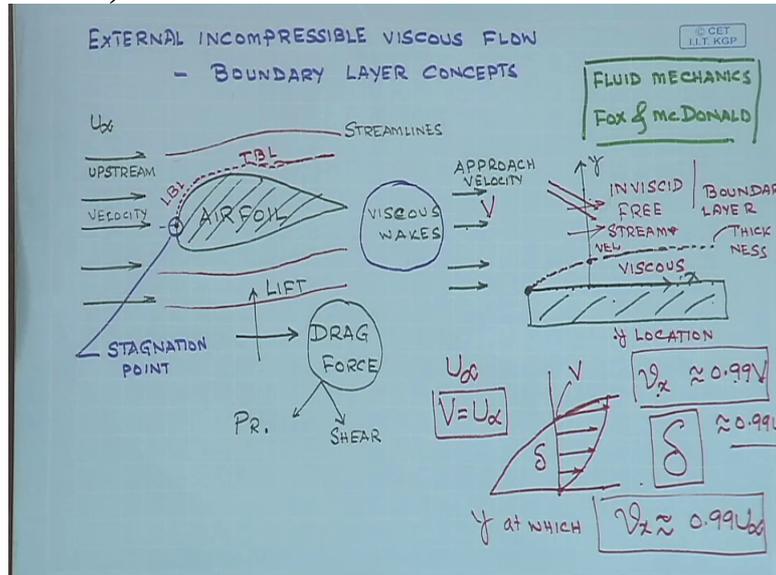


Transport Phenomena
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Boundary Layers (Continued)
Lecture 17

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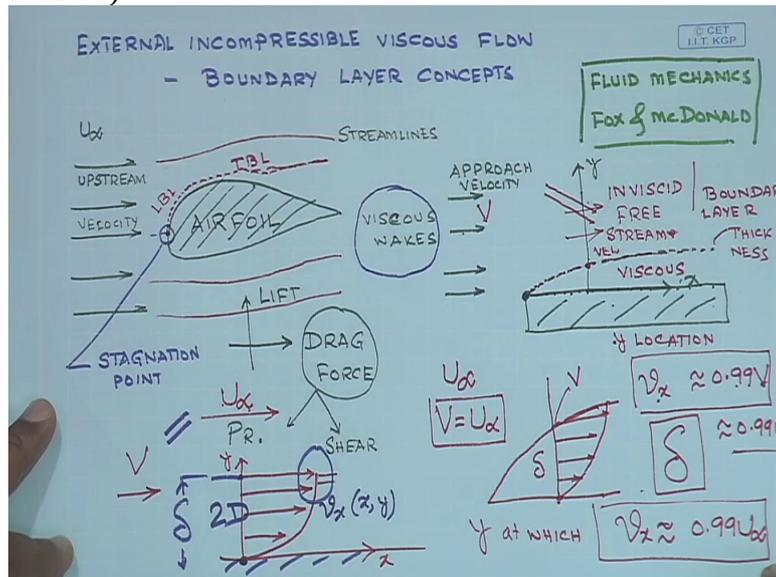


So we will continue with our introduction to boundary layers and the different concepts which are involved in this and I will refer back again to our discussion previous discussion in which we have seen that wherever there is a flow over a surface over a solid surface all the transport phenomena is going to be confined in a region very close to the surface. Beyond that region the flow will move unhindered unaware of the presence of the solid below and in that region the flow can be treated as inviscid that is without any viscosity. However in a region close to the solid surface where the velocity will vary from that of the solid plate, that of the solid surface to the free stream condition is known as a boundary layers.

So the concept of boundary layer which was the missing link between theory and experiments while design the shapes was introduced by Prandtl and what he had shown is the layer of inviscid fluid and the layer of where the layer of viscous forces are important. In the concepts find widespread use in the design any moving object be it a car, motor bike, cycle, bus, and aircraft and so on. And also in the field of sports intelligent use of the formation of boundary layer can make a person do wonderful things while bowling while playing any other games base ball and so on. So we are going to have our structured study of boundary layer in the coming classes.

But first of all we have to define what the thickness of boundary layer is. Because that is of paramount importance if we need to know how do we know that upto this distance the effect of viscous forces are present and beyond that point the viscous forces are not present. So we use the velocity at that location to be a pointer to decide whether or not we have hit the edge of the boundary layer. So the problem is that the velocity varies from that of the solid object to that of the free stream. And by free stream I mean where the fluid is free of the viscous forces. So they vary asymptotically. That means the velocity the gradient of velocity with y slowly decreases the gradient of velocity in the x direction with y slowly decreases.

