Flow Through Porous Media
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Lecture -11
Mass Continuity (Elementary Flow)

I welcome you once again to this course on Flow Through Porous Media. We were discussing about Continuity and various.

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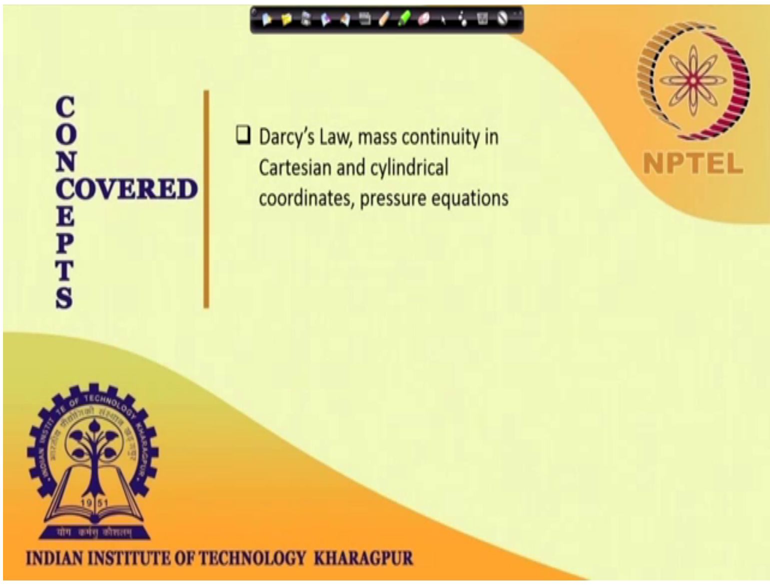

We were discussing about continuity equation and more specifically we were discussing about complex potential. We were discussing about potential function and stream function and their combination in terms of complex potential. Now we are going to see how we can use complex potential in our; in our; in our study of flow through porous media.

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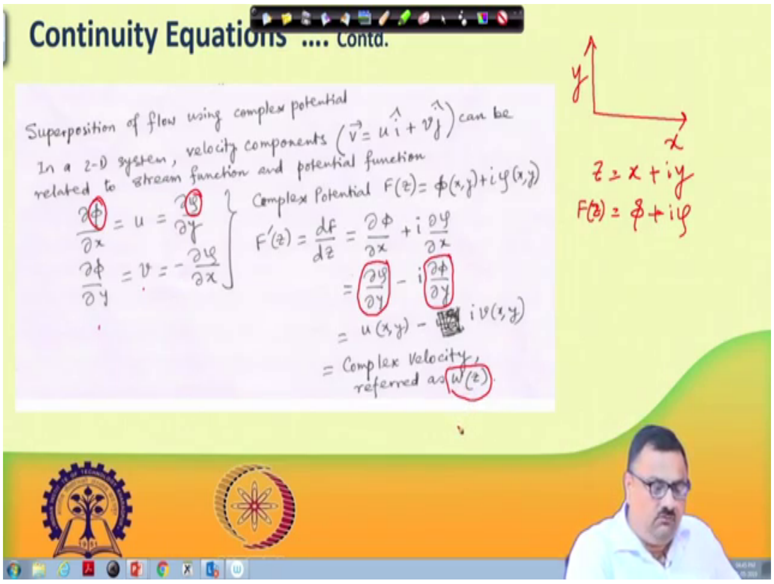

So, what we were discussing in the last lecture is that we have already defined what is called what is this stream function and what is this; what is this potential function and what is this stream function.

So, we said that in a 2D system velocity components can be related to stream function and potential function in the form that is mentioned here. Now we want to define here something called a complex potential which is phi plus i psi where phi and psi they are function of x and y. So, now, if this is the complex potential if we want to take a derivative of it and derivative. So, we are having x and y. So, that is these are the coordinates x and y. So, now, we are writing z as x plus iy ok. So, now, we can do we are writing F z as F z we are writing as phi plus i psi.

