Flow Through Porous Media

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Lecture - 10
Mass Continuity (Streamlines and Potential Lines)

I welcome you to this course on Flow Through Porous Media. What we are discussing is the continuity applied to porous media. How we can leverage this understanding of continuity as we see in fluid mechanics to solve, to get a better visual of what all are happening in porous medium assuming continuum to be valid. So, suppose in a porous medium, we have a well a nearby, we have a line source, then we have a boundary, then we have several things happening together ok.

And we are trying to find out flow will go in which direction. I mean Darcy's law is fine. We can find if you have a unidirectional system, you can find out what is pressure drop and all these, or the other ways we can solve this we can put these in ijk and go by we have already solved the continuity. But if you have several such you know if you have a combination of all these flow elements in one place, is there any better way can we do it with some amount of finish this handling.

Or, at least our the idea is that we want to get some field flow would be taking place from which direction to what direction in or what kind of twists and turns it can make. So, for this purpose, you must have noticed already that in fluid mechanics streamlines this half this purpose very well these streamlines, they call it stream tubes.

So, in porous media also these applications of streamlines and stream tubes, these are very common. In fact, this petroleum engineering and this oil and gas reservoir simulation people they are constantly they are trying to find out how these how the streamlines would be how the flow take place to get better visual, you know because these are I mean numbers they do not appeal to us if you get a better visual and that is that is very much and very much wanted. And if we have a way to connect those visuals with the digital information that is the best thing we need.

So that is why we will be defining these streamlines, potential lines, and of course, they are all tied with streamlines, streamline represents a particular value of a stream function, along the streamline the value of stream function is constant. So, we have always we can toggle between digital information and the visual, so this is something, which we are taking our continuity to we are taking our continuity to the next level which is in the form of stream function and a potential function. So, let me start today's lecture. This is we are working on this mass continuity in Cartesian and cylindrical coordinate system. And our objective is to find out pressure equations and velocities.

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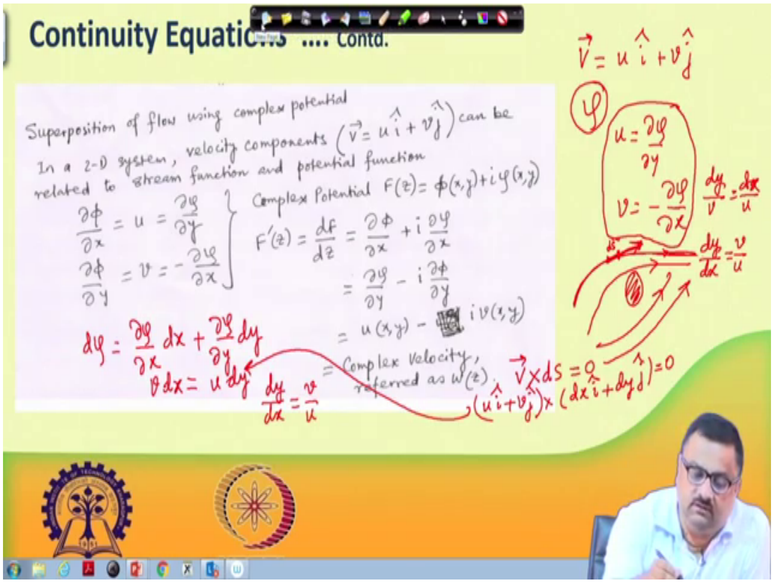

Now, we have this definition of a stream function and potential function. So, what is this definition? First of all the definition of a stream function is something like this. We have we have these we have already talked about a velocity field right, velocity field is u i hat plus v j hat. Let us say we are focusing on a two-dimensional velocity field. So, it is u i hat plus v j hat ok. So now, this stream function which we are calling is psi, this stream function is defined such that u is equal to del psi del y. And v is equal to minus del psi del x.

So, this is the so now, what we are trying to do is this velocity field with two values of u and v instead of that, we are trying to put this entire thing into one package, which is psi the stream function. Instead of calling u component, what is u component tell me, what is the v component u tell me, instead of this what is the stream function u tell me? So in one package, so the package is that u is equal to del psi del y and v is equal to minus del psi del x. What is the advantage of doing this? That advantage is that we have already said that, if we have we call this a streamline, I have an abstraction here and I am seeing the flow taking place like this.

So, we are having these, so this is so, let us say this is, the flow this, so these are, these are basically the streamlines, the lines along which a paper boat will travel. So, what we said is that the tangent to this stream line, defines the velocity at this point. Tangent to the stream line, defines the velocity at this point.

