Flow through Porous Media
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Lecture – 48
Interception of Suspended Solids (Contd.)

I welcome you to this lecture of Flow Through Porous Media. We have been discussing about these Interception of Suspended Solids and I we are trying to in the last lecture, at the end of last lecture we started discussing about the reason why this fines get migrated in a porous medium; that means, a part of the porous medium itself which was attached to the wall.

And now that is getting dislodged from the inner part inner wall and it is traveling downstream and essentially that is being intercepted at the downstream. So, we have been talking about fines migration in this lecture.

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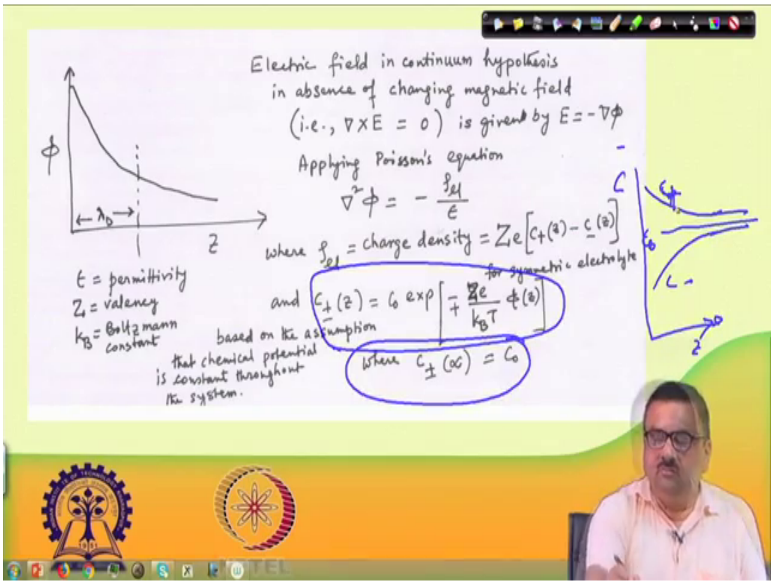

So, we discussed this potential as a function of Z and we said that this is the concentration profile one will end up for positive and negative ions. And we said that at infinity the concentration of this plus and minus they have to be C 0; that means, we had drawn this concentration profile as a function of Z and we said this is C 0 and plus and minus they are converging to C 0 value. This is Z this is C this is C plus and this is c minus and this value is C 0.

So, this is something which is which we have already discussed. Now if we take this Poisson's equation replace this rho el by this quantity ok.

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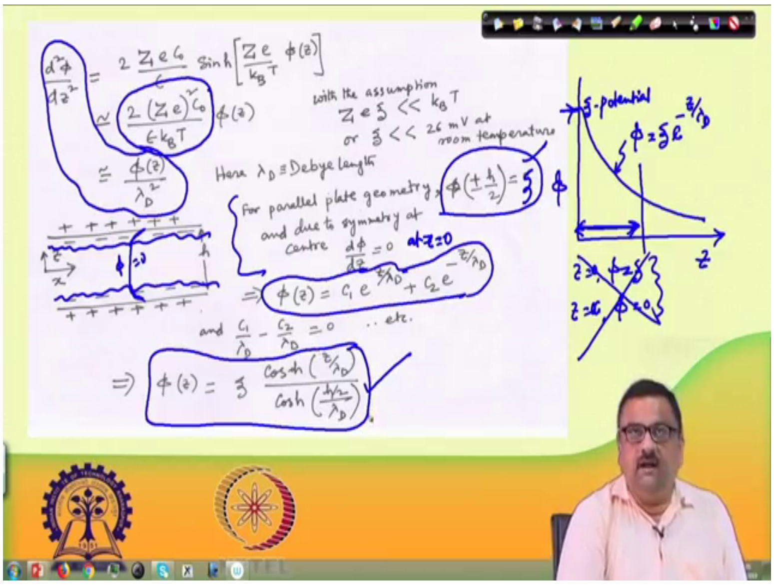

And try to solve this so, one ends up; I mean I am skipping few steps. I am just trying to give you the final result here, because this is it will be e 2 the c 1 e to the power mx that that will be this term and then one can write this in terms of sin hyperbolic this quantity ok.

