Flow Through Porous Media
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Lecture- 21
Flow Equations (Packed Bed)

I welcome you once again to this lecture of Flow Through Porous Media. What we were discussing where the Flow Equations we have already discussed about Darcy's law and its genesis. Now we are looking at other way of looking at flow through porous media. In particular the way the flow Navier Stokes equation and flow in a; flow in a conduit as it is handled in fluid mechanics. Here we are trying to extend that concept to porous media.

So, we are trying to define Reynolds number specifically for porous media and then trying to come up with some expression for friction factor, so that through friction factor we can get a pressure drop. So, this will be an alternative route to these this Darcy's law. So, we are trying we are heading towards this Kozeny Carman Blake Plummer type equation or a combination that Ergun equation. So, here we are heading towards that. So, this is the point where we ended I mean we mentioned about how to calculate Reynolds number and what is the implication of friction factor in the last lecture.

So, we will continue building on that and come up with a final expression for pressure drop through porous media using these concepts. And we arrive at the final form which is known as the Ergun equation. So, let us start. So, what we have here is.

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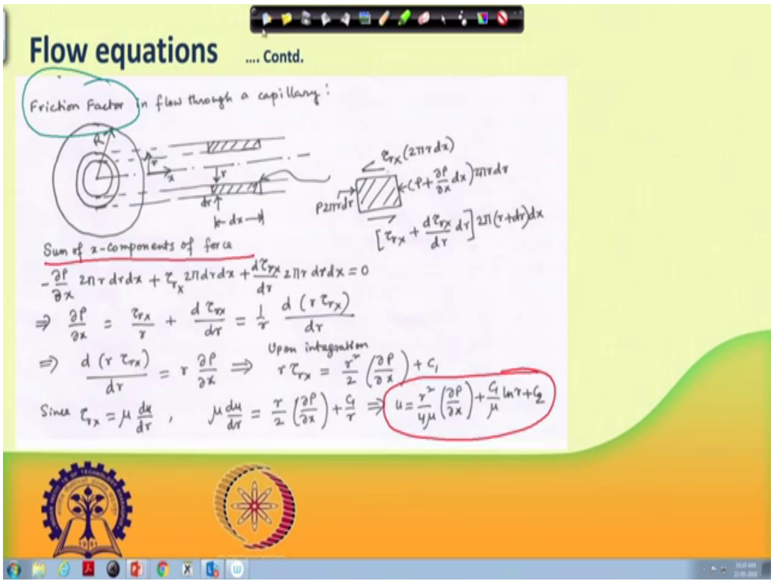

In the last class we discussed about these we discussed about the definition of friction factor, we discussed the definition of friction factor. In particular we had; we had; we had seen we have done a momentum we have done a force balance equation, sum of x component of force. And from there we arrive at the general equation for velocity profile.

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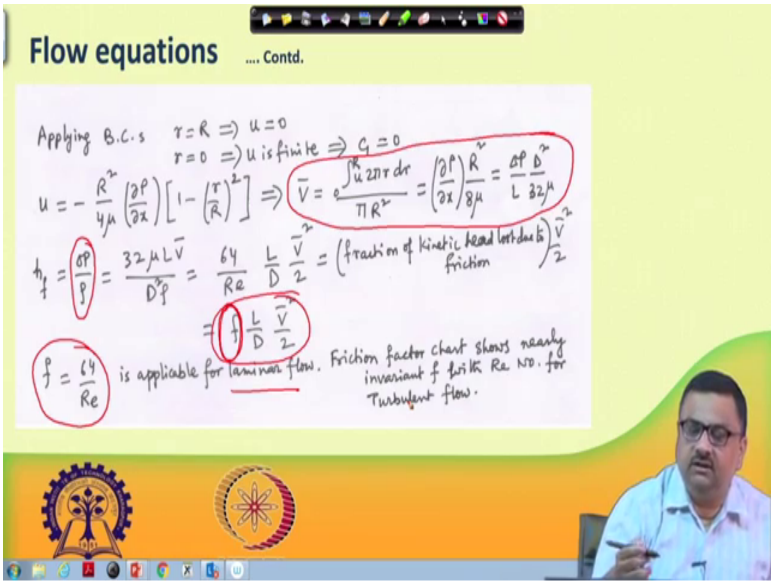

So, this is something which we had done in the last lecture and then further down what we did is we have found out that this is the final form of the velocity profile or in other words we were interested in the average velocity. So, this is the expression for average velocity.

