Flow through Porous Media
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Lecture – 42
Immiscible Flow (Contd.)

Welcome to this lecture of Flow Through Porous Media. We were discussing about Immiscible Flow, flow of two phases.

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And in this connection we had talked about relative permeability and two phase flow. And now what we are discussing at the last lecture was this Buckley Leverett formulation. And we had particularly discussed about a porous medium, which has containing only the which is containing wetting and nonwetting phase.

And it was at interstitial saturation; that means, wetting phase is at interstitial saturation and so it is primarily non wetting phase. And then we are injecting wetting phase. So, it is primarily oil 90 percent oil, 10 percent water and now we are injecting water from the left end. So, and we are trying to see how saturation changes within this porous media. And how what would be what we get at the outlet.

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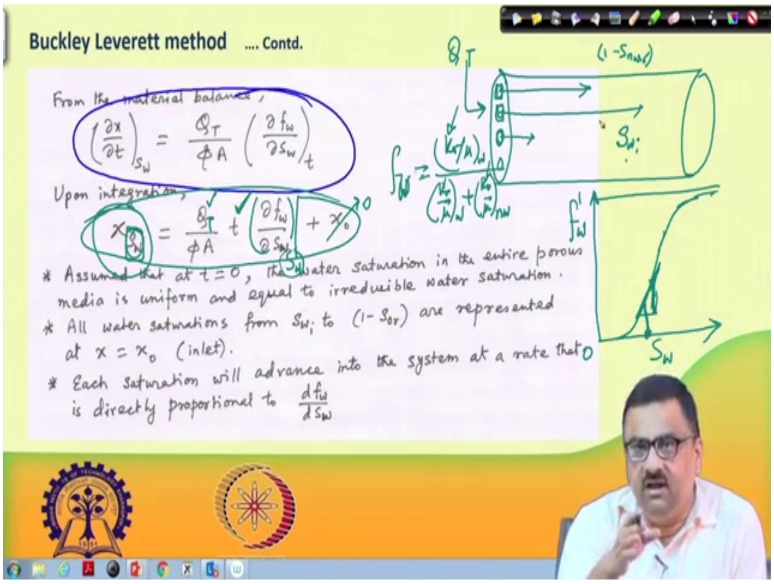

So, now we had discussed about this one form of Buckley Leverett equation. We had talked about these as we had talked about these as this was the. So, this gave me the our conceptual understanding is something like this. That if this is the porous plug, we have as if we have saturation packets at the inlet.

And these saturation packets are travelling into this medium some are travelling far some are traveling this much depending on what is the saturation value of this packet. So, this one is maybe having a saturation value of 0.4725 one is having 0.6321. So, depending on the saturation these X S W is the distance that particular saturation S W will travel. The depends on dell f W, del S W value for that particular saturation.

So, whatever saturation we are looking at that saturation whatever is the del f W del S W value. This saturation packet will travel at that same rate and how we get this del f W del S W? We said that, these we can we have to generate a curve of f W S W either a curve or a relation f W has a function of S w. We need to produce a curve like this, where this is the; this is the curve. So, f W will vary from 0 to 1.

And this curve we can obtain because, we know what is the definition of f W. Definition of f W is equal to k r by mu for wetting phase, divided by k r by mu for wetting plus kr by mu for non wetting. So, this is depending on some correlation that we put for kr wetting and k r non wetting. Depending on those these we can find out how f W versus S W curve we will come out.

And then for any r for any S W that we are looking at these S W will travel by one distance given the time t. So, we have to find out what is the corresponding S W? Let us say we are looking at this S W. So, we go up there this is the f W versus S W curve draw a tangent there. So, this gives me the slope of this gives me that del f W del S W measure the slope measure the slope.

And then put that slope here then you know this particular S W will travel up to this distance. X 0, is 0 at the inlet; that means, all saturation packets at t equal to 0 wherever that is. So, we assume all of them are at the inlet. So, then this is 0, this del f W del S W for this given S W we can find out from the curve that is W we find go up there, find this location draw it tangent find the slope put it here.

We want to know at a particular time how far this saturation front has traveled Q, Q T is known to us this is Q T and phi and A; these are known to us. So, for a given time how far this saturation bracket will travel depends on what is the slope of this f W S W curve at that saturation. Because slope is changing depending on what is the saturation right. So, and the for that particular saturation we are talking about this we have to find out that slope.

