Flow through Porous Media
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Lecture – 51
Interception of Suspended Solids (Contd.)

I welcome you to this module of lecture of on Flow Through Porous Media, we have been discussing about Interception of Suspended Solids.

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We have already discussed about fines migration, why it happens and how it gets deposited? And, how the permeability is going to; we have not talked about how permeability is going to change, but that is going that we are going to do these in this lecture. And, of course, the implication of this in environmental flow when in soil science or in deep filtration or in sandstone reservoirs or petroleum recovery.

So, these are what are the implications of that, we have already talked about those implications, but how this permeability would be changed, because of deposition. We started working with this continuity and we said that this deposition, rate volumetric deposition rate the sigma s dot that is going to be an important parameter. Now, but how sigma is dot would be, first of all how from continuity, how we can; how we can relate to what all experiments that are being done on sigma s dot?

And, how eventually, how it affects the permeability? Because once we know what is the effect on permeability, immediately we will know what is its effect on the pressure drop and that is something which we are interested in we started with a deep filtration process and we are having a flow and we are seeing suspended solids are getting plugging the pathways and we are happy that we are producing something at the effluent which does not contain the suspended particles. That is what we wanted in a deep filtration process.

But with time we will expect that the permeability will go down press so, permeability is going down means, pressure-pressure drop across the across this medium is increasing. So, it will increase at what rate? Because if you want to design a deep filtration process in a chemical engineering application, one must know how pressure-pressure drop will change with time or with flow, so, we are heading to that.

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At the end of last lecture we talked about this continuity equation and we have already mentioned that this is how we would be talking about these are the two terms here c p is we talked about the c p which is the volume fraction of the particulate. So; that means, volume of the particle divided by total volume of suspension and u as the combined I mean, u is the weighted average velocity, right.

So, it is basically u p c p plus u f c f. Where c p c f these are the weights given, because c p c f these are dimensional is these are simply fractions. So, these are the weights given

and u was obtained as weighted average velocity, which takes care of the velocity of particle as well as the velocity of the fluid.

And, then this is this sigma s dot is the volumetric deposition rate and these that is this diffusive flux which is more of a definition, which is not this is not and not this has nothing to do with diffusion, but just we have to define a term and so, we define this term basically this, this talks about what is the additional movement the particles are having with reference to the weighted average velocity?

So, with this understanding now, we are going to look at the cases where this c p is small, which we expect I mean c p we do not expect 40 percent particle and 60 percent fluid and flowing through a porous medium that is unlikely so, when c p is small what we can expect?

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So, when suspension is very dilute, then c p is much less than 1 and c f would mean that case almost equal to 1, because c p plus c f is 1. And, u which is the weighted average velocity would be same as the particle velocity as same as the fluid velocity. So, with this as assumption the quantity of particles, deposited from such dilute solution occupies a negligible space in pore. So, when we are working with such kind of dilute suspension, we have already talked about this that we can expect a very negligible reduction in pore volume.

So, porosity is not going to change only near the pore throat there would be some deposition so, that will change permeability so, that is the assumption. So, with that assumption; so, we have permeability impairment that is happening very much, but the porosity is not changing. So, by that logic earlier we had what we had phi into c p here inside.

So, phi was taken out so; that means, phi is not changing with time, phi is constant. Phi is constant, because c p is small so, very little particle volume fraction of particle is available. So, that is not going to change the pore volume here or that is not going to change the porosity here. Only they are depositing at the pore throat and because of that permeability is vary. So, phi is taken out here and c p instead of c p now, we start calling it c your c is the concentration of particle, ok. Because then c f is practically 1 for our purposes and then one can take this u also outside with this assumption.

So, this is the equation one has which subscript p is dropped. Further one may note here is that this sigma s dot; so, now, this is just a very standard equation I mean, how much of particle is going in u is remaining constant. So, any change in concentration of u, any change in concentration of particle of volumetric volume fraction of the particle is essentially, because any change sorry any change in the volume fraction of particle is essentially in minus out and a deposition. This is probably the simplest possible. So, everything is out u is constant and taken out phi is constant and taken out.

So, this is probably the simplest I mean, I mean to say in minus out it is just like bookkeeping. I   n this differential volume I have a pore volume and whatever is in minus out there would be change in particle fraction inside the volume and there would be some deposition. This is probably the most basic bookkeeping that is possible with the particle that is one thing.

So, this is this equation, ok. And, the second thing is if we look at this sigma s dot for that I mean if we want to know the character of sigma s dot, ok. So, if we want to know the character of sigma s dot this should be proportional to number of particles flowing through unit area per unit time. So, what that means? Is this is this should be proportional to u into number of particles divided by total volume.