So, if we solve this because we have the Z e c plus z minus c minus z and the c plus will have e to the power something minus c minus will have e to the power something. So, e to the power x minus e to the power minus x so, those that kind of relation will end up with sin hyperbolic expression for sin hyperbolic. So, this is what we have for grad square phi which is writing it as d square phi d z square. And then sin hyperbolic this quantity we can replace it by this quantity itself that is sin hyperbolic u can be replaced by u provided this is small.

And this is small; means this quantity Z e zeta what is zeta now. We said that potential would come down like this right with z. So, and this is the potential which is coming down. The potential at the wall this is known as zeta potential, this is defined as zeta potential and there are ways to measure this zeta potential. So, this is referred as zeta potential. Now instead of phi Z we can see that the phi would always be less than zeta value 0 potential.

So, instead of phi we replace these Z is zeta and as long as this Z is zeta the numerator is much less than k B T. Then one can replace the sin h u as u ok, that is what has been done in this step with this approximation sign. And one can show that for standard parameters for example, Z is considered 1 or 2, e is the charge of an electron, T is if you take ordinary room temperature k B value is that is a Boltzmann constant available. If you put those numbers one can get zeta as much less than 26 millivolt at room temperature.

So, if some is working with a system where zeta potential value is much less than 26 millivolt at room temperature. So, one can says that this approximation is valid so, then this approximation is now it is written as phi z divided by lambda D square where lambda D square takes into account these entire term this is called lambda D square. So, phi z by lambda d square, now lambda D if you look at it this if phi changes like this ok.

So, then one can write phi is equal to it is it can if you solve this equation D square phi D z square, phi z by lambda d square if one solves this with this assumption that at Z is equal to 0 with this boundary condition that at Z equal to 0 phi is equal to zeta. And at z equal to infinity phi is equal to 0, because it is tending to 0. If one uses this boundary condition and this D square phi D z square is equal to phi z by lambda D square then one can get phi is equal to zeta e to the power minus z by lambda D.

So, that is the form one can end up with. And then one can say that lambda D is some characteristic length ok, some characteristic length by which this potential value comes down to some x 2 to a significant extent and what is that significant extent that would; that would that you can find out from here. So, this lambda D is referred as Debye length and this lambda D is one measure of the thickness of these electric double layer. So, now lambda D is this so, this is though this is one way one can do it, but we are not exactly so, now this is the zeta potential and this is how we have defined it here.

We are not exactly interested in a one dimensional problem like these rather, if we work with a parallel plate geometry not like it is Z tending to infinity if we work with a x z system where x is in this direction and z is at the center of a channel and channel is comprising of two parallel plates. The distance between two parallel plates is h. So, if this is the case distance between two parallel plates is h and z starts from the center.

So, z goes from minus h by 2 to plus h by 2, if that is the configuration and then one has to solve this equation D square phi D square equal to phi z by lambda D square, but now with a different boundary condition not this boundary condition. Now if you work with this boundary condition of parallel plate geometry where phi plus minus h by 2 equal to 0. Phi at plus h by 2 is equal to 0 at the wall, sorry this has to be zeta.

Phi plus minus h by 2 is to be equal to zeta; that means, at z equal to plus h by 2 phi would be zeta z equal to minus h by 2 phi would be zeta and due to symmetry at the center at z equal to 0 so, at z equal to 0 d phi d z has to be equal to 0. So, this is something which is these are the boundary conditions now applicable if you are working looking at a channel, because gradually we are heading towards porous media; it means because we are not interested in a infinite media.

So, now, if one solves this equation this would be the c 1 e to the power z lambda; because this is the general solution here, solution of this equation is c 1 e to the power mx plus c 2 e to the power minus mx. So, that same equation we are that same equation we are solving here also.