So, we said that each saturation so we considered, when we inject water we are injecting water at rate Q T. And but we are assuming that it started with S W i and now so we said that there is a range possible for saturation within this porous medium. The lowest value is S Wi the highest value is 1 minus S n w r or 1 minus s o r. So, within this range whatever saturation packets are possible they will travel as per this equation ok. So, each packet we need to find out how far they travel and based on that we have to generate the curve, how saturation changes with distance.

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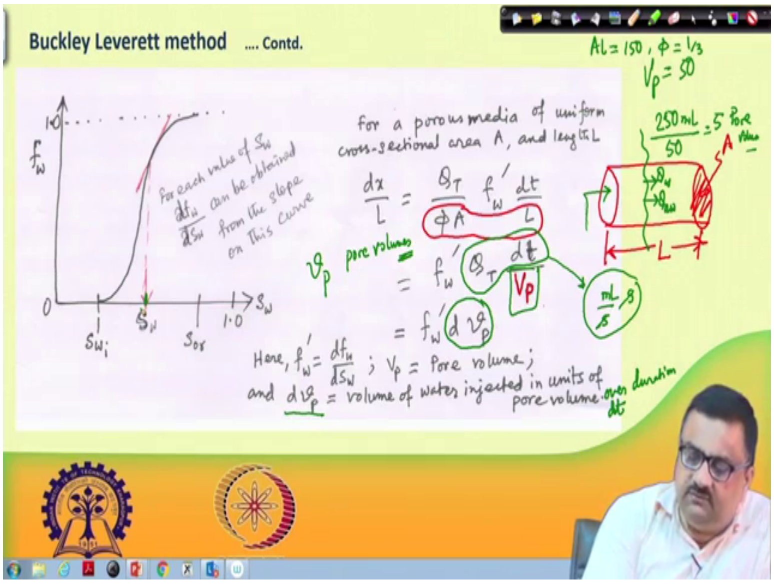

So, now, let us proceed further. So, now, if we look at this expression once again. Here we have dx, dx if we leave it here dx would be this side.

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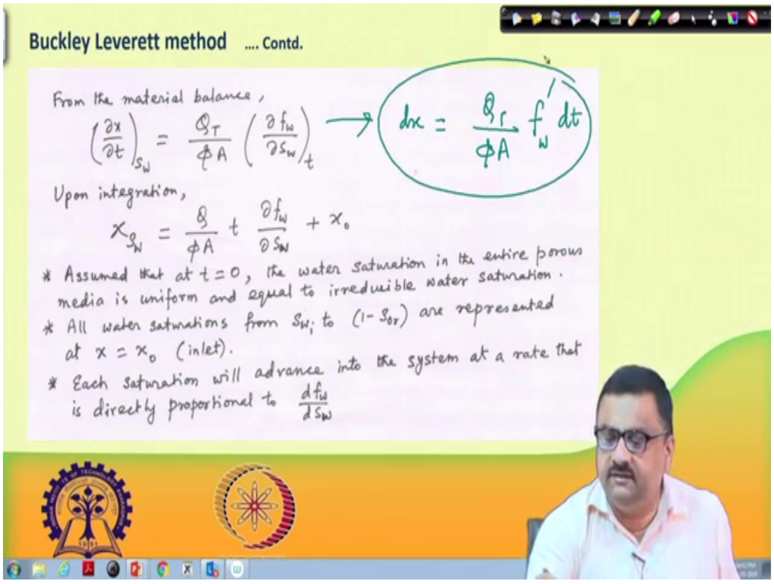

I mean when before integration, if we break it up in differential form it would be something like this. That dx would be equal to Q T divided by phi A, del f W del S W we can write this as f W prime but mind it f W prime does not mean del f W del t it is not a time derivative. It is the derivative with respect to saturation it is del f W del S W.

So, that we are writing it as f W and we write this as d t I mean from this equation we can write this right. So, now we this equation only these carried forward. So, dx is equal to Q T by phi A, f W prime dt this is what we said earlier right that just now I mentioned. So, only thing is now you have in the denominator we have one L on the left hand side and one L on the right hand side.