So, on the other hand, what do we see here, if we write this as u is equal to del psi del y and v is equal to minus del psi del x, What we see here is that what is d psi in this case? I can write d psi as del psi del x d x plus del psi del y d y, what is del psi del x? Del psi del x is equal to del psi del x is equal to minus v. So, minus v d x and del psi del y is equal to u plus u dy ok. So, what does d psi is equal to 0 implies?

So, if I force for the time being if you assume that let us force this d psi to be equal to 0, what does that mean if this become 0 then in that case, we can say that this v dx becomes equal to v dx, becomes equal to u dy or you can write dy dx is equal to dy dx is equal to v by u. So, dy dx is equal to v by u. So, what does this mean? I mean suppose, I tell you that streamline is a line along which psi is equal to constant. So that means, I can say streamline is the line along which d psi is equal to 0.

So, streamline is the line along which, streamline is the line along which dy dx is equal to v by u. And what is dy dx? The slope of this line and so every point the slope of this line or in other words you can write here as dy by v is equal to dx by u. So that means, dy by v is equal to dx by u. So, like; that means, let us say this is the streamline and I have a section, I have a section. Let us say which is d s. So, this d s is broken into dx i hat plus dy j hat right. So, this segment d s is dx i hat plus dy j hat on the other hand the velocity has u i hat v j hat.

So, we are saying dy by v is equal to dx by u. So, essentially we are saying that this tangent is same as it is showing the same direction as what is given by the velocity, there is other way to look at it also I mean if you want to look at it this way it is that these tangent to the streamline and the velocity they are in the same direction, you can write this as v cross d s has to be equal to 0. So, in that case you can write u i hat plus v j hat and take the cross product of dx, d x can be written as dx i hat plus dy j hat that is equal to 0. And i cross j is equal to k. So, if you follow this you will end up with this expression.

So essentially, one can show I mean my point is that these I am trying to say on one hand streamline is a line, along streamline is a line at any point if you draw it on tangent on this line, this shows me the velocity of that this, this tangent shows me the direction of the velocity at that point. On the other hand we are saying that streamline is a line, along which the stream function psi is constant, where psi is defined in such a way del psi del y is equal to u and minus del psi del x equal to v. So, this is the definition of stream function, we must make note of.

Now, let us close this discussion on stream function and instead we get to the potential function. Potential function generally the genesis of this potential function is that we have for the flow to take place one needs a gradient in some form. I mean just like that same by the same token, we have we need the temperature gradient for establishing a heat flow, we need a voltage gradient to establish a current flow. Similarly, I mean we said at the outside that this Darcy’s law and everything there we have a cause and effect relationship we need a pressure gradient to have a Darcy flow.

Now, what people have done in fluid mechanics or also in other place in porous media as well that instead of putting in instead of instead of relying on pressure, because pressure is a different kind of term.

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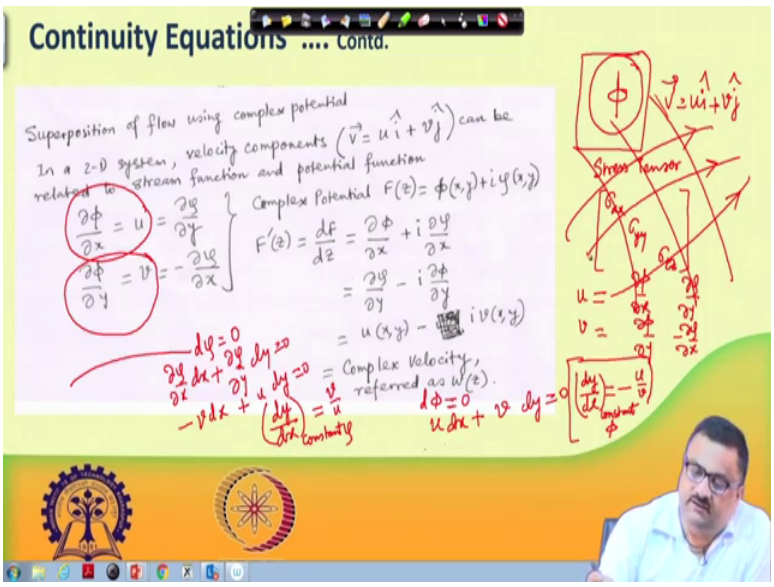

So, just for this purpose, they wanted to prefer to define something called a potential. Velocity potential just like you have voltage is electric in the electrical sense it is the potential. Similarly, for flow purpose you have a velocity potential and we want to call it phi. So, this is you will find that potential lines, which means that this potential phi is constant and isomeric lines, where the pressure is constant they in many cases you will find in many cases they are similar.

