Flow through Porous Media
Prof. Somenath Ganguly
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 57
Heat Transfer with Fluid Flow (Contd.)

I welcome you to this lecture of Flow through Porous Media and we were discussing about Heat Transfer with Fluid Flow.

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So, what we have here is Thermal Equilibrium in flow through porous medium which we have introduced already.

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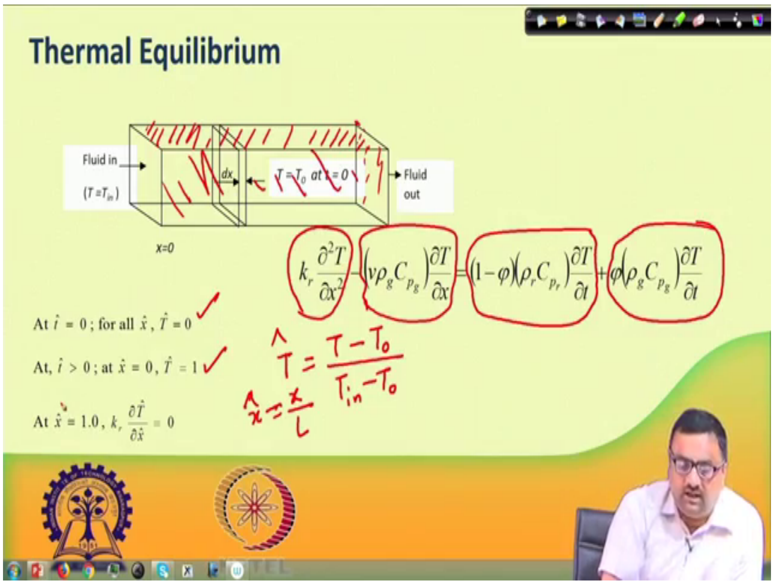

So, what we discussed there is if there is a flow through a porous medium and there is a differential element. So, then we have we can write this as the governing equation, where this is the heat conduction through the solid this term is responsible for heat conduction through the solid. This term is responsible for heat convection through the void phase this term is responsible for the heat accumulation in the solid phase, and this term is responsible for heat accumulation in the fluid phase. Here the fluid is given a subscript g which is basically gas is meant by g, and the solid is given a subscript r basically rock is meant instead of the solid.

So, now and we have discussed about what the implications of thermal equilibrium in this in arriving at this equation. We are talking about only single temperature field in this case which is continuous. Now if we look at the boundary conditions what we see here is that at time T equal to 0, one can write that the temperature which is basically this T hat is the here the temperature is made dimensionless. Dimensionless in the sense instead of working with T in and T out so, or let us say we have that initial temperature is T 0 everywhere T is equal to T 0 at time T equal to 0.

So, we write T hat is equal to T minus T 0 divided by T in minus T 0. So, in that case we can all this temperature inside this porous medium we can translate within a scale of 0 to 1. And that is preferred because then when you go for a numerical solution particularly then you do not your field becomes the range becomes 0 to 1, it does not affect your you know accuracy into the computation. Similarly here so, we can say that if this is the way we define then at time T equal to 0 everywhere in the porous medium the for all x T hat is equal to 0, and for x greater than 0 time greater than so, time greater than 0 at x equal to 0 T is T in

So, if this T becomes T in then this numerator and denominator they cancel out and T hat becomes 1. So, these are the two conditions one has. And then one condition we can bring in here, so, what condition we should put at the outlet ok. One condition one can you can put here is that there is no heat flow out from this no heat flow out by conduction. Suppose I insulate this face ok. So, by insulation I can apply. So, I can make sure that there is no conduction out from this phase. Basically you can see this since they were treating this as a one dimensional problem so that means, these layers are considered not only there is no flow out from the surface, these also this surface.