Here also we solve the same equation, but in this case since at z equal to infinity phi tends to 0 and z equal to infinity this term was blowing to infinity so, at that time we set c 1 as equal to 0 and so, automatically at z equal to 0 we used these other boundary condition at z equal to 0 phi is equal to zeta. So, from there we got these zeta is equal to c 2 into e to the power minus 0. So, c 2 so, that is why this c 2 is was replaced by zeta at that time, but now we have a different boundary conditions so, we have to follow.

So, this is the governing equation this remains same, but then you have to you put these as the boundary condition and you have to find out what is c 1 and c 2 for example, one condition will be c 1 by lambda D minus c 2 by lambda is equal to 0. So, you have to work with these boundary conditions, if one does these then one gets a solution out of it; which is phi z is equal to zeta cos hyperbolic, z by lambda D divided by cos hyperbolic h by 2 divided by lambda D.

So, this is the; this is the equation for phi z. So, one can see that this phi z would be one can see that it would be zeta at the wall and then it would it will change to 0 within a very short distance, I mean if we look at it this way. Here also we will have a Debye length on this side here and also we will have a Debye length there are although this would be much smaller. I mean here I am right drawing it much larger, if this h is 1 micrometer this is much smaller to that effect.

So, now they are here it would be from zeta to this will come down to 0. So, from zeta to it will come down to 0 and then it will remain 0 again it will go to zeta near the wall. So, that is how we are looking at the profile for phi in this case inside the channel and that profile is given by this equation ok. If you plot this equation you will get this kind of a plot that this would, be that phi is equal to 0 most of the places and near the wall it would go increase to zeta and here it will increase to zeta. So, this would be that type of profile one can get and this is exactly that is what you get here.

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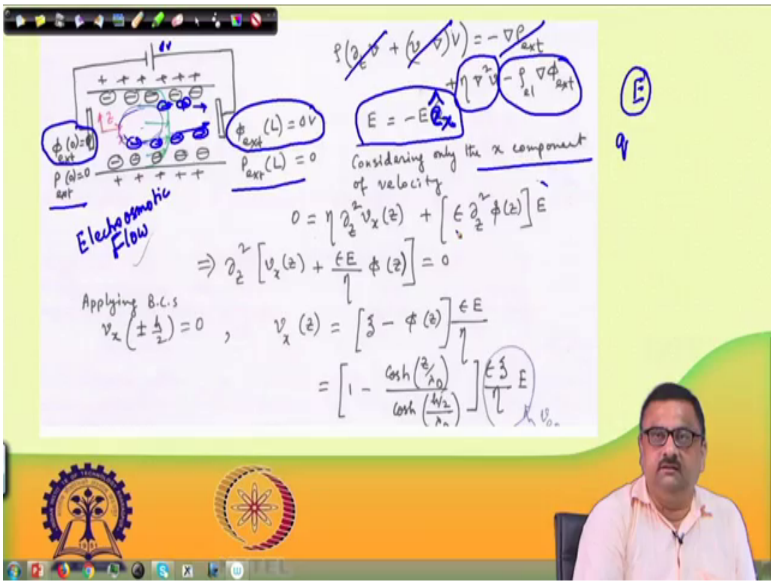

Now, with this phi z being known a situation where we have this wall all positive charges it has, now it is positive charges here and we cooled in let us say negative charges. I mean its not a deprotonation, deprotonation would have taken give us si o h and we said the deprotonation leaving s i o minus, but it does not it is some other it is made of some other material and some other charge transfer has taken place.

So, let us say now this is the situation here, you have all minus charges they are tucked here. Now suppose I have apply a voltage across these in this direction. So, if one applies a voltage this is positive and this is negative you can see the symbol here. If one applies the voltage there you can see that of what they have done here is that this, here the potential that is applied at this point is delta v against the potential on this end is 0 so; that means, delta v is the potential that is applied across this, and pressure on this side is 0 pressure on this side is 0.

