Transport Phenomena

ProfessorSunandoDasgupta
Department of Chemical Engineering
Indian Institute of Technology Kharagpur

Lecture 5

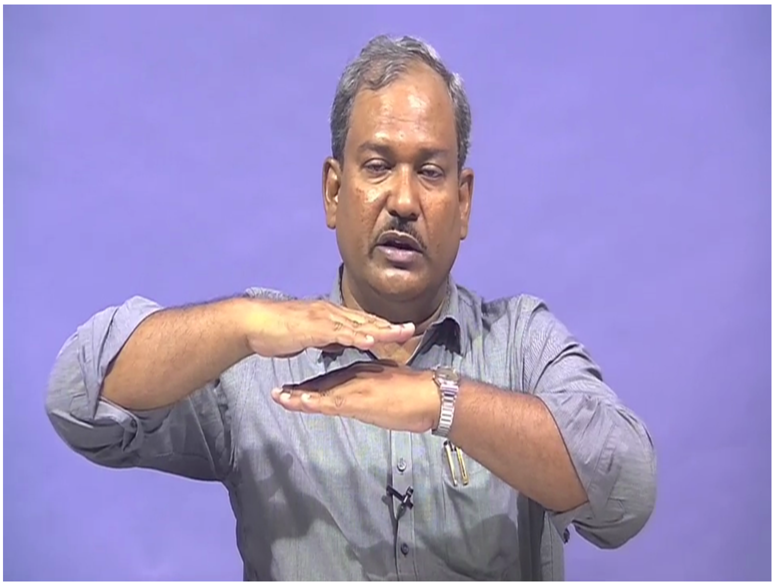
Example of Shell Momentum Balance (Contd.)

In our introductory classwe have seen how important viscosity is in transfer of momentum. Now viscosity plays a critical role (whene) whenever there is a difference in velocity between two adjacent layers. So when two adjacent layers of fluids pass by one another, there would be some amount of momentum transferred due to molecular motion. So molecules with higher velocity would jump from one layer to the slower moving layer thereby transferring the momentum from the faster moving layer to the slower moving layer.

And the same thing happens when a molecules from slower moving layer would come to the faster moving layer. This has given rise to a transfer of momentum in a direction perpendicular to the flow which we commonly call as the shear stress. And we have seen what is the how the shear stress is generally represented. It's represented with double subscript.

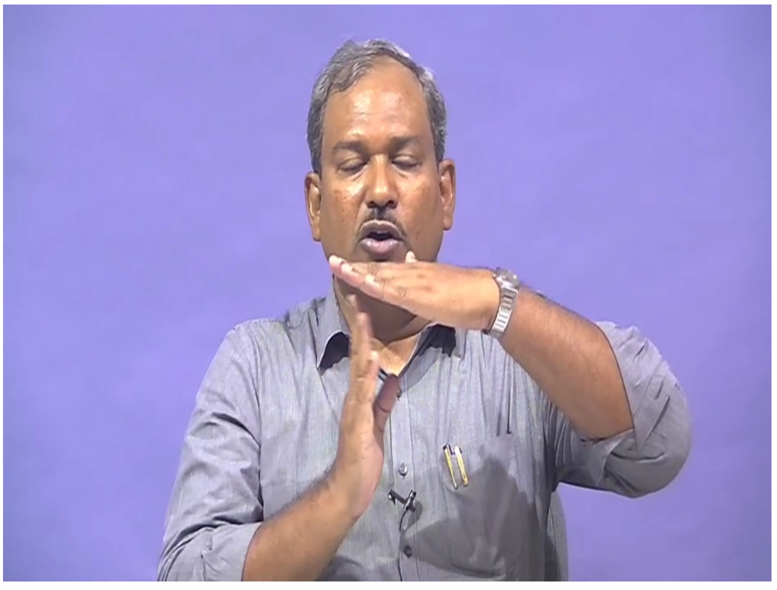
The first subscript refers to the direction of motion, the principal direction of motion and the second subscript refers to the direction perpendicular to the principal direction of motion in which the momentum gets transported. So if you think of a layer which is moving in the X direction and another layer on top of it which is also being dragged in the X direction but with a lower velocity, then the X momentum of flow gets transported because of viscosity in the Y direction.

(Refer Slide Time 1:44)



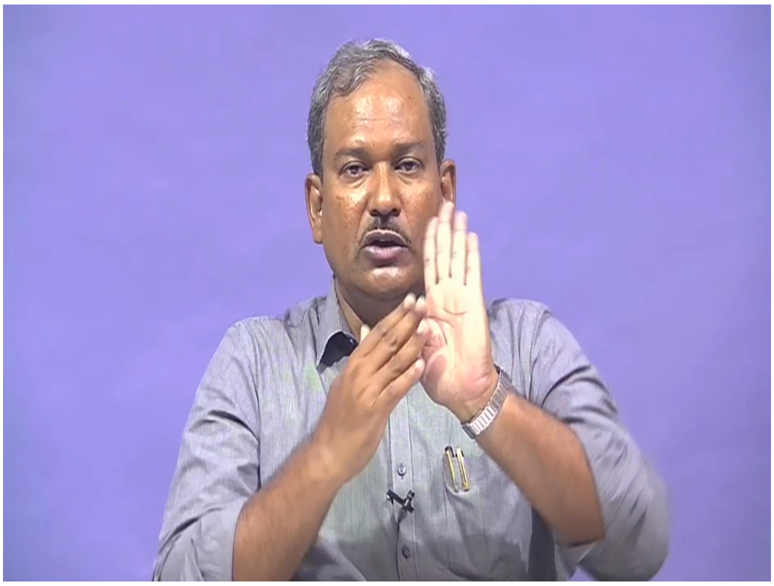
Solayers above the faster moving layer, then the next layer and the layer above that, all will start to move in the X direction as a result of the invisible string.The viscosity which binds these two layers.So (visc) the momentum even though it is in the X direction it gets transported in the Y Direction. This is also called the molecular transport of momentum and the double subscript is a very common way to represent the shear stress. That means the stress being felt being exerted by the moving fluid on the layer just above it.

(Refer Slide Time 2:28)



So this area which is in contact with the layer below it, it gets stressed from force per unit area which is a direct result of viscosity.

(Refer Slide Time 02:30)



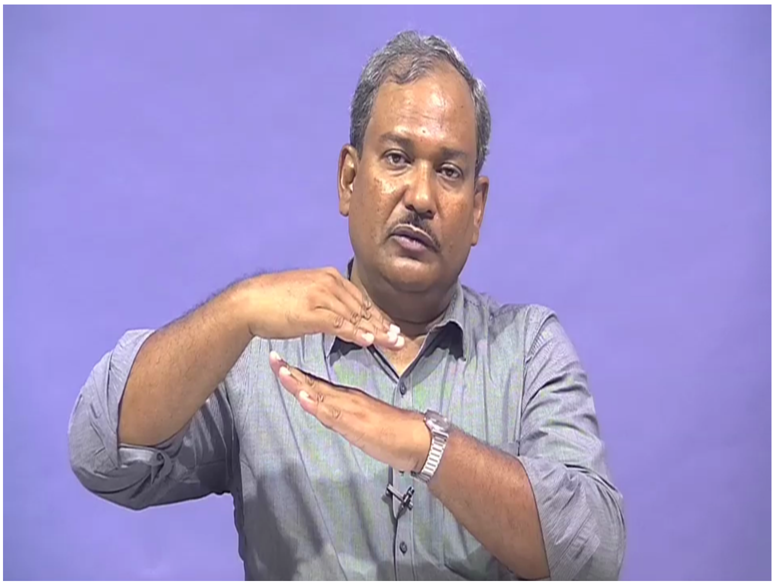
So the defining equation of viscosity as we have seen previously is the Newton's law of viscosity. So where the stress, the shear stress is directly proportional to the cause which is the velocity gradient. So the proportionality constant of this is known as the viscosity. So (Newto) (liq) the fluids which follow Newton's law of viscosity, that means tau is equal to minus times velocity gradient.

The minus signifies the momentum always gets transported in the direction of decreasing velocity. Those kind of fluids which follow this law are commonly known as Newtonian fluid. Examples of different behaviors are quite common. That could be some fluids which would stuck, which would first resist motion but once a threshold stress is applied on it, it will automatically start to move and from that point onwards the stress is going to be proportional to the velocity gradient.

So those kind of fluids which have a threshold stress which must be applied for it to start its motion are called Bingham plastics and common example of Bingham plastic is the toothpaste. You have to push thetube with a certain force. If we do not go beyond that force, nothing will come out of the tube. So the toothpaste is an example of Bingham plastic.

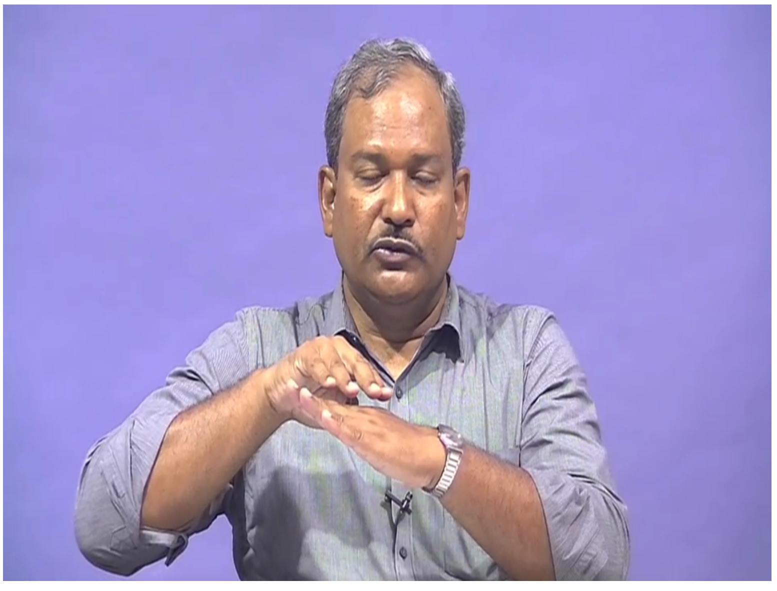
And then we have in the subsequent classes what we have seen is that it is useful to define a shell in a moving fluid and find out what are the forces, what are the momentum that are acting on that shell. So these shells are generally defined in the smaller dimension of the shell is the direction in which the velocity is changing. So in the previous class we saw the example of flow along a flat plate. So there would be a flow of liquid along the flat plate and so obviously the velocity is going to change in this direction.