And then we try to find out what would be the delta P by row pressure drop arising from friction factor. So, if the average velocity depends on pressure drop by this way. So, what would be the pressure drop in that case in terms of average velocity? This is the expression and then we finally; we express this delta P by rho in terms of a friction factor f. And then the kinetic energy basically V bar square by 2 average velocity square by 2.

So, this is some kind of kinetic energy per unit volume that is something which we are looking at. So, essentially we are trying to find out what we have some idea how to get this friction factor. And we found that this friction factor is 64 by Reynolds number, when it is flow is laminar. And when the flow is not laminar when flow is above the threshold value of Reynolds number we find that this f is supposed to be invariant with Reynolds number this we discussed in the last class. Now, if we try to build on this concept if we try to go further on this. So, what we would do in that case is.

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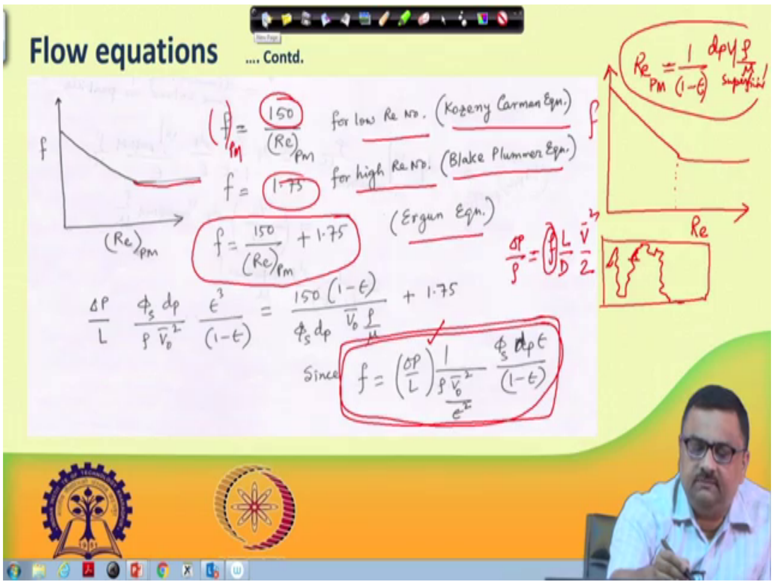

We will also write a very similar equation here, here you can see the friction factor versus Reynolds number would take this kind of plot if we are looking at fluid flow through a conduit. So, in that case if it is a general Reynolds number which is the diameter of the pipe velocity average velocity density divided by viscosity. And if this is the friction factor then we expect this f to be 64 by Reynolds number so; that means, f is proportional to one by Reynolds number so it is decreasing like this.

And then beyond the threshold value of Reynolds number it becomes; it becomes constant I mean for all practical purposes for I mean for some amount of if we may make some amount of approximation; obviously, then it can be treated as constant.

So, if we extend this same concept, so what is the difference? I can think of they say let us say in a porous medium we can think of a conduit, but this conduit if this is the length if this is the length of the bed. So, in that case even it was a pipe it was this entire this length was I mean when we wrote this delta P by L we considered this length. But when it comes to a porous medium it is basically it is taking a zigzag path right in some places it is occupied by solid like this. So, if one has to one has to make note that it is a longer path way they are taking and as a matter of fact they are taking that they are going at an angle not exactly along l.

