

Flow through Porous Media
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Lecture – 35
Immiscible Flow (Contd.)

I welcome you to this lecture of Flow through Porous Media. What we are discussing in the last lecture was Immiscible Flow, when two phases they are immiscible; that means, oil and water and if we want to have flow of two immiscible phases through a porous medium, what all the complications or what all physics evolves in that.

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So, in this context we are heading towards relative permeability and two phase flow and then we will build on it and further we look into other aspects.

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Surface Tension & Contact Angle Contd.

Non-spherical Cap

P is a point on the surface
 AB and CD are arbitrary pair of orthogonal lines, drawn along the surface.
 R_1 and R_2 are the radii of curvature at P.

At point 'A', an element δl of the boundary line is subjected to a force $\sigma \delta l$.
 Projection of this force along PN = $\sigma \delta l \sin \phi$
 = $\sigma \delta l \left(\frac{r}{R_1} \right)$ when ϕ is small.

Similar contribution of all points A, B, C, D
 = $\sigma \delta l \left[\frac{2r}{R_1} + \frac{2r}{R_2} \right] = 2r \sigma \delta l \left[\frac{1}{R_1} + \frac{1}{R_2} \right]$
 where r_1, r_2 are the principal radii of curvature, and follows Euler theorem $\frac{1}{r_1} + \frac{1}{r_2} = \frac{1}{R_1} + \frac{1}{R_2}$

What we discussed at the end of last class was that for a non spherical cap, how this Laplace pressure was defined. So, we were building on that. We showed in the last class that if we pick up a region around point P basically the distance of all this points of these of this line this they are same, if we follow the surface. So, it is basically a surface we are talking about, this surface has multiple radii of curvature. You can see if I draw CD it has the radius of curvature is R_2 if I draw AB at the same point P I have a radius of curvature R_1 .

And in this case we showed that for a $\sigma \delta l$; for δl element length on this curve the surface tension force that is σ into δl . For δl length of this element we have the force as $\sigma \delta l$. Now, what we need to do is, we need to integrate this over this entire curve to get the net force by which this surface; net force the surface, this surface is subjected to.

So, what we found was the contribution from point A point B similar δl element similar δl element point sorry, similar δl element point D, similar δl element point C. So, all these 4 δl elements if we sum them up we see the contribution of all points A B C D as $\sigma \delta l$ into $2r$ divided by capital R_1 plus $2r$ divided by capital R_2 . R_1 and R_2 are the two radii of curvature, and then we said that because of Euler theorem, these $\frac{1}{R_1} + \frac{1}{R_2}$ that is equal to $\frac{1}{r}$

plus 1 by small r 2, where r 1 and r 2 the small r 1 and small r 2 are the principal radii of curvature principal radii of curvature ok.

So, this is; so, what are the principal radii of curvature that is generally the maximum and the minimum because you can have multiple radii of curvature out of those the minimum and the maximum those 2 represented by small r 1 and small r 2.

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Surface Tension & Contact Angle Contd.

The forces due to Surface tension to be balanced by the pressure forces for mechanical equilibrium.

$$(P'' - P') \pi r^2 = \int_0^{\frac{2\pi}{4}} \left\{ 2 \sigma \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \right\} r dl = \sigma \left[\frac{1}{r_1} + \frac{1}{r_2} \right] \pi r^2$$

To avoid repetition of forces, integration was performed over one quarter of a revolution.

$$P'' - P' = \frac{2\sigma}{r_m}, \text{ where } r_m \text{ is the mean radius of curvature.}$$

$$\frac{1}{r_m} = \frac{1}{2} \left[\frac{1}{r_1} + \frac{1}{r_2} \right]$$

P'' is the pressure on concave side.

So, what we did in that case is we said that now what we are putting here is that the forces due to surface tension is to be balanced by the pressure forces for mechanical equilibrium; that means, we are holding a curve like this. So, inside the pressure is P double prime outside the pressure is P prime. So, difference between this P double prime and P prime these difference in pressure that would be equal to; these difference in pressure will support. I mean otherwise that the surface tension force will tend to reduce the volume of this sphere; this volume of this region. So, now, on only force that is compensating; only force which is opposing it countering it is the in the mechanical equilibrium that is this P double prime minus P prime these two; the difference in these two pressures, that is countering the surface tension force.

So, now if we go back to the; if we go back to this our original one here the area that we have area over which this is so, this is the radius is rho. So, inside we assume that the pressure is say P double prime and outside the pressure is P prime. So, these difference in pressure multiplied by this area of this surface. So, that is basically pi rho square, rho is

the radius of this curve. So, the curve is drawn such that every point on this curve is rho distance away from the point P, but you have to for the rho is following the surface ok.

