Chemical Process Instrumentation
Prof. Debasis Sarkar
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 10
Performance Characteristics of Instruments and Data Analysis – I (Contd.)

Welcome to week – 2, lecture – 10. So, this is the last lecture of second week and also the last lecture of Performance Characteristics of Instruments and Data Analysis part I.

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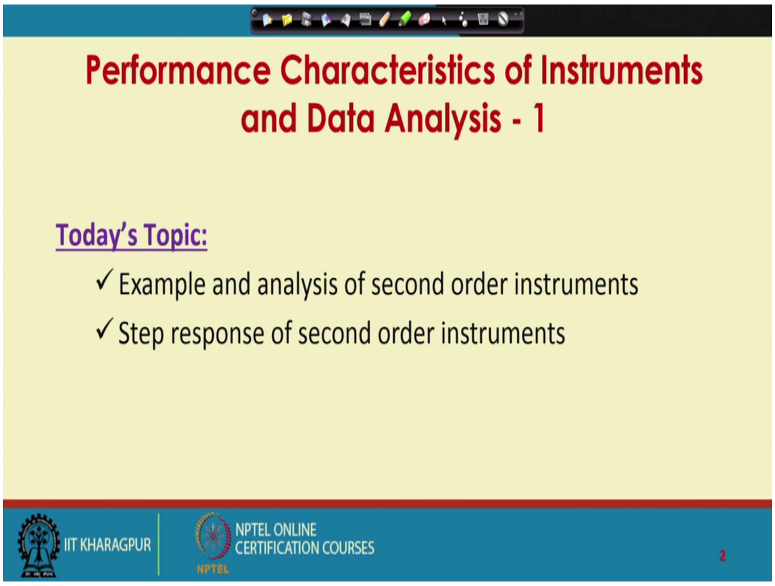

So, today will talk about analysis of second order instruments. We have talked about analysis of first order instruments in the previous class. So, in this class we will talk about analysis of second order instruments, we will take an example of second order instrument and analyze the instrument, write down the modeling equations and show that the instrument is second order in nature. We will also briefly talk about step response of second order instruments.

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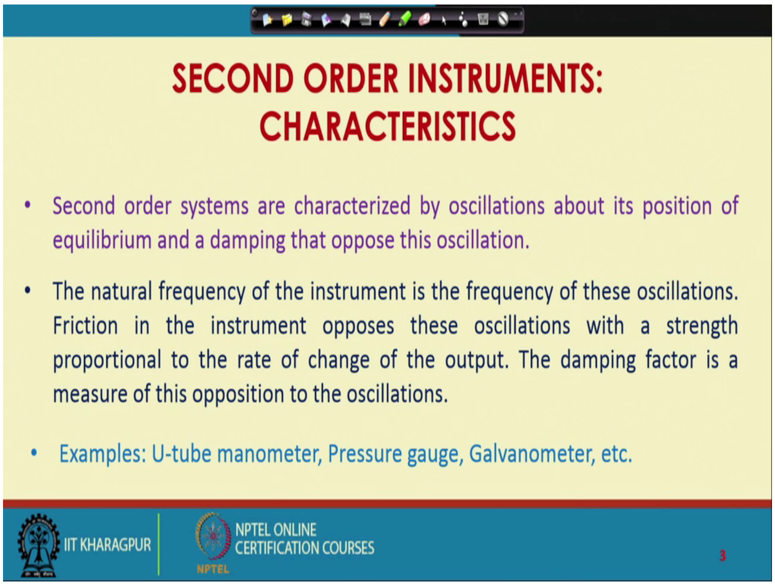

As we have seen the first order systems are characterized by capacitance and resistance. Similarly, second order instruments are characterized by oscillations about it is position of equilibrium and a damping that oppose this oscillations. So, in all second order systems there will be an oscillation about its position of equilibrium and there will be a damping that opposes this oscillation. So, depending on the damping factor the amount of oscillations will vary in second order systems. We will talk about more later in this class.

The natural frequency of the instrument is the frequency of these oscillations. Friction in the instrument opposes these oscillations with strength proportional to the rate of change of the output. The damping factor is a measure of this opposition to the oscillations. Some examples of second order instruments are U-tube manometer, pressure gauge, galvanometer etcetera.

