if when

we talked about this parallel model there we are assuming that thermal conductivity in all directions they are same.

But, if you have a structured model like this then in the x direction you have a thermal conductivity, y direction you have a thermal conductivity, z direction you know. But, you must note the thermal conductivity when it comes to that how would the heat flow through this system say let us say this is the solid part, this is the solid part. So, heat can flow like this then heat can go like this it can go like this, it can come like this heat can go like this heat can.

So, heat can mean flow like this and heat as if it can be it is flowing through a network which is different if you are heat is flowing in the x direction as against when the heat is flowing in y direction. So, heat is coming like this, heat can bend like this. So, so it can go like this. So, as if it is flowing in x direction we can think of as if we have some network through which heat can flow in x direction, heat can flow in y direction and z direction is perpendicular to the screen.

So, heat would be moving parallel heat conduction is taking place parallelly through these parallel through this porous medium in parallel to those capillaries that are running right perpendicular to the screen. So, then we have this k x x k y y and k z z, these are thermal conductivities that can definitely that will depend on what w f w s hf h s you are working with. So, if we have gone to this level of rigorousness one may like to go and define this thermal conductivity in the three directions.

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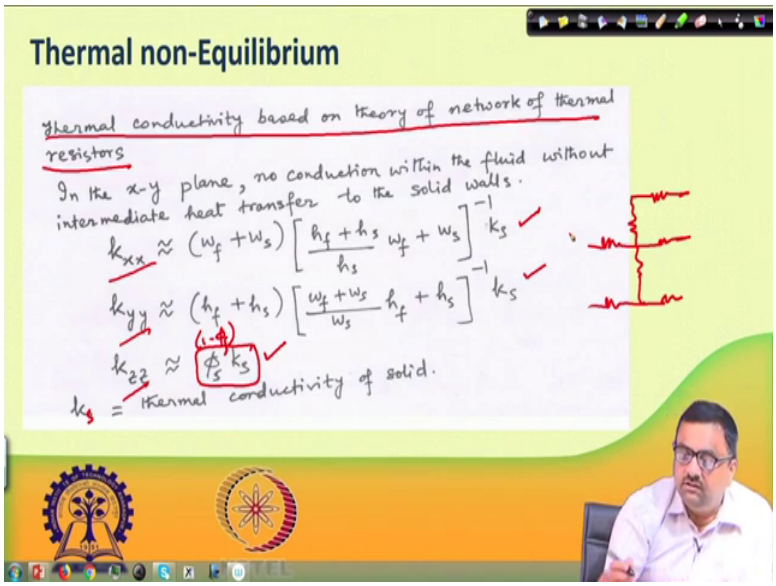

Few other terms I will go to this thermal conductivity in a moment. In fact, we can talk about this thermal conductivity first that the thermal conductivity is based on; thermal conductivity can be written based on theory of network of thermal resistors. So, as if we have a network of thermal resistors. So, you have a resistance here then you have a resistance there then you have a resistance there, you have a resistance there. So, current can flow as if I mean you.

So, if from one node to another node how the current will flow? So, there are certain theories existing in the area of network of thermal resistors. So, that was invoked by previous researchers to arrive at these values of k x x k y y and k z z. And, what are those? w f plus w s into h f plus h s by h s into w f plus w s inverse, that means, the reciprocal of it into k s, where k s is the solid thermal conductivity; that means, thermal conductivity of the solid and bulk level which is reported in literature.

Similarly k yy instead of w f now it is becoming h f and h s and instead of these terms it is becoming w f plus w s by w s; so, this is the change changes that are taking place. So, you can see these are coming here and k z z is simply phi s k s and phi s is basically 1 minus phi f, that means, this phi s is the solid fraction multiplied by thermal conductivity. So, this part we can recognize a little bit that the corresponding a solid fraction multiplied by the thermal conductivity of the bulk solid. So, k is the k s is the thermal conductivity of the solid. So, this part we can recognize from our earlier understanding.

And, these are we can accept that this is that these are these are arising I mean if it is and if it is shown by the previous researchers that this should be the x this should be the expression for thermal conductivity in x and y directions; if w f and in terms of w f and w s if one research to theory of network of thermal resistors.

