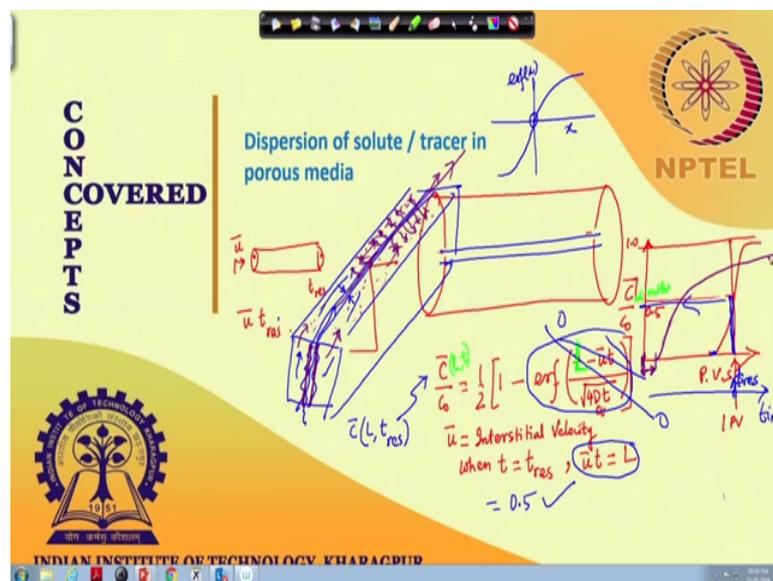


**Flow Through Porous Media**  
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**Lecture – 30**  
**Miscible Displacement (Step Change in Concentration) Contd.**

I welcome you to this lecture of Flow Through Porous Media. We were discussing about this Miscible Displacement.

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In particular we talked about this process of dispersion of solute of tracer in porous media. What we had been talking about is as the flow takes place in a porous media, how mixing takes place and particularly for last lecture, in the last lecture we talked about the step change in concentration. If one gives a step change in concentration, how this steep change evolves at the outlet I mean a step change in concentration how the concentration profile evolves and the outlet.

And by looking at the concentration profile what I mean how in a with some amount of (Refer Time: 01:11) how we can characterize the porous media or which porous medium is giving more mixing which one is giving less mixing. So, instead of looking at the experimental data, we can we try to put together these certain theories and we talked about these dispersion coefficient or Peclet number or the dimensionless time and those kind of terms we could arrive at.

So, in this regard I must point out that since we are discussing about this flow through porous medium, when there is a porous medium we said that there could be a stagnant; at the end of last lecture if you recall, we were talking about a stagnant zone. So, what could be the effect of stagnant zone? So, in this regard let us see if what would be the case, if there is a high permeability streak or a so-called fracture; that means, a crack exists inside a porous medium. Let us say a fracture runs through these porous medium. So, let us say a fracture runs through this porous medium and what you have given is a steep change in concentration and you are trying to plot the concentration at the outlet. You are writing it as  $C$  bar by  $C_0$  as a function of  $I$  am writing it as PVS number of pore volumes.

Had it been just a uniform core without any fracture, I would have observed around one pore volume there would be a breakthrough. So, this is called breakthrough this point is where first concentration is registered at the outlet. So, this is the breakthrough point and then at some point the concentration is reaching 0.5; concentration is reaching 0.5. So, we this is 0.5 and this is 1.0. So, at the end it would be reaching 1.0. So, if there is no fracture if we had talked, if we assume that there is no fracture, then we would have expected a plot something like this. And, we could have said that at now let us see we call this to be 1 pore volume 1 PV.

Why we are calling this to be 1 PV? Because, you recall this from equation that we arrived at for the step change; it was  $C$  bar by  $C_0$  is equal to half into 1 minus error function of  $z$  minus  $u$  bar  $t$  divided by  $I$  believe it is square root of  $4Dt$  or  $4D$  ctd. So, if this is the concentration profile so, then when we try to apply this for a porous medium, first of all I must point out that these  $u$  bar has to be equal to interstitial velocity not superficial;  $u$  bar has to be interstitial velocity because we are extending now instead of a capillary we are calling it the porous medium. So, instead in case of a porous medium this velocity was applicable for velocity through the capillary and porous medium we have considered porous medium as bundle of capillary. So, the velocity inside the capillary we are talking about not overall velocity.