No not only there is no flow out from the surface there is no heat loss from the surface as well. So, one so, if really if you want to establish this experimentally then you have to take a porous block and put an insulation around first of all put some wrapping that will make sure that no flow comes out through the surface. No flow permeates out from the surface and also there has to be an insulation so, that no heat can release from the surface. So, that is one assumption so, similarly we can put one another assumption that here we place at the outlet no heat flux through the solid phase

So, one can do that. So, then we will bring in k of the solid phase del T del x equal to 0; that means, heat flux is equal to 0 at the at x equal x hat equal to 1, x is again made dimensionless x is written as x divided by L let us say where L is the length of the porous media. So, x hat equal to 1 means the outlet so, at the outlet you can impose that heat flux is 0. So, no heat is leaving the no conduction is happening out from this flow can fluid can leave this can be one boundary condition another form of boundary condition could be that add the outlet you hold a particular temperature T out ok.

So, you can you can give them then you give a at exact equal to 1.0 T hat equal to so and so you can impose that as boundary condition. Another boundary condition would be that porous medium can be considered infinite ok, and definition of infinity is your choice I mean you what are the other dimensions depending on that if the other dimensions are in micrometer then centimeter would be or millimeter would be an infinite infinity. So, then this add infinity you can assume that this front will never reach so, add infinity one can one can say that add extending to infinity the temperature is equal to T naught ok. So, temperature is equal to T t naught means T hat is equal to 0.

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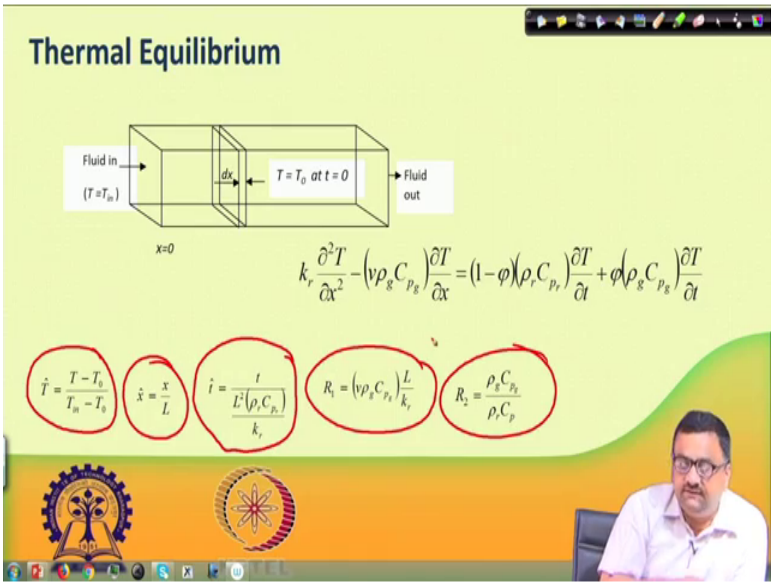

So, this is also another type of boundary condition one can set. So, now if we look at how the solutions are so, these are the this is how this was made dimensionless, I have it in the slide you can see T hat is equal to T in T minus T 0 by T in minus T 0 x hat equal to x by L. T hat time also was made dimensionless and this is the actual time divided by some way you can make dimensionless if you are looking for dimensionless time, and these are some of the parameters one can have dimensionless terms. And so, this equation can be written in terms of dimension dimensionless parameters.

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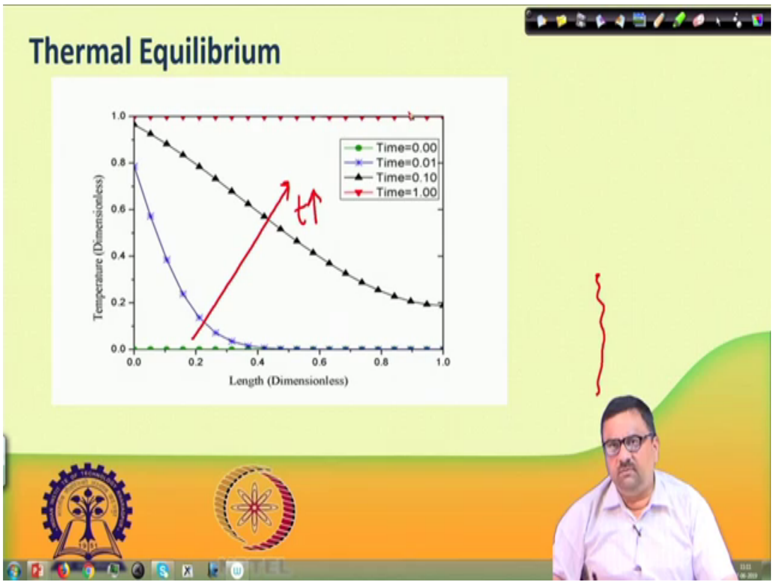

If we look at the temperature profile in this case we first of all we have a single temperature field ok.