So, then there were just there are arguments ok, let us say we go by a angle of 45 degree. So, it would because of I mean it is then 1 by root 2. So, you have to take into account those factors etcetera. So, instead of 64 they have come up with this number 150 this is what is Kozeny Carmen equation. We said that friction factor for porous media, if the flow is for low Reynolds number it would be written as f porous media would be 150 by Reynolds number for the porous media what is Reynolds number for the porous media? You must recall we had already discussed this in the earlier lecture.

We in fact, we derived it Reynolds number for pure porous media was 1 by 1 minus epsa where epsa is the porosity and then we had dp; dp is the diameter of the particle which porous media is the constituting the porous media. That means, if we if those particles are spherical the particles that are constituting the porous media. If those particles are spherical, then we recall this dp if they are non spherical then we have to take another attachment to it which is phi s this sphericity factor we have already defined those.

So, either dp or phi sdp then we have V superficial V superficial not interstitial because already we have taken out one V interstitial divided by phi. And then we had made were sorry we had; we had; we had taken V superficial we already had interstitial. So, that time we had there was an extra term and that if extra term was already taken out from here and these epsa was canceled. So, you remember. So, dp V superficial row divided by mu. So, this is the; this is the Reynolds number for porous media. So, this is so for this Reynolds number there is a friction factor for porous media which is 150 by Re Reynolds number for porous media.

Whereas when there is it is at high Reynolds number, the region where f is supposed to become invariant with Reynolds number they put a number 1.75 and this part was attributed to Blake Plummer Blake its called Blake Plummer equations. What Ergun equation? What Ergun's proposition is that, friction factor for porous media would be written as sum of these two. So, the idea is that when Reynolds number is small then this term will dominate 1.75 will be ignored when Reynolds number is high this term would be low in that case 1.7 will be will dominate.

So, automatically it will so it would be a either Kozeny Carmen or black plumber equation depending on which range and Reynolds number one is working with. So, if somebody wants to follow these Ergun equation then one can and particularly if one defines friction factor in this context as this quantity then one can write this automatically this will follow when you write this in expanded form. Because you know Reynolds number for porous media would be phi s dp Vo bar rho by mu 1 by 1 minus epsa. So, exactly these terms it is basically exactly 150 by R e pm you can see 1.75 it remains as it is.

And these term we are calling it the we are calling these the friction factor f. So, what is the what is friction factor? We can see here one delta P by L in that friction factor we have these kinetic energy term here see we had earlier what was the; what was the equation we had delta P by rho is equal to f L by D into V bar square by 2 right.

So, this was; this was; this was the equation this was the friction factor we had and this time we are trying to define friction factor. So, delta P by L is going there I can see rho V o bar square by epsa square this is going there and then 1 by epsa would be there and then phi s dp. So, this is; so, you have this L by D. So, that D has to be taken into account. So, this is this is something which is defined as a friction factor in this context.

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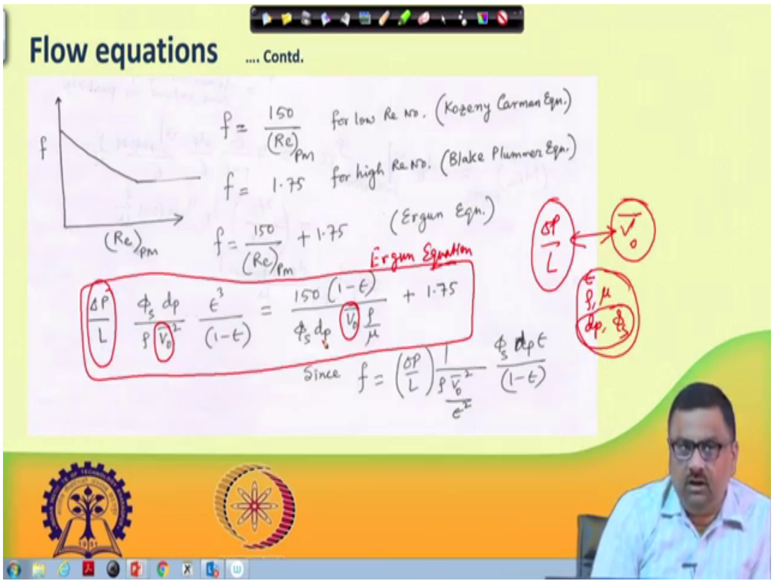