So, this  $u$  bar is interstitial velocity, this we must remember ok. So, when  $t$  is equal to residence time; when  $t$  is equal to  $t_{res}$ ; that means, when  $t$  is equal to the residence time then what would be  $u$  bar into residence time in this case? Think of a capillary; think of a capillary, I have this velocity is let us say  $u$  bar and residence time it takes it supposed to

take is  $t_{res}$  ok. So, then in that case what would be the  $\bar{u}$  divided by  $t_{res}$  or just one second what would be the  $\bar{u}$  multiplied by  $t_{res}$ , what does this give? First of all unit wise I can see  $\bar{u}$  is meter per second and  $t_{res}$  is second.

So, meter per second into second they will cancel out. So, this has a unit of meter. If you look at it  $\bar{u}$  is a time  $\bar{u}$  is the velocity; that means, the distance this fluid front travels, the center point of the fluid part or in an average sense the distance the fluid front travels over time of 1 second; let us see  $v$  what is so, and so, meter per second. So, many meter will be traveled in 1 second. So, over  $t_{res}$  that residence time  $\bar{u}$  into  $t_{res}$  is that this is the length this midpoint of this fluid front will travel so; that means, the midpoint of the fluid front would travel. So, these  $\bar{u}$  into  $t_{res}$  is the length scale.

So; that means, after  $t_{res}$  time, then midpoint of the fluid front would travel from this end to that end that is the idea right. So, then this  $\bar{u}$  into  $t$  then when  $t$  is equal to  $t_{res}$   $\bar{u}$  into  $t$  becomes equal to  $L$ , where  $L$  is the length of the capillary or in this case length of the porous medium. So, here what are we plotting by the way? We are plotting the concentration  $\bar{C}$  at outlet frankly speaking at outlet. So,  $\bar{C}$  at outlet we are plotting divided by  $C_0$ . So, at outlet we are plotting means, this  $z$  will be simply equal to the length we are plotting right because, we are interested in not any  $z$  anywhere are not interested though we can find out using this equation, but we are not doing that.

For a porous medium I have very little, I mean generally it would be easier for me to collect the sample from the outlet and measured its concentration. So, naturally this  $\bar{C}$  would be then as a function of  $L$   $t$  we are look not  $z$ ; so, automatically this instead of  $z$  now we will have  $L$ . So, now, we will have  $L$  and we said that this at  $t$  is equal to  $t_{res}$   $\bar{u}$   $t$  is equal to  $L$ . So, what that means, is when  $\bar{C}$   $L$  we said.

So, basically it is when we are finding out  $\bar{C}$  at  $L$  and  $t_{res}$  at residence time at the outlet when you are trying to find out then that would be equal to half into  $1 - \text{erf}$  this quantity, but then  $\bar{u} t$  is equal to  $L$ . So; that means,  $L - \bar{u} t$  this whole term becomes equal to 0 and  $\text{erf}$  of 0 we remember I have already talked about these erf function  $x$  versus erf this this is  $x$  this is erf of  $x$ . So, we know that it is going to be 1.0 it goes, but at  $x$  equal to 0 it would be 0. So that means, this whole error function; this whole error function term is becoming 0.

When we are talking about  $z$  as equal to  $L$  and time is equal to  $t_{res}$ ; that means, the residence time. So, then this if this term is equal to 0, then this becomes equal to this whole thing is becoming equal to half into  $1 - 0$  which is 0.5. So, what essentially it means is that after I injected 1 pore volume that is what is the definition of 1 pore volume right. The residence time the flow that has taken place till the residence time that is equal to 1 pore volume.

If we multiply simply these length multiplied by the area and multiplied by the porosity this gives me the pore volume. So, here what; that means, is; so, what I what I mean to say is that, if I had plotted instead of pore volume, if I had plotted these as time, then this 1 pore volume; then this 1 pore volume would have come, would have been synonymous to 1 residence time right. So, it is synonymous to one residence time. So, it is the same thing if on time scale residence time on a volume scale 1 pore volume.