So, there is no pressure gradient only there is a voltage gradient. So, because of these what this will do is, since these are negative charge and these are positive charge a positive this is the plus electrode positive electrode. So, these negative charges there would be more free negative charges here so, these negative charges would be pulled towards this positive electrode. So, because this negative charges are pulled towards the positive electrode so, they will impart a body force on the fluid the neutral fluid that is inside.

So, they will impart a body force on this neutral fluid and so, one can see there is fluid continuously flowing from left to right and this type of flow is given the name electro osmotic; electro osmotic flow. So, one is to solve the Navier Stokes equation to find out what would be the velocity profile etcetera. So, one is to solve Navier Stokes equation which is written here.

This is the transient term, this is the convective acceleration, this is the grad of P external which is 0 anyway in this problem, then you have that viscous terms here. And there is one additional term which was not considered earlier, this is minus rho el grad of phi extra phi external and this is the columbic; you remember what was the columbic suppose I have any in an electric field. If there is a charge q is placed so, by what force it would be pulled that is known as columbic force right.

So, E is basically minus grad of phi external right. So, E is grad of the potential electric field is grad of the potential, and there is this q is the charge ok. So, it is synonymous term means charge density here. So, because this terms in Navier Stokes equation they are on a per of per unit volume basis. So, one needs to consider these and these would be the body force term one has to consider in Navier Stokes equation. So, by considering these additional term and treating E as E has only ex hat ex hat. That is the only it has only x axis and that is E ok.

So, these electric field is not existing in e y hat direction or e z hat direction it has only ex hat direction electric field. And since I have written x has positive in this direction whereas, this is positive and this is negative so, that is why you are writing it here as minus E ex hat. So, now doing all these and if somebody is only looking at x component because there are other components will be gone.

So, if one focuses on this then they are left to it only this term would be left to it because this is there is no external force this is gone this is gone. So, if one will next left with this term and this term.

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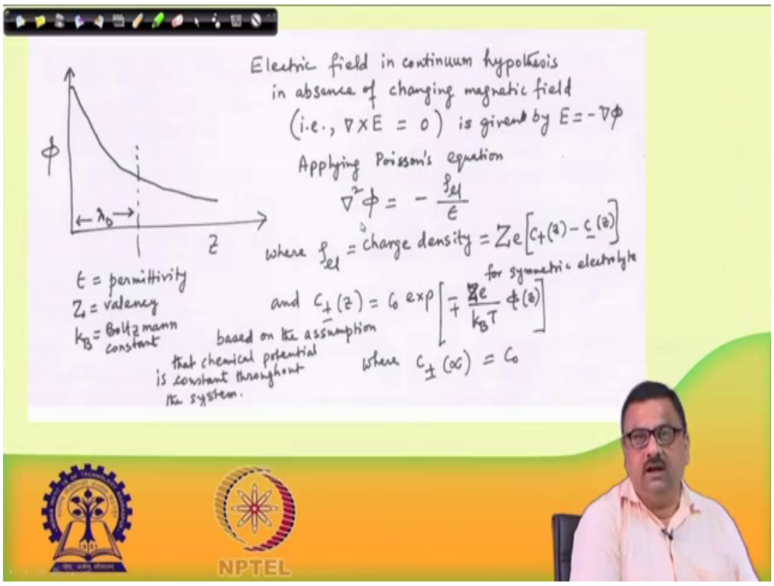

So, now this term here one uses Poisson’s equation once again and ended up with this you remember the Poisson’s equation what did you do at that time, this was the Poisson’s equation grad square phis phi is equal to minus rho el by epsa so; that means, what we did here is grad square phi is equal to minus rho el by epsa. So, rho el is given here as grad square phi into epsa from that equation Poisson’s equation and grad of phi is here as E. So, this is how this equation is changed using Poisson’s equation it came from here to there.