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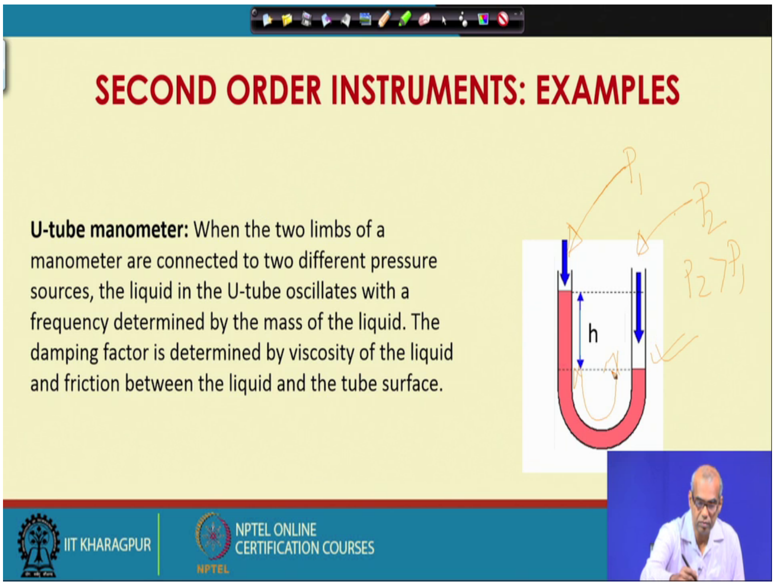

So, this is an example of our schematic of U-tube manometer. So, this is that U-tube manometer. Commonly, it is made of glass tubes and bent in the form of U. These are the 2 limbs of the U-tube manometer; these 2 limbs are connected to different pressure sources. If the limbs are connected to 2 different pressure sources the level of mercury in both the limbs will be different.

If both the limbs are connected to the same pressure source, let us say both the limbs are opened to atmosphere; that means, same pressure is being applied on both the limbs then there will not be any level difference in the U-tube manometer. So, the difference of level of mercury or manometer fluid within the manometer is the measure of difference in pressure in these 2 limbs.

So, when the 2 limbs of a manometer are connected to 2 different pressure sources, the liquid in the U-tube oscillates with a frequency determined by mass of the liquid. The damping factor is determined by viscosity of the liquid and friction between the liquid and the tube surface. So, when both the limbs in initially imagine the both the limbs were opened to atmosphere. So, they are connected to the same pressure source. Now, I connect this to a pressure source P 1 and this is another pressure source P 2. P 1 and P 2 are different, as the figure shows P 2 is greater than P 1. So, that is why this is pushed more.

So, when you do this there will be oscillations of this manometer liquid and this oscillations will be with the frequency that is determined by mass of the manometer liquid that is present within the U-tube manometer. This oscillation will be opposed by the viscosity of the liquid and also the friction of the liquid with the surface of the U-tube. So, the damping factor is determined by viscosity of the liquid and the friction between the liquid and the U-tube surface.

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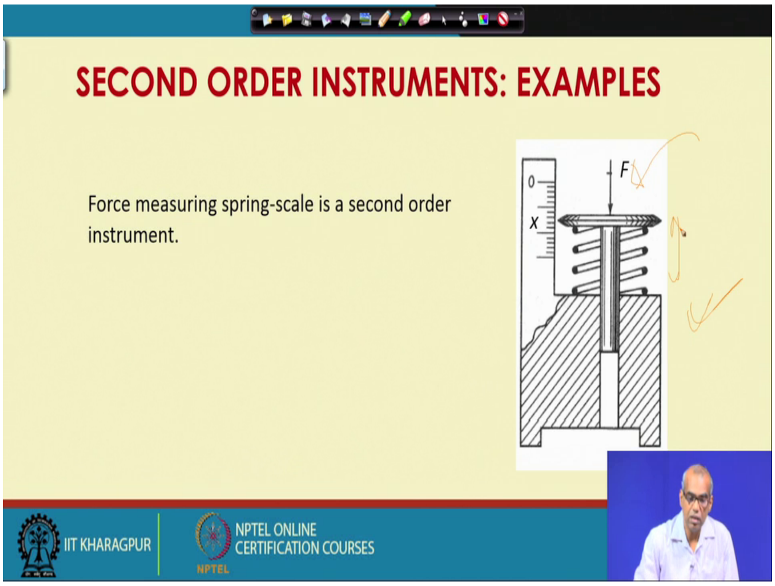

Force measuring spring-scale is another example of second order instrument. If I put a force here or if I put some weight here, this will oscillate. It is an example of second order instrument.