So, with these equation so these equation becomes; so these equation becomes these equation becomes the Ergun equation. And these gives me the pressure drop delta P by L this is the pressure gradient. As a function of what or we have on the other hand we have these V o bar square now you can see here it is a quadratic equation here we have only V o bar all other terms are known. Sphericity factor and dp; dp is the particle diameter of the particle equivalent diameter of the particle which is constituting the porous media.

So, this is this information should be known rho is the density of the fluid epsa is the porosity. So, this is the this is now these are only terms so basically it is relating delta P by L and V o bar these two are being related; these two are being related with the these two are being related through epsa porosity. Rho mu these are the characteristic of the fluids epsa is the porosity of the porous medium and we have dp and phi s phi s. So, these are the party diameter equivalent diameter of the particles and sphericity of the particles that are constituting the porous media.

So, this is the; this is the relation between these two and these are the through these through these; through these parameters. So, this equation is the famous Ergun equation and this is you can say this is an alternative to Darcy's law because we this is basically quadratic to V o bar bar V o bar square and V o bar. V o bar is basically the superficial velocity in this case. So, this is how Ergun equation is defined and you can see how it is an alternative to Darcy's law.

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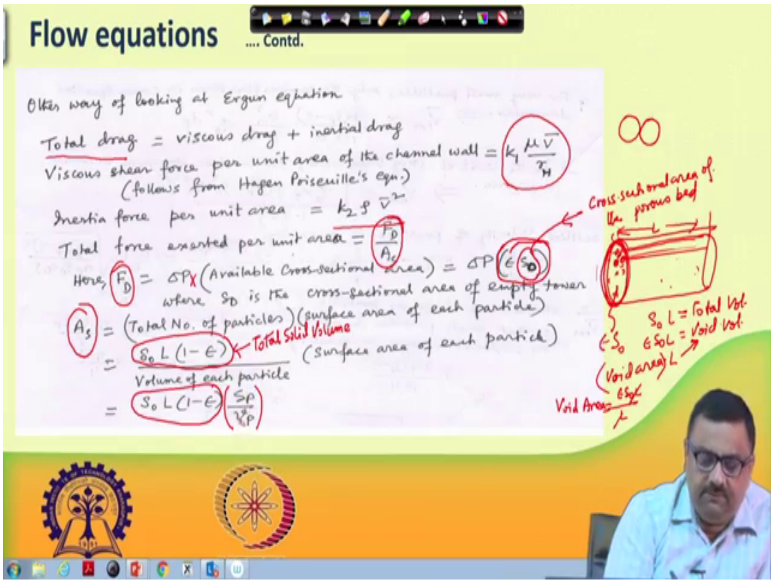

Now there could be other way of looking at this whole exercise that we have done here and which is given here other way of looking at Ergun equation. So, one can consider total drag as combination of viscous drag plus inertial drag and then viscous shear force per unit area of the channel wall is given by this quantity whereas, this follows directly from Hagen Poiseuille’s equation. Hagen Poiseuille’s equation is the Hagen Poiseuille’s equation is if one works with Navier stokes equation include mechanics it is a very similar treatment we have already shown when we arrived at the velocity profile for a flow through a pipe.

So, the one very into this is maybe a stokes equation and that is already so for. So, I mean similar time equations were defined. So, from that equation one can arrive at this expression which is viscous shear force per unit area of the channel wall. On the other hand intertiaforce per unit area is written as k 2 rho V bar square and total force exerted per unit area would be this drag force divided by projected area. So, here the FD is equal to in case of a porous media FD is equal to delta P into available cross sectional area, so delta P into available cross sectional area ok.