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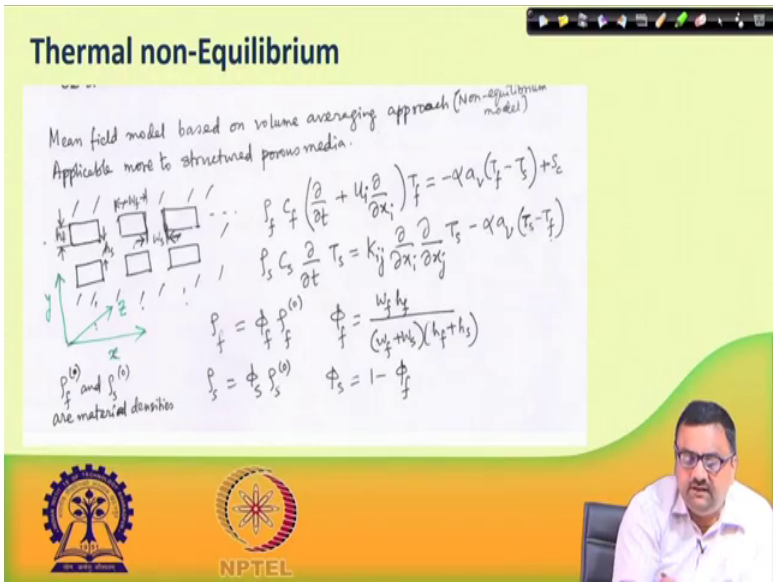

So, that is one thing, but if we if we go back and look at the other terms for example, we have this c f, c s, a v alpha. So, we have several terms we had talked about here. So, what are the definition of this terms these are given here.

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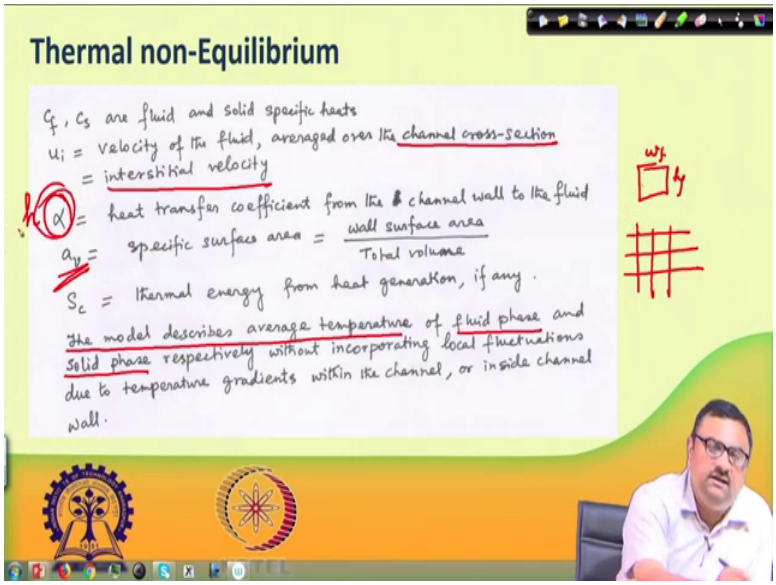

So, what we see here is c f and c s these are fluid and solid specific heats. So, one is a flow specific heat of fluid and the other one is the specific heat of solid; u i is velocity of the fluid averaged over the channel cross section which is basically the interstitial velocity. Now, in this regard I must point out that here we have, see here we are multiplying these u with the porosity term because this rho f; rho f contains phi f term so, that means, this term when you are going here.

So, automatically you are multiplying this u i with the porosity term. So, it is u i has to be the interstitial velocity and when you multiply by the porosity it gets reduced and it becomes superficial velocity.

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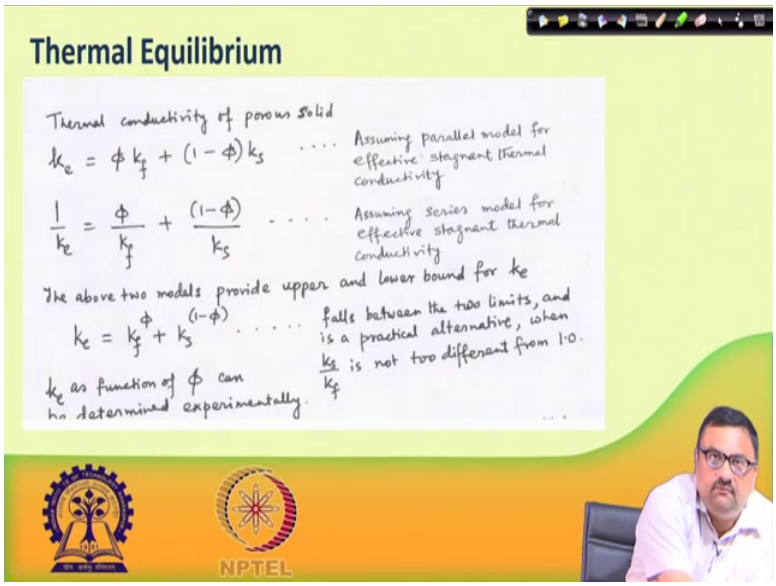