This is one additional the other term dx is equal to Q T divided by phi A, f W prime dt is following from the earlier expression that we put there. So, now let us see these Q T by phi A, QT divided by phi A; what they are doing is. Let us say we club these together we club this part together phi A and L, L is the length of that core.

We have this core that porous plug and on which we are doing this exercise. So, this is the length and A is the cross sectional area. So, A into L is that over total volume and multiplied by porosity phi into A into L. So, this gives me pore volume right, pore volume is what is a void volume of this core. So, that is written by capital V P, V is capital so, this is the pore the void volume ok.

It is the way that this I have a porous plug and it I have a porosity instead of that now I am talking about the void volume and then it is capital V P. So, instead of phi into A into L, we can write it as V P. And f W prime Q T and dt they remained as it is left hand side is dx by L. Now, you think of it, we are looking at Q T dt what is Q what does Q T dt mean? Q T dt means this Q T dt term Q T is the total flow rate, multiplied by d over dt time the time interval over dt over dt time step Q T dt is the total flow that has taken place over this dt time.

So, Q T is in meter cube per second, dt is some delta t small time step so, that is in second. So, this gives me so much of meter cube the second and second will cancel out. So, Q T dt gives me that so much of meter cube of flow injected into the system right. Q T was the total flow ok, total flow injected not since it is a steady state process. So, you can say that at any location. Let us say in this location, I have some amount of Q W flowing some amount of Q and W flowing. So, wetting phase flow rate is there, non wetting phase flow rate is there and they are some we are calling it Q T ok.

So, Q T dt is the total flow that has taken place at any location over time dt ok. So, now, if we divide it by the void volume what does that mean? We are writing it as d V P. I have flowed 100 meter cube of fluid at a particular location over certain time 100 meter cube of fluid. And I know that, the pore volume the original volume was 150 sorry not 100 meter Q.

100 milliliter 100 milliliter I have flowed at for over a particular time over this porous media. These porous medium the total volume is 150 ml out of that porosity is one third. So, when I multiplied by one third it becomes 50. So, 50 into 150 sorry, 150 into one third is 50. So, 50 milliliter is the void volume. So, for my V P in that case is 50 and A into L that gives me a volume which is 150.

I have arrived at V P is 50 because we assumed phi to be equal to one third. So, now, out of this now this is V P is equal to 50 suppose I have injected 250 milliliter at some point, let us say 250 milliliter let us say I am not talking meter cube per second we are talking about milliliter per second. So, 250 milliliter I have injected. So, 250 milliliter would be equal to 250 divided by 50 that is equal to 5 pore volumes that is what we had that is what we discussed before. The pore volume would be then a unit instead of milliliter. So, then what exactly we did? Whatever volume that has flowed, we divided it by V P write the void volume.

So, here also exactly the same thing Q T multiplied by dt that is the total volume that has flowed through some grid. And that divided by the V P, which is the total void volume this gives me d v p that is the pore. So, d v p here in on down d v p is volume of water injected in units of pore volume. So, d v p is volume of water injected in units of pore volume over this duration dt. Let us say we if we want to qualify this over duration over duration dt.

So, this is; so this is how. So, what do we get here dx divided by L is equal to f W prime d v p; where d v p is over this time dt I have injected some wetting phase, which is water over this time dt. And whatever I have injected if I convert it to this to pore volume. So, I say I have injected 250 ml over this time dt I am not going to express it in terms of 250 ml. Instead I will explain express this as 250 divided 50.

So, I will express this as 5 pore volume so, this instead of 5. So, this 5 is same as the d v p right, when we do the integration over d v p, when time t is finite time from 0 to some finite time. Then d v p when we do the integration this becomes v p, but that v p would be always small v p. So, mind it in subsequent exercise, we will write small v p this would be the pore volumes. I probably the best way to express it is you remember v p pore volumes. When you as when you see these volumes; that means, I am talking about these injecting 5 pore volumes.

Whereas, if somebody said only pore volume know S; that means, we are talking about capital V P, which is basically the void volume ok. So, void volume is fixed that is 50 ml, but pore volumes it could be 2 pore volumes, 5 for volume, 6 pore volumes. If somebody says I have injected 10 pore volumes; that means, that person has injected 50 into 10; that means, 500 milliliter.