Rho is not just joining two points as a straight line rho is one has to follow the surface and that distance is rho. So, this is the area and over this area the difference in this pressure P double prime and P prime that is acting so, what is the net force on this? So, net force is P double prime minus P prime into pi rho square, the pressure difference multiplied by the area over which this difference is acting that is equal to; what they doing is here we are looking at 0 to 2 pi rho by 4.

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Surface Tension & Contact Angle Contd.

The forces due to surface tension to be balanced by the pressure forces for mechanical equilibrium.

$$(P'' - P') \pi \rho^2 = \int_0^{\frac{2\pi\rho}{4}} 2 \rho \sigma \left(\frac{1}{r_1} + \frac{1}{r_2} \right) dL = \sigma \left[\frac{1}{r_1} + \frac{1}{r_2} \right] \pi \rho^2$$

To avoid repetition of forces, integration was performed over one quarter of a revolution.

$$P'' - P' = \frac{2\sigma}{r_m}, \text{ where } r_m \text{ is the mean radius of curvature.}$$

$$\frac{1}{r_m} = \frac{1}{2} \left[\frac{1}{r_1} + \frac{1}{r_2} \right]$$

P'' is the pressure on concave side.

So; that means, that the perimeter is given by 2 pi rho that is the perimeter this is the perimeter over which the force is acting here it is P double prime and here it is P prime. So, this perimeter is 2 pi rho now since you are doing the you are accounting A B C and D all 4 sides so, that is why this integration is done over this length. The integration is done over one-fourth of the total perimeter because you are already accounting a these four forces.

So, to avoid repetition of forces integration was performed over one quarter of the revolution and so, this; so, that is why the integration is between 0 to 2 pi rho by 4 only one quadrant. And so, that is why it is; so, accordingly this comes here as sigma into 1 by R 1 plus 1 by R 2 into pi rho square. So; obviously, this becomes so, one can write P double prime minus P prime that is equal to 2 sigma divided by r m. r m is the new term

we are coining here, r_m is the mean radius of curvature which where $1/r_m$ is half of $1/r_1$ plus $1/r_2$.

So, this is r_m is a mean radius of curvature. So, P'' is a pressure on concave side; that means, if the pressure on the concave side; that means, this side the pressure is P'' and the other side is P' . So, that is the pressure difference. So, last time we are already seen that if we look at capillary rise, we saw that it is simply if the capillary radius is r it is 2σ by r .

Now, for an arbitrary surface it would be $2\sigma/r_m$, where r_m is the mean radius of curvature; mean radius of curvature means it is $1/r_m$ is half of $1/r_1$ and $1/r_2$ what are r_1 and r_2 ? These are principal radii of curvature at that particular point P . So, principal radii of curvature; that means, the maximum radius of curvature and the minimum radius of curvature so, this is what we learnt now.

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The slide contains the following content:

- Diagram 1:** A concave meniscus of a liquid drop on a solid surface. Labels include 'Gas', ' σ_{lg} ', ' σ_{sl} ', ' σ_{sg} ', and 'Surface of solid'.
- Text:** 'Liquid drop, placed on a smooth planar solid surface. For equilibrium, σ_{lg} is countered by σ_{sg} and σ_{sl} '.
- Equation:** $\sigma_{lg} \cos \theta = \sigma_{sg} - \sigma_{sl}$ (referred as Young's Eqn).
- Diagram 2:** A sessile drop on a solid surface with contact angle θ . Labels include 'Vapor', 'Liquid', and 'Solid'.
- Text:** 'Measurement of contact angle: Sessile drop'.
- Text:** 'Tilting Plate Method: Plate rotated until the liquid remains flat upto ... at the plate.'

So, further we need some other some quick brushing up of our understanding of contact angle this you have already studied before, but I just very quickly I would like to go through it. There is this famous Young's equation which says that if there is a surface of the solid; surface of the solid and a liquid is sitting there then a force balance can be drawn to arrive at these expression ok. So, accordingly if one has a say for example, we have you remember when we looked at the capillary rise and this was the meniscus. So,

the surface force was acting in this direction and we have drawn the component of it in vertical direction. So, these treatments we could do with these this understanding.

So, now, measurement of contact angle this contact angle can be measured by taking a photograph of the contact angle on a surface; for example, we can see here this is here in this case the contact angle is less than 90 degree. So, this is considered more of waiting phase whereas, in this case the contact angle is the earlier case it was less than 90 degree now it is more than 90 degree. So, this is considered as non-waiting phase ok.