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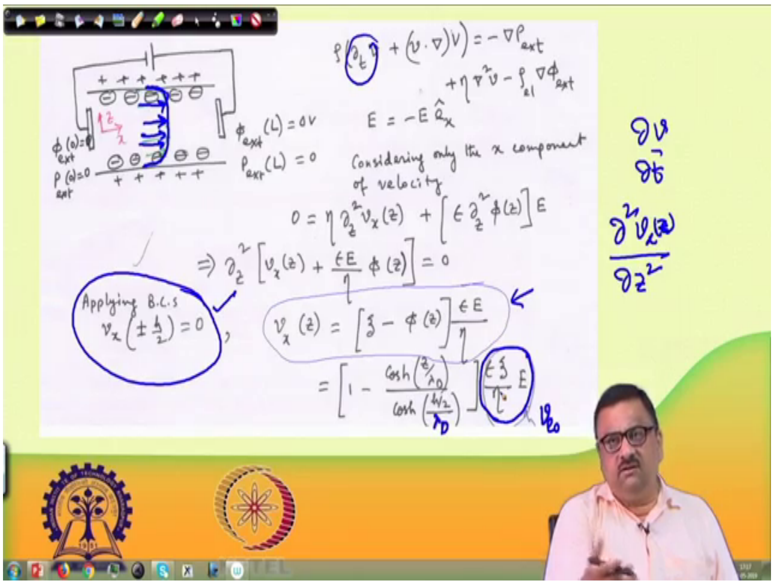

So, then you write this entire thing under del z, so, this is a particular nomenclature one must make note of here is that when we write del t v; that means, del v del t we are talking about ok. When we write let us say del z when I write del square z v x so; that means, we are talking about del square v x z v x z del z square. So, like this so this is so, we can write this as del z square into this quantity.

So, now if we take we have this as the boundary condition; that means, at wall the velocity wall means z equal to plus h by 2 and z equal to minus h by 2, so, at either of these the velocity would be 0. So, this is that this is one boundary condition we have. So, if we apply these boundary this boundary conditions if we take a solution of these first of all del z del square these del z square is equal to 0.

So, del del z of this is equal to constant right and then this quantity would be then constant multiplied by z like this. So, one can work with these and apply these boundary conditions then this is the solution one ends up with. So, this is the solution one ends up with. So, with this solution then in the last slide we have already derived this phi z as zeta into cos hyperbolic z by lambda D divided by cos hyperbolic h by 2 divided by lambda D, this is something which we have already established that phi z.

So, we are now just putting it here and zeta we are taking outside. So, we are taking this zeta outside so, this becomes. So, zeta comes outside and so, these becomes the expression for v x z. So, here in green you can see this v x z is plotted. This is the velocity profile if somebody plots these one can see that this velocity profile it develops within a very short distance from the wall and then it remained practically flat I mean this is the trend of this 1 minus cos hyperbolic this thing right.

So, if you look at these profile, if somebody plots this is what they will end up with and again near this wall there would be within a very short distance it turns to 0 and what is the velocity in most of these places. In this entire cross section that velocity is basically epsa zeta divided by eta into E this quantity. So, I mean if somebody wants if somebody can afford to ignore these development of velocity near the wall and if somebody is interested in; what is the average over this entire cross section.

Without considering this change within a very short distance near the wall then one can simply work with these as the velocity ok. Because most part of the cross section this is the velocity that would be operating, what are the terms it has, it has epsa which is permittivity which is which comes from electrostatics ok.

It has zeta, zeta is that same zeta potential we have a way to measure zeta potential experimentally and zeta potential is a property that is this of the system; that means, the wall material that you have chosen and the electrolyte that you have chosen. E is the electric field; that means, how much voltage you applied; that means, if the length of this channel is 1 centimeter and you have applied 10 kilo volt then e would be this 10 kilo volt divided by that, the magnitude of E would be 10 kilo volt divided by 1 centimeter. Because in unit of e is volt per centimeter.

So, this is the E and eta here is the viscosity, eta is here is a viscosity ok. So, this is known as a velocity electro osmotic. So, this is electro osmotic velocity and if somebody does not bother much about these they can very well say. That if I apply a voltage gradient then I can expect an electro osmotic velocity to develop within this fluid and magnitude of this electro osmotic velocity is given by epsa zeta y eta into E. Eta is the viscosity of the fluid; that means, sodium chloride solution if you are working with what is the viscosity of sodium grad. If it is near a it is the mostly aqueous solution it will be the viscosity of water.

