Flow through Porous Media

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Lecture – 44
Immiscible Flow (Contd.)

Welcome you to this lecture module of Flow Through Porous Media. We were discussing about the Immiscible Flow and to be specific we were talking about Buckley Leverett front and movement of Buckley Leverett front. We have already discussed about relative permeability and two phase flow and we are extending this discussion to Buckley Leverett formulation.

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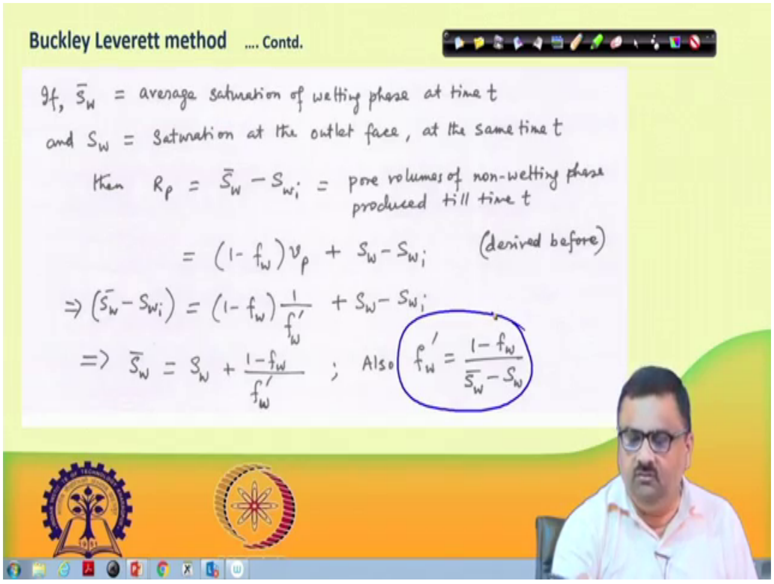

So, what we ended up in the last lecture was that this f W prime is equal to 1 minus f. So, the this is one expression that we ended up in the last lecture. What was the origin of this?.

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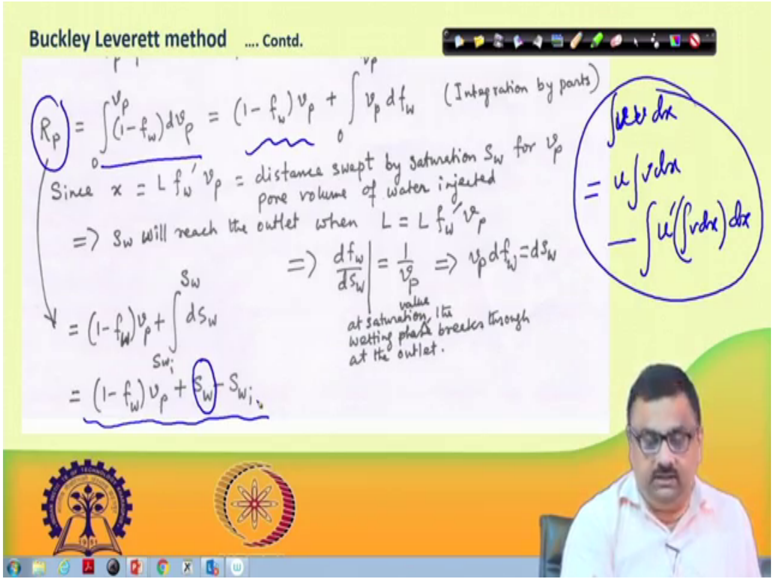

If we go quickly to these, we had earlier we were talking about the oil produced and then this expression we had integrated by parts. So, as I mentioned in earlier section if you write u v d x this integration is equal to u integral v dx that is the first term here and minus integration of u prime that is derivative. So, that is why we have minus of d f W. And integration of v dx this entire thing again integrated over dx.

So, this is this was the equation for by parts that I mentioned in the last lecture and this is something which is which we had done here. So, we arrived at the R P is equal to this quantity where this S W represents the saturation at the outlet.