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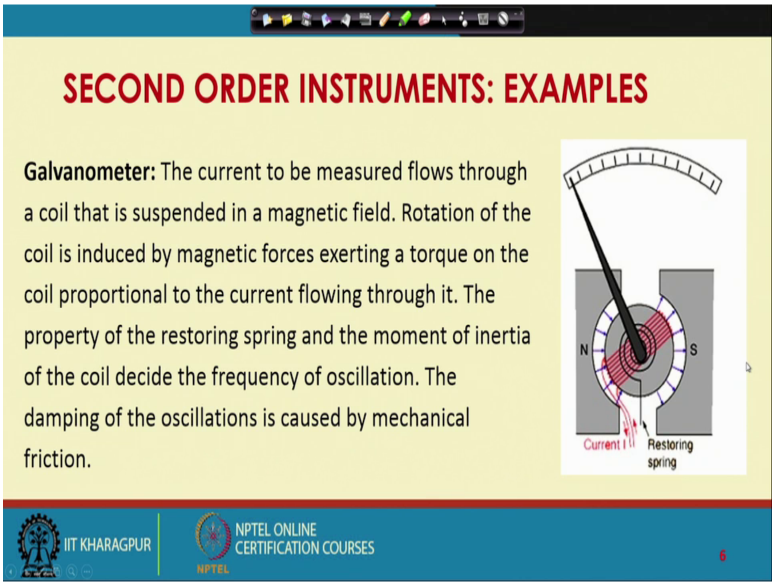

Galvanometer is another example of second order instrument. Galvanometer measures current. So, the current to be measured flows through a coil, this is the coil, that is suspended in a magnetic field. Rotation of the coil is induced by magnetic forces exerting a torque on the coil proportional to the current flowing through it. The property of the restoring spring and the moment of inertia of the coil decide the frequency of such oscillation. The damping of the oscillations is caused by mechanical friction.

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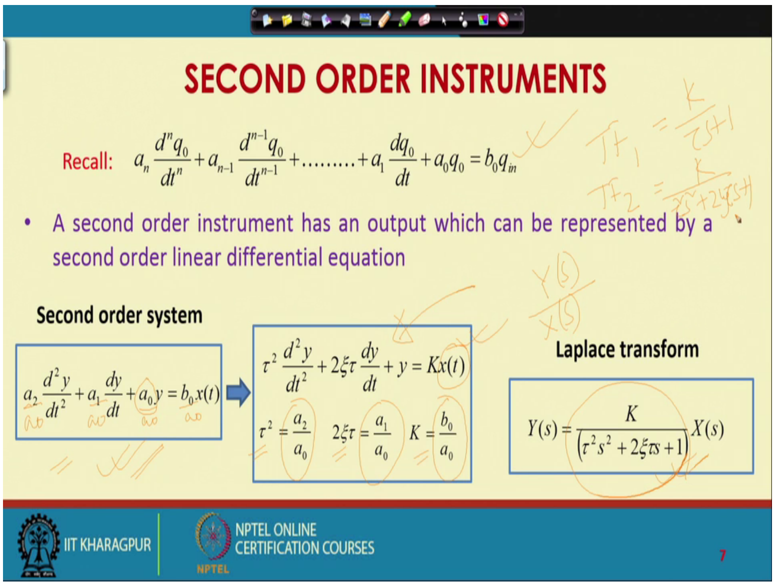

So, now you have seen few examples of second order instruments. Let us now talk about mathematical model of a second order system or second order instrument. So, will start with the generalized mathematical expression for an nth order instrument and will retain terms up to second order so that the second order differential equation that will get will represent a second order instrument.