So; that means, the available cross sectional area would be if a, so is the cross sectional area of the cross sectional area of the cross sectional area of the porous bed. So, what; that means, is if this is the porous bed we are talking about. So, we said that this porous bed can be thought of as multiple number of capillaries bundled together and in between there are some solid phase. So, one can think of this if S o is the cross sectional area this entire cross sectional area is called a S o this is S o. And if this S o is multiplied by epsa; epsa is the porosity.

So, then you get the cross sectional area that are available for flow; that means, the holes through which the fluid can flow how it is possible we have already discussed this once basically this is S o into L S o is the overall cross sectional area. So, overall cross sectional area multiplied by the length of the bed that gives me the total volume total volume. And this epsa into S o into L that gives me then void volume because porosity multiplied by the total volume is the void volume and then if this is the void volume. So, then the void area if I look at what is the void area this is the length this is the length over which this is this ports are running.

So, what is the void area? So, basically void area available for flow multiplied by L that gives me the void volume. So, void area multiplied by L would be equal to the void volume. So, void area multiplied by L is the void volume so; that means, void area would be in that case void area would be what is the void volume? Void volume is epsa is S o L divided by L. So, this L and this L they will cancel out. So, you are left to it only epsa. So, that is exactly what we had done here. So, epsa S o is the available cross sectional area for flow, multiplied by delta P which is the pressure draw across this length. So, these gives me the force FD.

Pressure drop across total so this is this gives me the FD the pressure drop. Now on top of this we see here that the surface area of particles which is A s which is because total force exerted per unit area would be F D divided by A s. So, if this is the drag force, but the idea over which this is applied is the area through which this flow is taking place this entire area.

So, that would be written as total number of particles that are present in the system total number of particles multiplied by surface area of each particle. That means, you are assuming that these particles they do not have any common area, two particles are sharing one common area they are there they are just and they are isolated; that means, one particle another particle they meeting but this common there is not any common area.

So, that is what your with that assumption one can write these total surface area total area over which this drag is applicable that is equal to total number of particles multiplied by surface area of each particle. So, then total number of particles on the total number of particles can be given as total volume of particle divided by volume of each particle.

So, that gives me total number of particles total volume of particle divided by volume of one particle that gives me total number of particle. So, total volume of particles would be the solid volume. Solid volume is total volume multiplied by 1 minus epsa because epsa into total volume is the void volume and 1 minus epsa multiplied by the total volume is the solid volume. So, 1 minus epsa multiplied by the total volume which is so the total area multiplied by L.

So, a S o into L is the total volume multiplied by 1 minus epsa. So, this gives me the total solid volume. So, this gives me total solid volume. So, these gives me total solid volume and this divided by volume of each particle. So, volume of each particle would be the volume of each particle and multiplied by surface area of each particle that is continuing because a says total number of particle multiplied by surface area of each particle that term is simply continuing.

So, now surface area of each particle divided by volume of each particle that is equal to Sp by Vp. So, that is so that is why you have this term which is the total solid volume multiplied by Sp by Vp that is what you have as A s.

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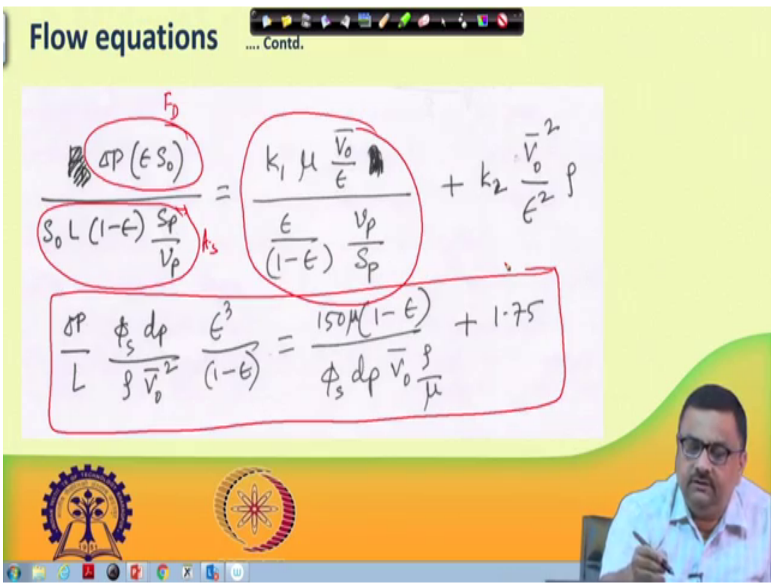