So, single temperature field means temperature of fluid and temperature solid at every grid at every point at every length at every distance from the inlet it would be a single value. So, now, we see that as time progresses, as time progresses first the temperature was 1 set as 1 at the inlet and rest of the places are all 0 and then as time progresses we see temperature is gradually rising. It does not mean see temperature has gone up to this place it does not mean that the fluid that I am injecting in through this porous medium that has gone up to this place, may be the fluid that I am injecting that has traveled up to this point it has it has traveled long a further far ahead, but the temperature front that is traveling that will typically that will travel at a much slower rate ok.

So, this is this is the temperature front position, fluid front might have traveled already. So, then you swine the temperature is increasing gradually and at much higher time and so, here the time was made dimensionless in such a way that the T hat at 1.0 u everywhere it would be same. So, temperature of the entire porous medium would be having the temperature at the inlet. So, this is how gradually the temperature will rise this will be the same case for heat conduction in a solid plate, but here the mechanism is different here it depends on the velocity, here depends on other several other parameters ok.

So, this is how I would expect the temperature to progress through a porous medium as the hot fluid is traveling through the porous medium. And this kind of there is this is a very common occurrence in many in many applications that that the inlet flow the injected fluid is at a different temperature, for example, if you go to this hydrocarbon applications there these injection of heated fluid is extremely common in it, because the crude oil it has a very high viscosity and so, it generally it tends to it there is some difficulty in flowing that crude oil out from the porous medium. So, for these reason they inject hot water, they inject steam, they do in situ combustion. They do all kinds of things to increase the temperature so, that because the viscosity decreases with temperature.

So, they have to increase the temperature surrounding the crude oil even they use electrical heating also, electro electromagnetic heating they have they use different forms of heating to make sure that the crude oil viscosity the crude oil gets reduced and. The hydrocarbons start flowing of course, that flow there are a lot of other mechanisms there, because there would be crude oil is a mixture and so, some part of the crude oil which is the lighter fractions that will travel we briefly talked about it earlier when we discussed in this Kelvin equation that the up a lighter part of the crude oil will travel ahead and the heavier part will remain there and those kind of thing

So, so but this temperature profile is that the flow of heat front along with the regular flow through porous media is very common in many different important applications.

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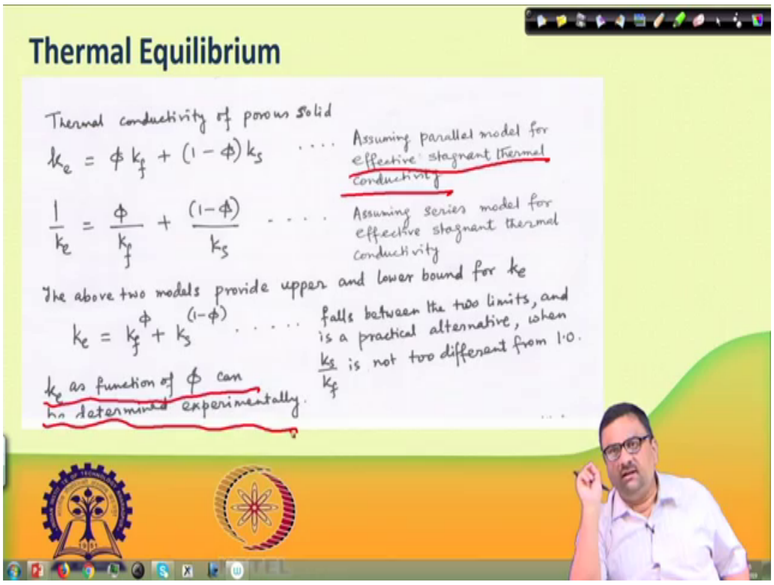

Now, when it comes to Thermal conductivity of porous solid I said that conductivity Thermal conductivity cannot be same as the conductivity in intact solid because, of though these reason because it is porous. So, had it not been porous had it been intact solid the grain to grain there is a much better contact. So, heat can flow much easily so, or not much higher rate. So, the Thermal conductivity would be completely different.