So, for 1 residence time at  $z$  distance at and outlet only for 1 residence time it should have been 0.5 that is what it says and that is what we see that it is 0.5. So, this is so, this mix; so, I mean if we by doing this I mean we get some better handle over this whole episode, that this  $C$  bar by  $C_0$  essentially though this is the equation we said  $C$  bar at  $z$  and  $t$ , but essentially when we are plotting this for a porous medium, we are plotting this for  $C$  bar at the outlet when we make this plot.

And then we said that at this is the time when first time the concentration starts recording, because it would be little less than 1 pore volume because of this dispersion. And then however, the midpoint of this curve will always this is this should correspond to 1 pore volume residence time and 0.5 concentration. So, this is intuitively understood and then it will go rise, it will be reaching 1.0. Now in this whole show if you have a fracture running through this; fracture means a crack through which a flow can take place.

I mean to say suppose I have a core or I have let us say a block let us say this is the block running like this and in this block there is a crack which is running. Of course, the dimension of the crack is very small, in the of the order of say half millimeter. So, it is we are talking about 500 micrometer whereas, when it comes to the pore size we are talking about 1 micrometer to 3 micrometer or. So, it is so, compared to the pore size this

dimension aperture is large. But, for our purposes for the purposes that we are looking at less than a millimeter is definitely very very small aperture.

So, now if one for fracture runs like this; so, naturally the fluid will have a tendency to flow through this because it is getting a much higher permeability through these crack through these fracture rather than flowing through this porous medium. So, what one would see in this case is, a very fast break through break through means this is break through that this is here the constitution started rising. So, in this case one would see the concentration rising and rising at here concentration will start rising from here.

So, concentration will start rising from here and then the concentration. So, that is there is flow taking place through these through these channel and the con the concentration is the fronde that they have injected this will start appearing at the outlet very soon. Only this part of this part of the cross section has to be filled, only this this crack or this part is to be filled and this volume of this crack is much smaller compared to the total pore volume of this body; had it been a uniform flow through the I mean uniform in the sense the uniform sweeping of the resident fluid, we would have expected that entire fluid of this porous medium has to be displaced.

So, then only the concentration front can (Refer Time: 15:42) out of that understanding we have multiplied  $u$  bar with  $t$  raised or we talked about this point  $u$  bar  $t$  is equal to  $L$  and arrive at this 0.5. But if there is a fracture running. So, naturally the fluid will be travelling very fast and of course, this particular this particular volume this particular volume in fact, is in a way this particular volume is an indication, what is the volume of the volume of this crack ok? So, this volume of the crack is small.

So, this is the time and then it starts breaking through, but after it starts breaking through, then it sort of this is called a tailing of the profile. So, this is tailing and it will take a long time to reach the 1.0 value these this whole thing will take a very long time to reach the 1.0 value. Because, these fluid that is traveling through these cross section these fluid would this what do we have in the surrounding space? This is the surrounding this is this is the adjoining space right adjoining space is the porous medium and this porous medium is having a concentration let us see 0 concentration is 0 and this  $C_0$  front is traveling through this very fast.

So, when it travels through this very fast, there would be diffusion of these diffusion not a flow it is just diffusion it is something like I am having a flow of a fluid and fluid flow I mean I am putting a jet inside a tank. So, tank is filled with certain fluid and then I am introducing a jet, jet is moving at a very high velocity so, but as it goes through the tank, it is the jet is mixing with the fluid already inside the tank and still it is reaching the outlet and I am collecting it from the from the other side, I will find it is this that the one that I am injecting that will definitely be much diluted.

So, that same effect; so, the here also it is it would be it would be there will be diffusion taking place. So, this concentration will be diluted. So, it would be appearing very fast, but then it turns and probably it will reach some value of 0.6 or so, and then beyond that it starts tailing you have been inject a lot of volume to reach this 1.0 which is much higher than the pore volume because this so, flow will continuously predominantly will take place through this ok.