So, it depends on what is the substrate or what is a liquid we are looking at. So, now, there are ways to measure this contact angle one is by taking directly the photograph and doing the image analysis there are other ways of doing it. For example, a solid material can be; solid plate can be placed inside a liquid and then can be gradually tilted, when it is seen that the plate has to be rotated; a plate has to be rotated until the liquid remains flat up to the phase of the plate.

So, liquid remains flat as long as this now this becomes a straight line these liquids gets flat here. So, then it becomes if one makes it more vertical, then you will find there is a rise in liquid level. So, it is as long as this remains flat so, that angle can be measured and this angle can be found out. So, this is referred as tilting plate method.

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Surface Tension & Contact Angle Contd.

Contact angle measurement for oil-water system

Crystal
oil
water
Crystal

Water advancing contact angle, measured as a function of time.

oil
Crystal

* Flat polished crystals of the mineral that is predominant in the rock surfaces are immersed in the sample of "formation water".

* A drop of "reservoir oil" is placed between two crystals

* The plates were displaced relative to each other

Contact angle **hysteresis**

Advancing angle >> Receding angle

Reasons

- (i) Contamination in liquid or surface of solid
- (ii) surface roughness
- (iii) immobility at the surface in macromolecular scale due to adsorption, presence of surfactant

Courtesy: Wikimedia Commons

In case of a oil water system since that is something which we have here in mind, I mean when we have for example, looking at a reservoir flow or two phases where oil water we are talking about. Generally this oil water contact a; for contact angle measurements what particularly for reservoir applications for hydrocarbon recovery applications, there is a way to measure contact angle which is where these flat polished crystals of the mineral that is predominant in the rock surfaces.

So, one makes this crystals and this is the crystal that is what you can see here. These crystals are placed the first is they are immersed in something called a formation water; that means, formation water means the water that is existing in a formation; formation means hydrocarbon bearing formation. So, and then a drop of reservoir oil is placed between two crystals. So, you can see outside there is water and inside there is oil and this is placed between two crystals this crystals are flat polished crystals of the same mineral that is predominant in the rock surface.

So, now a drop a reservoir oil is place between two crystals and then the plates were displaced relative to each other; that means, these plate remains fixed this plate moves in this direction. So, this plate has moved. So, you can see this is the arrow. So, this plate has moved in this direction. So, now, this is the shape of this droplet. So, from here one can measure the contact angle not only that this is then referred as water advancing contact angle because here in this case this plate is pulled. So, here water tends to advance and what in these regard something which is very unique which one must note here is something called contact angle hysteresis, which can be shown here in this picture.

When the fluid is introduced here, you can see the fluid is moving in this direction. So, this is the contact angle whereas, when fluid is drawn out from here this becomes the contact angle and you can see that this contact angle and this contact angle they are different. Generally these advancing angle; here it is the advancing angle, here it is the water advancing contact angle ok. So, because the water is advancing so, advancing angle is much greater than the receding angle. So, this is known as hysteresis ok, see that is how the; that is how hysteresis is defined.

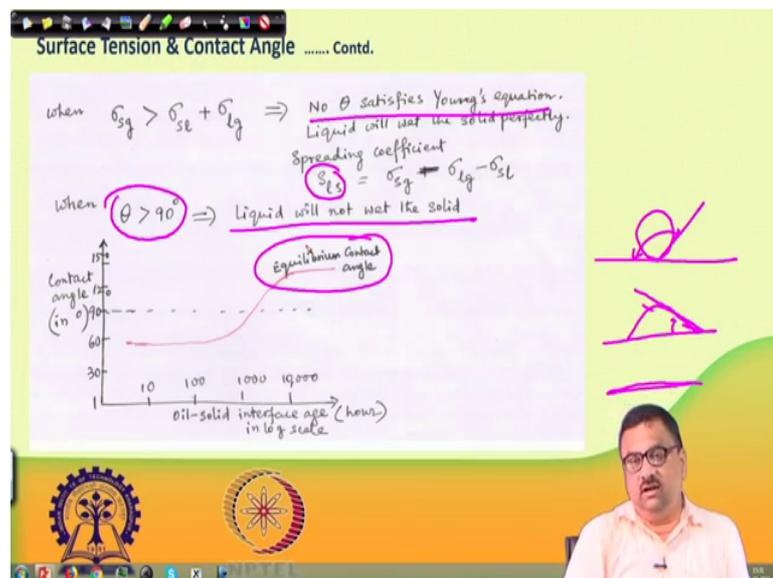
So, contact angle depends on history of how contact angle is created. The reasons for these hysteresis are; number i contamination in liquid or surface of solid, number ii there

could be surface roughness, number iii immobility at the surface in macromolecular scale due to adsorption or presence of surfactant. So, because of many reasons there could be such thing happening I mean whether liquid is being pushed or liquid is being pulled it depends on that the contact angle can change.