So, now if you introduce voids inside a solid matrix so, how Thermal conductivity changes, so in this regard these are some of the models that are being considered in earlier considered by the earlier academicians. This is one is k e Thermal conductivity of porous solid is written as, the porosity multiplied by Thermal conductivity of fluid plus 1 minus phi, which is the solid fraction multiplied by the solid Thermal conductivity. So; that means, here what is assumed is a parallel model for effective stagnant Thermal conductivity. So, you are assuming there are parallel model for effective stagnant Thermal conductivity. This is this is first of all so, few terms you are mentioning here.

So, let us do this effective stagnant Thermal conductivity. So, we are assuming there is no flow stagnant Thermal conductivity there could be from the heat can flow parallelly, one through the through the fluid which is occupying the void and the other part is through the solid. So, these are two parallel channels through which it is flowing so, effective permeability a sorry effective Thermal conductivity is given by phi into k f plus 1 minus phi into k s. There could as will be a series model for effective stagnant Thermal conductivity and which should be defined by 1 by k is equal to phi by k f plus 1 minus phi by k s.

So, now if we if we if somebody can measure this Thermal effective Thermal conductivity they will find at these are the two limits within which this Thermal conductivity can be found. So, above two model provide upper and lower bound for k e. There is another way to be to another way this Thermal conductivity is also presented which is, k e is equal to k f to the power phi and k s to the power 1 minus phi, this can be done when k s by k f is not to different from 1.0.

So, this is this is also another way of looking at it. Now there are k e as a function of phi can be determined experimentally. So, there is there is a way to measure determine Thermal conductivity and stagnant Thermal conductivity you can very well measure in the in using the same setup that is used to measure Thermal conductivity, but instead of intact solid you can place a porous solid.

So, there could be it could be measured and as porosity is varied ok. So, how Thermal conductivity changes with porosity. So, one can find out Thermal conductivity versus porosity plot and give some put a correlation in that to find out for an unknown porosity what should be the Thermal conductivity in that case. So, we must note that this Thermal conductivity is no reason to believe that this Thermal conductivity is simply the Thermal conductivity of the solid material that is reported in literature it is little different it is a function of porosity.

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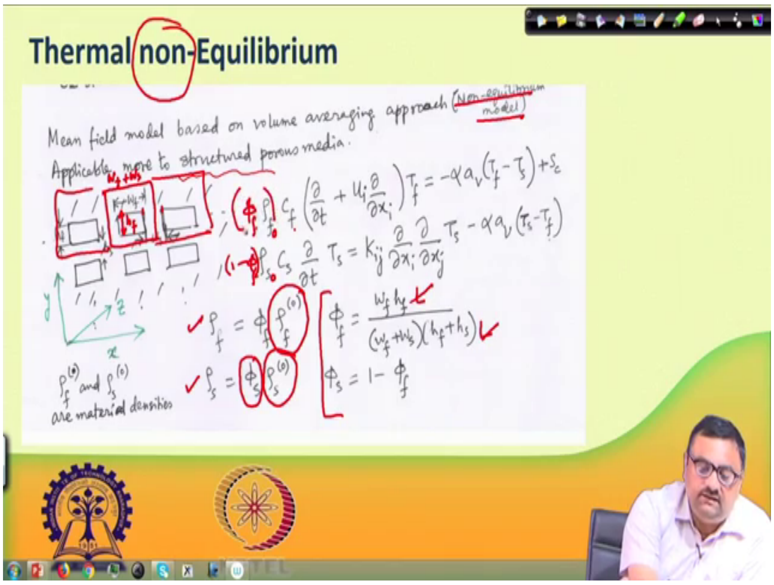

Next what we have here is, mean field model based on volume averaging approach. This is called non equilibrium models, non equilibrium model.