So, let us look at the nth order ordinary differential equation that we have talked about previously that represents an nth order instrument. So, recall this instrument an nth order ordinary differential equation with constant coefficients represents the variation of output with respect to time and on the right hand side I have the input function. A second order instrument has an output which can be represented by a second order linear ordinary differential equation.

So, if I retain up to second order terms I get this ordinary differential equation from this general nth order ordinary differential equation. So, this represents a general mathematical model for a second order instrument. As we have done previously, let us divide this equation throughout by the coefficient of the 0th order term. So, I get a 2 by a 0, a 1 by a 0, a 0 by a 0 and b 0 by a 0. I call this a 2 by a 0 is tau square, a 1 by a 0 as 2 zeta tau and b 0 by a 0 is K, the static sensitivity. So, there are 3 parameters in the second order instrument taus zeta and k. So, the equation can now be written as tau square d square y d t square plus 2 zeta tau d y d t plus y equal to K into x t.

Where, x t represents input function; such as step input, ramp input, sinusoidal input, so on and so forth. So, to get transfer function of the second order instrument, what I have to do is, I have to take Laplace transformation of this equation and then take the ratio of Laplace transform output by Laplace transform input see if I do this I will get these expression. So, the transfer function for second order instrument which is Y s by X s will be K by tau square s square plus 2 zeta tau s plus 1.

For a first order system, we have seen the transfer function for a first order system let us say transfer function for first order systems was K by tau s plus 1. So, there are 2 terms involved K and tau. In case of second order instrument the transfer function for a second order instrument can be written as K by tau square s square plus 2 zeta tau s plus 1.

Would like to make a point here, that sometimes you will see the transfer function for first order system is written as 1 by tau s plus 1 and also for second order system it is written as 1 by tau square s square plus 2 zeta tau s plus 1. In other words these 2 expressions I mentioned the value of K is equal to 1; say, for mercury-in-glass thermometer we have seen, the first order equation that represents a mercury-in-glass thermometer we have written in 2 different forms. In one form output was taken as x 0, x you remember that is the level of the mercury in the capillary. So, that is distance, a measure of distance. So, the relationship was there with input temperature and the distance which was taken as output from the thermometer.

We also wrote another equation which is related of course, where the input was the temperature and the output of the thermometer was again also taken as temperature. When the input was temperature, output was also temperature the transfer function will be written as for the first order system one by tau s plus 1.

So, in other words when you have inputs and outputs are physically same quantity, the nature of the signal is same, the value of K will be equal to 1. So, the transfer function for first order system can be K by tau s plus 1 in general, but K will be equal to 1 in certain occasions and this happens when the input and output are of same type of signals. For a mercury-in-glass thermometer input is temperature output was also temperature. So, the value of k there will be equal to 1, you can verify it from the equation that we have derived.

The same logic applies to second order system transfer function as well.

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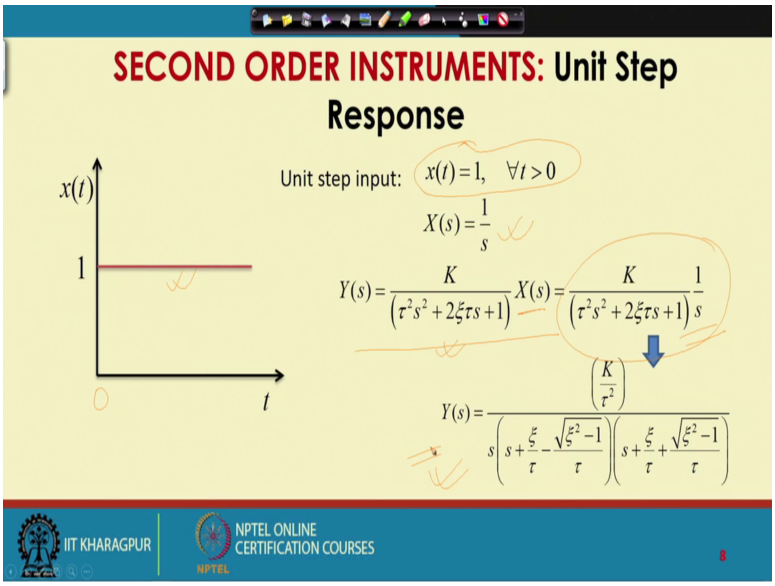