So; that means, it is u into volume of particles divided by total volume. Number of particles, number of particles is volume of particles divided by average volume of

particle; average volume of one particle total volume of particle divided by average volume of particle that gives me the number of particles. So, number of particles by total volume that gives me the particle concentration and that multiplied by the velocity.

So, sigma s dot; that means, to start with I have some deposition already has taken place, some deposition is there already. Now, I am trying to know that if I continue to run it for from t to we go to t plus delta t so, over this duration delta t there would be some additional deposition and that deposition is let us say delta sigma s, ok. So, there is this additional deposition which is delta sigma s. So, on what this delta sigma s will depend? So, delta sigma s will depend on what number of particles that has flown over this unit per unit area per unit time how many particles have flown through this medium?

So, it is so, amount of additional deposition over time delta t will depend on the number of particles that have flown through this flown here through this conduit, flown through this differential volume over this duration delta t. So, if we make this kind of an understanding that amount deposited is proportional to number of particles flowed through the system over that same time, ok. It is more like I have one deposition deposited phase and I have another flowing phase on top.

So, any deposition depends on how many particles are above in the bulk; if number of particles in the bulk increases deposition automatically increases. If the number of particle decreases in the bulk deposition automatically decreases. So, change in deposition delta sigma s over duration delta t will depend on over this duration how many particles have flowed through the system? So, any delta sigma s will depend on that value. So, that is why sigma s dot will depend on number of particles per unit area per unit time which is flown through this and flowing through this per unit time.

And, so, this is then it is automatically number of particles for total volume, which is volume of particles by total volume into 1 by average form of particle. It is coming, because number of particles would be volume of particle by average volume of one particle. And, so, this divided by total volume and that would be proportional to u; u remains as it is this is following the velocity and then instead of c p we are writing volume of particle by total volume is c and average volume of 1 particle is V p star, average volume of one particle is written as V p star.

So, u c divided by V p star and that becomes the sigma s dot, sigma s dot means, del sigma s del t.

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So with these understanding, for a set of particles V p star is constant; if I have a set of particles that are flowing through the system, V p star is average volume of one particle that is constant. So, one can write sigma s dot is proportional to u into c; u is the u is equal to u p is equal to u f. So, basically the Darcy velocity of the fluid that is flowing through this system and this is the same as the particle also is flowing at the same rate and sigma s dot is del sigma s del t and this is proportional to u into c.

So, one can write del sigma s del t, del sigma s del t is equal to k naught into u c. I mean, proportionality constant is let us say k naught; this k naught is also referred as filtration coefficient at constant rate in the area of deep filtration there is this k naught which is known as filtration coefficient at constant rate and this can be measured, but we take this one step further.

Now, if we bring in these u here on the denominator, this u from the right hand side numerator in the right hand side if we bring this to the denominator here. So, then it will becomes del sigma s into u del t and also we divide this both sides by A. So, A appears in denominator here, A appears in the denominator here. So, then we brought this del t on this side so, now, this u A del t that gives me, that we can write as del q; where q is the cumulative flow.

Think of it I have an average velocity I have a cross sectional area A and this average velocity is superficial over delta t time, so, u A delta t velocity multiplied by area gives me volumetric flow rate, that is meter per second into meter square gives me meter cube per second. So, u into A volume volumetric flow rate meter cube per second. And, then meter cube per second into delta t delta t is second.

So, meter cube per second into second that gives me meter cube. So, u A delta t gives me the dq, right. Over duration del del t the volumetric, flow cumulative flow that has taken place is del q; it is not a flow rate it is the flow over del t duration del q amount of flow del q meter cube of fluid has flown. So, now, this gives me del so, this becomes del q. So, automatically and this k naught by A into c instead of that we write this as gamma into c let us say.

Or, it is probably easier for me, if we work with something called if we we call it lambda instead of gamma I think gamma is it is better if we can; if we can call this lambda, lambda into C. So, del sigma s del q is equal to lambda into c. Where here q is the cumulative flow; so, del q is the cumulative flow over duration del t and sigma s is the total volume of particles we drawn from a location in the flow any particular location not total porous medium.

So, sigma s is the total volume of particles we drawn from a location in the flow and then deposited during this cumulative flow, ok. So, during this cumulative flow apart is withdrawn and deposited on the surface. And, this lambda; lambda is to be determined experimentally, this is if this we call it lambda then lambda is to be determined experimentally for a deep filtration process. So, lambda has to be determined experimentally deep filtration process. Lambda has to be determined experimentally.