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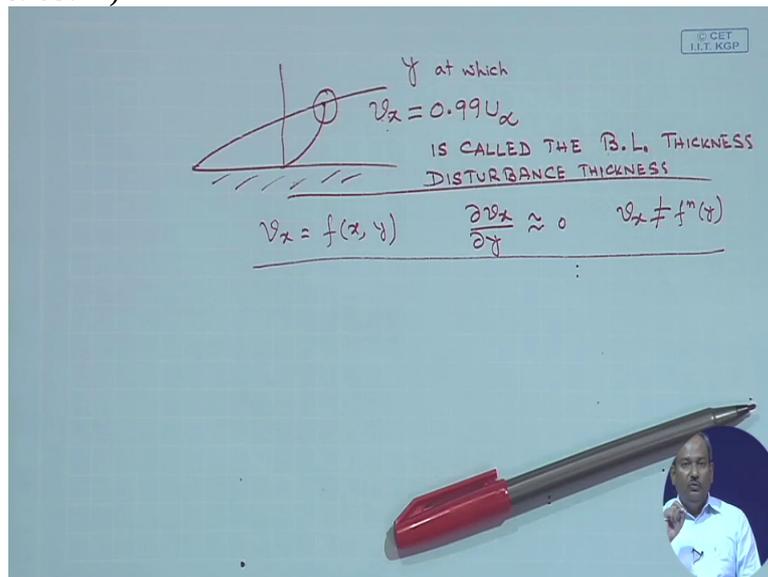
And the profile is something that I have drawn over here. So the profile of the boundary layer is something like this. So if I can draw the velocity in here the velocity profile it is going to start at value equal to 0 and then this is your approach velocity which is denoted by V and over here I have the free stream velocity which is denoted by U infinity. That means the velocity in the x direction at a distance at a very large distance from the plate this is a X direction and this is the Y direction. So inside this boundary layer the velocity in the boundary layer in the X direction is a function both of X and Y and outside the boundary layer the velocity is simply going to be a constant equal to the free stream velocity.

So it is 2D flow inside and it is inviscid flow outside. For this special case of flow over a flat plate the approach velocity is equal to the free stream velocity. But in order to keep our discussion general we will refer to the point at which the velocity inside the boundary layer becomes equal to the free stream velocity we call that as the thickness of the boundary layer generally denoted by δ . So δ is a distance over which the velocity changes from 0

relative velocity is 0 on the solid plate or the velocity of solid plate to the velocity of the free stream.

This is what delta is, but if you look at this region the nature at which the velocity approaches the free stream velocity its asymptotic in nature. So where I am going to draw the line is it this point where it has become equal to U_∞ ? or is it this point at which the velocity becomes equal to U_∞ . So each person probably will decide on the velocity the location at which the velocity becomes equal to U_∞ in a slightly different fashion.

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So the common practice of the boundary layer thickness is its the thickness at which the velocity that is V_x has is equal to 99 percent of the free stream velocity. So y at which V_x is $0.99 U$ is called the boundary layer thickness. So this is the standard definition of the boundary layer thickness. It is also called sometimes it is called the disturbance thickness because this is the extent to which the liquid fills the presence of the solid plate below it. So it is also commonly known as Disturbance thickness.

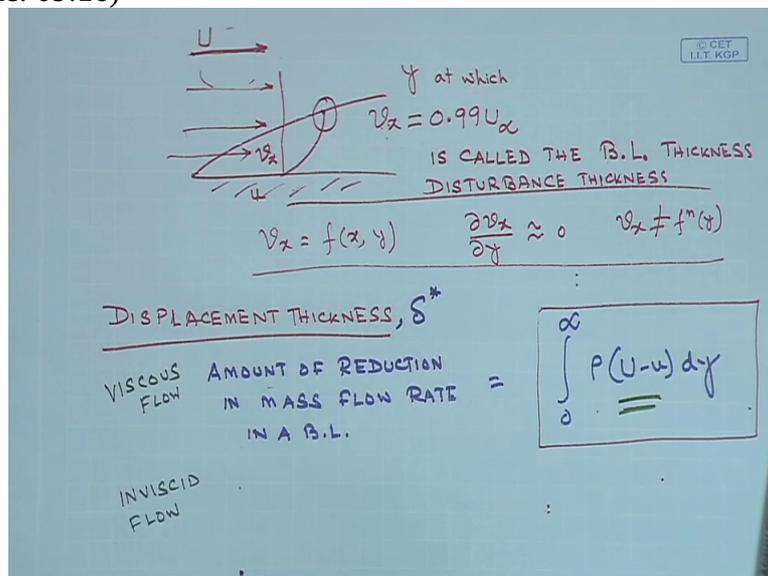
But even then it is not a full proof method, that the fact that where the location where the velocity is 99 percent of the free stream velocity is also prone to error because of the nature of variation of V_x with y . So I understand that V_x is a function of x and is a function of y . However near the boundary layer $\text{Del } V_x \text{ by Del } y$ approaches 0 that means beyond the boundary layer v_x is not a function of y V_x is not a function of y anymore. So these are mathematically the definition of the location of the boundary layer but it is so difficult to theoretically pin point where this happens.