(Refer Slide Time 04:44)



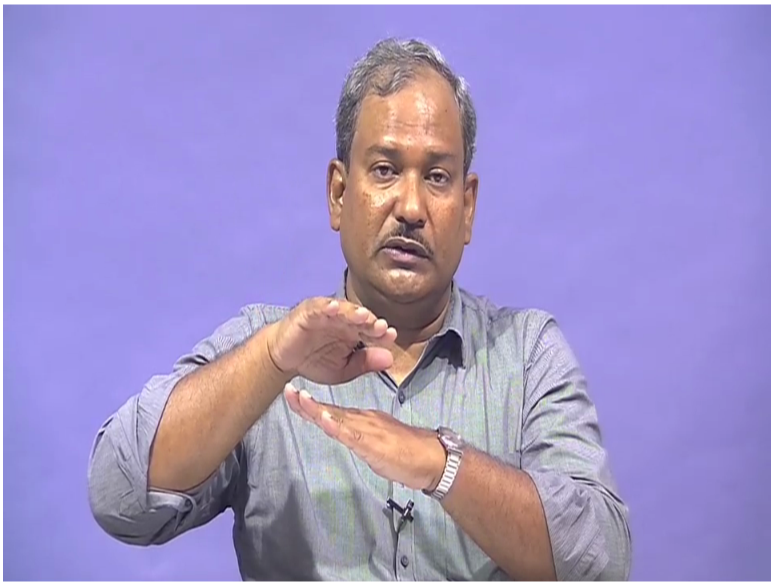
So at this point the boundary connection is going to be no slip condition which we know that at liquid-solid interface there cannot be any motion of the liquid molecules. So the liquid molecules adjacent to the solid boundary will have zero relative velocity. So that's called the no slip condition.

(Refer Slide Time 05:07)



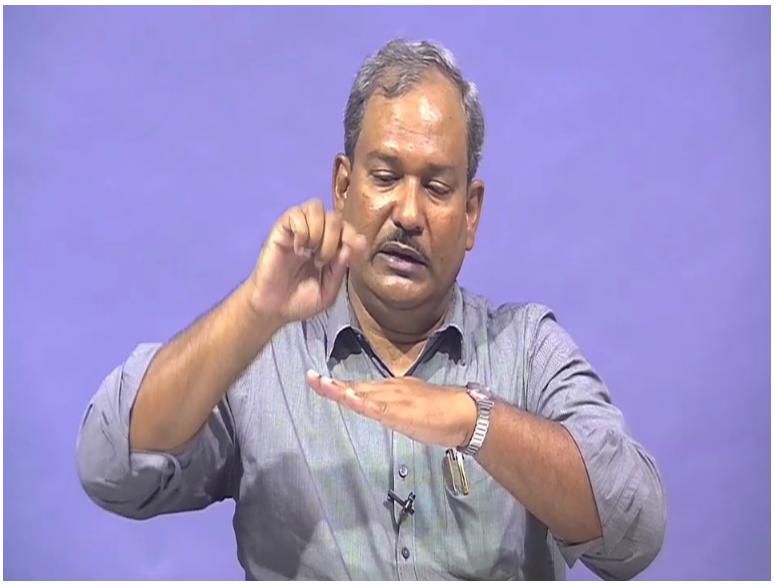
At the other end where we have the liquid vapor interface, the shear stress would be across the interface would be equal to zero.

(Refer Slide Time 05:18)



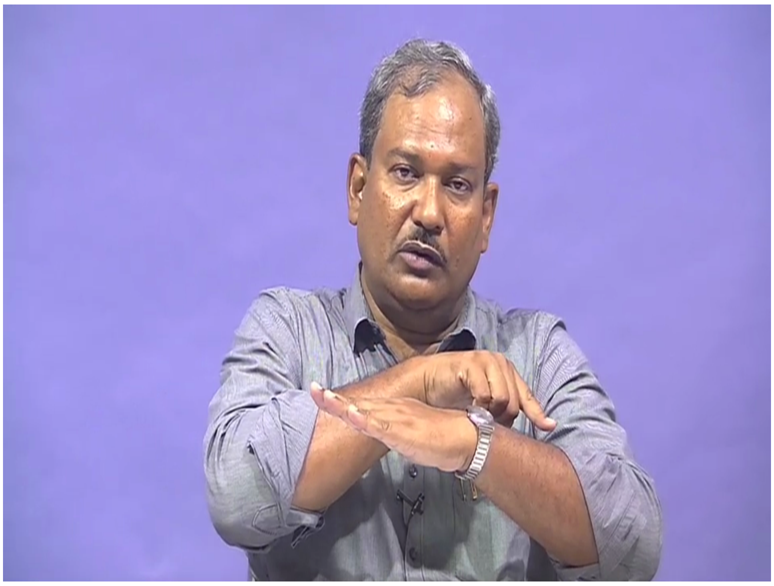
True for most of the cases, many cases when the velocity differences are not too great or the air is not flowing moving with a very high velocity creating waves and so on. There we have seen since the velocity vary in this direction the shell that we are going to define is going to have a smaller dimension of, let's say this is the X direction deltax.

(Refer Slide Time 05:42)



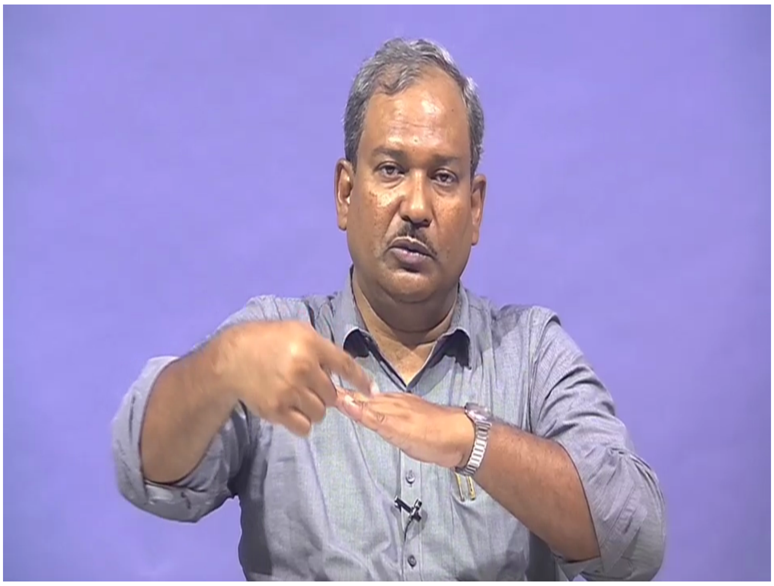
It can be any length and any with and since we are assuming its one dimensional laminar flow, incompressible flow, where the density remains constant the velocity is not going to be a function of the actual distance.

(Refer Slide Time 05:58)



It is not going to be a function of the width.

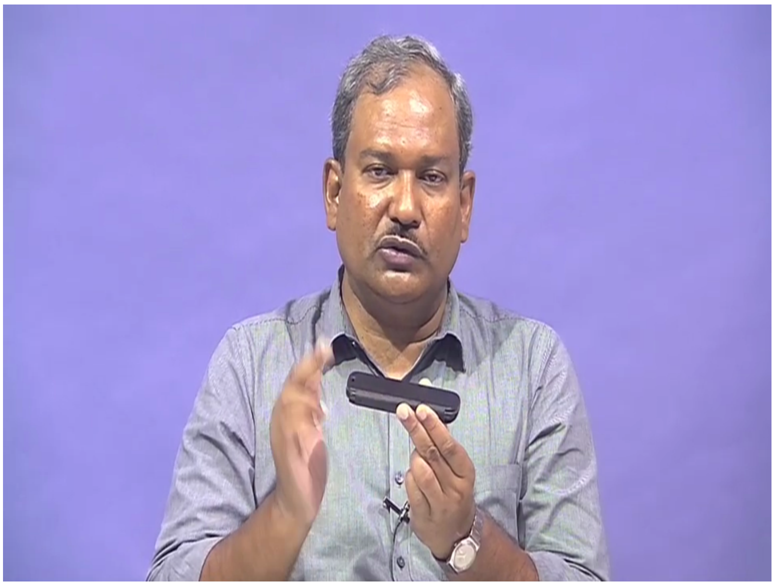
(Refer Slide Time 06:02)



It is only going to be the function of depth of the film. So for those one cases, we should always define the shell as having any length L, any width W, but this dimension is going to be delta x. Since between X and delta x the velocity can change substantially. So we write the physics, we express the physics in the form of a difference equation. So when we understand that there is going to be some amount of flow if this is my shell.

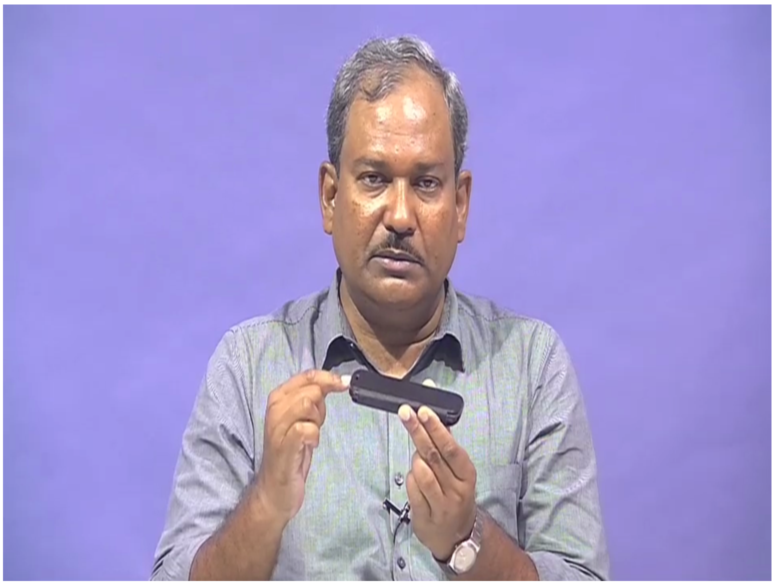
Let's say if this is the shell, then I am going to have some amount of fluid which comes into the shell because of its flow. Which comes into this surface but since I don't have any velocity component either in this direction or in this direction nothing comes in due to flow through these four surfaces.

(Refer Slide Time 07:00)



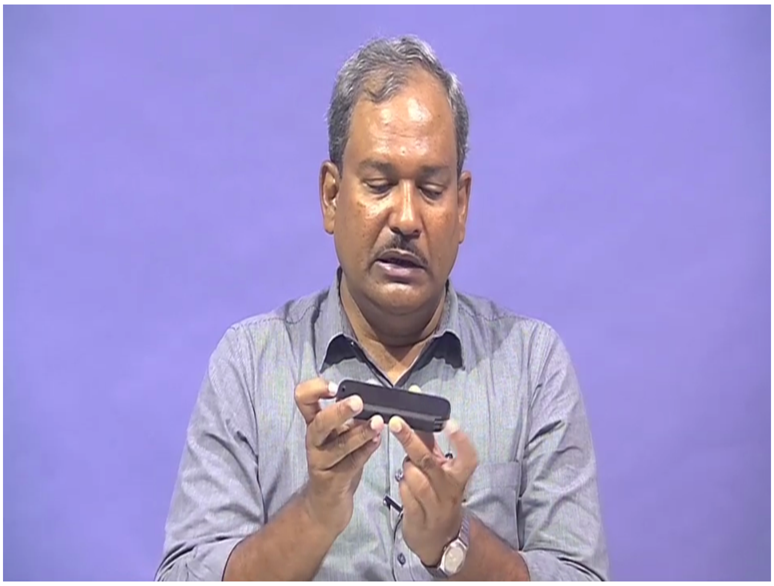
So flow in will carry some momentum along with it.

(Refer Slide Time 07:05)



And flow out will take some momentum out of the control volume.