So, now, if we now expand this; if we now expand this, we can see here the this term is see here they are writing it as k 1 little let us go back once again and revisit.

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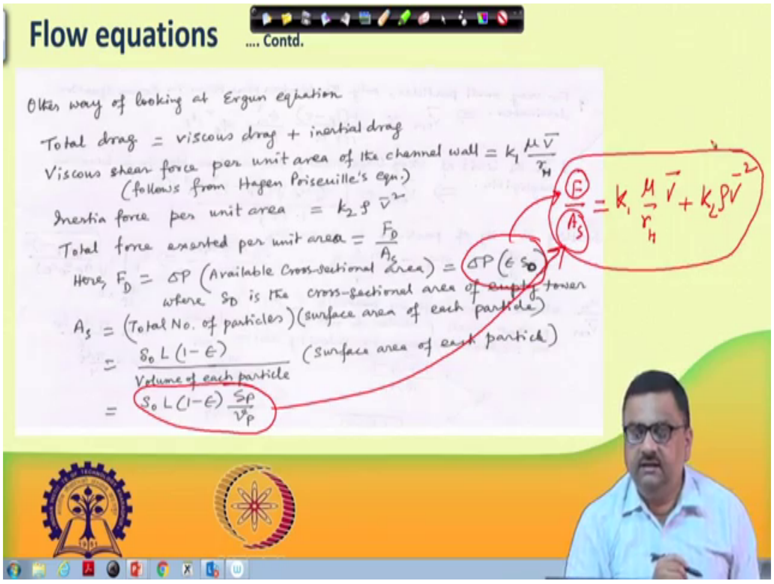

So, what we have here is total drag, so total drag is FD divided total force exerted per unit area is FD divided by A s and that is equal to viscous shear force per unit area which is k 1 mu by r H into V bar and inertia force per unit area which is k 2 rho V bar square. So, now this A s term is we have already obtained this as the A s term so, this will go there as A s. And these FD this term would be given by this quantity, so this will go there.

So, these would be equated with k 1 mu by r H V bar plus k 2 rho V bar square. So, this is how you would you would look at it and then you. So, remember this is the equation that we are expanding further in the next slide. So, this is going to be so this left hand side what we have here is delta P epsa. So, this is the drag force this is the total this is the total force and this is A s we had defined in the last slide.

And this is k 1 mu so this we have already talked about the hydraulic diameter r H is there. So, we put in put all those expressions there and this is k 2 V o bar square by epsa square because I think that was in interstitial velocity. So, you have to; you have to convert these two superficial. So, basically you have to write it in terms of superficial velocity. So, you end up with this k 1 and k 2 other than these two terms you end up with these.

So, now if you look at these if you place this k 1 as 150 and k 2 as 1.75 you see that you will arrive at that same equation that we had obtained earlier. So, this is the same equation. So, what did we do here we have if we just go back once more.

