Flow through Porous Media
Prof. Somenath Ganguly
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 29
Miscible Displacement (Step Change in Concentration) Contd.

I welcome you to this lecture on Flow through Porous Media; what we are discussing was Miscible Displacement. In particular we were talking about Dirac function, a pulse or a step change provided at the inlet and we wanted to see how the mixing takes place inside the porous medium. So that, at the outlet what is the fate of that step or what is the fate of that pulse how much that gate that is getting broadened? So, these are and from those signatures we try to understand how much mixing is taking place inside the porous medium and how we can theoretically characterize that mixing.

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So, in these regard we were; so, let us see what we have here, we were discussing about dispersion of solute or tracer in porous media.

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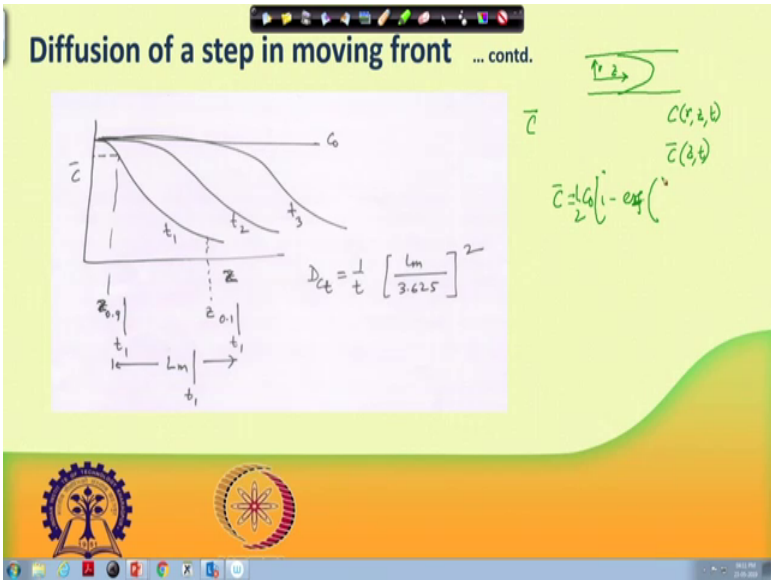

So, now we had talked about these dispersion process and we have given you this expression for C bar, we have given you this expression for C bar. Now, C bar is the average concentration that we see at the outlet; why we are putting it as C bar? Because this is over the entire cross section we are getting. That means, I am because it is you remember we said at the very outset that in case of a capillary we are, we could be having a parabolic velocity profile. And so because of that we have concentration as a function of r z and t; where r is the radial distance, z is the axial distance and t is the time.

But we since we cannot handle this radial concentration so, we said that we are interested in C bar z and t. So, now, these C bar as a z and t and we said that if we give a step change in concentration; if we give a step change in concentration then we have this C bar written as C half C 0 into 1 minus error function of. What was that exact expression we had? Let me see quickly.

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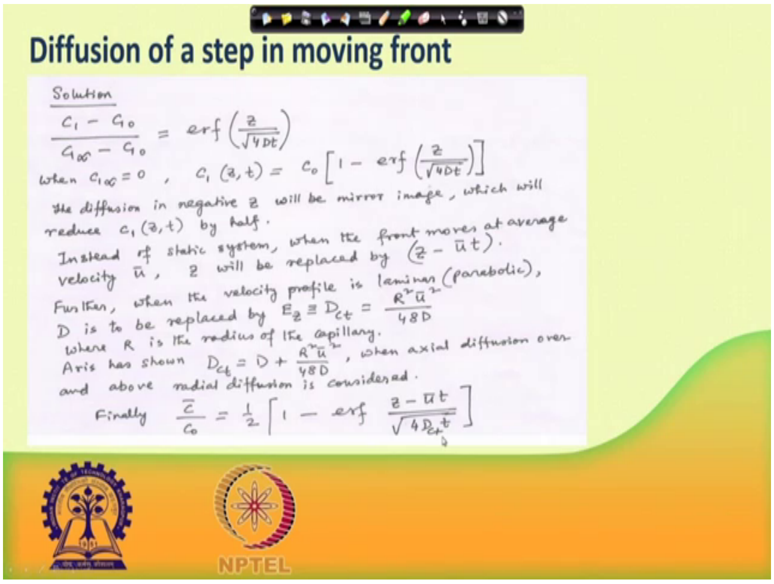

We had this error function of z minus u bar t divided by square root of 4 Dt.