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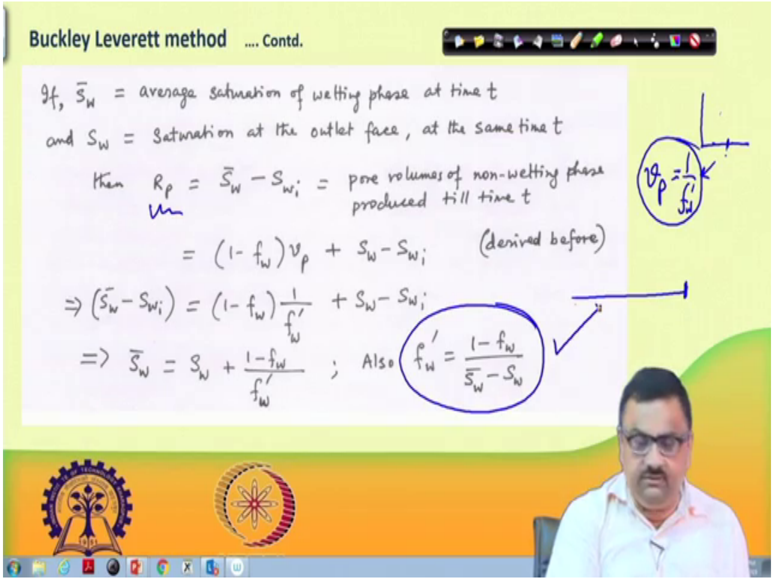

And next what we did is we had talked about these R P is equal to S W bar minus S W i and so here we had now equating these two; we ended up with this expression that f W prime is equal to 1 minus f W by S W R; S W bar minus S W. And mind it S W bar is the average saturation over the entire pore and S W is the saturation at the outlet. This is something we have in mind because for writing this equation this writing this part of the equation; writing this part of the equation we have assumed that v P is equal to 1 by f W prime or f W prime is 1 by v P.

So, this is only valid; if we measure this f W prime we estimate this how do you estimate f W prime? From f W S W curve. So, we can draw a tangent. So, this must be done this f W prime must be done at a particular S W value and that S W value corresponds to the saturation at the outlet, not any other S W values. So, then only this equation is valid. So, this S W is not any arbitrary S W; it is S W at the outlet. So, this is something which we got.

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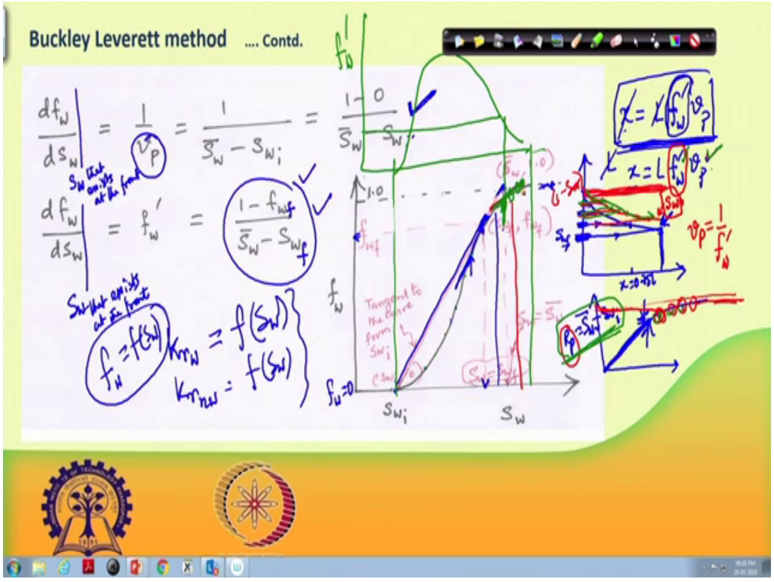

So, now, if we note further that d f W; d S W; d f W, d S W this is that slope of f W, S W curve at a particular S W that is equal to 1 by v P that is f W prime is 1 by v P when? When we had written x equal to L; f W prime v P and then instead of x, we put x equal to L and this L and this L was cancelled.

So; that means, we are looking we are talking about. So, when we defined x equal to L f W prime v P; f W prime is the is estimated at that saturation and we are looking at displacement of that saturation from the inlet end. So, now we are saying we are forcing that x equal to L; that means, displacement of that saturation packet is all the way to the outlet ok.

And so the situation that we are trying to simulate there; this f W prime corresponds to then the saturation at the outlet because these the same saturation for which f W prime is calculated we are talking about movement of that saturation packet and that displacement is x. So, that is why so this; so when we write d f W d S W or in other words f W prime.

So, when we write f W prime is equal to 1 by v P; we are talking about this case where d f W; d S W is measured at some value of saturation that exists at the that exists at the front when Buckley Leverett front breaks through. So, that is something which we are making here which we are assuming here. And this v P; v P is what? V P is the cumulative water injected and that is equal to till this point; that means, when Buckley

Leverett front has reached the outlet till this time what was produced? Only oil and what would be that value at that time?