But, people prefer to work with this potential function because pressure by in a true sense pressure. The definition of pressure is not that if you have a pressure gradient then there is a flow, if the pressure is not defined that way. Pressure is defined as if you look at the definition of stress tensor, you will find that at a point in the stress field you can generate the stress tensor and then the diagonal terms, which is basically sigma xx sigma yy sigma zz which are the normal component, these if you take minus of average of these diagonal terms that is defined as pressure and so.

So, pressure definition of pressure comes from a different view point it the definition of pressure did not come from if you have a pressure gradient then there will be a flow. So, for this purpose they prefer to have a different term all together which is potential. So, that is the origin of potential function and potential lines for that matter along which potential function is constant, just like you have streamlines along which the stream function is constant. Now, when it comes to this potential function I found there are some authors who prefer to work with prefer to write the velocity as, let us say I have u and v we said that v is the velocity field is u i hat plus v j hat

So, this velocity u and v, they prefer to write these as u as minus del p del x and v as minus del p del y, what was u? You remember u was del psi del y and v was del minus of del psi del x. So, we can see the difference between psi and a phi. Now, there are some people who argue that this since, you are following if you call the potential is the original potential is it is the gradient along which the flow will take place just like along the temperature gradient the heat transfer takes place.

So, you should impose a minus sign just like we have in Fourier’s law of heat conduction we have minus k del t del x. But, there are another substantial number of authors, who did not put this minus sign because the purpose is one and the same I mean you without minus sign still you can generate these plots. So, what we are doing in this analysis, we are not putting a minus sign you can see here we are writing u is equal to del phi del x and v is equal to del phi del y. So, these are just like we have streamlines by the same token here we are talking about potential lines along, which this potential function phi remains constant.

So, what is the relation between streamlines and potential lines? You can see here very well that on one hand you are talking about del phi del x and the other end you are talking about del psi del y, the line along which the stream function is constant. The line along which the stream along which the stream function is constant. So, that line is basically is giving me d psi is equal to 0.

So, d psi is equal to 0 means we have del psi del x dx plus del psi del y dy that is equal to 0. So, what we had here is del psi del x is equal to minus vdx plus udy is equal to 0. So, what; that means, is dy dx at constant psi dy dx at constant psi that is equal to v divided by u. Similarly, so slope of streamline, slope of a streamline, which is dy dx at constant psi with a streamline means constant psi that is equal to v by u the velocity component in y direction divided by velocity component in x direction. So, at any point slope of the line is v by u.

Whereas, if we look at the potential lines which says d phi is equal to 0 so that means, you have d phi is equal to 0 means, you have del phi del x dx plus del phi del y dy is equal to 0. So now, del phi del x is equal to u. So, instead of this, you are writing it as, instead of this you are writing it as u and instead of this you are writing it as v. So, what does this mean now? That this means dy dx at constant phi that means, the slope of potential line slope of potential lines is equal to dy dx is minus of u by v minus u by v. So, slope of streamline at a point is v by u, and slope of potential line at a point is minus u by v. So, if we take the product, the product of the two slopes is minus 1.

So, what that means, is if you look at our understanding of basic geometry and coordinate geometry, and we find that slope the product of the slope minus 1 means 1 is perpendicular to the other. So, that means, if these are the streamlines; if these are the streamlines, I would expect the potential lines will be perpendicular to the two these lines. In fact, this makes perfect sense because if these are the streamlines, let us say flow is taking place in this direction, so what that means, is that must be there must be a pressure the pressure gradient existing in this direction.

So, I can say the pressure here, pressure here and pressure here, they are changing ok. So, this is these are potential lines which is for many for most of the purposes, it is the isobaric lines. So, streamlines and potential lines they are perpendicular to each other. This is something which we have we are supposed to know at this time. So, with this understanding of streamlines and potential lines there is another major advantage of streamlines which is that you the flow the streamlines are drawn in such a way I mean let me just clarify this.

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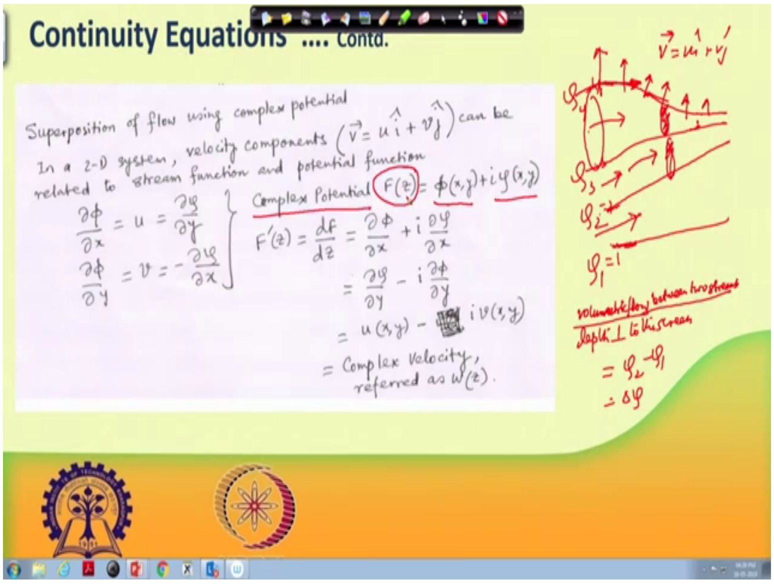