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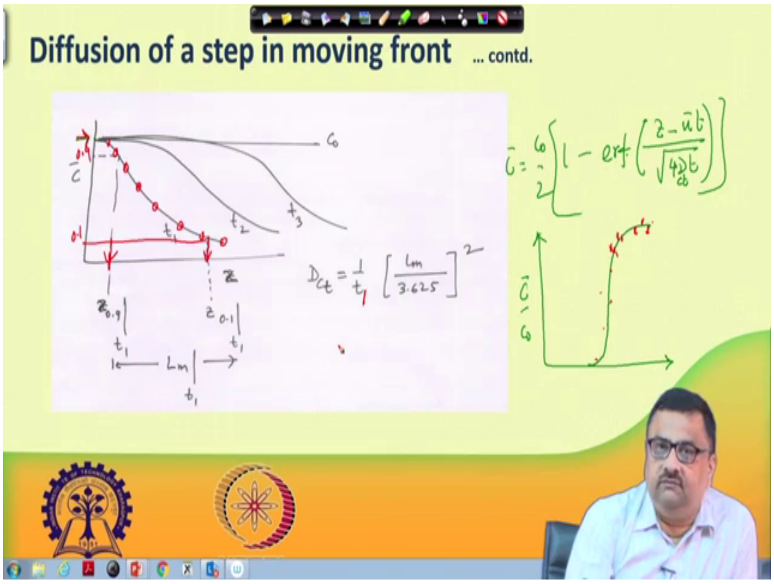

z minus u bar t divided by square root of 4 D t, 1minus this C naught by 2 that is equal to C bar.

So, we said that one can plot I mean the just like we had said in the last lecture that these C bar by C 0 which is supposed to change from 0 to 1 in normalized scale as a function of pore volume and at one pore volume, near about one pore volume we would have seen the concentration profile to be. There is some amount of smearing and by going backward; that means, this is the experimental data we have. Now, what should be the value of D c t such that this 4 Dt in here four D c t. So, what should be this value of this D ct such that this profile is matched? So, that is one way of looking at it.

So, but this in porous medium there is particularly the people who are doing it in subsurface applications, what they have; they tend to collect samples. Let us say a step change is provided at this location; step change is provided at this location and then the concentration is monitored at other locations ok. So, let us say one location is at the location, I mean at different locations let us say you are at some particular time you are finding out what are the concentrations at different locations. And then similarly at some other time so, it is at time t 1 at some other time what are the concentrations.

At some other time what are the concentration. So, these concentrations are noted and at in finite time one can expect this entire porous medium would be having a concentration C 0; it is applicable for any porous medium. So, any porous medium if there is a step change provided for in with an unidirectional flow one can expect. So, one can collect these points. So, here after plotting this if one can find out the location, the value of z at which the C bar has come down from 1.0 to 0.9.

So, this is that value, C bar is 0.9 and the correspondingly what is the z value at which the C bar has come down to 0.1 from 1.0. So, that if you note down those two location; so, this is one location, this is another location. And these difference in these two locations the distance between these two locations this is referred as L m; L m probably you can call it a mixing length.

So, L m at t 1 because all these data points were collected at time t 1 then D ct can be written as 1 by t, this is shown theoretically, but here you can just make note of it we do not want to get into that theory here. D c t is equal to 1 by t, t is in this case t 1; 1by t 1 lm you have just measured from experiment divided by 3.625 whole square. So, there are ways to quickly calculate D ct instead of; see if you collect these data points and then ask this equation tell me what should be the value of D ct such that these data points are fitted to a line.