So, where we are all heading to; we said that, we said that this is the most the simplest form of bookkeeping we have with the particles, particle I bigger differential volume particle is going in u is outside, phi is outside. So, only whatever is going in based on the velocity I have from outside no change in velocity nothing; so, it is within the differential element in minus out is the accumulation in the fluid phase, accumulation in the fluid phase and the deposition on the solid surface.

The most simple bookkeeping we have here and further if we want to delve into this what is this deposition rate; obviously, this equation will not give me the deposition rate. This is the this relates deposition rate to the change in particle concentration as it travels if somebody wants to know, how much would be the concentration of particle, how much it will decrease as it traveled? So, this is the bookkeeping one has to fall.

But we need to find out what is sigma s dot and or in del sigma s del t? But instead of writing it as del sigma s del t it is customary to write it as del sigma s del q. This make this is easier to think about; you can think of that this process most likely I will not be running it at constant velocity, ok. So, with time something is changing instead of that I am will and relating it with the cumulative flow is much simpler.

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Because we are looking at a porous medium we are looking at a porous medium and then we are having some deposition here and, because of this deposition there is. So, there is some deposition happening. So, how much instead of calling it with respect with respect to time, how this deposition is changing with respect to time instead of that I think this information is more useful to me if is if it is mentioned, if I have this much of flow how it is changing?

Because I may not be running it at constant velocity, one thing you have assumed here is this is proportional to u into c del sigma del t, but I may not be running it at constant velocity. I mean, my velocity may be changing continuously, but if I know sigma versus q; that means, how this what is the deposition particle deposition how much it will increase if I have an additional so, many volume of suspension flowed through the system that information is more useful.

Here I need not have to run it at constant velocity and generally this kind of processes it is very difficult to run at constant velocity, because to start with I do not have; I do not have that information a priori whether these this velocity will not lead to a very high pressure, because if I do not know if I do not have that control then I mean, I started with a velocity and after a couple of hours I see that the pressure is enormous and this entire porous medium it took fractures it cracks through.

So, it is not wise to work with a constant velocity in this case rather, one prefers to work with a constant pressure at the injection site. So, that initially there would be a high amount of flow, but then gradually the flow will decline, but the pressure ceiling is always set, ok. So, it is always better to have sigma s as a function of cumulative flow; del sigma s del q this information I can use wherever required very easily, ok.

And, say and we said that this is this we this we call , this we call a lambda and this we this; we this; we this we call lambda and this we called lambda and this we called lambda so, this is this lambda is to be determined experimentally for a deep filtration process.

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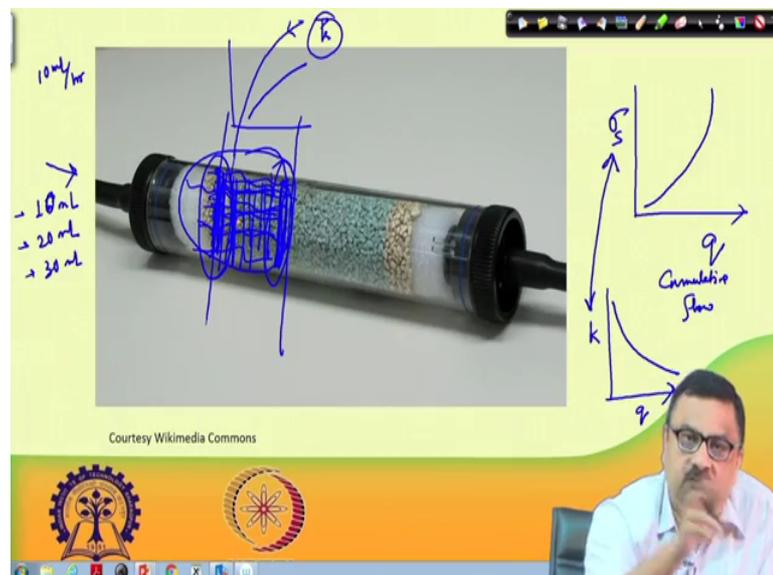
This is a picture of a porous medium; now, think of it this deep filtration process here, I can see that this is as the flow takes place we can I mean, I since we have been talking about all the time about these this sigma and q and all these. So, essentially what we are looking at here in this case is that I am having a flow here, ok. So, flow of fluid plus this is a basically a suspension, fluid plus some solid.

And, we said that is concentration or the volume fraction of this solid is small; obviously, if it is very large you cannot expect that suspension to flow through this. So, this is a there is a; there is a small there is a suspension which is flowing through this and then as it travels through as it travels; so, we expect that these areas there is getting deposition, there is deposition. And, because of this deposition we are having this; so, this deposition we are referring it as sigma s, what is the unit of sigma s now?

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If we go back to our original exercise, sigma s dot was volumetric rate of deposition.