Now, let us quickly take the look at step response of a second order instrument. Specifically, will give unit step input to a second order system and you will see the response when such unit step input is given to a second order instrument. So, unit step input is represented by this x t equal to 1 for all time t greater than 0. So, at t equal to 0, the input values taken to magnitude 1 and maintained there for all time to come. We have seen previously that the Laplace transformation of x t equal to 1 is 1 by s. So, let us may make now use of the transfer function equation which is this. So, I have to put X s equal to 1 by s, I have done it here and then I can rearrange to write this as this.

I will rearrange it so that it becomes easy for me to take inverse Laplace transformation, because if I now take inverse Laplace transformation I will get Y t; that means, how output Y changes with respect to time. So, output does the function of time will be obtained when I take inverse Laplace transformation of this.

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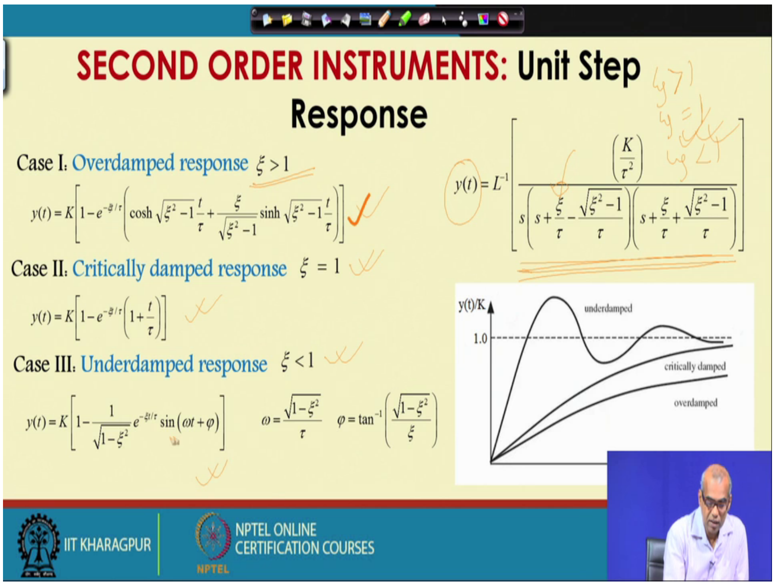

If you do that, you will get different types of response depending upon the value of zeta. So, this is what you obtained from the transfer function of second order instrument after putting X s equal to 1 by s and then we rearrange and we wrote in this form.

Now, you take inverse of this. Now, when you take inverse of this depending on the value of zeta we can have 3 different types of responses or the y t will have 3 different types of expressions depending on value of zeta. Zeta can be greater than 1, zeta can be equal to 1 and zeta can be less than 1. So, depending on 3 such values we can have 3 different types of expressions for y t. Now, when zeta is greater than 1, we get this expression and this is known as over damped response. Zeta is damping factor, so, in the value of this damping factor is greater than 1, the response I get is known as over damped response and this is given by this expression.

When zeta value is exactly equal to 1, it is known as critically damped response and we get this expression. When zeta is less than 1, what we get is known as under damped response which is represented by this equation. Look at here, this is the sine function, so, there will be oscillations. So, this is the radium frequency of this oscillation and this is the phi value. So, depending on the value of zeta we have 3 different types of responses accordingly we call over damped instruments, under damped instruments and critically damped instruments. So, for a critically damped secondary instrument you know the damping factor is equal to 1.

For under damped instruments, the damping factor zeta is less than 1 and for over damped instruments the damping factor zeta is greater than 1. So, they show responses which are known as over damped response. Zeta greater than 1, critically damped response, zeta equal to 1 and under damped response, zeta less than 1.

So, pictorially, they look as follows. Please look at here, this is under damped response. See, there are sustained oscillations because you have the sine function there. So, here zeta value is less than 1. So, you have oscillations the response will oscillate. For over damped, you will get this response and for critically damped you will get this. So, oscillations will be reduced using the increasing the value of zeta.