So, now F z we if we want to take a derivative of this F z. So, if we take a derivative of this F z what we get is d F if we take a derivative of a complex number there is a certain way to take a derivative of complex number this is available in standard mathematics book that either you take del phi del x plus i del psi del x this could be possible or we can take del psi del y minus i del phi del y. So, it has to be either in terms of x because here see the argument z it itself is x plus iy.

So, either you take the derivative with respect to x like this or you take the derivative with respect to y, but then you it becomes minus del psi del y minus i del phi del phi del x I am not take it this is; this is; this is available in mathematics book if you take a derivative of a complex number. Now we can see that these del psi del y we have already defined del psi del y as u and del phi del y we have already defined as v. So, we can write these as u minus iv u minus; u minus iv that is what is F prime z. Now u minus iv we define these u minus iv as complex velocity and we define we give this reference W z.

So, this is the complex potential and I mean see it is something like this and now earlier we had a potential which is phi and we had taken a derivative of it and we got the velocity, we have taken a derivative in y direction, we got the y component of the velocity now you have take calling it a complex potential and we are suppose it is basically consisting the part real part is only phi only.

So, when we take a derivative of it just by the same token we should get velocity. So, when we take a derivative of this complex potential we get instead of a real velocity as we had it here we get it get a complex velocity and this complex velocity is referred here as W; W as a function of z. So, now, with this understanding of complex potential what we are going to do is we are going to look at some elemental flows. What are these elemental flows?

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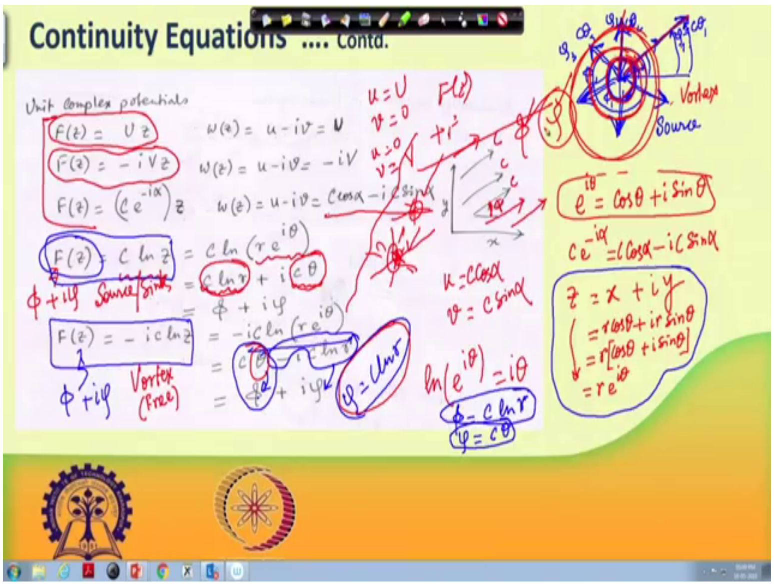

Let us say we work with some of the unit complex potentials. Let us say some examples one is F z is equal to U z in the later on we will take different forms of this F z. So, F z is equal to U z ok. So, then what would be W z? W z would be the derivative of it and W z we have already seen that in the last session.

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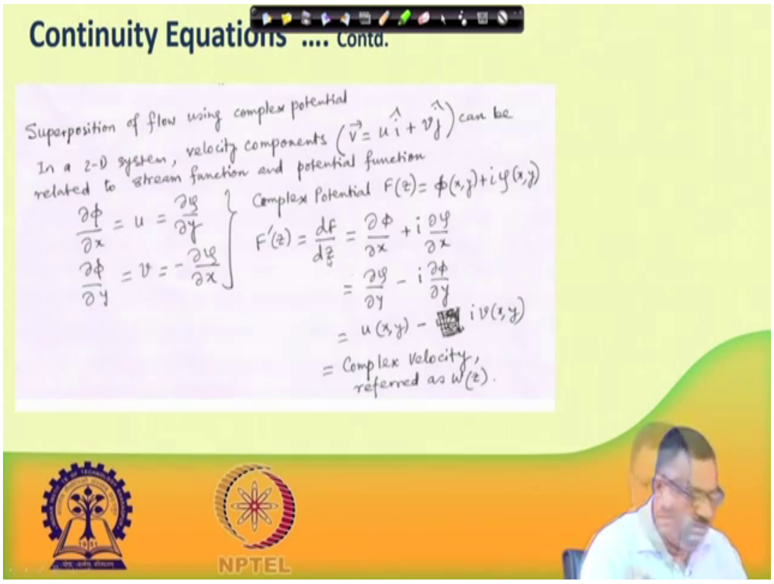