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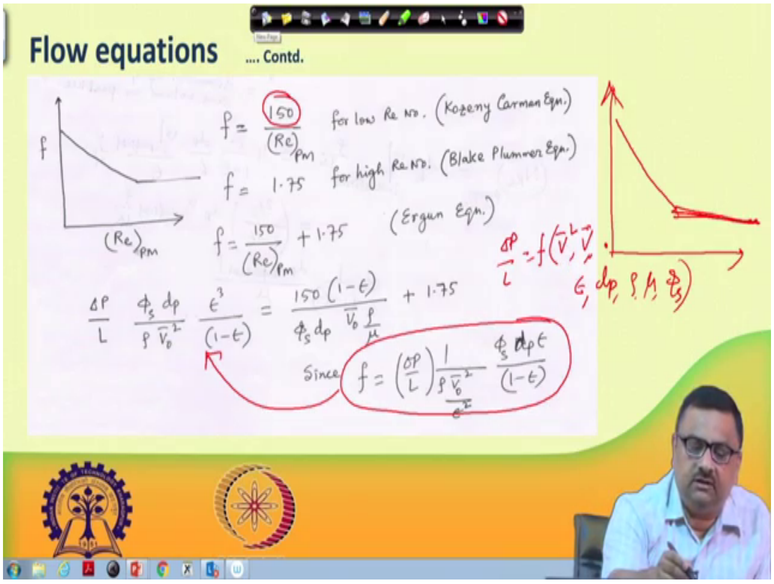

What we did here is we said that, the friction factor for a friction factor is given by it has certain behavior. And these behavior we can this part is inversely proportional to Reynolds number and here it can be considered practically constant. So, from this argument and then they said that actually flow through a pore is not a straight flow through a pipe. There are bends and there are twists and turns. So, those are to be accounted. And so, there are various factors I mean I do not want to get into those how those factors were arrived at.

But these through those factors they have changed they said that it is not 64 by Re. In fact, it was not done in one go it was proposed some other number was proposed and then there this was corrected it was close to 150 and then finally, it was rounded up to 150. So, 150 by Re pm instead of 64 by Re pm, which we derived for flow through a circular conduit of circular cross section. So, then and then said that the other part is 1.75 where it remains practically constant. So, then this friction factor and now what is this friction factor friction factor is defined in exactly similar way as it was done for flow through a pipe.

So, this is how they are they were defined and these friction factor was placed here and it was written as a friction factor is equal to 150 by Reynolds number plus 1.75. This equation happened to be giving relating delta P by L the pressure gradient as a function of it is a quadratic expression basically it depends on V bar square and V bar and it depends on other parameters such as epsa dp rho mu phi s. So, these are various parameters it depends on. So, this is a different type of equation altogether and this equation is referred as Ergun equation ok.

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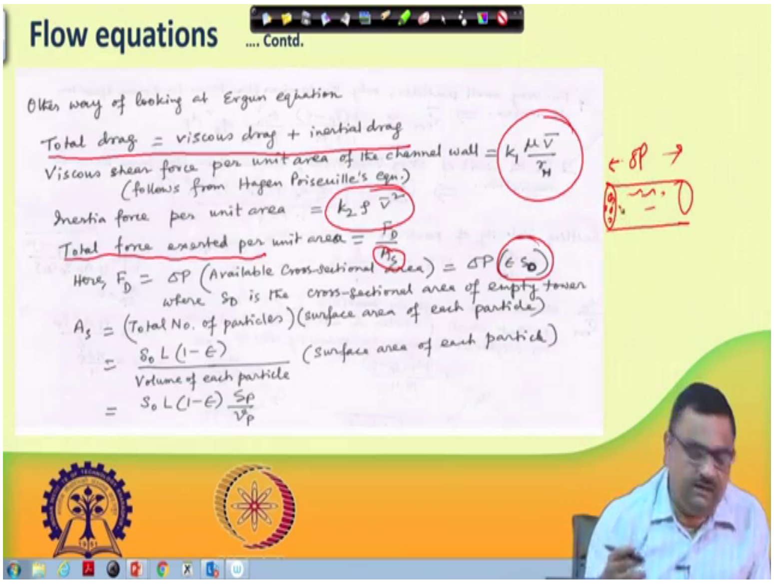

Similarly, we had talked about another way of looking at it and here what we did is we define these total drag force as sum of viscous drag per unit plus inertial drag. Viscous drag is basically the contribution which we are getting for 150 by Re pm. And the inertial drag is contributing to 1.75. So, essentially what we showed is this k 1 is 150 and k 2 is 1.75 and these are this these are the; these are the ways by which it was done. These are the ways we have written the cross sectional area A s is equal to a total number of; total number of particles multiplied by surface area of each particle.