So each one of us probably will decide that this is the location of the boundary layer. And we are also going to use some sort of a measurement technique a device an instrument to demark

the point where the velocity is 99 percent of the free stream velocity. Each of these instruments they have errors associated with it each one of us will make some errors the human errors. So these errors are taking into account of these errors the final error in the evaluation in the determination of the boundary layer thickness is extremely difficult.

And is prone to large errors. The problem comes more because we are trying to measure the velocity at a point or at every point and then try to project that to some sort of a thickness, some sort of a disturbance thickness. This is a differential approach; any differential approach is prone to experimental error. So there is an alternative method, if you try to think of an alternative method which is integral in nature then that code probably give us a fairly decent error free measurement of some sort of a boundary layer thickness. And that is what we are going to see in next.

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So the next approach is known as the displacement thickness. So what is displacement thickness that is what we are going to find next and it is generally denoted by the symbol δ^* . So we would see what it is. When I have flow in this the mass flow in this region where the velocity is moving with the free stream velocity is going to be more as compared to this case where the velocity where the fluid is moving with v_x or in some cases it is also denoted by small u . So v_x is the x component of velocity or which is also denoted by u .

So over here the fluid is moving with u over here the fluid is moving with v_x or u and u is definitely less than capital U the flow inside the boundary layer is less than the free stream condition. So the amount of mass which travels through the boundary layer is less than that of the free stream. So how much of less mass that we are going to have inside the boundary layer. The amount of the due to the influence of viscous forces the amount of mass would be

amount of reduction in mass flow rate in a boundary layer would simply be equal to 0 to infinity $\rho \times (U - u) \times dy$. Here we have assumed that the depth perpendicular to this paper is infinity so if I did not have the boundary layer the amount of mass flow rate would simply be $u \times dy$.

Since I have small u present in this part of the film so this capital U minus small u denotes the reduction in the mass flow rate as a result of the presence of the viscous forces in it. So if I integrate it from 0 to infinity though I understand that I do not have to integrate it from 0 to infinity I can simply integrate it from 0 to δ because by definition small u becomes equal to U once you cross δ . But any way in order to keep the generality we keep this definition which gives me the reduction in mass flow rate inside a boundary layer. So this is due to this denotes the due to the presence of the boundary layer. Now let us say that I would like to get the same reduction in the case of inviscid flow. So this is viscous flow and I am imagining an inviscid flow in which I would get the same reduction in mass flow rate.

So I have a plate I have the motion I have the formation of the boundary layer inside the boundary layer the liquid moves slowly as compared to the free stream. So the reduction in mass flow rate is simply going to be the difference in velocity between these two points multiplied by ρ multiplied by Area. So area is fixed with or unit with and this length scale is simply going to be 0 to infinity to be more correct mathematically more correct or 0 to δ the thickness of the boundary layer to make it practical.