In the last slide we have seen that W is basically u minus iv W is u minus iv and W is d F d z. If W is d F d F d z and W is u minus iv. So, here we can write W as u minus iv and these when we take a derivative of these with respect to z we get u. So, d F d z is u. So, W z is u minus iv and on the other hand if we take the derivative it is capital U. So, what; that means, is that for this type of flow here u is equal to capital U and v is equal to 0 right because on the right hand side I have only the real component U is some constant let us say U is some constant number.

So, then we have we can see if we match real with really imaginary with imaginary I see real is basically capital U and imaginary is there is no imaginary component here it is 0 ok. So, when you took the derivative it is U derivative of U z is U with respect to z it is U and this U is u minus iv it has to be. So, then what you get is u small u is equal to capital U and this v is equal to 0.

So, what kind of flow we are talking about? We are talking about a velocity field which is U i hat plus 0 j hat so; that means, it is the velocity in x direction. So, see if somebody plots this verses let us say this is x and that is this is y. So, we are talking about velocity this is the type of velocity we are looking at and the velocity is u and we are talking about velocity in x direction.

So, F z is equal to U z is an elemental flow which is the flow in x direction at uniform velocity capital U. Now we look at the velocity F z these elemental flow. What we are trying to do is we are trying to kind of different expressions for F z when F z represents some elemental flows. So, F z equal to minus iv z. So, in that case if we take the derivative of it becomes minus iv and on the other hand W z is u minus iv.

So, if we equate the two here in this case we see u is equal to 0 and v is equal to; v is equal to capital V. So, what kind of flow we are taking flow is taking place here. Here in this case here; in this case the flow is basically the velocity is 0 i hat plus V j hat and a velocity and the fluid flow that we are taking talking about is velocity in y direction uniform with the value V.

So, this is the this is this next elemental flow which is the flow in y direction. Next is we are looking at F z equal to C e to the power minus i alpha into z. So, this is another elemental flow. Now if we take the derivative of that if we take the derivative of this expression F z equal to this quantity C to the power minus i alpha z if we take the derivative what we end up seeing is that it is it would be see e to the power i theta; e to the power i theta is equal to cos theta plus i sin theta right e to the power; e to the power C e to the power minus i alpha z.

So, when we take the derivative with respect to z it would be C e to the power minus i alpha. So, you can write this as C e to the power minus i alpha can be written as C cos alpha minus i C sin alpha. So, this is what is happening here as W z is equal to u minus iv which is equal to C cos alpha minus i C sin alpha. So, what that means, is then in this case this it has to be u minus iv. So, u is equal to C cos alpha and v is equal to C sin alpha.

So, if that has to happen so that means, these this is talking about a velocity with magnitude C at an angle alpha to x axis. So, we are looking at velocity neither in x direction nor in y direction, but velocity at an angle alpha; velocity at an angle alpha to x axis. So, uniform velocity. So, this is referred as F z is equal to C e to the power minus i alpha into z. Similarly, you can have F z the elemental flow which is equal to C ln z C ln z can be written as C ln now z is equal to x plus i y.

So, x plus i y if you forget about the xy system if we forget about the xy system and work with the r theta system. So, in that case any point on these with let us say at distance radial distance r and this angle theta. So, then we can write these x component as x equal to r cos theta and the y component would be r 90 degree cos 90 degree minus theta or r sin theta. So, y component is r sin theta so; that means, z is equal to x plus iy if we want to shift from Cartesian system to the other I mean a say cylindrical system or r theta system then you can write these as r cos theta plus ir sin theta. So, if you take cost if you take r common you write these as cos theta plus i sin theta.