Then one has to go for some kind of regression right ok. So, there has to be they have to start with a guess of D ct and then run this and see for which value of D ct the error between the experimental point and profile from this equation the this difference square of this difference sum of all such squares that has to be minimum. So, something of that sort one has to do to find out what is the value of D ct. So, instead of doing that exercise and they have made the life a little simpler, they are saying that if take these measurements and this will be directly it will follow ok. So, this is one way of calculating dispersion coefficient.

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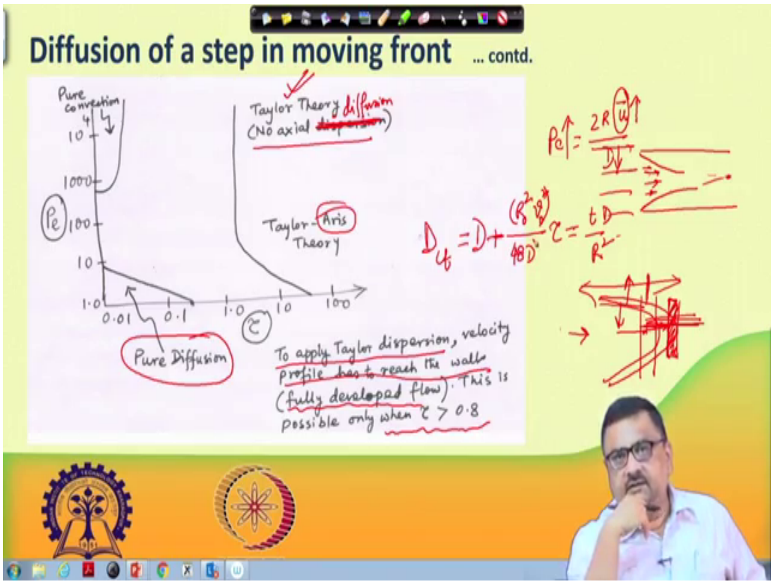

Now, I must point out we have discussed about these Peclet number and its importance and all this. Here we are talking about these Peclet number this is a log scale Peclet number plotted against tau; tau is that dimensionless time; dimensionless time in the sense we had talked about tau is equal to t D divided by R square right, t D divided by R square. So, this is the dimensionless time. So, on x axis a dimensionless time is placed and y axis is a Peclet number is placed.

So, then they have collected several data points of experimental measurements and from there they try to find out what would be their different regimes or where all these regimes are applicable. So, you can see here the Peclet number we defined was 2 R u bar divided by D. So, then Peclet number is small, Peclet number is small means, D is large and u is small; Peclet number is small means D is large and u is small. So, naturally one can say that the convection will not be dominating, the diffusion will be dominating.

What; that means, is suppose I have a flow; that the flow that is taking place inside, I have there is you might have seen that we have talked about Taylor dispersion where we have talked about these axial convection, we said velocity multiplied by the cross sectional area which was at that time you remember 2 pi R d R multiplied by v z. So, it is coming in similar term is going out and whatever is v velocity v z into 2 pi R d R that gave me the flow rate multiplied by the corresponding concentration ok.

And since C 1 is varying with z; so, similarly at the outlet we have v z 2 pi R d R C 1 at z plus delta z. So, this difference is in minus out. So, there is a convective term which is contributing to buildup of concentration within this place and similarly there is it always that diffusion can take place; diffusion will always lead to some amount of building of concentration or release of concentration. So, now in this context if the velocity term is very small and the diffusive term, diffusion coefficient is much larger in that case one can say that the diffusion will dominate the dispersion process.

So, the velocity I mean; obviously, I am having a flow through a porous media when I am having a flow through a capillary, the velocity is very small on the other end it tends to diffuse very fast. So, naturally diffusion will dominate the convection will not have that significance. So, that is why this particular regime where Peclet number is small this particular portion is shown as pure diffusion. On the other hand when Peclet number is much larger beyond 10 to the power three beyond 1000 so, naturally it gets reversed we can then say that these Peclet number is large in this case. So, Peclet number is large means u bar is large and D is small. So, then the diffusion is; then the diffusion is small whereas, convection is large.