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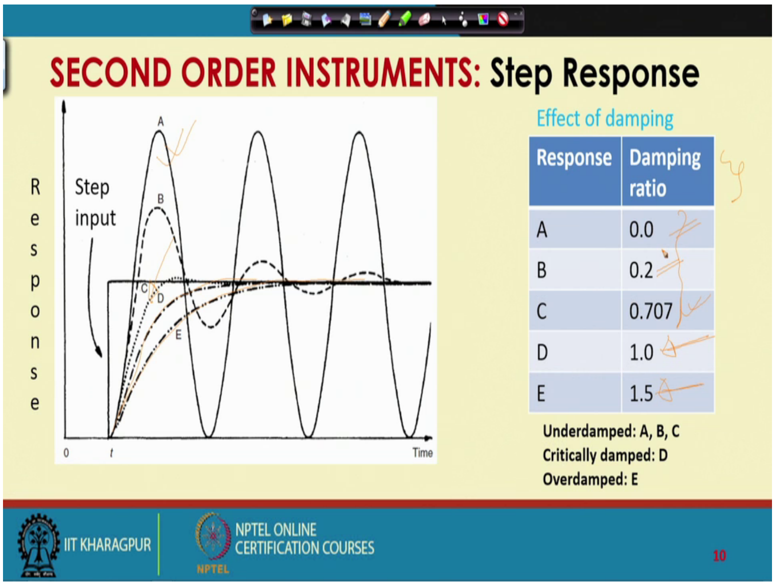

So, this once again shows the effect of this damping factor or damping ratio zeta on the step response for a second order instrument. If there is no damping, so, zeta equal to 0. You will have sustained oscillations like sine wave. When we have some non-zero positive damping ratios you will have oscillations, but of course, reduced. See, you have 0.707 you see the oscillation has reduced now to a great extent.

This is critically damped. So, this is the response and this is over damped. So, this even runs below the critically damped curve. So, this 3 are under damped response, critically damped response, over damped response. Please note one thing, that for under damped instruments there are oscillations, but it reaches the final value much more quickly than a critically damped or over damped instrument will do.

So, second order instrument such as pressure gauge is generally designed as an under damped instrument with zeta value maybe around say 0.7 or so. So, then what will happen is the speed of response will be reasonably good and there will be oscillations, but there will be not much oscillations, you can look at this figure.

See, this one represents zeta equal to 0.7; see, this is the time by reach it attains the final response. So, this was the step input given at this time and this, my instrument shows instrument should show this response finally, with zeta equal to 0.2, it takes very long time. Zeta equal to 0.7 it takes somewhere here; zeta equal to 1, it takes somewhere here; zeta less than 1 takes even longer time. So, the pressure gauge which is a second order instrument is quite typically designed as an under damped systems with zeta value around 0.7 or so.

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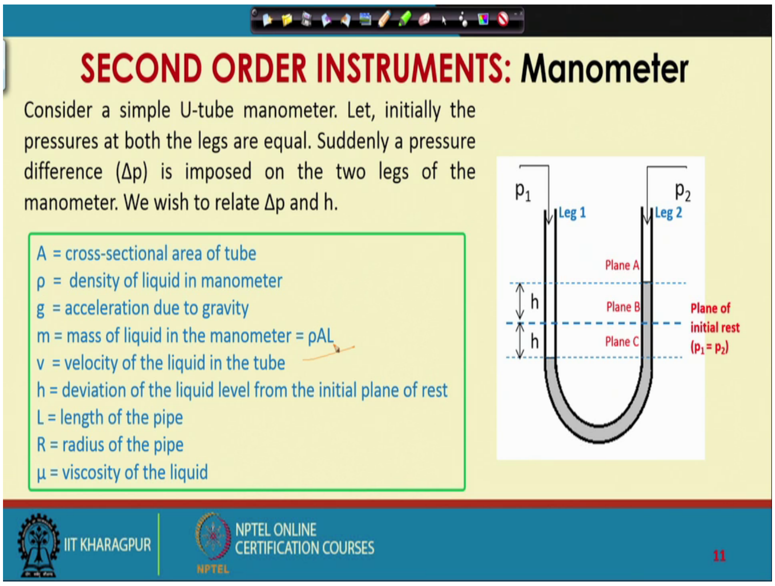

Now, let us quickly take an example of U-tube manometer and I would like to develop the mathematical model that describes the behavior or the dynamics of this manometer. So, basically we will try to develop an ordinary differential equation that relates input to the manometer and output on the manometer. So, let us consider a simple U-tube manometer.