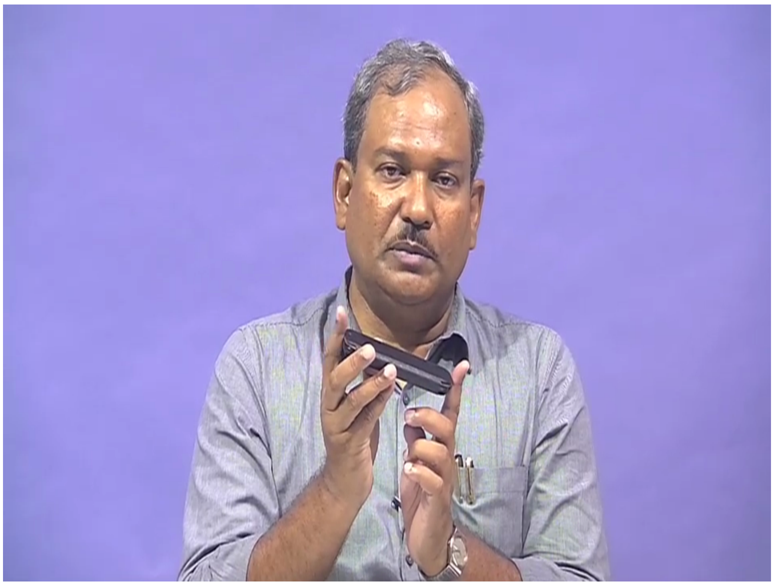
(Refer Slide Time 07:06)



Since the velocity is changing in this direction so some shear stress will be felt by the bottom surface and some shear stress would be felt by the top surface. So the forces due to the share which would act on the two sides of this of this shell would be tau with appropriate subscript multiplied with the bottom area and tau with again with right subscript multipliedby the top area. So these are the momentum in by convectionand net momentum in by conduction, by molecular means, by viscous transport of momentum.

There may be other forces acting on it. For example this is freely falling so there is no pressure difference between these two points.

(Refer Slide Time 07:56)



So the only there is no surface force present in it. However since it is inclined the component of gravity would act on the volume of the liquid contained in this control volume. So the only force which is acting on it would be the gravity force, the body force, gravity force. In steady state, the sum of all these would be zero. So the fundamental equation that we are going to write for any shell is, rate of momentum in minus rate of momentum out plus sum of all forces acting on it must be equal to zero, at steady state.

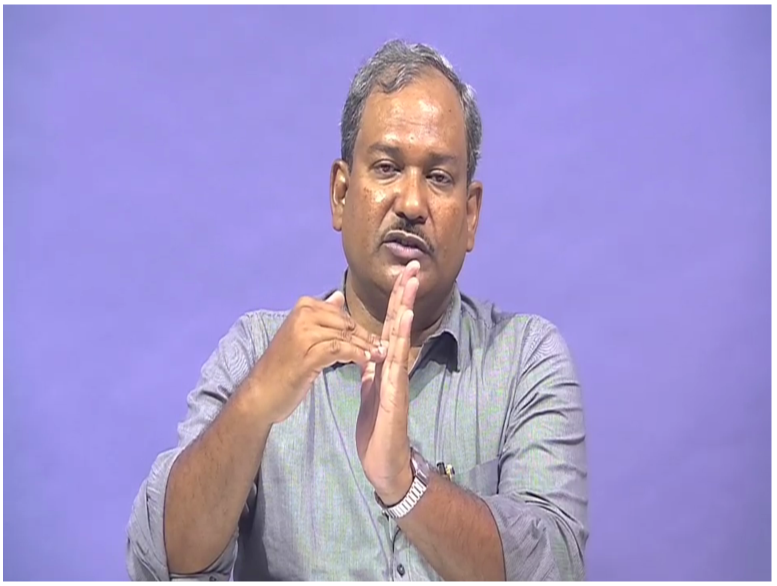
And then we identify that the momentum can come in as a result of convection and as a result of conduction. Convection is with the flow, conduction is perpendicular to the flow do the presence of the shear stress. So once we have the difference equation then we are successful in (ex) explaining or in expressing the physics of the problem in the form of difference equation which contains the smaller dimension, let's say delx. If x is the direction in which the velocity is changing.

So the next step would be to divide both sides by del x and taking the limit when del x tends to zero. So that would essentially be using the definition of the first derivative and out of the difference equation you get the differential equation. So that was the fundamental of shell balance which we have discussed in the previous class. And wehave used an example of flow along (fla) an inclined flat plate to clarify some of the concepts.

Once we have the (go) governing differential equation we need boundary conditions. Since we are going to integrate that in order to obtain the expression for velocity for certain case. The two boundary conditions, the common boundary condition that one would expect in momentum transport is no slip at the solid-liquid interface and no shear at the liquid-vapor interface. So we used those two conditions to obtain a compact expression for the velocity as a function of depth.

Once we have the expression for velocity, we would able to obtain the, what is the maximum velocity and then we should also be able to express not the point value of velocity but we are more interested into the average velocity. All such kind of averages are always done across the flow cross sections. So across the face which is perpendicular to the flow direction. So if I have flow in this direction, then I need to find out what is the velocity at every point in an area that is perpendicular to the direction of flow.

(Refer Slide Time 10:50)

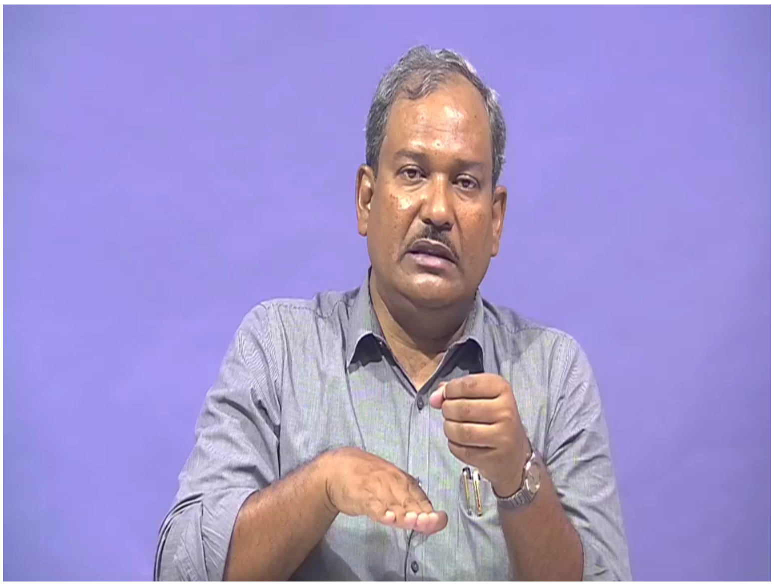


So with that we have obtained what is the expression for average velocity and once we have the average velocity then we cancalculate whatis the flow rate volume and the mass flow rate of the following film along the incline. in two or three classes we would see some examples of the use of shell momentum balance in everyday problems, in everyday situations that we know of.

So the most common fluid flow phenomena that you canyou see almost everyday is, flow of water let's say, through a pipe. Through a circular pipe. So you would like to see in this exercise how we can use the shell momentum balance to obtain an expression for velocity (shu) in a tube through which the liquid is flowing. Now we are going to assume in this case is that the pipe is the tube is vertical. So there will be effect of gravity. Gravity would try topull the liquid in this direction.

And there is also pressure of gradient. There would be some pressure over here and a slightly lower pressure at the bottom. So the pressure force the pressure gradient the pressure force is acting forcing the liquid to move downwards. There would be gravity force which is going topullthe liquid downwards. So the effect of pressure gradient and gravity is to create a flow in the downward direction.

(Refer Slide Time 12:24)



Now as the fluid starts to flow, it is going to interact with the walls of thetube and the way it is going to interact is through viscous forces. When it reaches steady state, the sum of all forces acting on the control volume suitably define for flow in a pipe must be equal to zero. So if I can define a control volume for flow in a pipe then we are going to find out what is the rate of momentum into the control volume by convection and by conduction.

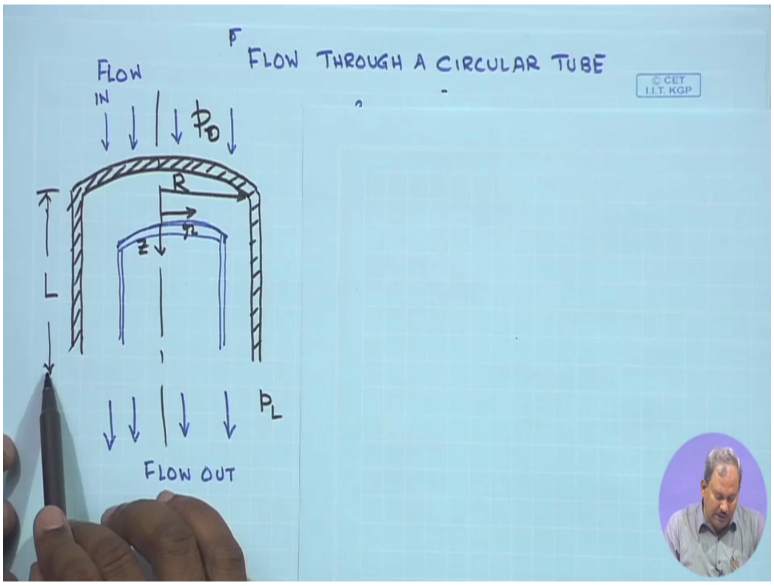
What is the surface force that is acting on the control volume? Namely pressure in this case. What is the force due to gravity that is acting on it? So it's a total amount of liquid contained in this control volume multiplied by rho and xi. So, at steady state the sum of all these would be equal to zero. So with that difference equation we should be able to obtain a differential equation and we should be able to use appropriate boundary condition for this case to obtain what is the velocity of distribution of a fluid flowing in a tube subjected to pressure gradient and subjected to gravity.

So that is what we are going to look in this class and in the next classes. Because this would give rise to some of the equations which are quite known to us. And some equations which are going to play a very important role infinding out what's going to be the, let's say if you are designing an experiment to measure the viscosity of a liquid. All of you probably have used capital viscometer.

So what is the principle? What is the principle on which capital viscometer work? So how do you design the experiment? How do you find out? How you can relate viscosity with the flow rate? So there will be many such examples which would be able tounderstand once we solve the problem of flow through a pipe.

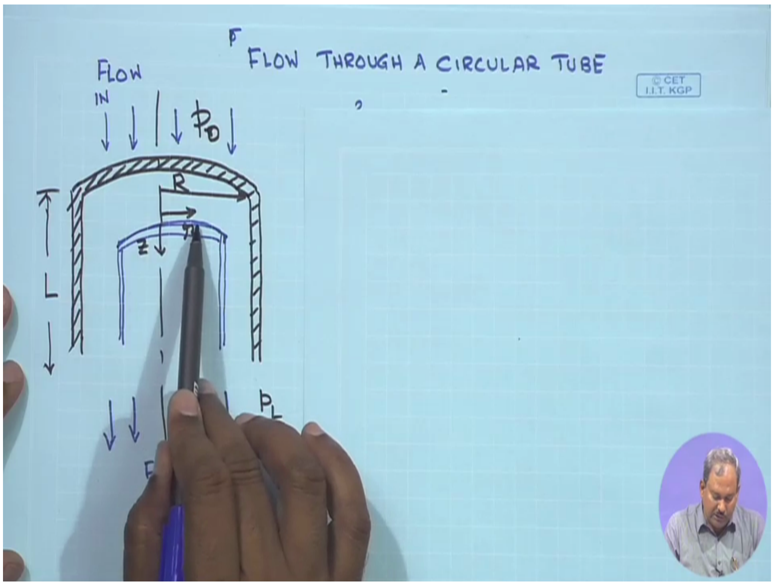
So that's the first problem that we are going to look at in this classes, flow through a circular tube. So what you see here is the pipe. Sowhich has a radius equal to R.There is flow from the top. The pressure at the top is p0 and pressure at the bottom is pL. The length of the tube is L and the R the radial and the Z coordinates are shown in here.

