Advanced Mathematical Techniques in Chemical Engineering

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Lecture No. # 40
Laplace Transform

Very good afternoon everyone. So, we are looking into the solution of partial differential equation using Laplace transform. So, we have looked into the solution of partial differential equation using separation of variable type of method; we developed the theory for that and then we have looked into several categories of partial differential equation, which can be solved by using separation of variable types of solution. Then, we looked into the solution of non-homogeneous partial differential equations using Green's function method, we developed the theory and applied that theory for the solution of partial differential equation.

Then, we looked into the similarities of transformation and similarity variable method by which we able to solve the parabolic partial differential equations, where one boundary condition is residing at infinity. And then, we looked also into the integral type of method solution, which can basically be applied to the boundary layer problems and then, we looked into the mathematical transform.

For example, we took up will take up two transforms calculus: one is the Laplace, another is the Fourier. In the last class, we have started the Laplace transformation and we were looking into the properties of the various Laplace operators; and then, we were almost starting a problem to be solved by using Laplace transform. In the next class, I will be going through the Fourier transform.

So, let me solve some of the, let me refresh some of the properties of the Laplace transform and look into some of the examples of the problems, which can be solved by using Laplace transformation.

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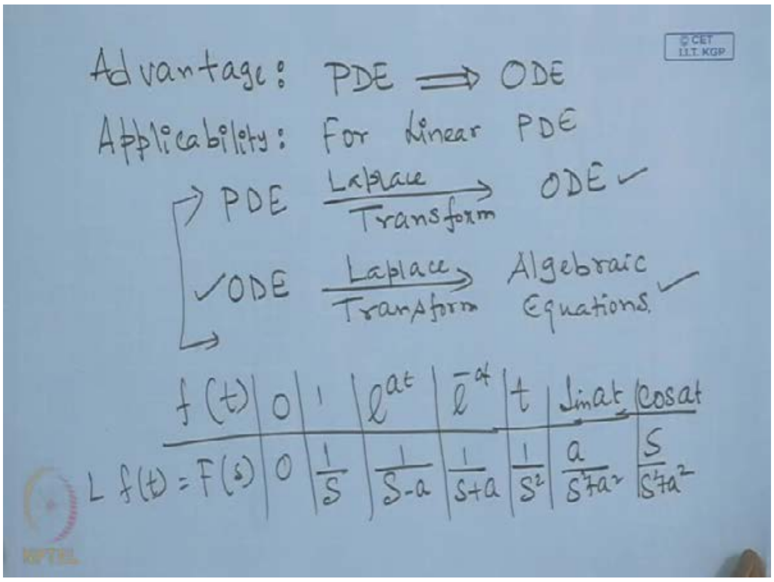

The major advantage of Laplace transform or any kind of mathematical transform is that basically, one can reduce partial differential equation into an ordinary differential equation, that is advantage. And the applicability of mathematical transform is for linear partial differential equation.

So, if the governing equation is linear, if you have a partial differential equation, by using Laplace transform, it can be boiled down to ordinary differential equation. If ordinary differential equation is the governing equation, then by using Laplace transform, we will boil down ordinary differential equation into algebraic equations. So, that is the advantage of Laplace transform; from PDE we get ODE, from ODE we will be getting the algebraic equation. In both the cases, ODE are easier to solve compared to PDE and algebraic equations are easier to solve compared to ordinary differential equation.

So, we have looked into some of the typical functions whose Laplace transform is domain are given; for example, f of t and Laplace of f of t, which are basically transformations in s domain. So, there will be, if f of t is 0, this Laplace transform is 0; if it is 1, this is 1 upon s; if it is e to the power at, then it will be s minus a; if it is e to the power minus at, it will be 1 over s plus a; if f of t is t, then transform is 1 over s square; if it is sin at, then this will be a divided by s square plus a square; if it is cosine at, then it will be s divided by s square plus a square. These are some typical functions, whose Laplace transform in s domain are given.

If you can look into the book of Carsick or any Schaum series of transformer algebra, then there will be a book on Laplace transform by Schaum series itself. So, in that book, there is a big table where the different functions f of t of the various Laplace transforms are given in a tabular form. So, one can look into those things, those tables and can get Laplace transform of any known function f of t.