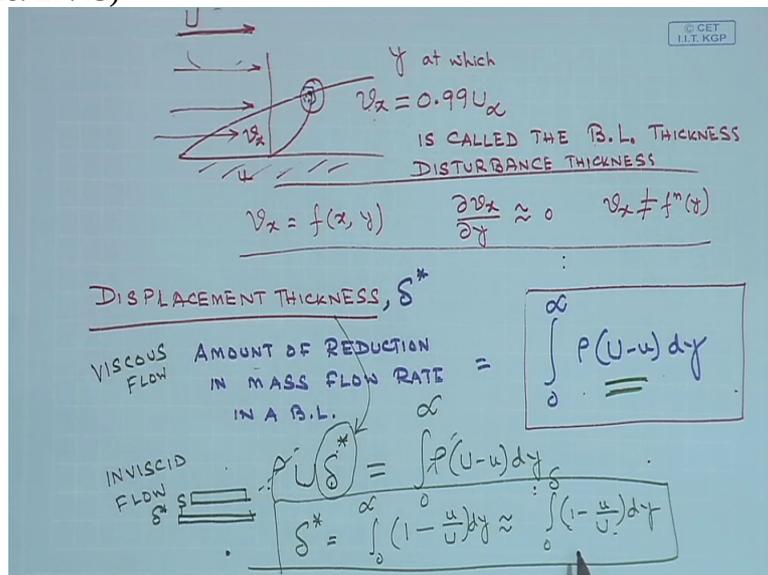
So this is in viscous flow the liquid slows down. Now let us think of that the entire fluid flows again flows over the flat plate but this time the fluid is inviscid. But I would like to have the same difference in mass flow rate and as in the case of viscous flow. So in order to reduce the flow fluid over the solid plate in inviscid flow the only option available to me to reduce the flow rate is simply by restricting the flow area, simply by putting the solid plate up by a certain distance I restrict the flow area and therefore whatever was flowing below whatever was flowing on top of this by which I have restate that area is no longer available to the flowing fluid any more.

So in a inviscid flow if I raise the solid plate by a distance equal to δ^* then I am essentially making that much of area unavailable to liquid flow. So what is the area which is unavailable to liquid flow now. One is this δ^* multiplied by 1 because I am talking about unit depth I have taken unit depth. So δ^* amount of area this $\delta^* \times 1$ this is no longer available to flow right now. And what is the mass that is moving that could have moved through this area $\delta^* \times 1$, the mass which would move through this is $\delta^* \times 1$ which is the unit depth multiplied by the velocity with which

the fluid was supposed to flow through this. What was the velocity with which the fluid was supposed to flow through that blocked area since it is inviscid flow the velocity everywhere is u infinity.

So if I raise it by a distance δ^* the amount of area that I have blocked the velocity of the liquid through that area could have been same u infinity. Since it is inviscid flow. So by raising it up by a distance δ^* the volumetric flow rate reduction is simply going to be area times the velocity. Area is $\delta^* \times 1$ and the velocity is going to be u infinity the free stream velocity. It is an inviscid flow so there is no question of any change in velocity so the volumetric flow reduction on account of raising the solid plate by a distance δ^* would be U infinity times $\delta^* \times 1$. What is the corresponding reduction in mass flow rate simply multiplied by ρ . So the mass flow rate reduction because of my raising the platform by a distance of δ^* in an inviscid flow would simply be $\rho \times \delta^* \times 1 \times u$. When the reduction in mass flow rate in the inviscid flow is going to be equal to the reduction in mass flow rate in the actual viscous flow that we have the expression for which we have already evaluated when these two are equal δ^* is known as the displacement thickness.

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So let us see what is the amount of reduction in mass flow rate in a boundary layer where viscous forces are present is this now in an inviscid flow the reduction in mass flow rate as we have decided as we have discussed is going to be $\rho \times u \times \delta^*$ when the platform is raised by a distance which is δ^* . So this is the reduction in mass flow rate inviscid flow when you raise the platform by δ^* . When these two are equal this δ^* is known as displacement thickness. So what is going to be the final expression of δ^* ,

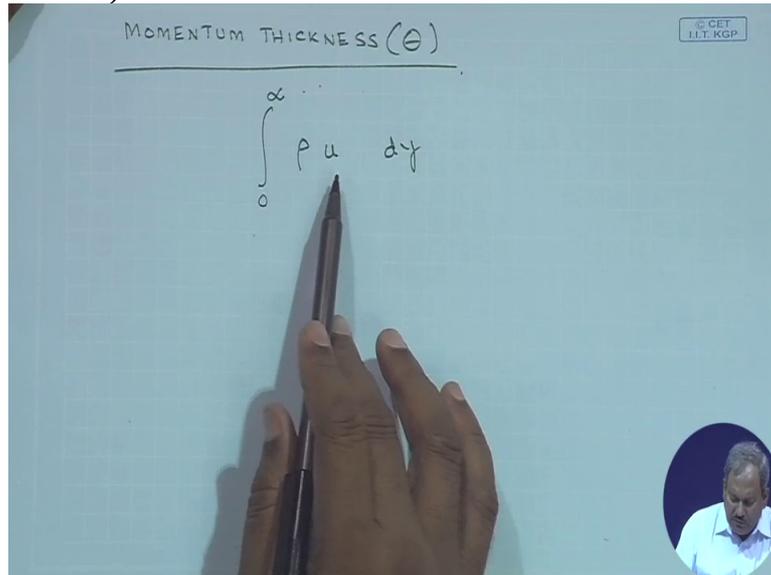
delta star would simply its an incompressible fluid so ρ will simply cancel out so it is from 0 to infinity and if I divide it by capital U 1 minus small u by capital U times dy or it can be practical purposes it can be equated to 0 to delta because nothing happens beyond delta by u into dy. So this is the definition of this displacement thickness.