(Refer Slide Time 14:58)



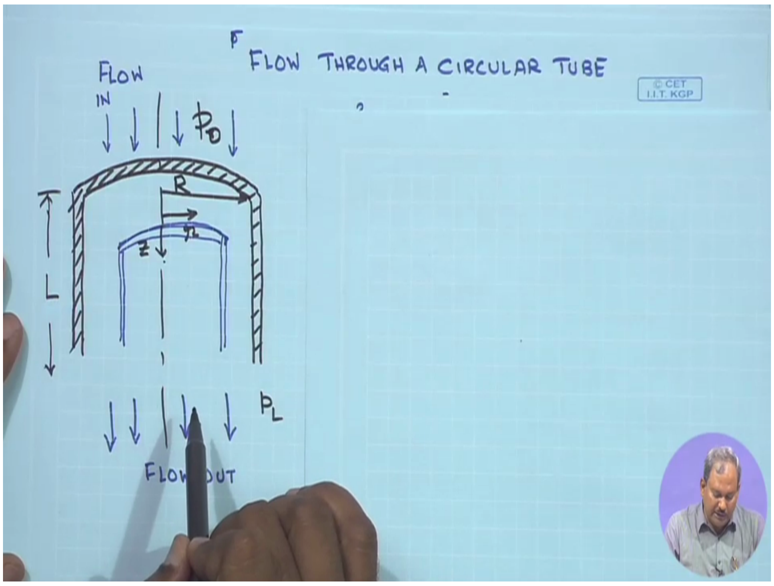
Now we understand that as the flow takes place in the tube, the velocity at steady state and for (impro) in incompressible cases, the velocity is a function of R only.

(Refer Slide Time 15:19)



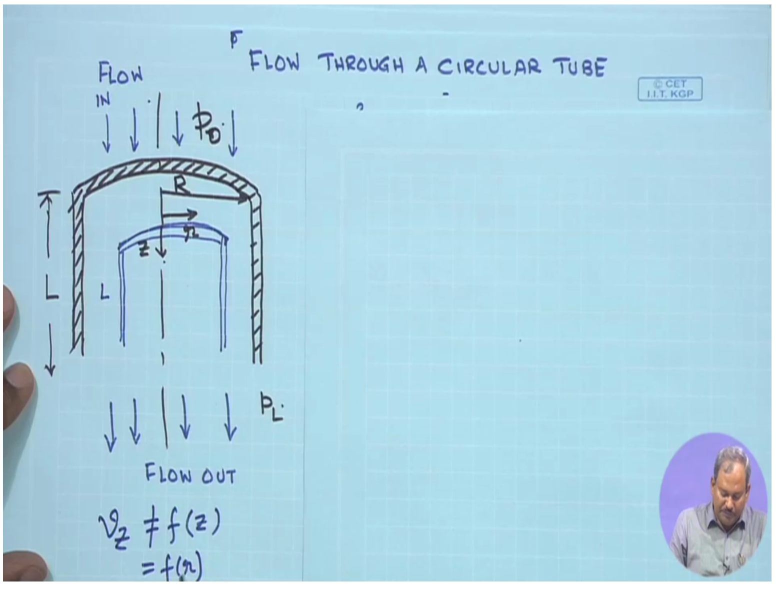
The closer to the wall the liquid layer is, the lower is going to be its velocity and as it moves progressively towards the center, the velocity will increase. Now whether this increase is going to be a straight line increase or some other form? That's what we are going to find out in here.

(Refer Slide Time 15:38)



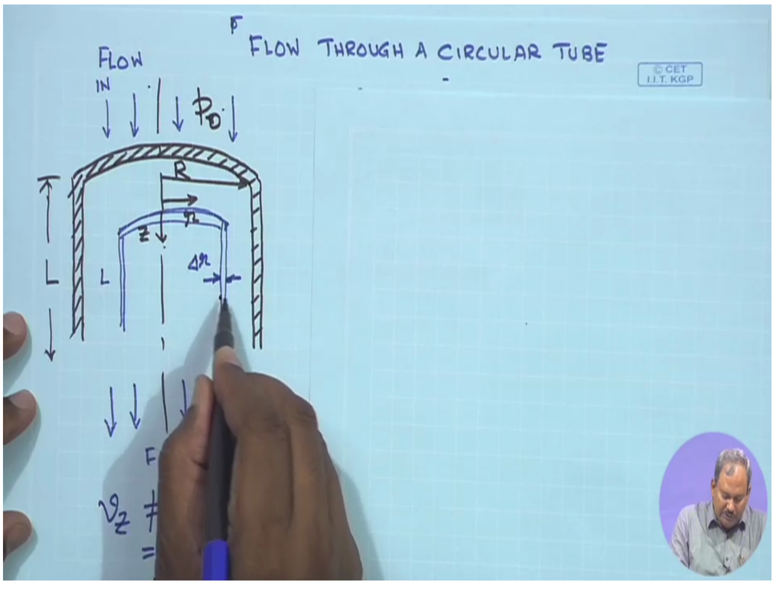
So the first job for this case where I have flow from the top as a result of pressure difference and as a result of gravity is to define a shell. And across that shell, we are going to make the momentum balance. So what would this shell look like? It could be of any length L, it does not matter since the velocity is not a function of L. Velocity vZ in the Z direction is not a function of Z. It is definitely a function of R.

(Refer Slide Time 16:16)



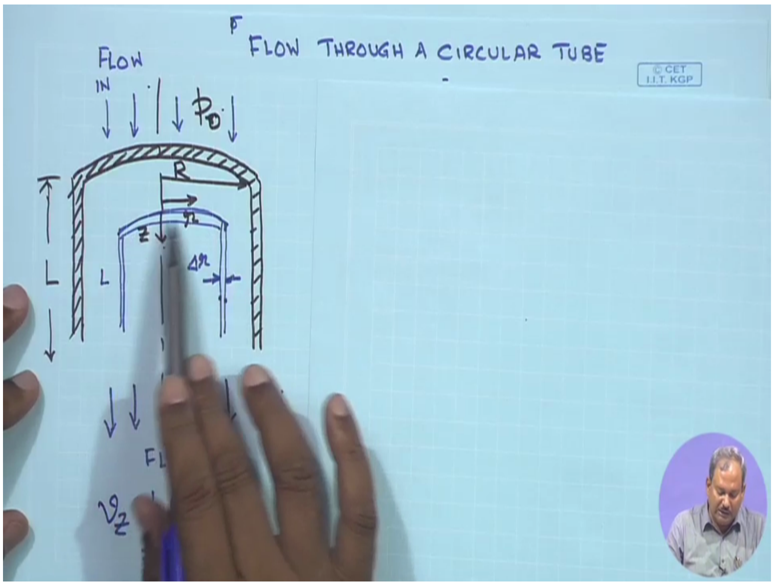
Since it a function of R, then my shell will have the smaller dimension as delta R. The way that I have shown here. So this is delta R. This point is R and this is at R plus delta R.

(Refer Slide Time 16:28)



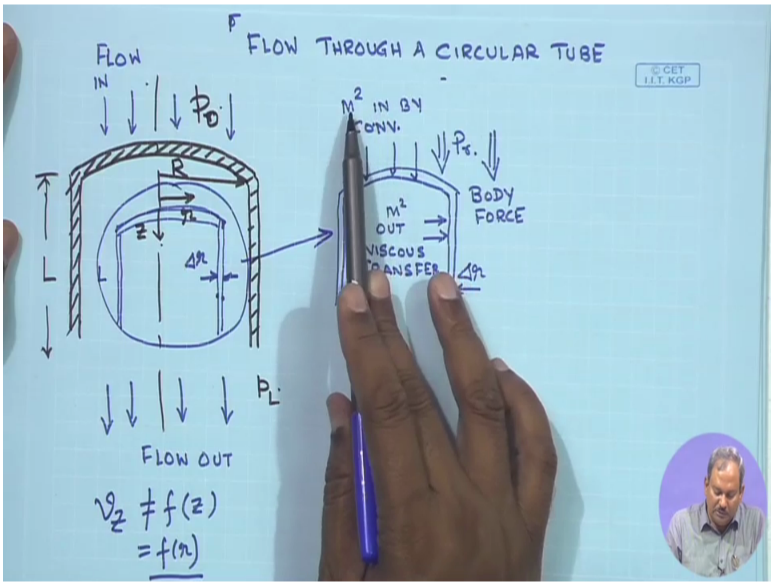
It could be of any length. It does not really does not matter since the velocity is not a function of Z. So I think of this shell, the one in blue and would try to see what would be the momentum in term, momentum out term. Then all the forces due to the surface force and due to the body force acting on the liquid contained in this shell.

(Refer Slide Time 16:56)



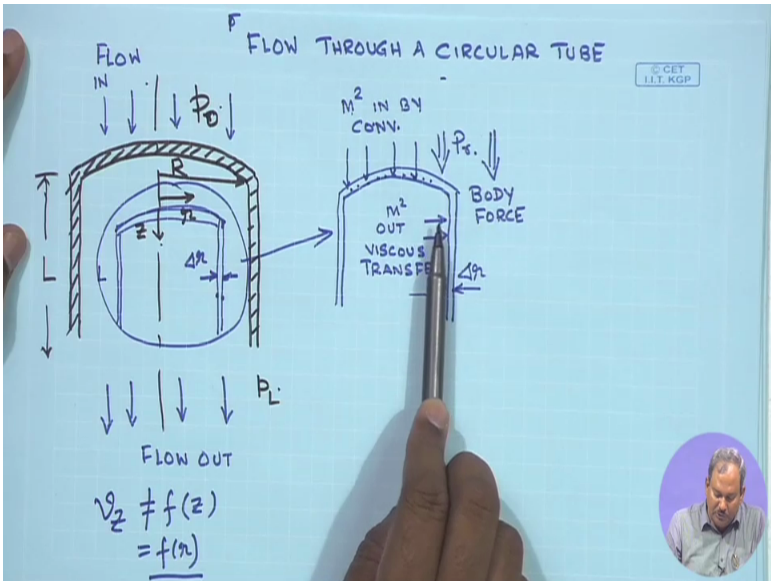
So here I havejust enlarged this section in here and what you see here is that through the top surface of the imaginary shell, momentum gets in by convection.