So, now we have we are talking about thermal non equilibrium. Thermal equilibrium was that temperature of solid and fluid they are same in this and temporaries, there is a single temperature continuum which is assumed in it in that case. Now we are bringing in two different temperatures, one is temperature of the fluid another is temperature of the solid.

So, generally this is applicable more to structured porous media ok, and this is an example of structured porous media little think of an think of a honeycomb type structure where these capillaries they are running perpendicular to the screen ok. The capillaries they are of size let us w f by h f width versus height; w f is the width of the capillary through which the fluid is flowing and h f is the height of the capillary and these same size capillary these are embedded inside a solid matrix. And the embedding is such that the distance between the two capillaries is on this in the y axis it is h s and in the x axis it is w s. So, w s h s these are distances between two.

So, this is basically the solid part dimensions and w f and h f these are the fluid the conduits through which the fluid is flowing the dimensions of that is given by w f and h f. So, the x axis is given in this direction y axis is in this direction and z direction is perpendicular to the screen.

So, in this situation if you assume that there are; first of all quickly we need to find out here the porosity of the flowing fraction, or that is what is the porosity. The porosity here is w f h f into let us say, I have l is the perpendicular to the screen we have let us say l is the length of this porous medium. So, then the total volume if you look at, that is I mean if I if I pick up, if I pick up one representative section out of it so; that means, if I pick up one representative section out of this. So, then these becomes w f plus half of h a half of sorry this is w f this is w s distance between this and this is w s. So, this is half of w s to this side this is half of w s to this side. So, this length would be w f plus half of w s plus half of w s.

So, this length would be w f plus w s and by the same token this side would be again h f is this dimension of this, plus half of h s on that side half of h s on this side. So, this side dimension would be h f plus h s so, that side is w f plus w s h f plus h s and if l is the length perpendicular to the screen. So, then we have the total volume is w f plus w s multiplied by h f plus h s multiplied by l. So, that is the volume of the total volume of the representative element. So, I multiply this by L and out of this representative element the void volume is this is the void basically, through which the flow is taking place

So, it is basically w f into w s multiplied by L. So, it is sorry w f w f into h f this side is h f this is this side is h f. So, w f multiplied by h f. So, that gives me and multiplied by L, L is perpendicular to the screen. So, that gives me the volume of the void in this total representative volume. So, this representative volume is repeated everywhere so, that same porosity would be repeated. So, total that void volume in the representative element divided by total volume. So, this L will cancel out. So, w f h f by w f plus w s into h f plus h s these gives me the porosity of the porosity the void part and then the solid part would be simply 1 minus phi f

So, these becomes the definition of phi f and phi s, the porosity of the void part and solid fraction. Now when it comes to the definition of rho f and rho s density of fluid and density of solid what is assumed here is, first of all it is bulk density of fluid multiplied by the porosity and it is bulk density of the solid multiplied by the solid fraction, this is required because you can see here they have not multiplied this part by the porosity term. I mean this would have been much simpler if you had if we had multiplied these by phi f and would have written it as rho f 0. So, rho f 0 c f 0 that would have been so,. so, so this is earlier in earlier case you remember in case of Thermal Equilibrium, we had written del del T del T del T multiplied by rho f that there we had rho g c g and multiplied by the porosity term.

Because porosity term came because there is this accumulation happening, but accumulation is only happening over the void part, because it is within the fluid accumulation is happening. Similarly this accumulation is happening with the solid part in this case del T s del t. So, naturally they had taken into account 1 minus phi f term, but instead of that instead of putting is 1 minus phi and considering this to be the bulk property, what they have done is they have incorporated it here itself and put rho f is equal to phi f rho f 0 and instead of rho f 0 phi f rho f 0 instead they are using rho f. Similarly 1 minus phi f into rho s 0 instead of that they are writing directly the rho s.