Now if you look carefully here the delta star the displacement thickness can be expressed since its an integral thickness it can be expressed as a sum of many such terms now the problem that we had in evaluating the current value of delta is right over here where the velocity essentially asymptotically merges to that of the free stream. What happens to those summation terms when we reach delta. As I reach delta u starts to become close to capital U and therefore the contribution of the terms near delta the contribution of all these terms near delta essentially becomes insignificant.

So the integration has been provided us with an opportunity in which the integrand vanishes in the free stream or near the free stream. Since most of our errors were associated with the evaluation of the exact velocity near the free stream here we have a method in which the integrand itself vanishes near the free stream. So therefore any possibility any error that you may have in the correct evaluation of velocity at a distance close to that or near that of the free stream. The effect of that error will be insignificant in the final form of delta star so that is why that is the beauty of an integral thickness over a differential approach over delta so delta star is going to give you much more accurate value with enough confidence as compared to delta which is differential in nature.

So displacement thickness is always safer to use than the boundary layer thickness or the disturbance thickness. Even then people still refer to the people still use boundary layer thickness delta or the disturbance thickness delta whereas it gives you a nice pictorial view of what happens inside view of what happens inside the boundary layer. Inside the boundary layer the velocity changes from 0 to that of the free stream. It gives you a picture which is very easy to understand on the other hand it is prone to error but the alternate is delta star is integral in nature. You need to know the concept before you can truly appreciate the usefulness of delta star. So the more common representation of boundary layer are they are always in terms of delta not in terms of delta star or if you go for some accuracy and we would see later in our subsequent classes that we use delta star as well in our calculations.

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There is one more definition which we need to cover before we close this class which is known as the momentum thickness which is denoted by the symbol Theta and the momentum thickness is also an integral thickness and what we do is we are first going to find out what is the actual flow that is taking place inside the boundary layer, so the actual flow which is taking place inside the boundary layer is the integration Rho times u times dy where y lies from 0 to infinity and the infinity is put to mathematically it is the only at infinite distance the velocity becomes equal to the free stream velocity.

But we artificially put the condition where the velocity reaches 99 percent of the free stream velocity. So we can later on convert this integral to 0.99 around. So will you think of the flow inside the boundary layer. We are trying to find out what is the mass flow rate. so the mass flow rate must be equal to integration since my small u is a function of y so I must integrate. So Rho times u dy y varies from 0 to infinity that is going to be the amount of mass flow rate inside the boundary layer.

What is the reduction in mass flow rate of this amount of flow because we have a boundary layer because we have viscous flow if this amount of fluid given by integration of 0 to infinity Rho small u dy had this been moving in an inviscid flow it will have a momentum associated with it and in order to find the momentum I must multiply this mass flow rate with the prevailing velocity so the velocity in inviscid flow is simply this mass flow rate multiply by capital U or u infinity where infinity is the free stream velocity.

So the mass of fluid that actually flows through the boundary layer had it been moving in an inviscid flow it would carry a momentum which would be equal to integration 0 to infinity and Rho u times U dy. But its not moving in the free stream condition its not moving in

inviscid flow its moving in viscous flow. Since it is moving in viscous flow its velocity is not capital U its velocity is small u and we understand that small u is a function of strong function of y. So what is the reduction in mass flow rate in this quantity of fluid because we have a viscous boundary layer the mass is $\rho u dy$. What is the reduction in momentum because of the presence of the boundary layer so that is what I am going to write here.