So, from this way we could find out what is the total force? Because total force is delta P across say let us say across the porous medium you have given a total force of delta P over a length L and these delta P was applicable only through these holes. So, the force is force that is force is basically drag force is delta P available cross sectional area multiplied by this delta P and available cross sectional area was epsa into a S o.

And then these force was divided by this A s where A s is the total area over which the flow is taking place. So, that is basically the total number of particles into surface area of each particle. So, from there we have arrived at this.

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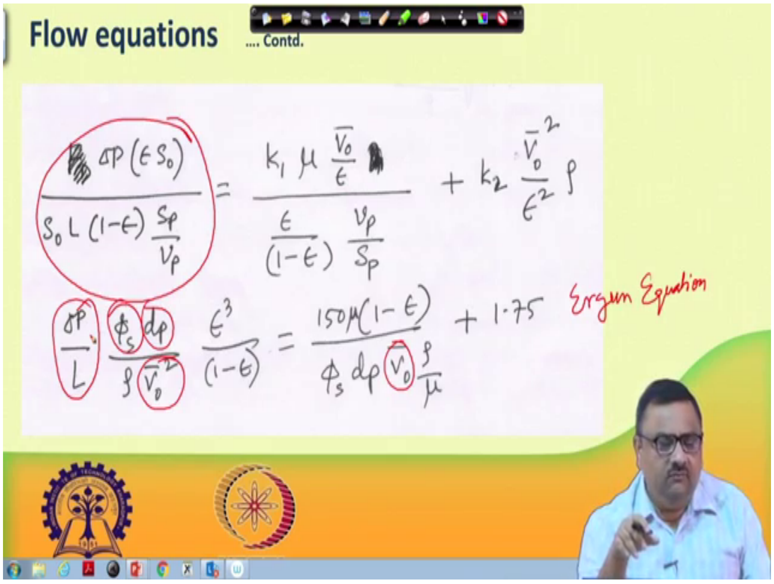

So, these from this understanding we have gone to say that this is the force per area and from here we have derived we can derive delta P by L once again as a function of V over square V o bar. And the same exactly same equation we arrived at which is known as Ergun equation.

So, when it comes to flow through a packed bed and particularly we are going to when people extend this flow through porous made flow through a packed bed to a condition where fluidization occurs etcetera it is very common to follow Ergun equation. So, so it is basically you need one needs to have good understanding of dp and phi s which is the sphericity factor and particle diameter.

If someone has good understanding of these in the if someone has this information then it is good to use this equation. So, when it comes to a packed bed where the packing one has full control over what they are packing what is the what are the dimensions of the particles, what are their characteristics of those particles whether it is spherical or non spherical? When one has that full handle over those then its Ergun equation is probably the best one, but when it is a porous medium where you do not have control or do not have full knowledge of what are the pathways.

Then it is better treat these not better not to get into this rather depend on Darcy's law. So, this is how; this is how we have the generally it is followed. So, when it becomes to packed bed flow through packed bed pressure drop Ergun equation is common. And what we have here is further these equation would be would be next extended in my next lecture you will find the these this equation. This Ergun equation would be the starting point where a packed bed may act not exactly as a packed bed it depends on certain velocity and we will touch upon that very briefly in my next lecture.

So, remember this equation that we derived Ergun equation and in the next lecture we will build on this. And particularly look at a case where the packed bed may not remain exactly a packed bed. So, it is a little different kind of porous medium, we will discuss about that in the next lecture. So, that is all as far as that the this model of the lecture is concerned.

Thank you very much.