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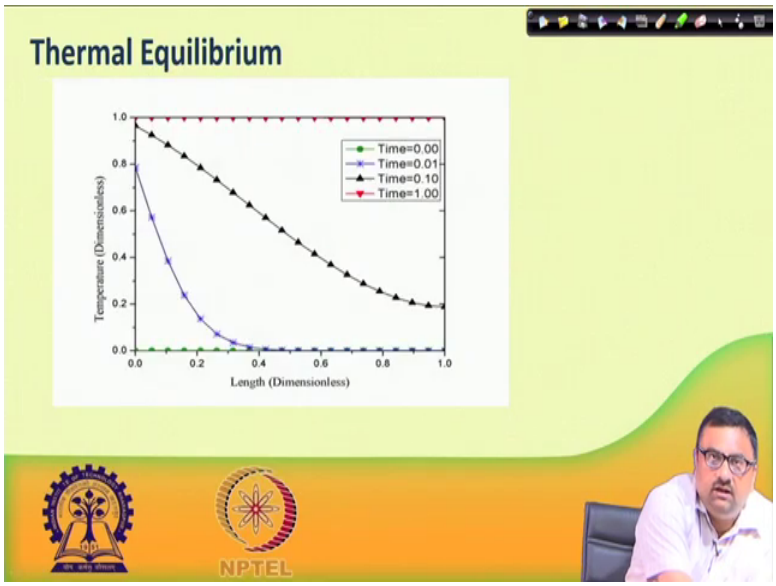

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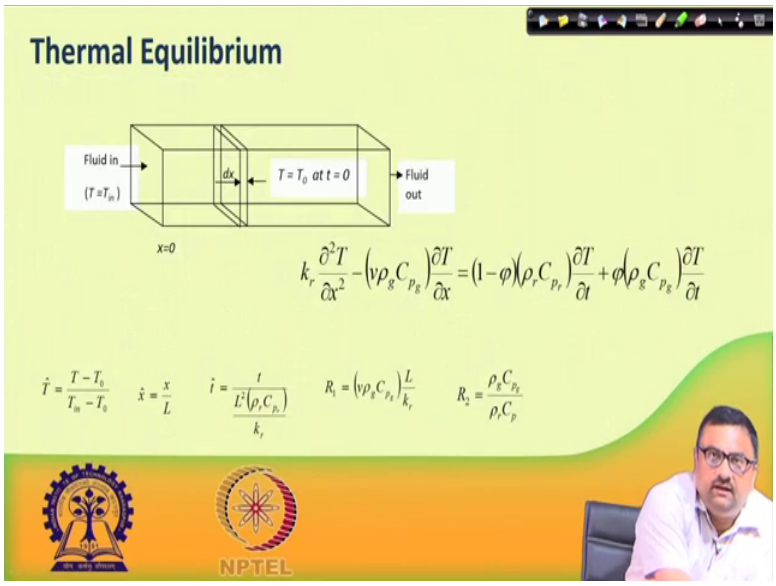

On the other hand when we talked about the thermal equilibrium model there we had we did not have any porosity term here, that is because we are working directly with the superficial velocity. So, one must make note here. So, then next thing is; so, this is the velocity of the fluid averaged over the channel cross section; that means, we have talked about w f w s.

So, over this channel cross section they are talking about; that means, over w f and h f this channel over this channel cross section it is averaged. So, this is the velocity we are

considering you can see actually if you really resolved to some computational fluid dynamics approach there this velocity profile it is expected to be parabolic

And so, velocity probably it is not, obviously, it is not a constant velocity, but that is what is assumed here. So, this is what I mean I said that is how we are you know we are working with two temperature fields, but we are not fully rigorous. So, this is the interstitial velocity, alpha is equal to heat transfer coefficient from the channel wall to the fluid.