So, this is how the arithmetic works here. So, for each value of S W; now for each value of S W we are looking at this so, we said that suppose I so, what is what this expression is all about.

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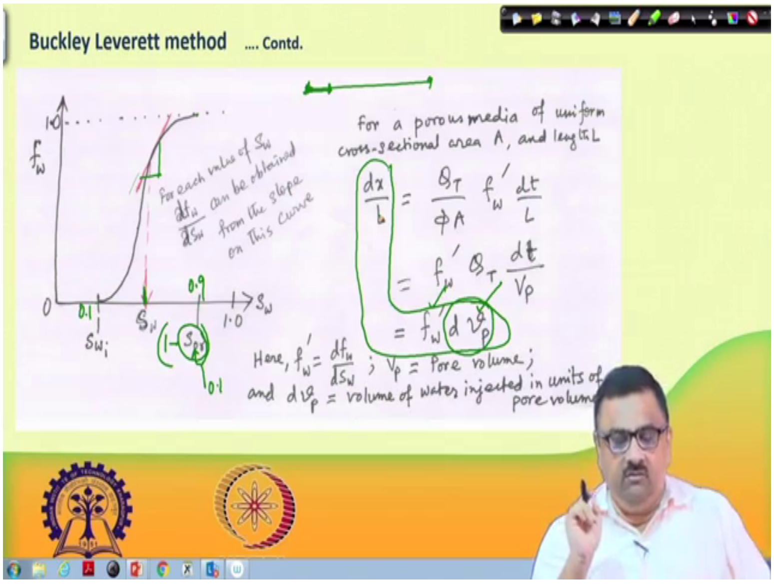

Let me clear this what this expression says is dx by L is equal to f W prime d v p, this is the final expression we have. So, now, we here we note that at for each value of S W let us say we pick up this value of S W. So, these value of S W we go there on these f W versus S W curve. You can see f W is within this limit S W i and we should be writing it as this corresponds to a S o r or S n w r.

But this is actually basically this is that the value here if you put it in one scale this is 1 minus S n w r. So, basically this is let us say we are talking about 0.1. Here if this s o r or a sin wr is also 0.1. So, then this we are looking at 0.9 ok. So, this our play of f W is between 0.1 and 0.9 and at 0.1 f W is equal to 0 at 0.9 f W is equal to 1 ok.

So; that means, here when the saturation reaches here all flow that takes place the entire QT is comprising of Q W. Because, f W is equal to 1 and f W is equal to Q W by Q T. Whereas, when it comes to the saturation is S W i then here I can see by looking at this curve, that corresponding f W is 0.

So, f W is 0 means Q W by Q T is 0 so; that means, there is no wetting phase flow. So, only phase that is flowing in these cases non wetting phase. And in between at any in between point you look at. So, it would be flow off both phases taking place both wetting and non wetting phase. And what is the saturation as the flow continues this way what would be the saturation?.

That, a particular saturation S W will travel by a distance dx or dx by L is the fraction of total distance. Total distal distance is L; total distance is L out of that a saturation packet starting from the inlet it would travel a distance dx that dx by L would be given by that for that particular saturation packet what is the f W prime value; that means, the slope of this line.

And over this I mean this to this displays for this displacement to happen. You must have injected some amount of water or some amount of wetting phase from the left end of this the from the at the inlet. And that amount here is d v p. So, you have injected d v p. So, now, you are trying to find out the saturation how far it will travel, which saturation? Some saturation you have in mind. So, simply draw that, find out f W prime find out f W prime d v p you already know you have imposed it. You can say that this much of distance this saturation front will travel. So, dx by L would be this.