(Refer Slide Time 17:13)



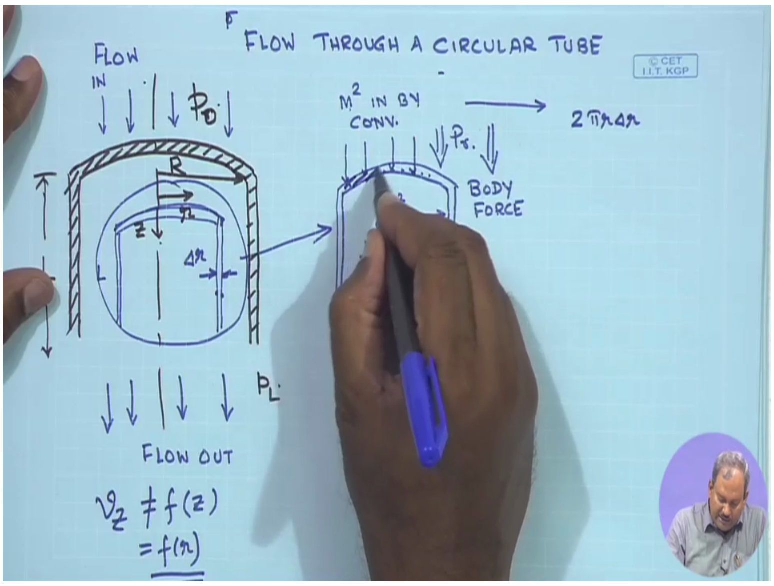
So the liquid is crossing thesepoints carrying away with it some momentum alongwith it. But since the velocity is changing in the R direction, there would be a gradient in velocity between this point, this point and so on.

(Refer Slide Time 17:28)



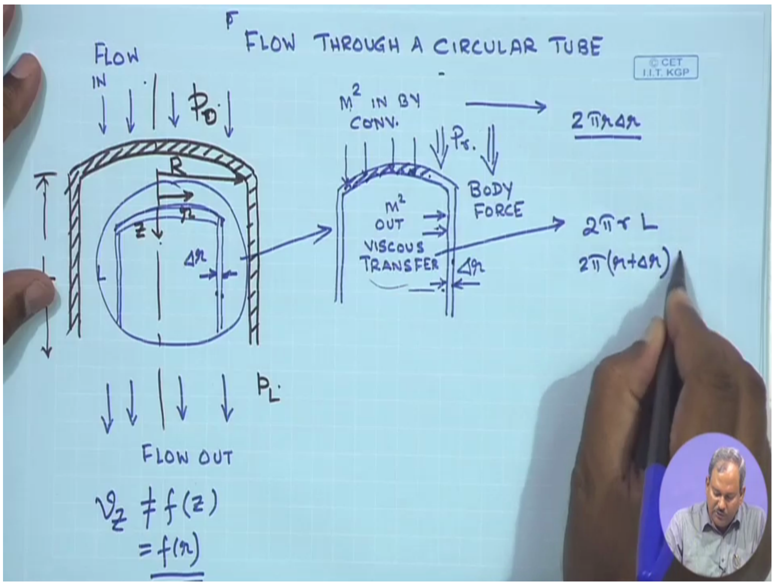
Since there is a velocity gradient, there would be viscous transport of momentum. So momentum in and out by viscous transfer is taking place to the sidewalls. So if you think of the momentum which is in by convection the area on which it is acting on must be equal to twice pie R times del R. So it’s this area that I'm talking about which is twice pie R into del R.

(Refer Slide Time 17:55)



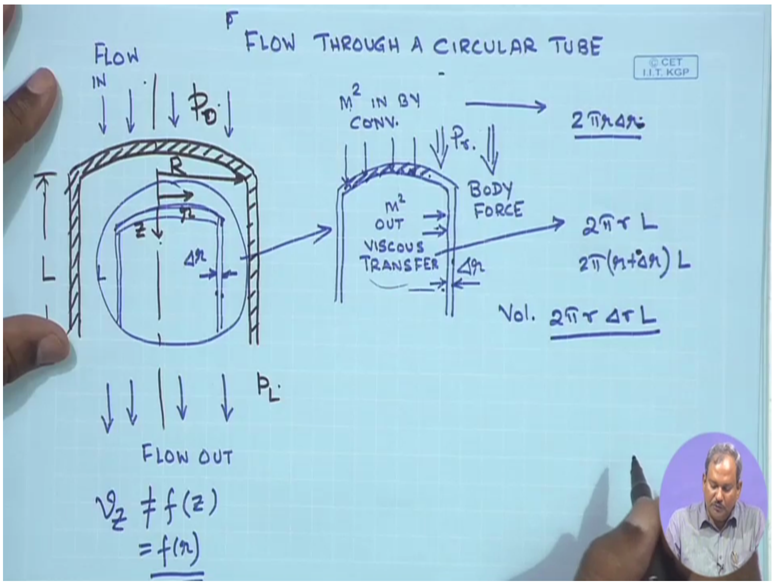
Whereas when we talk about viscous transport, its acting at the inner area and the corresponding area for that would simply be equal to twice pie R L or if you think about the outer one twice pie R plus delta R into L. So these are the two different areas through which the convictive transport of momentum and conductive and transport of momentum is taking place.

(Refer Slide Time 18:18)



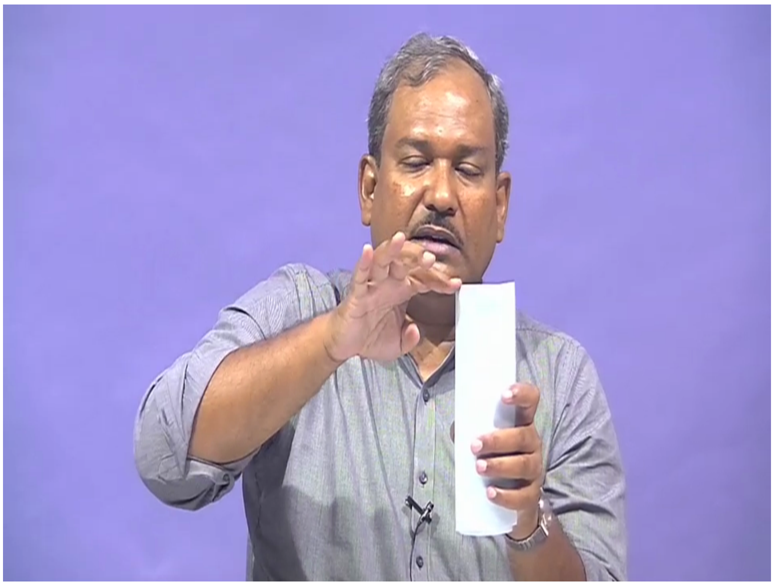
So if this, what is the volume of this? It would be simply equal to twice pie R delta R times L. So this is the volume of the fluid which is contained in this shell. I think it is clear to all of you by now that what I am trying to do is to see what is the area on which the fluid is coming in? What is the side wall of this through which the momentum is going out?

(Refer Slide Time 18:42)



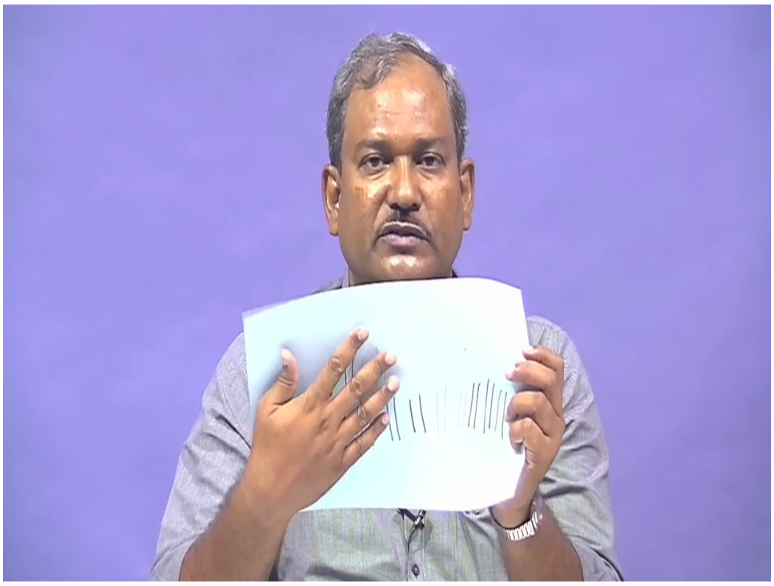
So if you think of this is my shell which is vertical, then this top area is the one throughwhich the convictive momentum is coming in. So whatever would be the velocity multipliedby the area would give me the amount of mass which is coming in. The mass of fluid that comes in carries with it some amount of momentum. So what is going to be the momentum in it? It is this area multiplied byvZ evaluated at Z equals to zero multiplied by another vZ in order to make it momentum.

(Refer Slide Time 19:38)



So that is the rate of momentum coming in through the top surface. Whereas when you think of the inside area, if I open it up it is simply going to be twice pie R times L. So when I open this up this is the area which encounters the molecular transport of momentum. So twice pie R L, this area is the one through which viscous transport of momentum is taking place.

(Refer Slide Time 20:06)

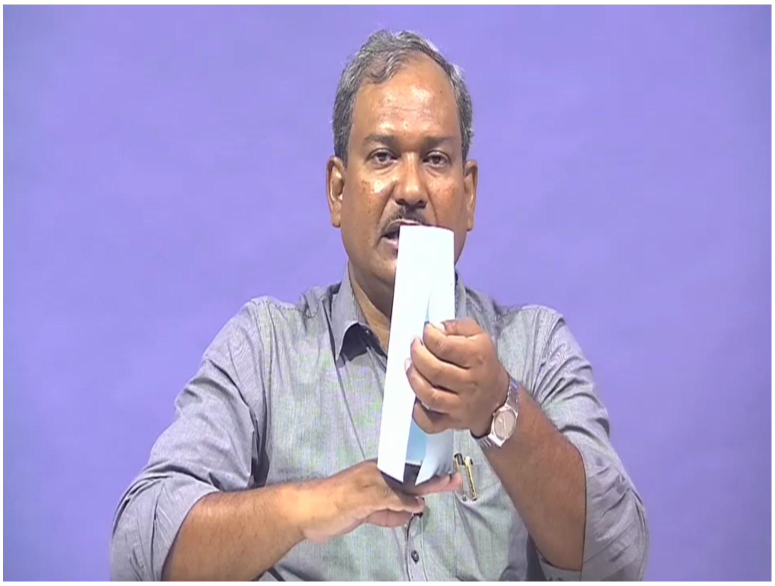


When you think of pressure it's this top surface on which the pressure force is acting. One the pressure p0 is acting over here.

(Refer Slide Time 20:15)

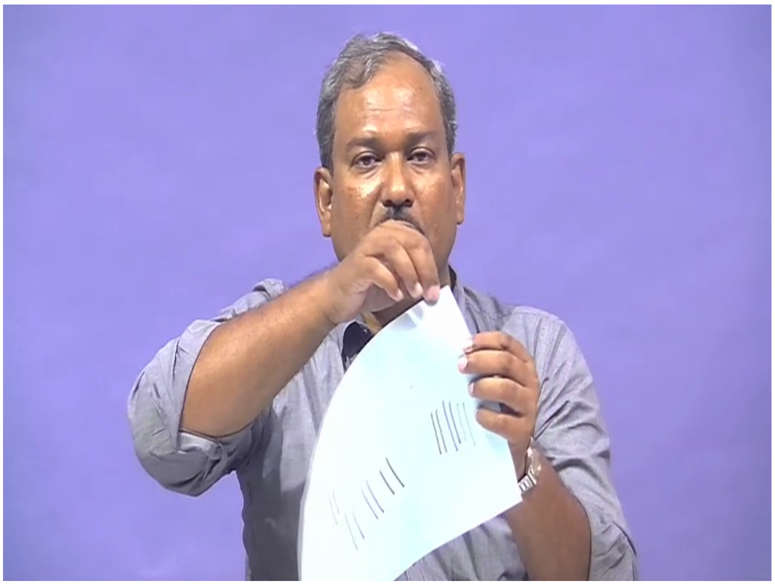
PL is acting at the bottom and whatever be the liquid contained in this thin strip of imaginary shell that is also being acted on by the gravity force.