So, we are talking about the situation just think of it; I am talking about a situation when Buckley Leverett front has reached the outlet ok. Earlier case was somewhere in between and then we there was a drop to S W i and continuing, but at this time Buckley Leverett front has reach to the outlet and so at this till this point the this thing was linear the cumulative oil produced versus water injected that is linear.

So, whatever water was injected that same amount of oil would be produced and that would be what? S W bar minus S W i till this point, beyond this point this will change beyond this point v P would be greater than S W bar minus S W i. Because S W bar minus S W i represents the amount of oil that is pushed out of the system; S W bar is the average saturation, S W i is the initial water saturation; so these difference arose because some oil I have pushed out of the system.

So, these difference is the total oil produced and since we are talking about the end point of this linear regime. So, we can assume till this point the amount of water injected is equal to oil produced. So, that is why v P is equal to S W bar minus S W i and instead of 1, I can write it; I can extend it to 1 minus 0; I can extend it to 1 minus 0. So, 1 minus 0 divided by S W bar minus S W i; this can be written for d f W; d S W at the way S W that exists here.

So, at this value of S W this should be true. On the other hand, d f W, d S W if we look at just the previous slide we have already shown that d f W; d S W is equal to 1 minus f W by S W bar minus S W; this is just in the previous slide we showed that this is true ok. So, now, you think of the curve f W versus S W and this is again valid and that S W that exists S W that exists at the front.

Now, you think of this curve this was the curve right. So, we are saying that we are looking at some kind of some S W value for the front and I am saying on one hand this S W say let us say I am talking about an arbitrary point; I say let us say this is S W ok. So, this is that S W at the front; so I am drawing a slope here.

So, we are saying on one hand this slope is equal to 1 minus 0 by S W bar minus S W i and on the other end the same slope would be 1 minus f W by S W bar minus S W ok.

And to be to separate things out let us say I put another subscript 1 minus f W f because this f W is calculated at this value of L S W; del f W, del S W is calculated at this value of S W. So, instead of recalling it S W that exists at the front let us write it as S W f and let us call this S W f; W f ok.

So, this slope has to satisfy these two conditions simultaneously and how you can satisfy these two conditions? Now, let us look at the limits here; f W the limit here is between 0 to 1 and S W we have some limit of S W i and further on that side. So, suppose I proposed something here that S W f is that point which is; if I draw from S W i, a tangent to this curve if I draw a tangent to this curve which is this; this tangent.

So, if I draw a tangent to this curve wherever hit; I mean if we try to draw from S W i; wherever it hits the curve that point is S W this; this corresponds to S W f and this corresponds to f W f; I mean I can say anything, where is the proof to it? So, let us see the slope of this line the slope of this line is and farther if I say on another step ahead and say wherever it cuts this line of f W is equal to 1.0; that means, this line wherever it cuts this line; I call this the corresponding saturation as S W bar.

I mean I can say anything, but what is the what is; where is the proof to it? First of all, what is the slope of this line? Slope of this line is here the f W f is 1. So, the coordinate of this point is x value is S W bar I said that is my proposition and 1.0; y right. So, this is the point and this is the point I said x value is S W f; the front saturation again that is a proposition and the corresponding value of f W this is f W f.

What is the slope of this line? It is this y value minus this y value divided by this x value minus this x value. So, what is the y value? Y value is 1; here the y value is f W f right. So, 1 minus f W f; I can see it is 1 minus f W f and S W bar minus S W f; S W bar minus S W f. So, this particular slope is same as this and if I look at the overall slope; if I look at the overall slope that would be how much? The overall slope would be 1 minus 0; I know here f W is equal to 0; f W starts from 0 at this S W equal to S W i and f W is equal to 1 here when S W is equal to 1 minus; so r.

So, here it is if I look at the slope of this entire line it is 1 minus 0 the change delta y is 1 minus 0 and delta x is S W bar; S W bar minus S W i. So, slope of this entire line if I look at the slope of this entire line is 1 minus 0 divided by S W bar minus S W i and that is exactly this one.

So, I can see that from S W i if I draw a line which is tangent to this curve; s wherever it hits the curve as tangent that point is S W f; correspondingly we have f W f. And wherever if we extend this tangent; wherever it hits the f W is equal to 1.0 line that is going to be the S W bar. Then that fits perfectly to this definition because both these f W prime values I have to satisfy and this is only possible if we call this S W f and further extend and wherever it hits; we call that as S W bar.

So, this is going to be a very crucial figure all together right because suppose I give you the expression for k r o and k r w or k r wetting phase and k r non wetting phase. So, some I have to give you some form of equations say let us say k r w is equal to some correlation I have to give in terms of S W. And similarly k r non wetting; I have to give you some as a function of S W.