There could be some flow which will take place through these adjoining medium also. In fact, that depends on the permeability ratio this aperture, it will have some permeability this permeability would be let us say we have as I said the aperture is  $b$ . Then if you have then permeability to be given by  $b^2$  by 12 one can find out it would be of the order of thousands of Darcy whereas, this is of the order of milli Darcy. So, then this is there could be there could be a permeability contrast; so, depending on this permeability ratio and the area ratio.

So, some flow will take place always there would be some minimal flow that will take place through this side very slowly ok, but main flow would be through these crack and then as it travels as the concentrate as the solute travels through this, the this solute will continuously be diffusing into the adjoining medium adjoining pore space which is filled with liquid of zero concentration. So, that is why because of this diffusion there would be dilutions of the front. So, it would be less much less than 1.0.

And one has to continue to flow whatever you are injecting the flow will immediately take place through the through these through these through this channel, but this entire thing has to be fade entire concentration has to be changed by diffusion. And, that would take long time it is the same situation what I discussed in a previous lecture as stagnant zone a very similar situation.

This part of the fluid is not you are not letting this part of the fluid to flow instead you are slowly feeding to this part of the fluid with the solute, but that is only by diffusion. So, this flow will take place very fast, but because of this diffusion it will take a long time to reach C by C 0. So, these are some of the aspects we must make note off as we are gradually making the transition from dispersion in a capillary to dispersion in a porous medium.

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**Dispersion with chemical reaction**

One dimensional dispersion with adsorption or Reaction

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2} - v \frac{\partial C}{\partial x} + r(C,t)$$

Here,  $v$  is the interstitial velocity  
 $r(C,t)$  is the rate of production of solute per unit volume of solution

Now, dispersion we said that if you recall the if we have one dimensional flow if we have one dimensional flow and then we have talked about this dispersion if you did I mean one can think in this line. And, this is this common in case of porous medium to put these as put these as longitudinal dispersion  $D_L$  longitudinal dispersion; that means, dispersion that is that will take place in the longitudinal direction ok.

So in fact there is a I mean when there is a flow taking place we can divide we can consider this dispersion to be longitudinal and transverse, but let us focus on only longitudinal dispersion because that is what we have studied in Taylor dispersion or that is what we have studied so, far. So, this what I am trying to say is that this is not diffusion anymore, this is dispersion this is dispersion; so, this term we are using now as dispersion.

So, what we had in Ficks second law we have been talking about all the time?  $\text{Del}^2 C / \text{del} t$  is equal sorry  $\text{del} C / \text{del} t$  is equal to diffusivity multiplied by  $\text{del}^2 C / \text{del} x$

square type term, but here in this case. So, this this when we when we arrived at this what did we do? We had this we had picked we picked up a differential element and then e in minus out is accumulation and we arrived at this  $\frac{\partial C}{\partial t}$  is equal to diffusion coefficient multiplied by  $\frac{\partial^2 C}{\partial x^2}$  if this is this direction is x.

So, this is concentration. So, here we have directly we are putting we are saying that there is a flow taking place. So, this is not this is not going to be a diffusion. So, let us directly put this as dispersion coefficient this is; that means, this account for diffusion and the n and effect of velocity n because of that there is there is mirroring of the front and so, all those are included. So, instead of diffusion coefficient now we are talking about dispersion coefficient and so, with that that is given the name L and since we are looking at dispersion in this direction this way according longitudinal dispersion.

Further we have this term you might have noticed already when we defined a Taylor dispersion that, there would always be some component of convective some convective term has to be there; that means, I pick up a differential element I pick up a differential element perpendicular to the screen area A. So, what I know that if this is A then we are always putting some  $v \cdot d f$  is the interstitial velocity in this case. So, v some velocity which is entering and these has some cross sectional area over which.

So, this is the volumetric flow rate multiplied by the concentration at x and here you have same v ac at x plus delta x and so, we can take the difference between the two. And, when we write this write the mass continuity; that means, when writing the Ficks second law if see when we arrived at Fick's second law we considered only diffusion; that means, at the inlet the flow flux is minus  $D \frac{\partial C}{\partial x}$  at x and here it is minus  $D \frac{\partial C}{\partial x}$  at x plus delta x and the difference is accumulation.