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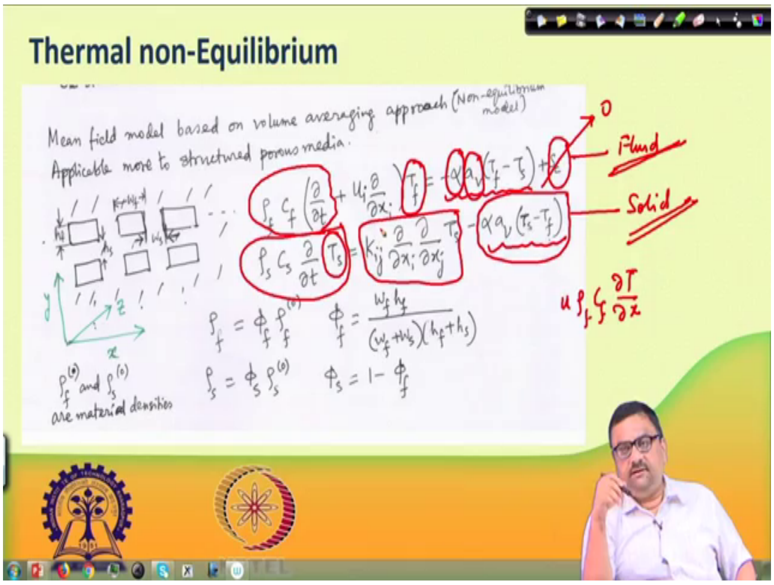

So, now if we look at, what we have here we have two equations right, one for the fluid phase and another for the solid phase. So, one for the fluid another for the solid, and there is some exchange happening. So, first of all there are two different temperatures here one is T f another is T s ok. And there is some exchange term here, this is the exchange of heat from solid to the fluid or vice versa right. In this case it is T f minus T s in this case it is T s minus T f ok. So, there is something called the heat transfer coefficient which is important here.

So, it is placed and this is surface area per unit volume, this is how it is the heat transfer is addressed from one phase to other. And there is one S c term dangling here because this S c is in case there is any generation happening, most likely we assume that there is no generation so, this term is 0. And this is a way we have there writing this notation it is rho del del t of the so, this quantity is basically del T del del capital T del small t. This we are already familiar with and this rho f already has the phi term I have discussed. So, this is basically this term is when we bring in this T here inside, this term is basically talking about the accumulation accumulation in the fluid phase and this term u del u i del del x i of T f ok.

This term is basically the convection term we had a convection term there. So, that same convection term we are putting it here. So, this term is basically now outside it is rho f c f. So, we are talking about u rho f c f del T del x. So, this is this is the very similar term we had with convection at that time. And then we had and mind it last time the accumulation was on the right hand side, so, it has a plus sign, but now we have taken all of them on the one side. So, this is now then that time this was plus sign on the right hand side and this was minus.

So, now we have brought everything together so, now it became plus and then this is the exchange term, this is the term that is arising because this is the heat that is given from the fluid phase to the solid phase. And similarly when we look at the corresponding solid phase, we see that this is the accumulation term as before within the solid phase and this is the heat conduction term. So, here also you may note that heat conduction the fluid is not considered ok, only the conviction is considered here. So, this is the heat conduction in the solid term. And this is the heat that has been received due to exchange from fluid to solid.

So, essentially now we are talking about a fluid phase temperature and a solid phase temperature T f and T s we have two continuum existing one is the fluid phase and other is the solid phase, and as if there is exchange between fluid and solid and this is that exchange term and that exchange depends on what is the temperature difference between solid and fluid or the corresponding temperature at that particular grid ok. So, that defines the exchange, so, this is I mean if you if you want to complicate this Thermal Equilibrium if you do not want to consider Thermal Equilibrium probably this is the first tip. I will there are of course, there are issues; I mean I have not discussed this term fully I will do that in definitely in the next lecture, because I will continue this to the next lecture. So, this is all I have as far as this particular lecture module is concerned.

Thank you.