See once S W is the if since we are talking about two phase flow; if S W is defined automatically in other phase other saturation is defined, it would be simply 1 minus that quantity. So, let us say we have; I have I give you these expressions the; if you have these expressions immediately you can calculate what is f W; f W as a function of S w. So, you can construct this curve; moment you can construct this curve and locate this S W i, you can simply draw a tangent to this curve; you do not have to do anything just locate; these points using these expression you get these points, fit a line through them and then simply draw a tangent from S W i and wherever it cuts you call this S W bar.

So, now the problem is resolved at what point this will happen? It will happen what is the corresponding R P? R P is simply S W bar minus S W i; S W i, I have already; I know the system the porous medium; it may be having to say 0.1 is S W i is 0.1 or S W is 0.2. So, that the porous medium that any experiment on that porous medium the experiment that we mentioned; initially it was all water and then we inject to oil for 10 to 24 volumes; see what is the end point saturation, then we inject again instead of oil we inject water again and 10 to 24 volumes see what is the end point saturation.

So, there I mean S W i can be done separately from the experiment that would be provided. And S W bar already i have drawn the curve from S W i; I will draw a tangent wherever it hits the f W equal to 1.0 line; so that is as W bar. So, moment I have S W bar momentum S W i; I already have, I subtract these two that would be the oil produced the pore volume of oil produced till the breakthrough, I can immediately get that nothing no; no more calculation.

So, this is the; this is the; this is the immediate calculation we can do. So, till this time there would be it; it would be 45 degree line and beyond this time it will start turning, but this point we have already located ok. And what is this S W f? If we try to find out what is the saturation profile. So, this S W f would be this point; I mean we draw a tangent. So, when the Buckley Leverett front breaks through; that means, I started injecting water and that water; here I am injecting pure water no oil and pure water in the sense only what; not water plus oil. So, water is injected and that means, wetting phase is injected. So, here I can maximum I can go is 1 minus; so r.

So, that is the maximum we can go 1 minus; so r because minimum if so are amount of oil has to remain in the pore space. And in between there would be saturation will decrease slowly and at this point these is; just the time when the Buckley Leverett front has broken through. So, this is the corresponding S W f we have.

Now, suppose I want to know what is the at this distance let us say at x equal to 0.75 L; that means, three fourth distance; three fourth of the length if we go three fourth of the length from the inlet what would be the corresponding S W. How will I find out? I can; I need to construct this curve right. So, to construct this curve what we have to do is first of all; I pick up another saturation, see mind one thing these particular saturation is the front saturation.

So, we are talking about this part of the curve which is important to me because I can see beyond S W f; it is only it is water saturation is increasing inside the pore. So, we do not need to bother about this lower part of the curve below S W f; we are concerned with the upper part of the curve. So, at any point; I draw a tangent, here I can draw a tangent. So, any point you can draw a tangent and that.

So, suppose I draw a tangent; I can find out immediately what is the corresponding f W prime ok. So, suppose I want to know this particular saturation; I mean suppose some other saturation here; so, what does this curve mean? This curve means I have this S W f saturation and given this after v P volume of water injection; these S W f saturation has travelled by distance x equal to L.

I am talking about a higher saturation packet with higher saturation. So, that will also travel by some distance which is x, but not L. So, how far that saturation will travel? So, how do we get that? Now, we have to use this equation once again x equal to L; f W prime v P that is that same mother equation; one has to use.

And in that same mother equation we see v P is already known because I am it is that curve that is frozen this is after v P pore volume of water injection. Now, I am just interested in the saturation at any intermediate location. So, the v P is fixed; v P is fixed I am just for the same v P we are looking at different saturation packets how far they have travelled. So, v P is already fixed; L is fixed and f W prime; so for corresponding saturations, I have to draw a tangent.

So, for this saturation I have to go up, draw a tangent, find out the slope use that as the f W prime because that would be the governing f W prime and with that f W prime with the given v P and L is known how far that saturation front will travel? I find this will travel up to this much; this saturation packet will travel up to this much this saturation packet will travel up to this much right like this.

And the saturation packet which is having 1 minus so r which is near the end this will not travel any distance. So, because the f x x is equal to 0 in that case; that means, f W prime has to be infinity in that case. So, one; so this is how this curve has to be read and from this curve for tan drawing tangent at different locations; now drawing tangent in the sense that we are doing it graphically just to get a better feel for this. If somebody wants to do it analytically that is also perfectly acceptable.