Let initially the pressure at both the legs are equal. So, initially, you will have this as plane of initial rest. Suddenly, a pressure difference delta p is imposed on the 2 legs of the manometer. So, now, there will be differences of manometer liquid level, that is, output of the manometer and that is this 2h or h around the plane of initial rest.

Let us introduce these terminologies; A equal to cross sectional area of the tube, rho is density of liquid in manometer, g – acceleration due to gravity, m equal to mass of liquid in the manometers which can be taken as rho AL because rho time say is the sorry A times L is the volume and rho times AL is the mass of the mercury. A times L is volume, so, we multiply volume with density so you get mass of liquid in the manometer. v is velocity of the liquid in the tube, h is deviation of the liquid level from the initial plane of rest, L is length of the pipe, R is radius of the pipe or the U-tube and mu is viscosity of the manometer liquid.

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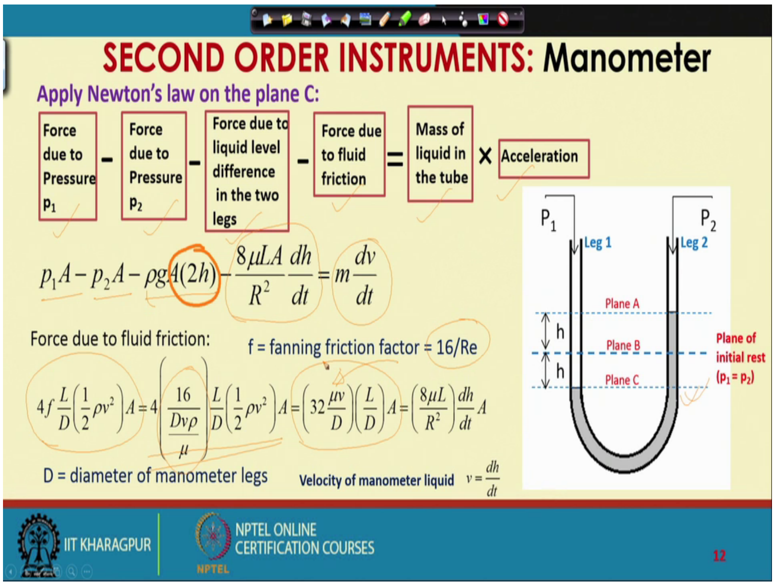

So, now let us apply Newton's law on the plane C. So, we can write the force balance equation as force due to pressure p 1 minus force due to pressure p 2 minus force due to liquid level difference in the 2 legs minus force due to fluid friction equal to mass of liquid in the tube into acceleration. So, some of all these forces is equal to m into f. So, force due to pressure p 1 is p 1 times cross sectional area of the tube, so, p 1 into A.

Similarly, force due to pressure p 2 is p 2 times cross sectional area p 2 A. Force due to liquid level difference in the 2 legs is rho g A into 2h; A into 2h times density, volume times density times acceleration due to gravity and this is force due to fluid friction, is mass into acceleration which is rate of change of velocity. So, force due to fluid friction can be written as this from fluid mechanics where small f is the fanning friction factor and if we assume laminar flow, the fanning friction factor can be written as 16 divided by Reynolds number.

So, if I put the expression for the Reynolds number, I will get this and this finally, will appear as this. Now v, which is the velocity of the manometer liquid, can be taken as rate of change of h with respect to time. So, v equal to dh dt. So, finally, the force due to liquid level force due to fluid friction can be written as this.