(Refer Slide Time 20:19)



So the mass of the fluid contained in this script, which is twice pie R delta R times L times rho times xi, would give me the total body force which is acting on this shell.

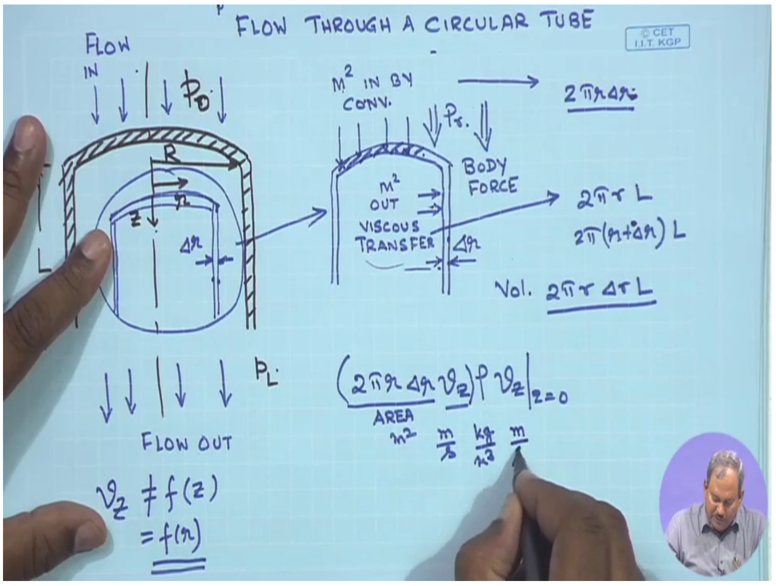
(Refer Slide Time 20:32)



So what I am going to do now is to write each of this terms and then see if it is possible for me from the basic physics or our basic understanding to cancel the terms that are not important or that are equal to each other. So that is what I am going to do next. So when Iwrite this momentum in term, one is going to be twice pie R delta R vZ rho vZ at Z equals to 0.

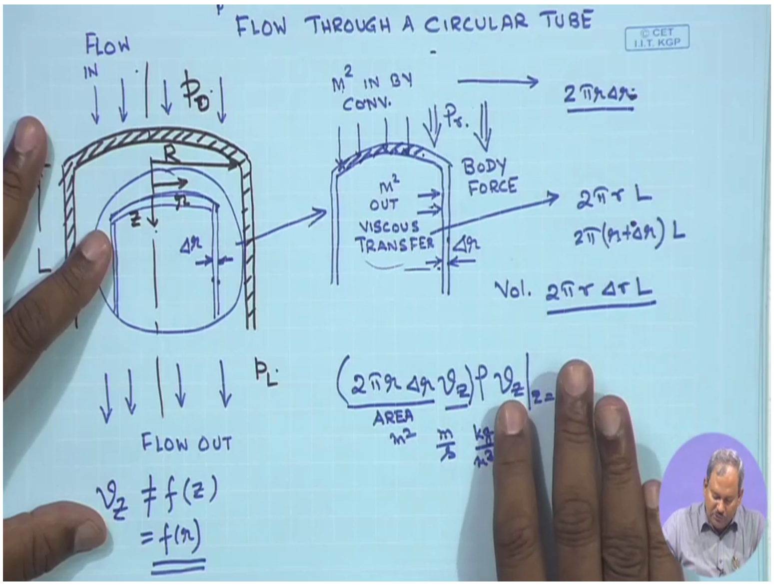
So this is the area which is in meter square, this is velocity which is meter per second. So this makes it meter cube per second. That means it's the volumetric flow rate. Up to this point is the volumetric flow rate meter cube per second. I will multiply that with rho, which is kg per meter cube. So that makes it, if I think of this mark, its kg per second.So that means up to this point thisis nothing but the mass flow rate. This mass flow rate is again multiplied by a velocity with units of meter per second.

(Refer Slide Time 21:54)



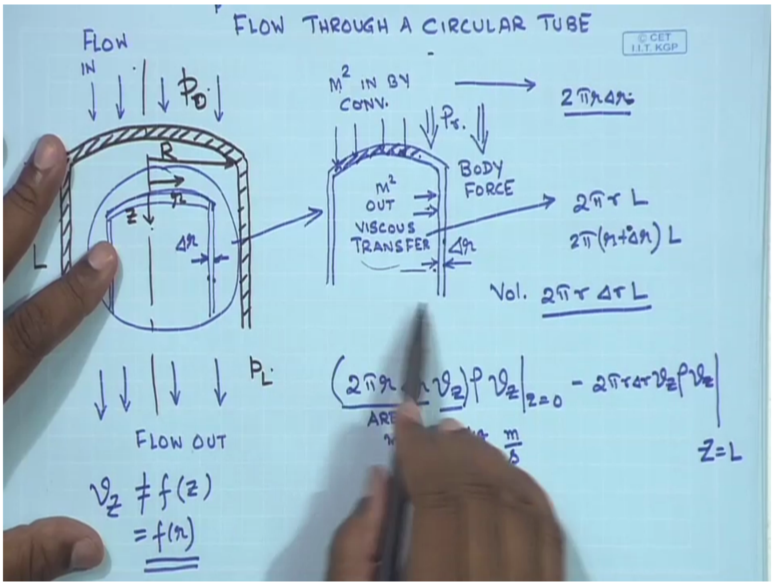
So what I have then here is,the total amount of convictive momentum which is coming into this.

(Refer Slide Time 22:00)



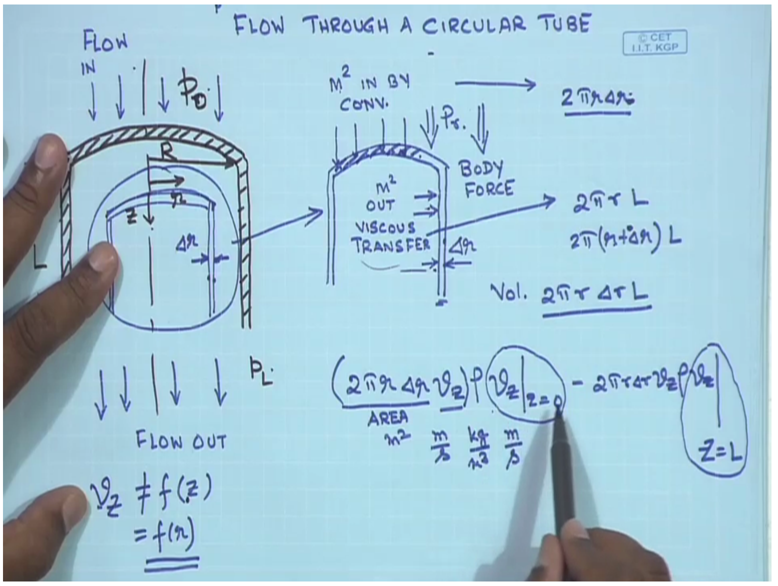
So this is mass times meterper second square. So this has unit of force. So this is the convictive momentum into the system and the convective momentum out of the system wouldsimply be equal to again twice pie R delta R vZ rho vZ. But this time it is evaluated at Z equals to L. That means at this point.

(Refer Slide Time 22:31)



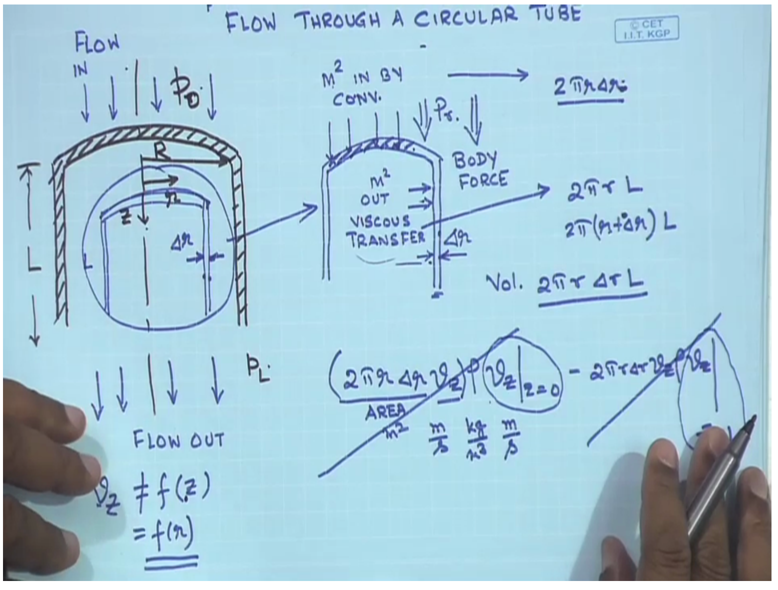
Now if you see thesethe variation of the velocity,vZis not a function of Z. So whatever be the value of vZ at Z equal to 0, must be equal to the value ofvZ at Z equals to L. VZ does not vary with Z.

(Refer Slide Time 22:47)



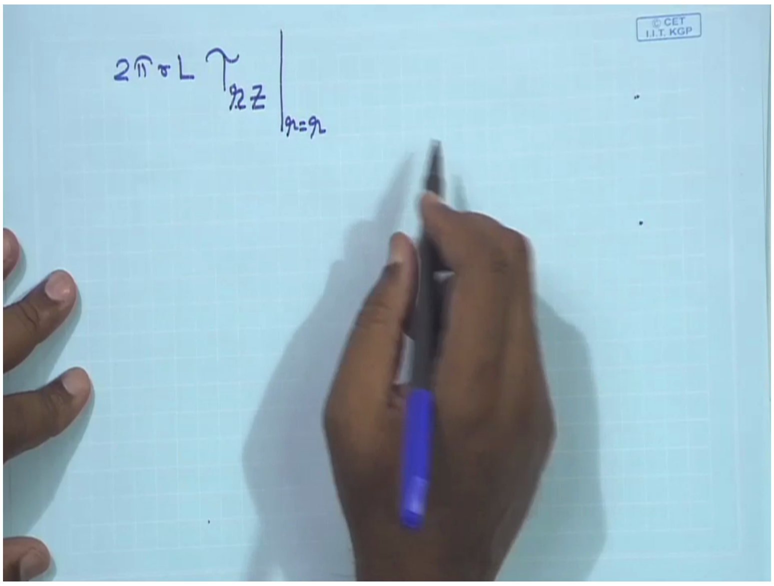
And therefore this term and this term will cancel out each other. So the convictive momentum in and the convective momentum out would simply be equal in this case.