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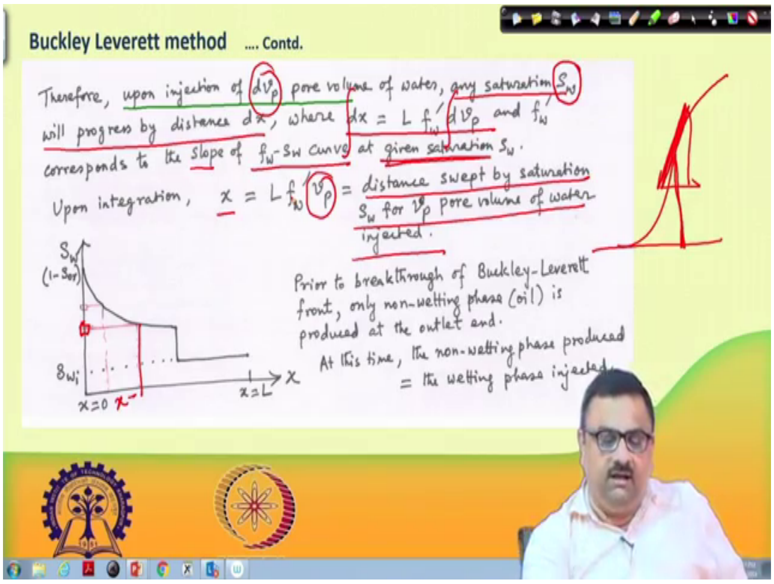

Now, so what do we have here? If we now think of that upon injection; upon injection of d v p pore volume, upon injection of d v p pore volume of water we have injected d v p pore volume of water we just now discussed. Any saturation S W, that we are talking about saturation packet ok. Saturation packet S W will progress by distance dx, where these dx in earlier page we had dx by L is equal to f W prime d v p or L.

We can take this to the right hand side. So, where dx this dx amount that movement that takes place is equal to L f W prime d v p. And f W prime corresponds to the slope of f W S W curve at given saturation. So, we are talking about a particular saturation here and for that saturation what is the slope of f W S W curve.

So, we have drawn this, we have found out what is this f W S W curve for this saturation we went up and found out for that saturation what is the slope of this line ok. So, from this we had drawn a tangent at that point on this curve and found out the slope. Now, if somebody does an integration of this dx equal to L f W prime d v p.

Then upon integration what they get is x is equal to L f W prime v p. because, if you do this integration on dx then you have to do this integration on d v p. So, this would be the so, what do we get upon integration distance swept by saturation S W for v p pore volume of water injected. So, we are talking about dvp now we have integrated it.

Now we are talking about a finite volume. So, v p pore volume of wetting phases injected from the left end. And we are trying to find out any particular saturation any saturation which lies within this range of S Wi and 1 minus S o r or 1 minus S W, S n w r. So, within this range any saturation packet how far it will travel so, that distance is x.

So, these distance is x as far as this particular saturation is concerned. So, it will travel up to this distance x; how would you get this? This x is equal to L multiplied by f W prime multiplied by v p; v p is the pore volume of water or pore volume of wetting phase injected at the left end. So, what we note here is prior to breakthrough of Buckley Leverett front only non wetting phase is produced at the outlet.

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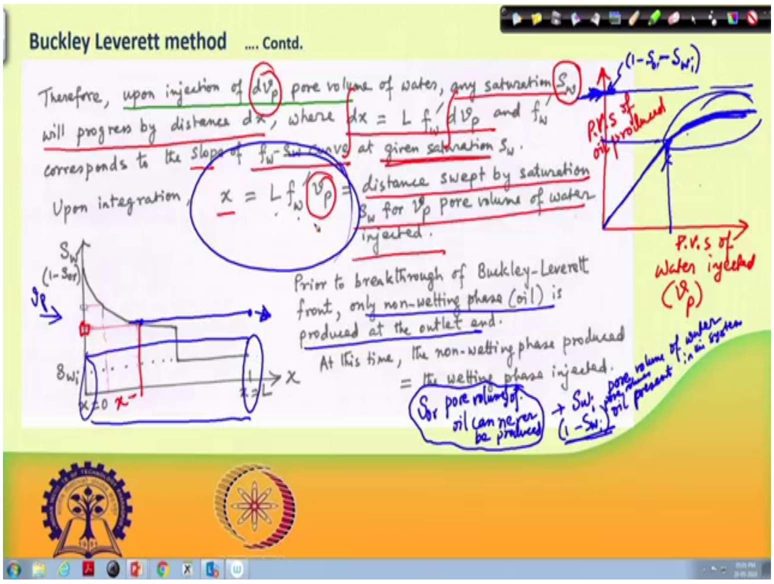

So, what we discuss we briefly talked about these. This is something like this that, if you have if you are plotting pore volume P Vs of water injected water and wetting phase synonymous water injected. So, what does this mean? So, this is essentially v p right. So, this is essentially v p. And here if we write pore volumes of; pore volumes of oil produced I write oil to just synonymously with non wetting phase oil produced let us see.