So, this aspect one must remember when we are going to now define the two phase flow. So, now, we had talked about this σ_{sl} $\sigma_{solid\ liquid}$, σ_{lg} $\sigma_{liquid\ gas}$; σ_{sg} $\sigma_{solid\ gas}$ and they have a balance here, $\sigma_{lg} \cos \theta$ is equal to σ_{sg} minus σ_{sl} . Now in the event σ_{sg} is greater than σ_{sl} plus σ_{lg} ; σ_{sg} is greater than σ_{sl} plus σ_{lg} . σ_{sg} is greater than σ_{sl} plus σ_{lg} ; σ_{lg} means $\cos \theta$ and this can have the maximum value of 1 and that will happen when θ is equal to 0 ok.

So, $\cos 0$ is 1; So, σ_{lg} plus σ_{sl} is equal to σ_{sg} I mean if we take σ_{sl} from right to left then you write σ_{sg} is equal to σ_{lg} into 1 plus σ_{sl} .

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So, now here we are saying that when it is σ_{sg} is not only equal, but the greater than this. So, when σ_{sg} is greater than σ_{sl} plus σ_{lg} so, naturally it is beyond $\cos \theta$ is equal to 1. So, that is why it is written here that no θ satisfies Young's equation. So, essentially what this means is liquid will wet the solid perfectly, I mean it is beyond θ is equal to 0 degree θ equal to 0 degree means it is completely wetting

the surface theta greater than 90 degree means it is not wetting the surface it is sitting on the surface as a blob ok.

So, now, but theta is less than 0 degree means it is; I mean it is practically no theta satisfies Young's equation and liquid will wet the solid perfectly. Under such situation one defines a term called spreading coefficient S_{ls} which is equal to σ_{sg} , how much it is greater; σ_{sg} is greater than σ_{sl} plus σ_{lg} , but how greater? So, that term one generally looks at spreading coefficient ok. So, this will definitely spread there is no doubt about it, it is a 100 percent waiting, but how strong would be this wetting tendency.

So, it is say S_{ls} spreading coefficient is defined as σ_{sg} minus σ_{lg} minus σ_{sl} . On the other hand so, this is the case when theta equal to 0 degree or even less no theta can satisfy. On the other hand when theta is equal to; when theta is greater than 90 degree, then we say that the liquid will not wet the solid because if this is the solid and then if theta is equal to 90 degree means this is the angle; and this is the angle it makes which is greater than 90 degree; so, theta is equal to less than 90 degree means; so, this is the angle and this angle is less than 90 degree. So, it is wetting.

This angle when it tends to 0; that means, the film is almost spreading like as much as possible. So, now in this case there is some issue with this oil solid interface age, this is something which researchers have observed while working with a reservoir with in case of hydrocarbon reservoir, what they observed is that after the solid and oil they are in contact for ages one can see that this wettability can also change. So, what they have observed is for when this oil solid interface age increases a lot, then at some point of time from 60 degree contact angle it can go beyond 90 degree.

So, there could be a shift from little less than 90 degree to little above 90 degree. So, that is also not them; I mean that is also something which is possible do not strike it off as something impossible. So, the overage if something is in contact then there is a this equilibrium contact angle is; so in this regard I think you one is to look at this definition of equilibrium contact angle and how it is different from advancing and receding contact angle. But this equilibrium contact angle can shift from less than 90 degree to greater than 90 degree.