(Refer Slide Time 22:59)



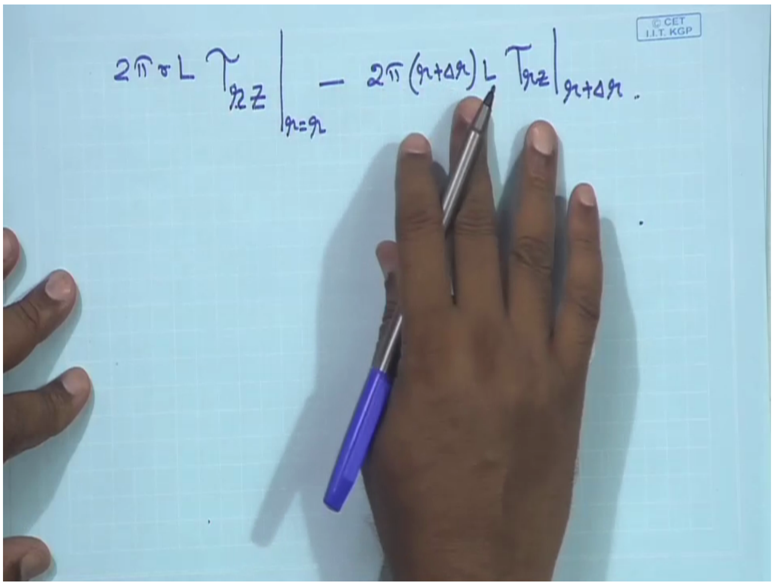
So with that then we are going to write what is going to be the molecular transport of momentum? So the area on which it is working on is twice pie R L and the shear stress. It's the Z component of momentum due to viscosity gets transported in the R direction. So therefore the subscript of tau should be equal to rZ. Principal direction of motion, the direction in which the momentum gets transported due to the presence of viscosity and this is evaluated at R equals (sub) at R.

(Refer Slide Time 23:39)



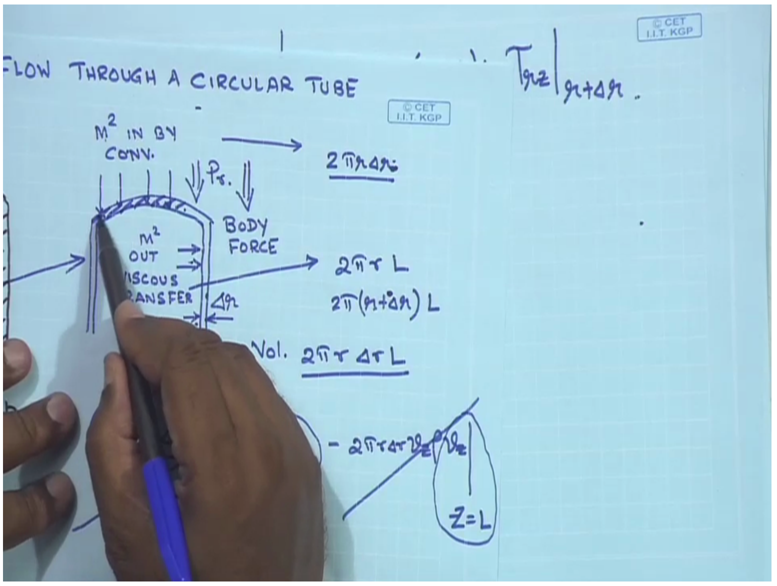
The one that goes out would be twice pie, the radius has now changed to R plus delta r, the length remain same, tau rZ is going to be evaluated at R plus delta r.So this is the viscous momentum in, viscous momentum out.

(Refer Slide Time 23:56)



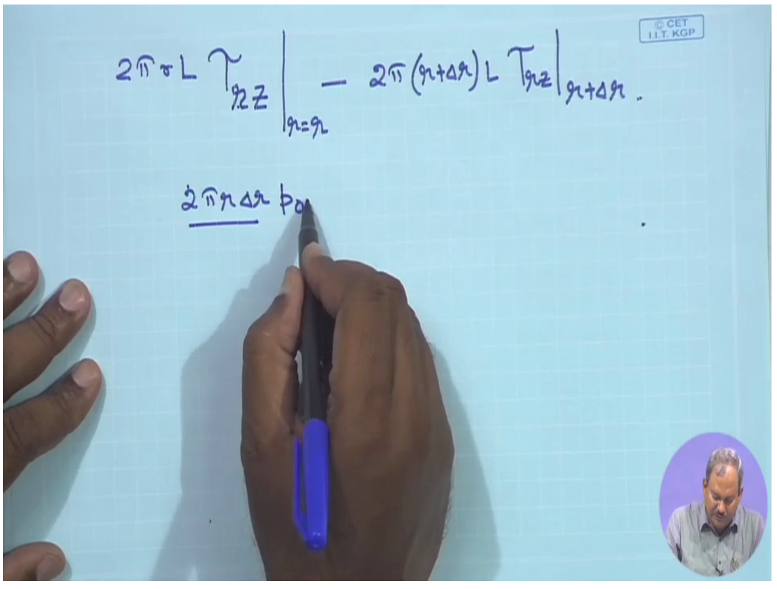
What is remaining is then, what is the pressure force? The pressure force acting at the top. At the top of this would simply be the area multiplied by whatever be the pressure that is p0.

(Refer Slide Time 24:08)



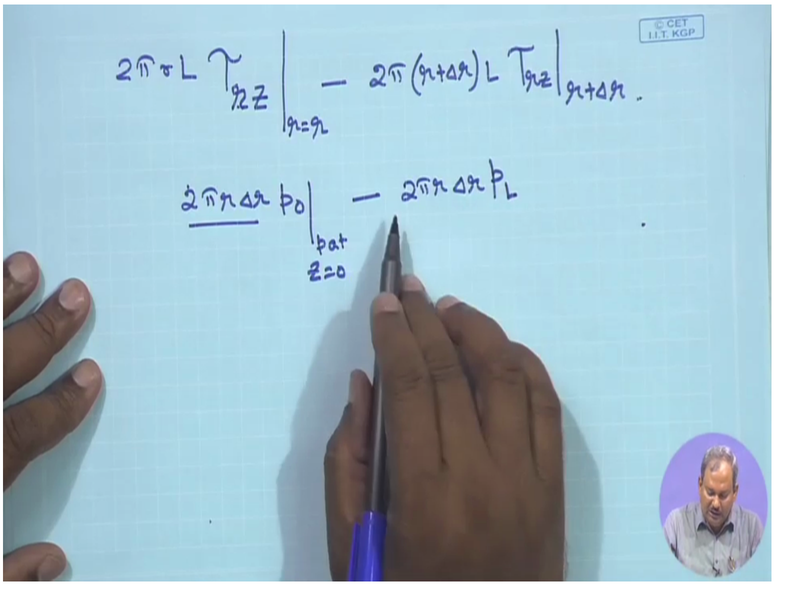
So the area that we have in this case would be twice pie R delta r times p0. So this is the area at the top, this is whatever be the pressure acting at Z equals to 0.

(Refer Slide Time 24:33)



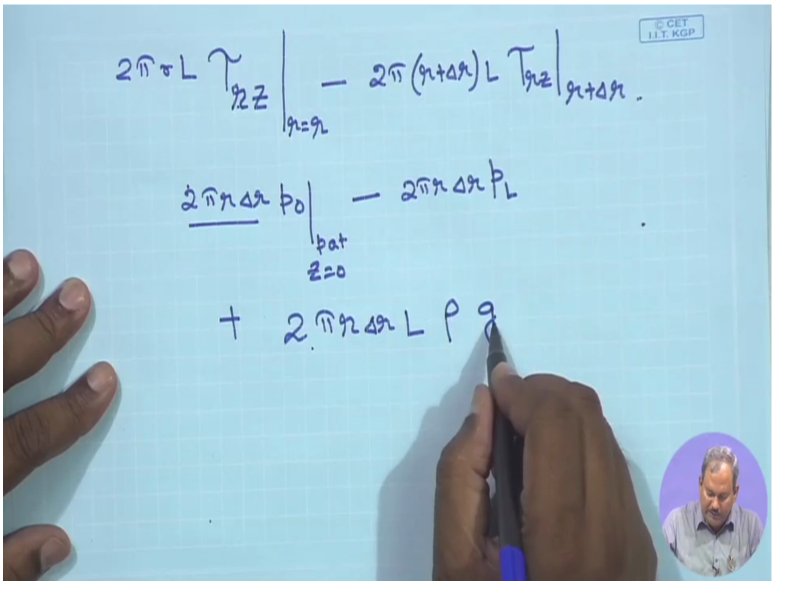
So this is the pressures at Z equals 0 and the one at the bottom would be the twice pie R times delta r, the pressure at the bottom is L.So it is going to be twice pie R L.

(Refer Slide Time 24:46)



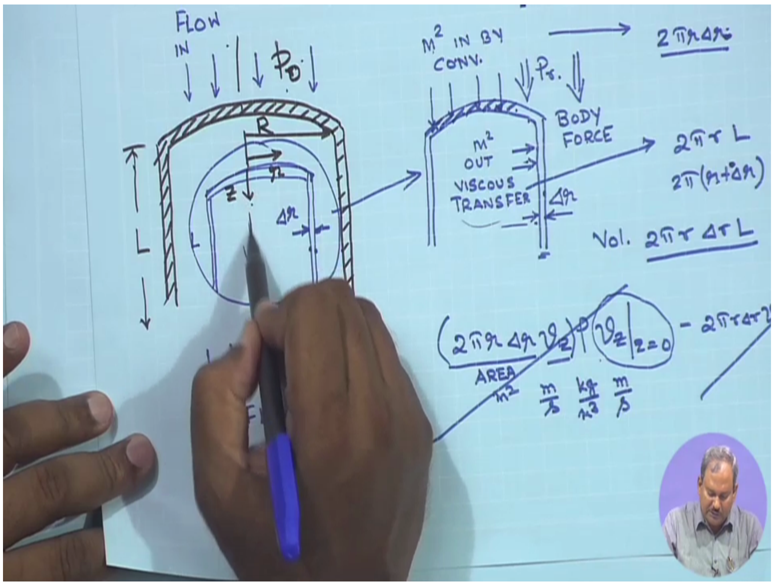
And I also have body force which is acting on it. In order to obtain the body force, I need to first find out what is the total volume of fluid that is present in here. The total volume is twice pie R delta r. This is the area multiplied by length. So this becomes meter cube, the volume multiplied by rho. So this becomes kg multiplied by G. In this case the gravity component is simply going to be plus G.