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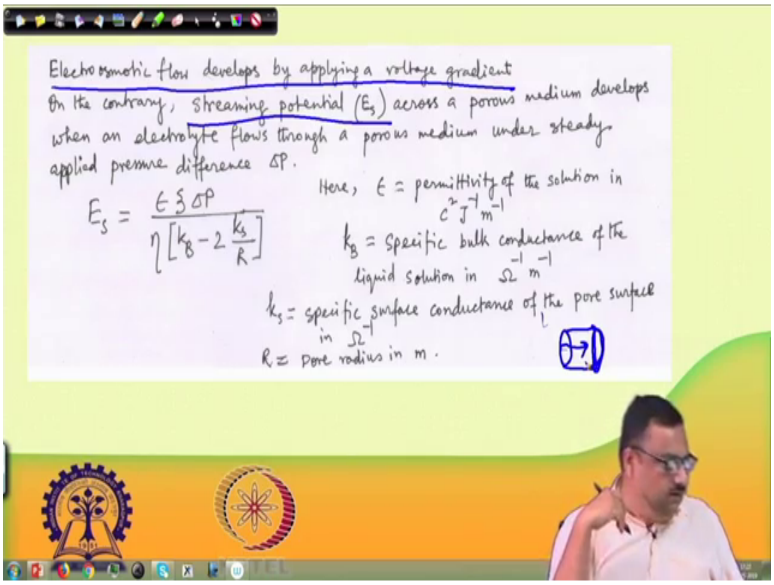

Now since I can apply a voltage and I could get away with I can produce a flow. So, now you can think of a situation where I have not applied a voltage, but now I have applied a pressure gradient. Here the pressure gradient is 0 here the pressure gradient is 0 and the voltage is applied and I am seeing a fluid flow. So, now suppose I do not apply a voltage, but I apply a pressure gradient. So, I am having a flow against all these in a potential developing, charge a electric double layer developing. So, then what would be the implication to the voltage in that case.

So, that would; that would be the next question immediately people will ask right. So, here I must point out that electro osmotic flow develops by applying a voltage gradient. On the contrary a streaming potential which is given by the name es streaming potential across a porous medium develops, when an electrolyte flows through a porous medium under steady applied pressure difference delta P ok. So, in this case now the I am applying a delta P across the porous medium and because of this delta P there is a flow taking place.

But because of this electric double layer and this charge you can charge developing near the wall, charge building up near the wall, zeta potential and all these there would be there would be a streaming potential developing across the pore now. And that magnitude of the streaming potential is given by in this case it is epsa zeta delta P divided by eta into k B minus 2 k s by r ok. I do not give that derivation here I mean I just added the slide because since we are talking about this zeta potential to understand why there is a dislodging or why there is a migration of particle.

So, I thought that this is also an important information you must keep in mind here in this case. So, here epsa is the permittivity of the solution that is flowing through the porous medium, whose unit is coulomb square Joule inverse meter inverse and k B in this case is specific bulk conductance of the liquid solution in ohm inverse meter inverse. Specific bulk conductance of the liquid solution k s is the specific surface conductance of the pore structure in ohm inverse and r is equal to pore radius in meter.

So, this streaming potential is very much; very much an important concept I mean when particularly when people are working with micro fluidics. When one is working with a porous medium in a micro fluidic sense suppose, I mean I can give you a small example quickly that suppose there is in case of a micro fluidics let us see there is a porous medium. And some fluid I do not know say; let us say I have a mixture and I know that I have a mixture here out of that some fluid would be traveling through this porous media, which fluid is traveling. So, I have a way to measure these E s the streaming potential.

So, from these I can get some idea what fluid is flowing through this porous layer For example, one is talking about a cell membrane through which there is some flow taking place. So, cell membrane I mean to say biological cell membrane. So, one can measure the streaming potential and from there one can think of going backward and see what was what is trans what is trans transporting through that membrane ok.