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Capillary Pressure

$r_{m1} = \frac{R_1}{\cos(\theta - \phi)}$
 $r_{m2} = \frac{R_2}{\cos(\theta + \phi)}$

Pressure inside wetting liquid is atmospheric
 For any penetration of non-wetting fluid, the pressure in the non-wetting fluid to be increased to $P_{atm} + P_c$, where
 $P_c = P'' - P' = 2 \frac{\sigma}{r_m} = 2 \frac{\sigma}{R_i} \cos(\theta \pm \phi)$
 $= 2 \frac{\sigma}{R_2} \cos(\theta + \phi)$ when flow is from left to right
 $= 2 \frac{\sigma}{R_1} \cos(\theta - \phi)$ when flow is from right to left

$P_{oil} = P_{water} + \frac{2\sigma \cos\theta}{r}$
 oil

Now, suppose we have a situation we had already talked about a situation where we have a pore cylindrical pore ok. So, when we have a cylindrical pore, at that time we have let us say we have a; this is a cylindrical capillary. So, inside let us say we have water which is the wetting phase; that means, the capillary is something similar to glass. So, it is water wetting. So, the substrate the capillary wall is water wetting and water is inside at let us say at a pressure of let us say one atmosphere pressure. And outside; so, it is filled with the water and outside let us say we have oil I mean water and oil these are; I mean we are talking about one is the wetting phase another is a non-wetting phase.

So, wetting phase is inside at pressure of one atmosphere and outside it is all non-wetting phase. So, now, we will have in that case there will be a meniscus developing and this would be the meniscus because you one has a contact angle and that contact angle water will make a contact angle which is less than 90 degree that is why you are calling it water wet, had it been oil inside and water outside, this would have been a different curvature all together you must understand that. Had it been oil inside and water outside then the curvature would have been different.

So, now this is water and this is outside it is oil. So, there is an angle, there is there is this angle here which is the contact angle. So, this is how the surface tension would be acting. So, we have already talked about these in case of capillary rise; in case of capillary rise

gravity played some role, in this case gravity does not matter since it is horizontal, I mean then the dimension of the capillary is small so, gravity does not come into play.

So, now, we have if r is the radius of the capillary cylindrical capillary so, in that case we know $2\pi r$ is the length of the perimeter of this cylinder and 2 inner perimeter; inner wall and then that multiplied by σ Newton per meter into meter. That gives me the force by which the surface tension is acting maybe you multiply it with $\cos\theta$ to talk about this angle would be this angle is θ , then this angle would be θ and then you can take the component in vertical direction which is $2\pi r\sigma\cos\theta$ ok.

So, this is the force by which the surface; this is the surface tension force and these surface tension force would have to be; it has to be compensated by what we have seen already? $P'' - P'$ is equal to this is the force. So, $P'' - P'$ would be force divided by the area that is exactly what we had done earlier in case of non-spherical cap and earlier even for the spherical cap. So, this would be the area would be πr^2 . So, $P'' - P'$ or in other words we can think of P'' ; I mean of what we had done earlier was probably a little different earlier we had multiplied this πr^2 to this side.

So; that means, we had multiplied these by πr^2 . So, πr^2 into $P'' - P'$ that gives me the force pressure difference therefore, mechanical equilibrium, the pressure difference across this which side is P'' ? It is P'' on this side you remember if just match it with what we had done earlier and this is P' . So, $P'' - P'$ over this area πr^2 that gives me the force and that force is ensuring that this interface is not squeezing the liquid to 0 volume. So, then that that is; so, this is this force has to be compensated by $\pi r^2 P'' - P'$.

So, accordingly this $P'' - P'$ would come to $2\sigma\cos\theta$ divided by r . So, that is what; so, this is what we referred as Laplace pressure in case of a sphere, this we are going to refer as capillary pressure. Our logic here is that oil will never enter into water phase unless you provide this difference so; that means, P' is one atmosphere outside oil is one atmosphere. So, let us say I have increased this pressure of oil by a little bit nothing will happen, there will be no flow though I have a pressure gradient from this side to that side there will be no flow. Only at the flow will

only set in only when this pressure on the oil side is the pressure in water plus $2\sigma \cos \theta$ by r then only there will be flow from this side to that side.

So, I would expect the flow to take place from this way the flow would take place only when the pressure on the oil side; when the pressure on the oil side is pressure on the water side which we have probably assumed as one atmosphere plus $2\sigma \cos \theta$ divided by r . This $2\sigma \cos \theta$ divided by r that is referred as the capillary pressure. So, one must; so, if we have oil and water flowing simultaneously through this pore one thing is for certain that the oil pressure has to be higher than the water pressure and how much higher by this amount and that amount is known as capillary pressure.

So, this is something which we must keep in mind from now on that I am having oil and water flow, water is flowing in its own pressure, but oil has to be at a higher pressure then only oil will get into this pore. Now we will look at this definition of capillary pressure when the pore that we are talking about in porous medium that may not have to be cylindrical that may be conical there may be undulations; so, how capillary pressure will be defined in those cases. So, there is something which you are going to take up in the next lecture, that is all I have for this module.

Thank you.