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So, volumetric rate of deposition which is mu s dot by rho p and volumetric rate of deposition would be then volume withdrawn from the streaming flow due to clogging per unit volume of porous medium, per unit time, ok. So, basically sigma s dot would is volume; when we talk about sigma s dot that is volume withdrawn so, we are picking we are having some deposition here and if we look at this deposition this is the volume

withdrawn per unit volume of porous medium. So, that is; that is, that is what is sigma s, ok.

Sigma s is volume withdrawn per unit volume of porous medium at a particular location; volume of solid withdrawn from the fluid and getting deposited on the surface per unit volume of porous medium. So, that is what is sigma s ok. Now, when we talk about sigma s dot; that means, del sigma del t when it is how it is changing with time and we said we are not bothered with del sigma, del t rather we are happy with del sigma s del t we are happy with del sigma s del q.

So, as so, at a particular location I mean, I am having a flow, so, this is cumulative flow; so, flow is continuously flow is happening. So, as the flow continues as the flow continues at a particular location I expect sigma s to increase. So, at this particular location I will find the sigma there will be more and more deposition taking place I am having some more flow, there is more deposition taking place.

So, this way we can see that this there would be sigma s is changing with the q. So, as so, I have injected let us say in first hour I injected 1 meter cube so, say or let us say first hour I injected 10 ml ok, then this is in the first hour. Second hour I injected 10 ml plus 10 ml so, cumulative volume; so, it is let us say I am injecting 10 ml per hour. I mean, that is the flow rate. So, then first hour I injected 10 ml, second hour I injected 20 ml so, third hour I injected 30 ml write this so, this is the cumulative flow. Every hour I am injecting 10 ml but cumulatively I have injected much higher volume.

So, I am now, plotting this I can say how sigma changes with q, how sigma s changes? Sigma s is volume deposited per unit volume of porous medium and how sigma s changes with q? As you increase the q, q is the cumulative flow; so, as you have the cumulative flow how much the sigma s changes? Now, sigma s at a particular location ok; now, so, if sigma s changes automatically what will happen is the permeability will also change. So, permeability will also change with cumulative flow, ok.

So, I expect that as cumulative flow occurs permeability at a particular location mind it, k at a particular location; not k of the overall medium k at a particular location will change with cumulative flow. And, I expect the permeability to decrease with cumulative flow right. So, permeability will decrease with cumulative flow as flow takes place permeability will decrease whereas, sigma s will increase with cumulative flow; that

means, as cumulative flow continues I expect more and more deposition. So; that means, more and more deposition means, the permeability will decrease.

So, there is a relation between permeability to sigma s, which is the deposition. So, that is one thing, but this whole exercise I said is the at a particular location, but we are not exactly interested in one particular location we are interested in is; let us say the fluid that is traveling fluid has traveled up to these location , ok. So, when it comes to the flow process then beyond this location it is all just the regular medium the original porous medium that I had with our original permeability and this part there would be a average permeability k bar, ok.

Which would be in that case the harmonic mean of permeability of individual slices and you must note that each individual slice has fist, so, to say less and less cumulative flow for example, this particular size has got least cumulative flow whereas, this particular slice near the inlet got the maximum cumulative flow. So, I expect the permeability also we the through this distance the permeability will also vary I mean, if you plot the permeability at this time you will find permeability is also lowest near this inlet, because this one has seen the highest amount of cumulative flow this one has seen the lowest amount of cumulative.

So, the deposition would be highest here lowest here. So, I have a permeability variation over this length, ok. And, one can find out some average permeability over this length by taking writely averaging the all the individual slices, ok. So, now, now our aim would be to use this sigma s this information; sigma s we have seen sigma s as a function of cumulative flow we have some constant, we have we said del sigma del q is equal to lambda into c where lambda is some constant.

So, now, with those information and also we have a bookkeeping formula that if sigma s this much of deposition take place in at this location then next location and next location how much would be the concentration, how it is traveled? So, now, it will be the interplay of that bookkeeping equation that I have and the sigma s with respect to q through lambda that relation that we have and then sigma s somehow has to be related to permeability, which is the local permeability and then local permeability will also have a variation over length, we can have average permeability.

Once we know all these information then I can predict how what would be the flow rate? So, this is something which we will had to in the next lecture. And, so, you have gotten enough introductions to how these a particle deposition and everything is taking place? So, in the next lecture what we do is now, we relate this sigma k sigma s and these this the individual permeability and the average permeability and how it will affect the total pressure drop? I mean if you have such kind of variation in permeability. So, then it will probably we will complete the full circuit.

So, with this I am closing this particular lecture module.

Thank you very much.