So, in that case you will find I mean then what you would see is that if this is how this is getting stretched then you are getting slices of these; slices of these you are collecting from the outlet then this slice this mixing that has taken place here also you I mean suppose diffusion is very insignificant. Still just because of the stretching of these you can then assume there would be good amount of spreading of this and that Dirac function; isn't it? Spreading of that pulse, you have put a pulse there diffusion may be in significant, but the spreading can be because of this parabolic velocity profile.

And then finally, when you collect it in a bottle then it is mixed because you are giving enough time there or that is a different thing. So, it is miscible, but not strongly miscible still just because of velocity profile they could they could stretch much and the pulse can broaden much. So, that is why in this part the convection dominates, diffusion is less significant with the convection dominant. So, this is the regime where pure convection is the order of the. So, here it is pure diffusion and here in this region you have Taylor theory, no axial dispersion and you have Taylor Aris theory. Taylor Aris means; Taylor Aris assume that there is over and above the radial diffusion, radial diffusion means these over and above the radial diffusion there was axial diffusion also.

So, you have convection you have axial diffusion you have radial diffusion by three modes there the transport three modes the mixing is taking place. Now, one thing one must note here is that to apply Taylor dispersion; to apply Taylor dispersion velocity profile has to reach the wall; that means, if somebody says that the velocity profile is developing. So, velocity profile is developing means it is a capillary and then first when the flow enters let us say it was all uniform; it was all uniform flow and I mean the velocity when the flow enters into the capillary then there is a boundary layer development it develops.

And at after some distance these boundary layers they merge and you considered this flow to be fully developed. So, one must provide that much of residence time in that capillary for the flow to be fully developed, and if the flow is not fully developed; that means, outside this boundary layer it would be all fluid they are moving at the same velocity right. So, if the flow is not fully developed; obviously, you cannot expect that the effect of a Taylor dispersion; that means, effect of that pulse getting stretched that issue does not arise. So, one; so that is why to apply Taylor dispersion velocity profile has to reach the wall and flow; that means, what; that means, is flow has to be fully developed then only this can happen. So, this is possible only when I mean this is a typical you know empirical information that the researchers are providing only when tau is greater than 0.8.

So, that is why you can see here this part there is a line almost and they said that if tau is greater than this number then only this Taylor theory, Taylor Aris theory, Taylor dispersion all these things they will matter if tau is less than this they do not matter. And out of these then further they are saying that if diffusion is strong and velocity part is weak then; obviously, it would be pure diffusion, here it would be pure convection. And here now we are talking about diffusion plus convection out of this this part again the Peclet number is small. Peclet number is small means diffusion where D is large; D is large compared to the effect of convection. So, if D is large then they said that ignoring axial diffusion will not be a good idea.

We had radial diffusion, but Taylor theory said no axial dispersion. So, sorry no axial diffusion; so, no axial diffusion. So, Taylor theory says there is no axial diffusion so; obviously, this is applicable when D is not that large, D is not that significant and this is happening when Peclet number is large so; obviously, when Peclet number is large Taylor dispersion Taylors theory is good enough. But when Peclet number is small; that means, diffusion coefficient is large it would not be wise to ignore axial diffusion.

So, at that time let it let us say better do the correction of Aris that arises provide. Because Aris has said that the diffusion D ct is equal to the contribution from axial diffusion there is same as the diffusion coefficient and then you have R square, the radius of the capillary some places we write it as R naught square the same thing.

Similarly, v naught square let us write it as r naught square if we write it and then divide it by 48D. So, this is how the tailor Taylor; this is how the various regimes they work out.