So, we said here that this will follow a straight line, because prior to breakthrough a Buckley Leverett front only non wetting phase is produced at the outlet end. Because this is say let us say this is x equal to L. So, the plug runs from so, this is the plug. So, this is the plug. So, plug runs from x equal to 0 to x equal to L. So, the front has come the Buckley Leverett front has come up to this point.

Beyond this point the saturation is interstitial saturation S W i. So, this S W i can never move S W i is all trapped. So, though I am injecting v p from this side I am injecting pore volumes of water here, but what is moving at this location is basically oil and what I am producing here is oil and system is all incompressible. So, if I inject v p amount of water I will be producing v p amount of oil.

So; that means, that this would be a 45 degree line whatever pore volumes of water I am injecting the same pore volumes of oil is produced. But the moment this front reaches the end here, beyond that point it would be producing both oil and water because its let us say beyond this point let us say this saturation would come here and arrive. So, this saturation means here now this saturation is not S W i.

So, when this saturation breaks through so; that means, it is a part is oil and part of it is water. So, now, so when this saturation breaks through then we have both water and oil producing at the outlet. So, from this point on this will start curving. And we said that the limit here that how much is the total pore volume we can produce is limited because, this is the total pore volumes of oil we can produce is 1 minus is S o r.

Because we have to leave say I have one pore volume of fluid in that system out of that I have to leave a S o r amount of oil in the system ok. So, naturally it is I have to start with, how much of pore volume of oil we had? We had and to start with we had S W i pore volume of S W i pore volume of water because S W i is just a fraction. Suppose we are talking about out of 50 milliliter, we have 10 milliliter of water and suppose we have out of 50 milliliter we have 10 million liter of water and 40 milliliter of oil ok. And 10 milliliter is the limit.

So, S W i is the pore volume of; pore volume of water present in the system ok. So, remaining 1 minus S W i was the oil present in the system; oil present in the system. 1 minus S W i was the oil present in the system and S W i amount of water was present in the system. And when at the end when we are when this place is completely flooded with water.

So, then also we have to leave some amount of oil and that oil is the residual oil so, that is known as S o r ok. So, at that time, what would be the fraction of oil that is remaining in the system? That is S o r. So, S o r pore volume of oil, S o r pore volume of oil can never be produced; can never be produced. Because that is the system independence; S o r pore volume I have to leave there.

And to start with I have 1 minus S W of oil present; so, out of 1 minus so 1 minus S W pore volumes. So, 1 minus S W pore volumes of oil was present in the system and at the end S o r pore volumes of oil can never be produced. So, then how much pore volume of oil I have produced? Essentially I have produced if you look at it should be 1 minus S o r S o r pore volume of oil will remain.

So, 1 minus S o r is the pore volume of oil that has been; that has been produced and we had initially we did not have 1 minus S o r. So, we had 1 minus S W i this much of pore volume of oil was present and out of this oil again S o r we have to leave. So, basically we have to leave S W i. So, we have a limit here these we have a limit here and that limit we have to leave that much of S o r amount of oil and to start with we have 1 minus S W of oil present.

So, this difference this much of pore volume of oil we can produce theoretically from this. Out of that, this linear part would be till the front reaches the outlet and beyond that it will start curving and producing both water and oil. And asymptotically it will reach the limit when you have injected 10 or 100 pore volume of water. So, this is the overall picture.

Now, the message that I am going to carry forward is these particular equations. This particular equation is going to be a key equation now. So, in the next lecture you will see that we will build on this equation to work further out. Because, we if somebody wants to know how much of oil will be produced in this region one has to find out how to predict this part of the curve.

This part of the curve is very easy I inject v p I inject whatever I inject to I inject not two, whatever pore volume I inject let us say point to pore volume of water I have injected I will produce point to pore volume of oil, but beyond some value it would start curving. So, this part is cannot be. So, at what point this breakthrough takes place and beyond the breakthrough how this curve changes? So, that needs to be understood ok.

So, this is something which you will head to. And this is the equation which is going to be a key equation you need to remember this x equal to L f W prime v p. That is all I have for this module of the lecture.

Thank you very much.