Generally this alpha with this is this term is used as h; I mean in literature in heat transfer literature they refer it as h, h is the heat transfer coefficient and a v is the specific surface area which is wall surface area by total volume. This is a standard heat transfer literature whenever there is some exchange happening generally it is attributed to remember it is expressed as heat transfer coefficient multiplied by the temperature difference ok.

If a heat transfer takes place from a solid to a flowing fluid it typically in heat exchanger calculations this definition of heat transfer coefficient is very much there. And, this is s c is the thermal energy from heat generation if any which would be 0, if there is no reaction or no heat generation term as such which is for a regular flow through porous media then there is no such possibility.

If there is a combustion taking place for example, then there will be heat generation. So, that has to be accounted. So, to summarize the model describes average temperature of fluid phase and solid phase respectively without incorporating local fluctuations due to temperature gradients within the channel or inside the channel wall.

So, within the channel no temperature fluctuation it is only a single temperature T f single temperature field T f and single temperature field T s. So that means, I am assuming that at every grid there is one T f value one T s value. It is immaterial whether there at that location the only fluid or solid can exist they cannot exist together.

But we are assuming that both fluid and solid are existing and they are at temperature T f and T s. And, there is exchange happening depending on the difference between T f and T s there is some heat exchange happening and that heat exchange is governed by this heat transfer coefficient.

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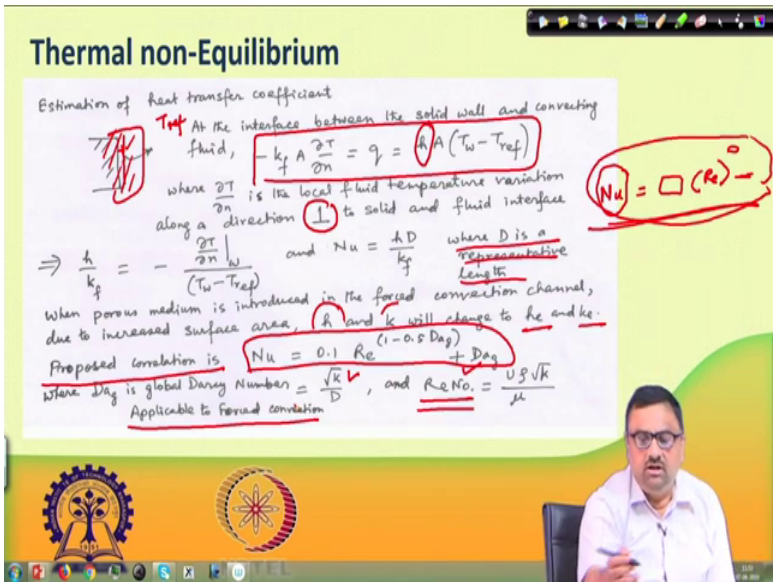

Now, we have talked about how to find out the k x x k y y and all these thermal conductivity terms in x y direction. But, we have not talked anything talked so far about these heat transfer coefficient. Now, this heat transfer coefficient when it comes to heat transfer coefficient there are some unique ways to do it particularly for porous medium that is if you have I mean first of all what is how these heat transfer coefficient, how it is arising here?

At the interface between the solid wall and a convicting fluid; suppose I have a solid wall and the convecting fluid, a fluid that is flowing over a solid wall. Generally if we try to write the heat conduction term that would be k f if as if the fluid is being there is a heat conduction within the fluid phase and that conduction that has to be equal to the heat transfer that is taking place. So, if we try to build in this way.

So, then this would be minus k f a del t del n minus k del t del n n is here the n n is n is basically the normal n n is del T del n is the local fluid temperature variation along a direction perpendicular to; along a direction perpendicular to solid and fluid interface. So, along a direction perpendicular to solid fluid interface del T del n the temperature gradient. So, minus k a del T del n that would be equal to the heat transfer coefficient h into T w whatever is the temperature here and T reference let us say T reference is a temperature which is existing far away so, T w minus T ref.

So, that is what is the definition of heat transfer coefficient that whatever is heat conduction that is taking place because of this temperature gradient that has to be equal to the heat transfer that takes place. So, now, with this definition of h which earlier we have talked about that in our earlier case we have talked about alpha; so, this it is this heat transfer coefficient you can write here if you follow this particular equation.