So, that is we are considering only diffusion at that time. If we have on top of this diffusion if we have some flow taking place and because of this flow something is being carried to this differential element by this by this volumetric flow rate and something is leaving from the other side. So, if we take that balance; that means, this difference if we take and then if we take if we if when you divide by delta x; that means, the way we had divided by delta x and set limit delta x tending to 0; when we do that exercise of limit delta x tending to 0 and then  $v \text{ ac at } x \text{ minus } v \text{ ac at } x \text{ plus } \Delta x \text{ divided by } \Delta x$

when we do this and we take  $v$  outside as a constant then we end up with minus  $v \frac{dF}{dx}$ .

Here we have a minus sign because in this case we have we started with a minus sign, but here we do not start with a minus sign here this minus sign got cancelled out. So, that is why we have this term, but here this minus sign there is a minus sign. So, here there would be a minus sign. So, this is called a convective term now what we are saying is suppose there is a chemical reaction. So, if there is a chemical reaction then one must consider another reaction term; that means, within this within this volume if there is a reaction taking place. So, within this volume since it is I am writing it as plus. So,  $r_c$  is the rate of production of solute per unit volume of solution.

So, now and  $vDF$  mind it when we are applying this to a porous medium  $vDF$  has to be the interstitial velocity. So, what; that means, is that this becomes the governing equation when somebody has a reaction happening inside the porous medium. If it is only flow taking place. So, when there is a flow plus diffusion taking place Taylor dispersion was about a capillary Taylor dispersion was Taylor dispersion their also we worked with  $\frac{dc}{dt}$  term and there we had a radial diffusion and there was axial convection.

But, now if we when we are talking about porous medium we have we have taken that experience we have taken that wisdom all those informations are in our are with us, but now what we are doing is if we for the time being let us if we leave out this if we leave out this reaction term, we are considering this part. That means, this is the accumulation term, this is that this is the diffusion, but not exactly  $D$  is not the diffusion we are directly putting dispersion coefficient and this is the convection; that means, the solute that is carried because of the velocity term.

So, these are the; so, you one has to work with this interplace. So, this is and minded this  $vDF$  is now considered interstitial velocity. So, now, we are no longer in capillary no longer in a channel, we are directly into porous medium these  $v$  has been replaced as  $v$  is now  $vDF$  it is the interstitial velocity this  $D$  is not a diffusion coefficient  $D$  is dispersion coefficient. So, it takes into account all aspects that there would be mixing inside the porous medium and what should we what one should have. So, now, this would be the equation that is governing and on top of that you have when there is reaction on top of that you have these reaction term.

So, this is the expression that one must follow if there is simultaneously a flow a reaction a reactive flow inside the porous medium there is some mixing taking place because of this flow come flow as well as diffusion process and then there is some reaction happening. So, you can see we have taken all these wisdom that we had from the processes in capillary and then gradually took the good they did the transition and now we are talking about this entire process as for porous medium.

And, now this dispersion coefficient that we talked about now this down now this dispersion coefficient we can dispersion coefficient you can define if you have some idea of what is the radius of the pore you can go by  $r^2 d + r^2 v^2$  by  $48 d$  or simply  $r^2 v^2$  by  $48 d$  one needs to have the understanding of what is  $r$  and what is the diffusion true diffusion by diffusion coefficient. So, those informations are required or if you have some a some experimental data one can fit for that same porous medium and can find out what is the dispersion coefficient by following that plot and fitting through that through the data points through that the equations that we discussed so, far.

So, I want to stop here at this point I will continue this discussion of dispersion in the next lecture, we in particularly we will particularly we would look at what kind of knowledge we have with regard to this dispersion in porous medium, what kind of experimental data people have got gathered so, far. They have taken various samples of porous medium, they have conducted these experiments that I mentioned just now with step change and everything and they found out what are the dispersion values they got depending on what are the grain sizes and all those things.

So, we will touch upon those in the next lecture. So, that is all I have for this lecture.

Thank you very much.