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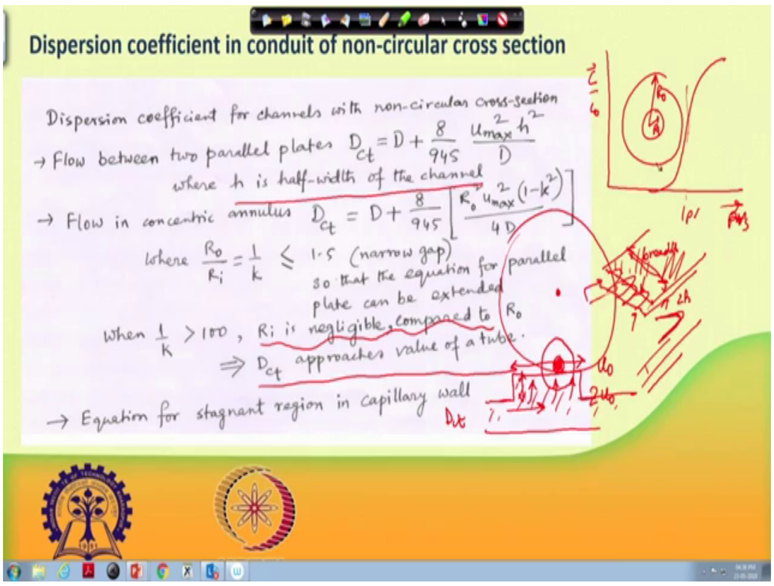

Now when it comes to this dispersion coefficient for the way we said these are square v naught square R square by 48D where R is the radius of the capillary. So, porous medium always we can go for a porous medium as bundle of capillary and bundle of capillary and we can then always. So, if you think of it how; what is the implication of these dispersion coefficient to us, one is we can get for example, some experimental data.

We can get some experimental data now and at one pore volume this is one pore volume. So, far says pore volumes pv s let us say pore volumes versus C bar by C 0. So, let us say we have a plot like this and we can find out what is the corresponding dispersion coefficient that is one way of looking at it.

Second thing is I mean there could be ways to think that why we are always going by a circular cross section will there be any better definition of dispersion coefficient if we use non circular cross section of the capillary? So, out of that interest what has been done is flow between two parallel plates, instead of flow through a circular cross section. So, flow between two parallel plates means if you have a parallel, if you have a plate like these and another plate like this.

And flow is taking place; let us say between these here you have one plate and you have below there is another plate and flow is taking place through this way. So; that means, you will see here the velocity profile would be; velocity profile would be these are the velocity profile. So, if I look at it from this side this would be the velocity profile, it would be again parabolic. So, velocity profile and near the edge there would be some aberration, but that is generally ignored; flow between parallel plates means this, this, this part is very that the distance between two parallel plates is small compared to this particular dimension which is called; so if this is called aperture and if this is called width.

So, then width is much larger compared to aperture. So, we can consider this to be just a parabolic velocity profile that is there inside everywhere. So, then the flow is taking place between two parallel plates. So, now, here in these case it has been shown that flow between two parallel plates the D ct that dispersion coefficient can be written as D, the contribution from axial diffusion plus 8 by 945 u max square h square divided by D; u max square. What was u max? u max is the maximum velocity; maximum velocity is typically the velocity at the center in case of in case of circular cross section we write if u naught is the average velocity u max is 2 u 0.

But in case of flow between two parallel plates it need not have to be 2 u 0 that that then u max is the velocity at the center line then we know that the velocity has to be maximum. So, that u max square h square, h is the half width of the channel; that means, generally you what they do is there; right, they put a center line and work only with the upper part of the problem. Assuming the lower part is just a mirror image or you can put the centre at 0 0 then it is upper part. So, basically, the aperture is considered as 2 h ok. So, if this is the centre line the upper plate is at h distance away and lower plate is h distance below.

So, this is; so, that is why 2 h is the distance between two parallel plates or in other words h is the half width of the channel. So, say if they call it width then I must call this as breadth, because this there if you call it width and then this must be breath. So, this is breadth and this is width. So, now, this D ct is D plus 8 by 945 u max square h square by D. So, this is something which the researchers have arrived at I mean doing these theories.