Then one can write h by k f can be written as a will cancel from both sides it is minus del t del n at wall divided by T w minus T ref; so, that would be h by k f. Now, there is a dimensionless number by which typical heat transfer in a fluid, in the context of fluid mechanics in the context of fluid flow in a macro scale pipe it is defined is there is a dimensionless number known as Nusselt number. Nusselt number is h D divided by k f, where D is a representative length, it could be diameter of the pipe, it could be some are depending on the geometry that one is working on.

So, this is traditionally this is the Nusselt number and in case of fluid flow in a pipe and the consequent heat transfer there generally this Nusselt number is related to then them correlations where analytically the one can derive definitely. And, also that has been tested empirically some constant some number multiplied by Reynolds number to the power some exponent and then there are other dimensionless numbers for example, Prandtl number. And so, these so, one can come up with a correlation in terms of dimensionless numbers, number sets.

So, this is common in case of flow through a pipe and this Nusselt number once you find out what is the Nusselt number, then you can go ahead and find out what is the heat corresponding heat transfer coefficient. And, this heat transfer coefficient will tell you what is the how much heat will be removed from the system, how much heat will be transported from one side to another. So, this is how this works out in a macro scale for a macro scale flow in a pipe.

Now, here in this case in a for a porous medium one can come up with an Nusselt number as well, but this Nusselt number would be then modified because, these h and k they will modify, they will be modified. So, what is the modification? When porous medium is introduced in the forced convection channel; when instead this whole thing was happening in an in a solid wall and a convicting fluid; now if you instead of these if you have a porous medium here with pores and everything. So, then in that case so, when

porous medium is introduced in a forced convection channel then due to increase the surface area; now there is an increased surface area h and k, the interplay of h and k that we talked about here they will change to h e and k e.

So, they would change there would be a different h and different k when it comes to porous medium as compared to the h and k that we have talked about from these correlations for flow through a pipe. So, these correlations will not be valid anymore because now the h e and h and k they are going to change. So, the Nusselt number for porous medium that is going to change. So now, a proposed correlation in this regard is something like this that Nusselt number is equal to 0.1 Reynolds number to the power 1 minus 0.5 Darcy number plus Darcy number.

So, D a g it is basically global Darcy number is given by square root of k by D, D is a representative length it is not diffusivity do not confuse with that. So, D is a representative length square root of k you remember unit of k was meter square. So, square root of k means it is the unit of meter. So, meter and divide by characteristic or representative length is also meter. So, they cancel out so, it becomes a dimensionless number.

So, Darcy number is a global Darcy number is square root of k by D and Reynolds number in this context though we had defined already Reynolds number for porous medium. But, the researchers who have developed this correlation they did they are into using this Reynolds number as velocity density into square root of k by mu; instead of diameter we are right he they are writing it here a square root of k basically the dimension remains same. So, this becomes the modified Reynolds number in this context.

And, then based on this Reynolds number and based on these global Darcy number they used they have gotten a correlation here. And, they said this correlation will give you the right Nusselt number and based on this Nusselt number you find out the corresponding heat transfer coefficient. So, this is that this would be the source of heat transfer coefficient, if somebody wants to work with a non-equilibrium model this is applicable to force convection.

Now, if there is a natural convection you understand what natural convection is where there is no it is inside a porous medium because, of density difference that the fluid of

lower density that tends to go up and higher density comes down. So, something is heated means if it upon heating if the density decreases. So, that goes up and then the surrounding fluid which is cooler and if the corresponding density is higher that you come down.

So, one can develop such kind of a convection as well, but this equation is not applicable for natural convection, this is equation is applicable to force convection. So, now essentially this is and all these things that we talked about is the non-equilibrium model, that means, here we are having heat transfer. But, we are assuming that these we do not have equilibrium existing between the solid or we are not assuming that there is a single temperature field existing all over the porous medium. Single temperature continuum field existing.

Rather we talked about two different temperature field. Liquid and the solid and that required some understanding of the heat transfer coefficient, because heat transfer coefficient will define how much of heat will travel from fluid to the solid phase. So, one has to then bring in the correlations in the same way as one has done for a fluid flow through a pipe wherein Nusselt number is expressed as Reynolds number and other dimensionless numbers.