Let us say I see the streamlines are going and then coming close to each other. So, here I see the streamlines they are coming close to each other. The streamlines are drawn in such a way say let us say this is psi, psi is let us say this is psi 1, this is psi 2, this is psi 3, this is psi 4. So, the streamlines are drawn in such a way that the differ basically if one can find a one can one can see that the volumetric flow that takes place between the two streamlines is basically that difference between the two stream functions.

So, volumetric flow per unit depth, depth perpendicular to the screen between two streamlines, volumetric flow between two streamlines; between two streamlines, the per unit depth perpendicular to the screen, this is equal to the difference between the two streamlines; that means, in this case it would be psi 2 minus psi 1, basically it is delta psi. So, this the flow that is taking place between psi 2 and psi 1 that is the volumetric flow rates that is taking place per unit depth perpendicular to this to the screen, so that is equal to this difference in this stream function.

So, if this is a value of psi 1, this is a value of psi 2, the difference between these two that gives you the amount of flow taking place. Similarly, between psi 2 and psi 3, there would be some flow is taking place, some flow taking place and that is the difference between the two stream function values; similarly, for psi 3 and psi 4.

So, what they generally do when they draw the streamlines they make them intentionally they make them let us say the psi 1 the value is 1, 1 meter cube per second per meter perpendicular to the board let us say I have that kind of unit. So, this psi 2 would be 2; psi 3 would be 3; psi 4 would be 4 like this. So, they are all equispaced, so if the difference there is always 1. So, I expect that it is the same flow that was that is taking supposed to take place. Now, when we see that these lines are converging to each other what that means, is that these lines that. So, there is there the velocity is increasing because it is the same Q that is flowing 4 minus 3; here also the same Q that is flowing 3 minus 2.

So, it is the same volumetric flow rate, but now I can see over a smaller cross section compared to these, so that means, when I look from this point at this point the velocity is increasing. This point at this point, velocity is even in a further increasing because I see the cross section is decreasing here, but same flow is taking place. So, when I see the two streamlines they are coming close to each other; that means, the velocity they are inside the velocity is increasing. Whereas, when I see the streamlines they are diverging from each other that means, the velocity is decreasing.

So, by looking at the streamlines, one can make this kind of intuitive understanding of what is the what kind of flow is taking place there, and how or where the velocity is increasing, and where the velocity is decreasing. So, this is one thing, one advantage of these visual. Another important point here, one must make note is that no flow can cross a streamline, why because we said that the tangent to this streamline gives me, the direction of velocity at this point.

So, if the velocity vector I have a velocity vector right velocity field which is u i hat plus v j hat. And that velocity field has the net the resultant is these. And this resultant gives me the tangent to this point. So, if this is the resultant you cannot expect any component existing perpendicular to the resultant that that is idea right. So, at in every point, this is true, that means, the tangent to this line and the velocity direction. So, you cannot expect any component existing perpendicular to this. So, no flow can ever cross a streamline at 90 degree.

So, streamline cannot, so streamline is a boundary line I mean no flow can be there perpendicular to a streamline at any point ok. So, that is also another take home message here from this. So, now, what I what we are going to do with these understanding of stream function and potential function, we are going to merge the two, we are going to define something called a complex potential, which is a complex number we could this is called F z, this is a complex number which is equal to the put value of potential function plus i, i is basically i square equal to minus 1, this is a complex number plus i psi so phi plus i psi. So, this is defined as complex potential.

And we need to use this complex potential to find out the flow if we have different elemental flows present in one place. For example, a source, a sink a line source in finite line source of where various types if we have all together what would be the net result, how, what would be the velocity what would be the stream, how the streamlines look flowed be from which direction to which direction. So, we get visual as well as digital information visual as well as analytical information from this study of this complex potential.

So, I will continue this exercise in the next lecture. That is all I have for today and for that is all I have for this lecture. I will continue this exercise of I will continue working on this complex potential in my next lecture.

Thank you very much, that is all I have for this lecture.