What should have been streaming potential had these been the fluid with this bulk conductance etcetera. So, this is this streaming potential is extremely important in from the concept of micro fluidics. In fact, this electro osmotic flow that I mentioned just now this is also very very important in the concept in the context of micro fluidics.

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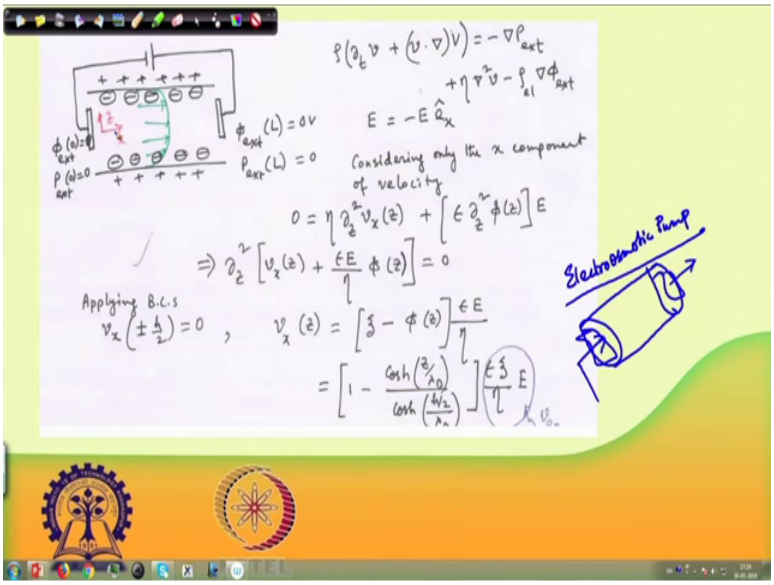

So, electro osmotic pump operates on this principle that this there is a channel and I mean it is not; it is not a micro channel as such they construct a porous medium and with the understanding that inside porous medium such type of channel will develop and such type of; such type of ion this electric double layer etcetera they will form and so, if I apply a voltage across these I will simply have a flow very precise flow.

So, there is a channel filled with glass beads has been considered for making this electro osmotic pump in this pump can deliver a flow of course, very, but did this flow would be very precise. And this pump will operate only by application of a voltage. So, only by application of a voltage this pump will operate and I mean whatever voltage is applied a very precise flow would be generated at the outlet. So, no moving a parts or no I mean what conventionally we see in a pump for example, piston impeller.

So, those will be or even in a micro fluidic sense there would be piezo piezo operated there would be; there would be; there would be other I mean even if there is not an impeller, but a similar mechanism would be in place, but that needs huge amount of investment in terms of micro electromechanical systems, but here by a simple channel filled with glass beads and by applying a voltage one can create a pumping action where the fluid will flow this is known as electro osmotic pump and for this a porous medium is used as this to create this kind of a; create this kind of a channel.

And so, this is the electro osmotic pump and on the other hand it could be the other if a porous medium I mean say for example, a cell membrane where you see this is the streaming potential then from that one can go backward and find out what is transferring transmitting through these through this membrane. So, these are some of the micro fluidic applications. So, and I thought I must mention this as part of this lecture since i am discussing about these electro osmotic flow.

And since I am discussing, about this zeta potential. So, essentially this zeta potential is there will be an overlap of electric double layer which will cause the repulsion and this fines will be then dislodged from the surface. So, what I will do in the next lecture is I will close this discussion on this why this fines get migrated and then I will get into immediately how to how what kind of what kind of conceptual models we have when it comes to these interception process; that means, I have a pathway through which some particles are flowing.

So, what are what is exactly happening there how the plugging takes place and how we keep track of it for example, are we supposed to draw are you supposed to write a continuity equation now on the particle itself or what is it how we characterize that because essentially at the end of the day we are interested to know how pressure drop changes. So, if there is a plugging pressure drop will increase. So, at what rate it will increase can we predict this. So, that is something which we are heading to in the next lecture. So, that is all I have as far as today is as far as this lecture module is concerned and.

Thank you very much.