Similarly, if somebody works with and concentric annulus; that means, you have flow taking place through this annulus. So, this part is blocked, flow is taking place through this annulus. So, what kind of mixing will take place it is not a circular cross section it is not a flow between two parallel plates either; flow between two circular flow through an annulus to be precise. So, in this case a flow in concentric annulus this is the expression one ends of it.

In fact, these expression is just obtained from that expression which was obtained for a flow between parallel plates; what they assumed here is that R o by R I; R o is if this is the annulus. If this is the annulus this is R i and this is R o outer radius of the outer cylinder and the radius of the inner cylinder. So, R o by R i is equal to this ratio if it is less or equal to 1.5, then one considers this to be a narrow gap; that means, the gap is very narrow. And in that case this narrow gap can be considered as flow between two parallel plates, this narrow gap one can stretch it up and consider these just hypothesize it as just the flow between two parallel plates and that is what they have exactly done.

So, the equation for a plate can be extended and that is what they have done, they have extended it. So, this is R naught square u max square into 1 minus k square where k is R i by R or 1 by k is R o by R i ok. So, this is the expression one gets in on the other hand when 1 by k is greater than 100; that means, R o by R i is greater than 100; that means, R o is greater than 100 R i; that means, R i is very small and R o is large in other case R i and R o they are comparable. So, that is why there you are talking about a thin gap. So, that is why you extended this way, but in the other case this R i is very small and ro is very large.

So, in this case the assumption they said is that R i can be neglected compared to R o. So, then one can say this D ct approaches the value of a tube, then it is as good as whether you block this pathway it does not matter if R i is if it is the ratio is greater than 100 ok. So, this is not the end of it there are some other exercise that has been done in this regard is that, if you have a capillary flow is taking place through a capillary and then you have a stagnant zone. If you have a stagnant zone then this dispersion process can be affected in a very big way, because this flow is taking place. So, you this originally the concentration was all at beginning to start with let us say all concentration was 0, and then now you have given a concentration.

So, now concentration of C 0; so, now, it is penetrating there. Now, C 0 here the flow was taking place through these through this cross section. So, flow was taking place flow was going here, but here there is a static pool sitting there. So, as the fluid travels through the static pool there would be some amount of diffusion taking place with this stagnant zone. So, essentially what you would see in this case is the flow is arriving at outlet, but they it will not be immediately reaching; it will not be reaching the final concentration because it will be always less than C 0. So, it will take a much longer time to reach the C by R i C 0 as 1.

Because one has to change the concentration of the stagnant zone, changed the concentration from 0 to C 0 that changes to happen and in other places in the conduit; let us say this was the conduit in other these places you are simply flowing. So, the fluid is simply pushed out, but here it fluid is not being pushed out only by diffusion molecule can go in there by Brownian motion and change the concentration.

So, it takes a longer time. So, what scientists have done is they have found out the expression for D c t given these dimensions let us say what is these dimension, what is these dimension. So, if these dimensions are given what would be the corresponding D c t. So, they have taken this trouble to find out the analytical form of this expression.

I just want you to mention that such type of analytical exercise and researchers have done from the other side of this. So, one side was that you get experimental data and you try to feed that experimental data and find some value of D c t. And so now, you compared D ct this one is a good D c t or bad D c t this core is better that code is more homogeneous etcetera.

There is one way; the other way is if we look at it from completely from the first principle that I have a circular cross section of capillary. Now instead of circular I have annular a concentric annulus; I instead of circular I have flow between two parallel plates instead of circular I have circular cross section, but attached to it and be attached with this a stagnant zone. So, how these what kind of D ct one will end up with.

So, this is also a parallel exercise researchers have done; then they try to relate the experimental data with this. So, I just wanted to try, I try to give you the overview of this various approaches that people have taken to arrive at better understanding of these D ct term. This is all I have as far as this module is concerned. I will talk about some other forms of dispersion in the next lecture.

Thank you very much.