So, in this case also they have attempted the same thing, but for porous medium they have specifically they have other correlations applicable because of, because this in when you put in a porous medium when you introduce a porous medium in a forced convection channel then there is an increase in surface area. So, automatically the h and k they will change ok. So, this is what we have here.

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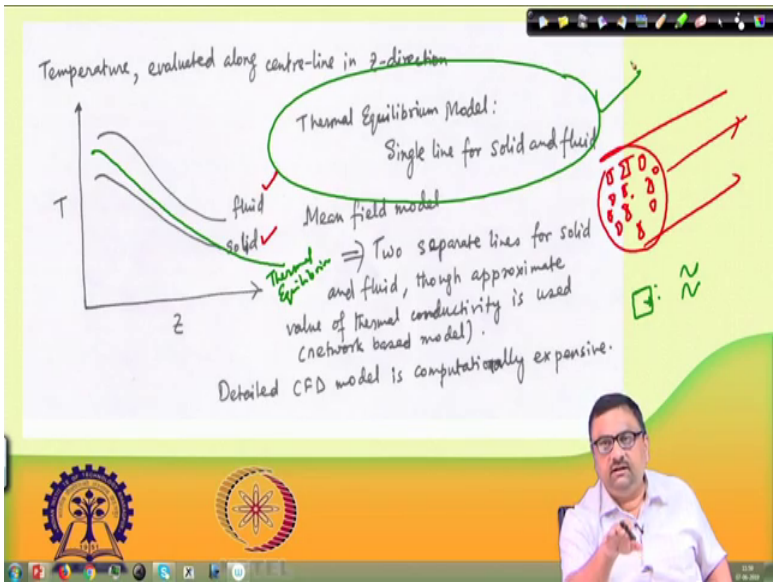

And now, if we try to compare what we have see temperature evaluated along centerline in z direction; if somebody if one has a porous medium and then there is a temperature field and then it is we have let us say mean field model with volume averaging that non-equilibrium model we have. So, temperature is changing with the perpendicular to the screen in the z direction.

So, at the centerline so that means, I have a porous, I have a porous structure and then from along the centerline the temperature is changing in z direction. So, inside this porous structure we have those what we call; we have those honeycomb type structure. We have those straight capillaries running with dimensions of w f w s and let us say along the center centerline in z direction if I try to find out the temperature profile.

So, what we find is that we have now considered temperature of fluid and temperature of solid as two different lines ok. So, that is what we have temperature of fluid is here temperature of solid is here. So, we have only advantage. So, when we are assume thermal equilibrium the first model that we considered single temperature field that time this temperature of solid and fluid they would have they are merged.

So, we have only a single temperature fluid that is probably running in between somewhere. So, this is the equilibrium thermal equilibrium model only a one single temperature field ok. And, this is probably the assumption you have that there is a variation between solid and fluid, but you can ignore that variation and you can work

with the thermal equilibrium model; single line for solid and fluid. On the other hand the next non equilibrium model would be mean field model where still we are holding treating them as homogeneous and only two temperature field T fluid and T solid and then we have assumed that there is some exchange happening between them.

So, then we will have two separate lines for solid and fluid though it is approximate. Because, you are assuming an approximate value of thermal conductivity look at it how we have done k x x k y y k z z and that is using theory of network of thermal resistors, but actually that is not supposed to be done this way right.

If there is a channel then if we are looking at the k in x direction, x direction here thermal conductivity is basically thermal conductivity of fluid right. And, then when it comes to the next when you are into the solid part it would be thermal conductivity of the solid itself ok. So, it depends on where the grid is, we are not talking we are not looking into that level of details, but we have by some means.

So, that is why you must note that when it when it is defined k xx k yy there is this sign not equal sign they are approximate. So, this thermal conductivity based on network based model that we used here these are approximate models ok; so, but at least some improvement over a single field model.

But if every really somebody wants to do the rigorous most rigorous exercise one has to go to a detailed CFD: Computational Fluid Dynamics model treating each capillary individually. So, one has to or inside the capillary the temperature gradient has to be taken into account, the velocity profile has to be taken into here the velocity is assumed a single value for every grid.

So, those are not done here; obviously, the reason is they are computationally expensive. But the fact is for all for I would say most of the cases this thermal equilibrium model is good enough. So, obviously, why would one go to these computationally expensive processes. So, this is all I have as far as the heat transfer through porous medium is concerned.

So, I am closing this chapter here, that is all for this module.

Thank you very much.