(Refer Slide Time 25:14)



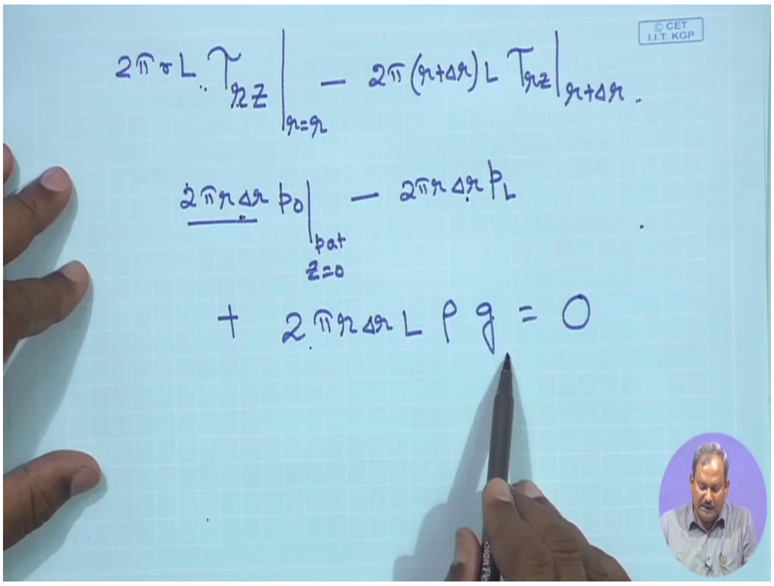
Because of the co-ordinate system that we have used in here. So in this direction it's the increase in Z. So G in this case is going to be positive. The gravity force is going to pull the liquid in the direction of increase in Z.

(Refer Slide Time 25:24)



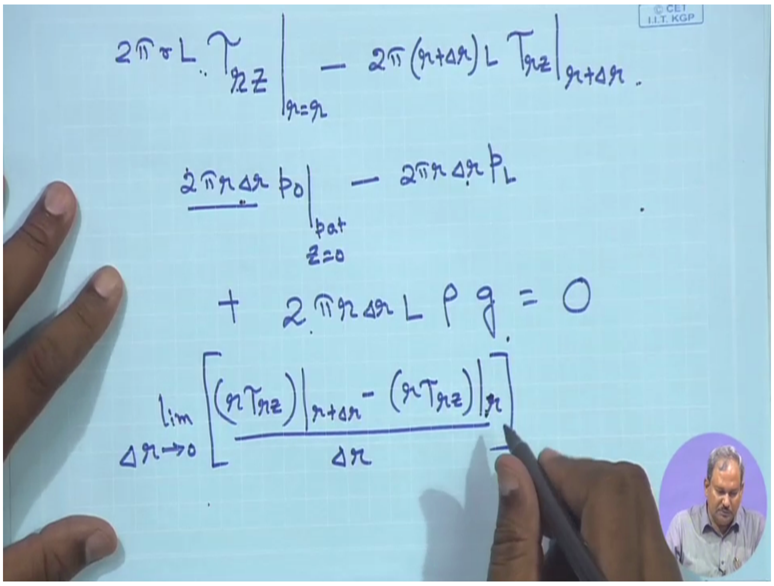
So the governing equation, the difference equation would simply be net momentum in by convection. Net (fo) the pressure which is acting at the top. The pressure which is acting at the bottom and the gravitational force which is acting on it.

(Refer Slide Time 25:53)



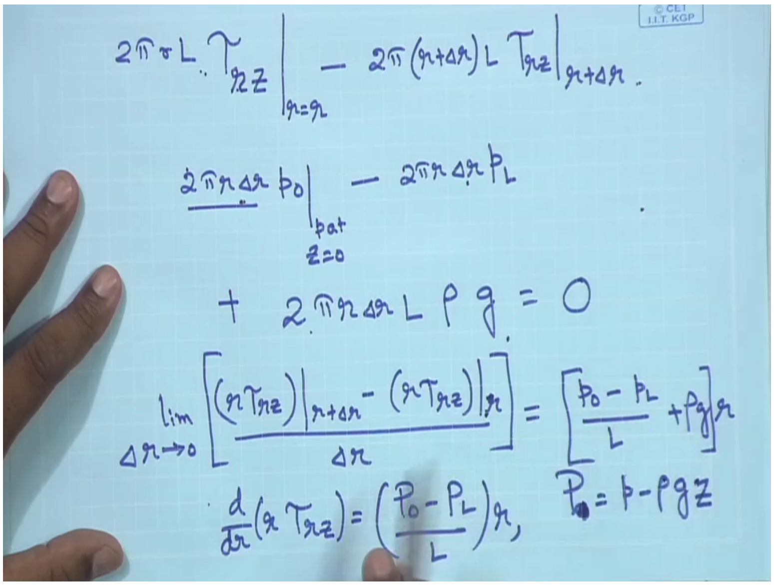
Now the next step as we have done in the case of difference equations is to divide both sides by the smaller dimension which in this case is delta r. So we divide both sides of the equation by delta r and taking the limit when delta r approach is zero and what we get out of it is a differential equation. So if I divide both sides and take the limit my equation would be limit as delta r tends to 0. R tau rZ, which is, this is evaluated at R plus delta r minus R tau rZ, which is evaluated at R, divided by delta r. So this is going to be the first two terms.

(Refer Slide Time 26:44)



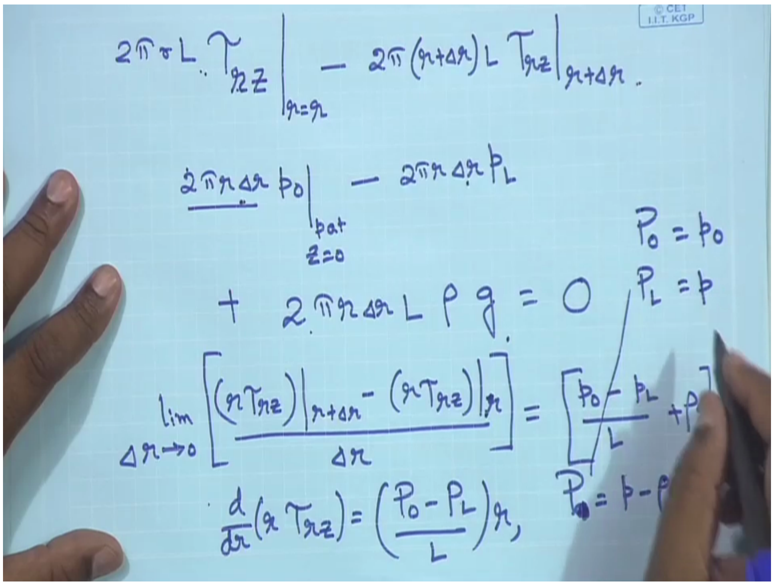
Then I am going to have is p0 minus pL by L plus rho G times R. So if you use a definition of the first derivative, what you are going to get is D dR of R tau rZ is equal to p0 minus pL by L times R. Where simply p0 or P is defined, P is defined as small p minus rho G Z. So if you think of this term, it simply can be expressed as capital p0 minus p0 by L. There is no additional significance to this P. This capital P is introduced so to give it just compact shape.

(Refer Slide Time 27:41)



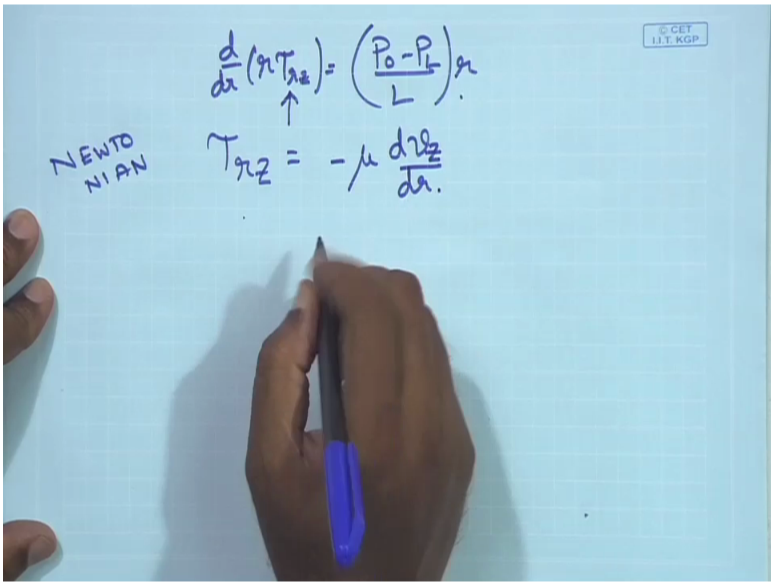
Because we understand that from here p0 is simply going to be equal to p0 and P times L is going to be pL minus rho G L. So now if you look at this term and this term, they are identical. I have just clubbed rho G in here. That's all. Thereis no other additional information, additional concept which are required. This is just the definition of capital P in order to have a compact expression.

(Refer Slide Time 27:53)



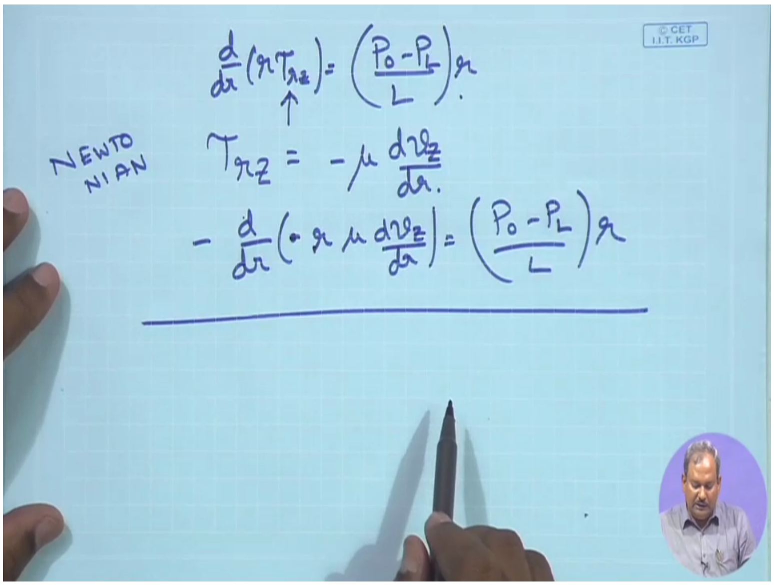
So then what I have as my governing equation is D dR or R tau rZ equals p0 minus pL by L times R. Once I have this equation, the governing equation, it is integrated to obtain the velocity expression. That's what we would like to do. Butbefore we do that, this tau rZ must be substituted in order to express it in terms of velocity. And we understand that this tau rZ, if we assume the fluid to be Newtonian, then tau rZwould simply be equal to minus mu with variation in Z with R.

(Refer Slide Time 29:02)



So once you substitute that in here then what you have here is D dR of minus of R times minus mu times D vZdR is equal to,p0 minus pL by L times R. This equation can now be independent and that's what we are going to do in the next class.

(Refer Slide Time 29:28)



So what we have done in this present exercise which we will continue in the next class is, we have found out simply by making a shell balance, we would be able to account for all the factors, all the different (meth) ways by which momentum can come into the system or leave the system. We have also correctly identified what are the different forces, surface and body, that are acting on the control volume. At steady state the algebraic sum would be zero. That difference equation can be converted into a differential equation.

And once we have the differential equation and plugging the expression for Newton's law of viscosity, then we have a differential equation of velocity in terms of the physical property mu, in terms of the impose pressure impose condition operating condition that is the pressure gradient, p0 minus pL by L. And the body force field present which is G.

So this expression we willnow integrate in order to obtain a very compact useful relation for the velocity and subsequently the flow rate of fluid through a tube in which there is a pressure gradient and in which there is the effect of gravity. So that relation we are going to do in the next class.