So, you can write these as equal to r e to the power i theta. This is the we have used these already in these context before. This is Euler's formula that cos theta plus i sin theta can be written as e to the power i theta. So, it is re to the power i theta. So, instead of C ln z instead of C ln z we can write C ln and instead of z we can write r e to the power i theta. So, first of all these are of one types of flow. These first three that we are discussing U z minus iv z and this quantity C to the power minus i alpha z. These we have already mentioned one is horizontal flow, another is vertical flow, another is flow at an angle.

So, these three are off one particular type, but C ln z is a new type. So, C ln z first of all what kind of flow we are looking at. So, to find this instead of finding out the velocity let us first look at how the potential function and stream function how or potential lines and streamlines how do they look. So, we see here that F z is equal to C ln z. So, if z is equal to C ln z leads to C ln re to the power i theta and if we break it up it becomes C ln r and C ln of e to the power i theta. So, ln of e to the power i theta would be simply i theta ln of e to the power i theta that would be simply equal to i theta. So, C ln e to the; so, ln of a into b that is ln a plus ln b, so, ln a is ln r and ln b is i theta.

So, it is C ln r plus i C theta. Why we are doing this because we can see here this is the real component C ln r and C theta is the imaginary component and we know that our original definition of F z was phi plus i psi phi plus i psi that was a definition of F z. So, then in that case the real component is basically the phi and the imaginary component is basically the psi.

So, we are talking about now we are talking about stream function phi is equal to C ln r and potential function sorry we have this we have the potential function which is C lnr and the stream function which is equal to C theta. So, what is potential function potential function or what is potential lines? Potential lines are the lines along which potential function is constant and stream function streamlines are the lines along which stream function is constant. So, I am looking at lines along which stream function is constant. So, or in other words I am looking at lines along which C theta is constant. So, what would be this line in this; in this; in this context what would be this line; what would be this line? In this context where the C theta will be constant.

So, I can we are talking about some line along which theta is constant C is C is some arbitrary constant. So, we are looking at some line along which theta is constant. What is that line in a r theta system along which theta is constant. See in r theta system we can have if this is the point this is the center we can have these lines radially outward or we can have these lines which are circles in r theta system. So, the line along which the stream function is constant or the lines along which theta is constant I see that the theta as far as this line is constant this is the theta. So, wherever you go this theta remains same.

So, I can say that this is a line along which theta remains constant this is also another line along which theta remains constant. So, we can say that these are the lines along with C theta remains constant. It is may be C theta 1, this may be C theta 2, this may be C theta 3, but these are the lines along which this r comes constant. So, I can say these are the lines along which the psi is constant maybe I call this psi is equal to psi 1, this is psi is equal to psi 2, this is psi is equal to psi 3. So, along these lines the psi would be constant. So, these are then the streamlines. So, what kind of flow we are talking about where these are the radial lines coming out from a point and these lines are the streamlines.

So, naturally we are talking about a source and what are the potential lines in this case. Potential lines are the lines along which the potential function remains constant so; that means, the C ln r remains constant C ln of any way C is a constant. So, the lines along which r remains constant. So, line along which r remains constant in these set up is this is one line this circle around this center these as r as constant some fixed r is there then this next line where the r is constant some other r it is r 1, this is r 2 that is r 3. So, this is. So, I can say this line has C ln r1 is constant, this line has C ln r2 has constant, this line has C ln r 3 has constant or in other words I can say that this is the line along which the potential function phi is constant and let us say this is phi 1 this is phi 2 and we can draw another line which is phi 3.

So, the potential lines are the circles and they make perfect sense because the radius makes perpendicular radius is perpendicular to the circle the radius is perpendicular to these line to these; to these; to these circles. So, the circles are the potential lines and the radial lines coming out from that point these are the streamlines.

So, you can see that when we talk about F z this F z as C ln z we are talking about a source elemental flow of a source. It may not have to be source alone, it can be it can also be sink depending on the sin of C. C can be negative if C is positive it is a source I can I agree, but if C is negative in that case we have to talk about sink there so; that means, the flow the stream lines would be inward steam lines instead of coming out from a point emerging out from a point they will be directed towards a point. So, then it will act as a sink. The next one that we have here is another it is; another it is another elemental flow which is F z equal to is minus i C ln z instead of C ln z now we put a put this as an imaginary component. So, a real we are putting it as imaginary the C ln z is put as an imaginary component.