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MOMENTUM THICKNESS (θ)

$$\int_0^{\infty} \rho u (U_{\infty} - u) dy = \text{REDUCTION IN } M^2 \text{ FOR THE FLUID THAT ACTUALLY FLOWS IN THE B.L.}$$

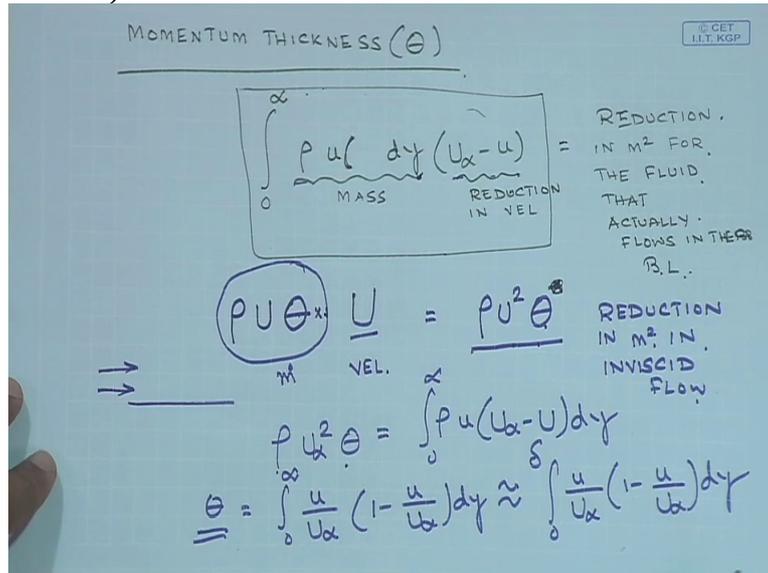
MASS REDUCTION IN VEL

So I must multiply this with $U_{\infty} - u$. so $U_{\infty} - u$ this is dealing with the reduction in mass flow rate reduction in momentum flow of this mass. So this gives me the mass and this is the reduction in velocity. So together this integrant gives me the reduction in momentum for the fluid that actually flows in the boundary layer. So this essentially is then the reduction in momentum for the fluid that actually flows in the boundary layer.

Now I would like to as in the previous case I have evaluated what is the reduction in mass flow rate of the actual fluid that flows in the boundary layer. Now I am trying to see what do I have to do in order to what do I have to do in an inviscid flow to get the same reduction in momentum? I have a flow some amount is flowing over this I understand that the reduction in momentum is what we have decided but I need to do something with the flow area such that the reduction in momentum flow in an inviscid flow is exactly the same as in the momentum that we have derived in previously.

So what I do is I raise it by a distance δ^* , I raise it by a distance θ sorry I raise it by a distance θ , whenever I raise it by a distance θ some amount of area is no longer available for flow which could result in a reduction in mass flow rate.

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So let us see what that is going to be when you raise the platform by distance equal to theta, so when you raise it by theta the reduction in mass flow rate is simply going to be Rho u times theta. So I am again showing that the depth is equal to 1 so theta times 1 the unity is already there. So u times the area multiplied by the Rho gives me the mass flow rate that an inviscid flow on a solid plate this could be the inviscid flow of mass over it, what is the momentum associated with it another u so this is the mass flow rate and this is the velocity that would have existed in an inviscid flow.

So the inviscid flow reduction in momentum since I have raised the platform by a distance Theta is this. So this is the reduction in momentum in inviscid flow. So this is the reduction in momentum of the actual mass of fluid that passes through the boundary layer and this is reduction in velocity so this is the reduction in momentum for the fluid that actually flows to the boundary layer and when I write Rho square theta this is reduction in momentum in inviscid flow when you raise the platform by distance equal to theta.

Theta is called the momentum thickness when these two are equal. So theta is turned as the momentum thickness when the reduction in momentum flow in an inviscid flow case is equal to the reduction in momentum of the actual mass flow that is taking place in a viscous boundary layer. So with this as previously we can obtain an expression for theta as 0 to infinity small u by U infinity 1 minus small u by U infinity dy or which for all practical purposes can be approximated by 0 to delta small u by U infinity 1 minus small u by U infinity times dy.

So this is what you would get when you this is what is the definition of the momentum thickness. So the momentum thickness as in the case of displacement thickness is also an

integral thickness and the integrand vanishes in the free stream. So momentum thickness will also be able to avoid the errors associated with displacement thickness on normal boundary layer thickness if you use either the momentum thickness or the displacement thickness. One denotes the reduction of in mass flow rate because of the presence boundary layer the other denotes the reduction in momentum because of the presence of the viscous boundary layer. So these two concepts will be instrumental in all understanding of the theory of boundary layers in subsequent classes.