Because you can have an expression analytical expression of f W versus f S f S W; f W that fraction fractional flow curve as a function of saturation and that function can be simply differentiated ok. So, this is something which one needs to do for finding out these locations. And in fact, now when this thing exits now suppose this you have reached the outlet; you have already found these curves. Now, when you go beyond this point; let us say when you; when you; when you when this front has traveled out from this; further water was injected. So, at that time; at that time the saturation front will take the shape something like this.

So, here the S W will be higher; now this would be the S W, but this S W would be higher not S W f; S W f is unique for this system and that is when Buckley Leverett front has reached the outlet. But now if you have another S W, so you go to that S W, find out what is the f W prime for that S W ok. So, with that f W; you can find out then v P is equal to for when; when S W is that same that at the outlet then one will have v P is equal to 1 by f W prime.

So, now for that particular S W we are looking at so, some other S W at a higher value of S W this is the new S W because some more injection has taken place; so the v P has changed. So, now we have to draw another tangent to this curve here; find out what is the corresponding f W prime, take the reciprocal. So, that would be the v P ok. So, for that v p; so that would be the v P; v P of water injected and then you one has to find out. Then correspondingly these tangent wherever it hits this curve that gives you the S W bar at that time.

So, again you have to repeat that same exercise of S W bar minus S W i and then you one has to; one has to; one has to repeat that same exercise to find out the corresponding value of once you get S W bar with S W i you can get the corresponding R P and then again you repeat these points for various saturations how far it traveled. So, you can get this curve and you can generate these points here.

And once you once this so this; so if this when this process continues at the end; we expect that this entire porous medium will have 1 minus; s o r that saturation. So, entire porous medium will be having single situation. So, this is how this saturation will continuously grow to a single value of the saturation of 1 minus s o r the endpoint saturation for this porous medium. And that time at that time you would say asymptotically this production cumulatively it is not changing anymore; it is the horizontal this line.

So, it is not changing anymore. So, these points you can track it down, I presume; I mean you can track these down. One can very well take some value which is higher than this S W, draw a tangent at this place; find out the corresponding S W bar, find out this is the S W against that this is this W bar. So, find out the difference to find out what is the R P produced at that time ok.

Against what is the v P 1 by f W prime is the v P. So, against this v P how much is the R P produced. So, from there with that v P; now you give again sent several packets to this end and then at that time your; your f W prime would be for all points; all points; all points higher than these. So, let us say this point; this point; so this is the outlet saturation at that time. So, beyond this outlet saturation whatever saturation values are there.

So, there this f W prime will be different again the f W prime will be different you have to put this corresponding f W prime; v P is same as 1 by f W prime corresponding to the outlet saturation that you have chosen to start with. So, that v P will remain there f W prime is based on these new saturation values; you have to draw a tangent and find out and you can locate what would be how far it would travel, how far the saturation packet will travel.

So, you can generate this curve and you can continue to generate these curves and based on the this they to start with wherever you are corresponding S W bar that S W bar minus S W i will give you the for that particular v P which is 1 by f W prime for this S W. So, for that particular v P how much of oil produced is given by the corresponding S W bar here minus S W i so that so you can get; so even generate these points as well.

So, one can use this ex; one can perform this exercise to generate these curves. So, this is more of a graphical construction and this construction would be made much easier if parallely one draws instead of f W S W curve, but the same S W for the same S W; if somebody draws f W prime curve also. So, it starts from S W i ends at s o r and in between it goes like these then comes out like this. So, this is f W prime goes like this.

So, instead of drawing tangent at every point and then doing this calculation; instead of that if first you pick up some points here and generate this f W prime curve; then it would be easier. I am talking about this saturation we do not have to draw tangent to find out what is the slope; directly we can read what is the corresponding f W prime. So, generally this is customary to draw f W prime curve also just against the same saturation against the; this just I mean you draw the saturation axis.

So, that they correspond to each other. So, that you can simply vertically you can raise these lines and find out the corresponding f W prime curve. So, the probably this becomes somewhat easier and if somebody can solve this analytically the even they can generate this curve analytically also; then you do not have to draw tangent either.

So, this is all I have as far as this particular lecture module is concerned and I more or less discussed about this Buckley Leverett method. And then I mean my next aim would be to conclude this Buckley Leverett method and then extend this whole understanding. Now, if we have we are talking about oil and water now instead of that if we have a three phase system and further moment we have three phase system when oil and gas; both are involved then one may have to consider something called a pvt relationship between oil and gas.

Because some components from oil will go to the gas phase that I mean oil then water they are immiscible, but moment you bring in gas you can never exclude that possibility. So, that also there I mean how to address that I will very briefly touch upon that in the next lecture. That is all I have as far as this module is concerned these lecture model is concerned.

Thank you very much.