So, then we do the same exercise here instead of ln z we are writing ln r; ln r of e to the power i theta. So, r e to the power i theta means this z is equal to r e to the power i theta we have already seen this z is x plus iy and x is r cos theta for this system y is r sin theta for this system effort for r theta when we convert from Cartesian to r theta system and then we said r we have taken out common which is then making it cos theta plus i sin theta and then we said that it is then further r because theta plus i sin theta can be using Euler's theorem we can write it as r e to the power i theta.

So, the same thing applies here instead of C ln i C ln z minus i C ln z we are writing minus i C ln r e to the power minus i theta then we again break it ln of a into b is ln a plus ln b. So, it would be a minus i C ln r minus i C ln e to the power i theta and ln e to the power i theta is simply i theta as we have seen. So, if we break this up we get C theta minus i C ln r if we go to the next step. So, now, again we are writing this F z as we had done earlier F z we are writing as phi plus i psi. So, then we have this C theta as phi and minus C ln r as psi minus C ln r as psi and C theta as phi. So, in this case we are talking about stream function psi psi is equal to C ln r.

So; that means, stream function stream lines of the lines along which stream function is constant so; that means, stream lines are the lines along which C ln r would be constant or in other words stream lines of the lines along which r is constant whereas, in case of potential lines along which the potential function is constant or in other words potential lines are the lines along which theta is constant.

So, I think it is just the opposite of what we what we have seen earlier in C ln z. In case of C ln z we saw stream lines are the lines that are coming out stream lines are the lines that are coming out, but this time we see that the stream lines are the lines we see that the stream lines are the lines along which the r is constant so that means, this is the new streamline this is the new streamline this is the new streamline. So, these are the streamlines.

Now, not potential lines and what are the potential lines potential lines are the lines along which theta is constant; so that means, putting these are the now the new potential lines. So, the streamlines are they are these lines now the circles. So, what kind of flow we are talking about where streamlines are circles we are talking about the flow of vortex. So, of course, when it comes to vortex and people generally talk about free vortex or forced vortex free this we are we are referring to free vortex there is no there is no viscosity involved here, no layer is sliding against the other.

Generally whenever there is a vortex at the near the core of the vortex it is called forced vortex and their viscosity plays a role, but away from the center of the vortex one can get a free vortex where the flow is in circular streamlines. So, we are here we are referring to free vortex. So, this is the this is a vortex. So, essentially this equation the F z equal to C ln z we v is basically source or sink and this is vortex to be vortex is good enough, but if still if you are little fussy about it let us call this free vortex. So, these are the elemental flows first there where three types of you know one is horizontal, the other is vertical another is at an angle.

So, these are that kind of velocity whereas, the C ln z this is the first time we saw that it is basically source or sink and this one here minus i C ln z here we see this is the equation for a vortex. So, these are the elemental flows. What we are going to do next is we will use these elemental flows to come up with see the advantage of working with these elemental flows is if the two flows are superposed; that means, I have a source I have a source where I have the flow is coming out. Let us say I have a well something is producing I have another well nearby where it is acting as a sink and then I have some line source available on the side.

So, if you have a situation like these and if you want to plot the streamlines, if you want to see where the streamlines are coming close to each other or they are diverging; that means, where the velocity is more and where the velocity is less what you will do is simply take the elemental flow of a source elemental flow of a sink element. So, different types of elemental flows and simply superpose them; that means, F of z of source plus F.

So, you just simply add them and find out what is the net phi, what is the net psi and net psi gives you the streamline and based on this net phi and net psi you can find out what are the velocities for in condition or cylindrical. So, that is the advantage of getting into this exercise of complex potential. I will continue with this exercise of superposing these elemental flows in my next lecture.

So, better you do some revision of these concepts before we get into the next lecture and this is all I have for today for this for this lecture. Let us meet again in the next class where we start playing with these superposed situations of these elemental flows that is all I have for this lecture.

Thank you very much.