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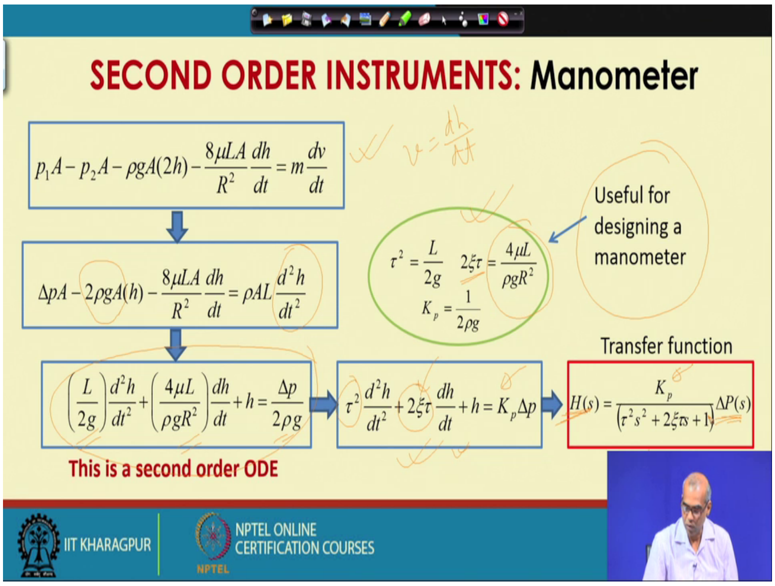

So, this is the equation we obtained in the previous slide. Now, v is dh dt, so, dv dt can be written as d 2 h dt square and if you rearrange this equation, how do you rearrange? We divide this equation throughout by this coefficient which is coefficient of 0th order. So, if we do that I will get this equation. Look at this equation. This equation is a second order ordinary differential equation. So, if I call this tau square this is tau square d square h dt square, if I call this 2 zeta tau 2 zeta tau dh dt plus h and if I call 1 by 2 rho g s K this is K into rho p. So, this is the equation we finally, get.

So, this parameter, this parameter zeta, tau, zeta and K are obtained from this and this. So, these become very useful for designing a manometer. So, you know if you want to make zeta less than 1, what sort of parameter value you should have? The transfer function can be written by taking Laplace transformation of this, again this is output by input is equal to K tau square s square plus 2 zeta tau s plus 1.

Note here, K is not 1, because your output is measure of length, but input is pressure difference. So, they are different signals. So, K cannot be 1.

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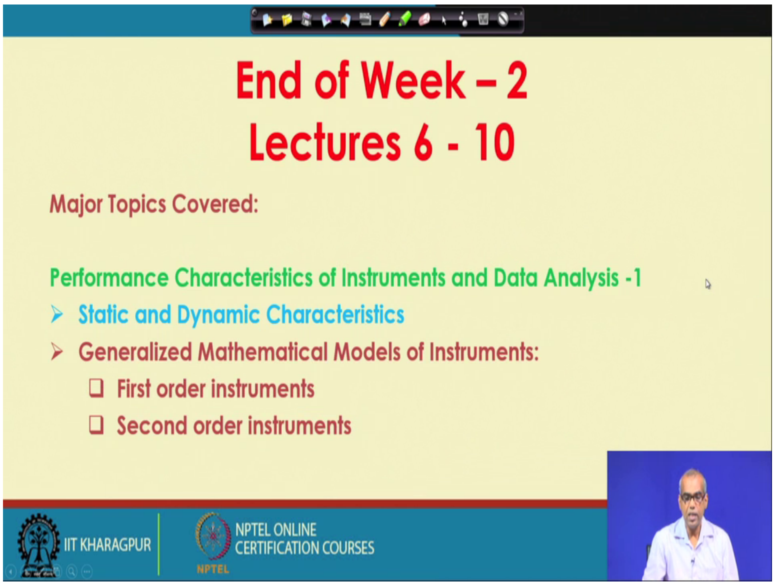

So, this is the end of week – 2 and lecture 6 to 10. So, the major topics we covered are performance characteristics of instruments and data analysis part – 1, under this we have talked about static characteristics and dynamic characteristics. We have also talked about general mathematical models of instruments, we have talked about first order instruments, we have taken examples and analyzed it, talked about step responses, we have also talked about second order instruments, we have taken examples, developed the mathematical expressions representing a second order instrument, analyzed it, also analyzed second order response.

So, this ends lecture – 10 or the last lecture of week – 2. In the next lecture, we will talk about performance characteristics of instruments and data analysis part – 2.