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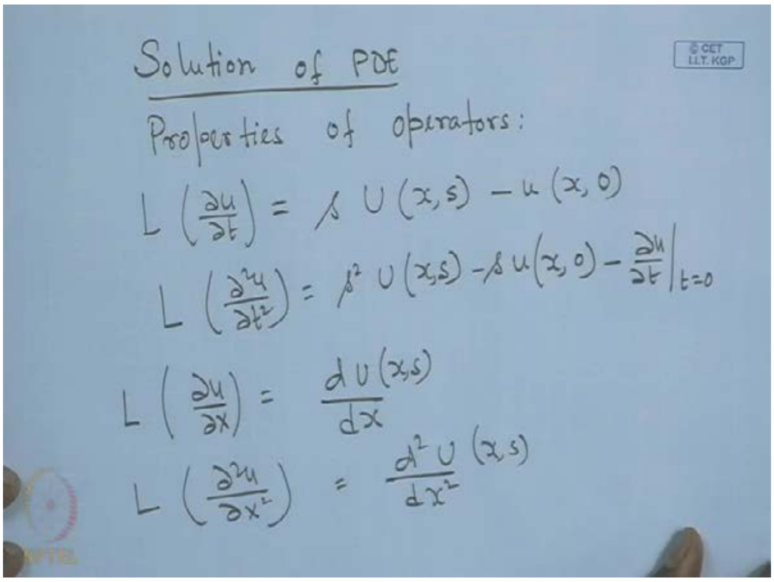

Now, we will be basically looking into the solution of partial differential equation. Now, for that, some of the properties of the operators are needed to be known. So, Laplace of del u del t, Laplace transform is always in the time domain, Laplace of del u del t is nothing but u of x and s minus u at x, t is equal to 0. Laplace of del square u del t square is nothing but s square u x and s minus s u at t is equal to 0 minus del u del t, at t is equal to 0. Laplace of del u del x will be du - this is transform variable - x s divided by dx. And Laplace of del square u del x square is nothing but d square u s dx square.

So, these are the properties of various operators in the Laplace domain. So, once we know, we brush up these properties of various Laplace operators and we know different Laplace transforms or Schaum's known function f of t, then we are in a position to solve an actual problem. Let us look into an exercise problem first.

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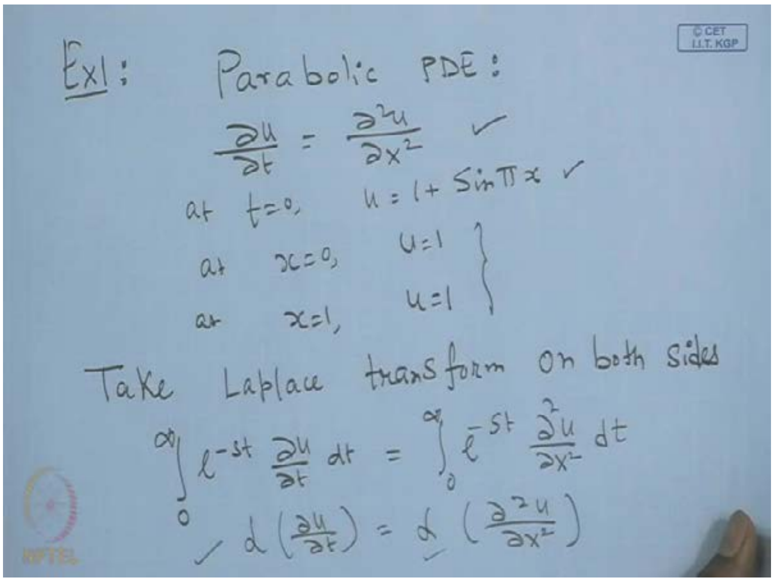

Example 1: let us look into a parabolic partial differential equation. So, del u del t is equal to del square u del x square; at t is equal to 0, let us say u is equal to 1 plus sin pi x; at x is equal to 0, u is equal to 1; at x is equal to 1, u is equal to 1. So, this is a typical two-dimensional parabolic partial differential equation, one dimension in time and another dimension is a spatial dimension. This is the boundary condition specified.

If you look into the initial and boundary condition, we need to have two boundary conditions to be specified in x direction, because it is order two with respect to x and we have only one initial condition, because with respect to t, it is order 1.

So therefore, if you look into the initial and the boundary conditions, you will see the initial condition is in non-homogeneous, the boundary conditions are also non-homogeneous. So, what do we do? We need not to in this case, we need not to break down this problem into sub problems to consider one non-homogeneity type, the way we have done in the separation of variable type of solution.

On the other hand, in this case, in a Laplace transform, we will be taking Laplace transform on both the sides; so, we take Laplace transform on both sides. So what do we do? We multiply both side by e to the power minus st del u del t dt from 0 to infinity; that is the domain of Laplace transform. We have already looked into the kernel and the limits of various transform - Laplace sine Fourier, Fourier, Cosine Fourier, Mellin and Hankle transform. So, 0 to infinity e to the power minus st del square u del x square dt;

so, this is because, basically Laplace of del u del t is equal to Laplace of del square u del x square.

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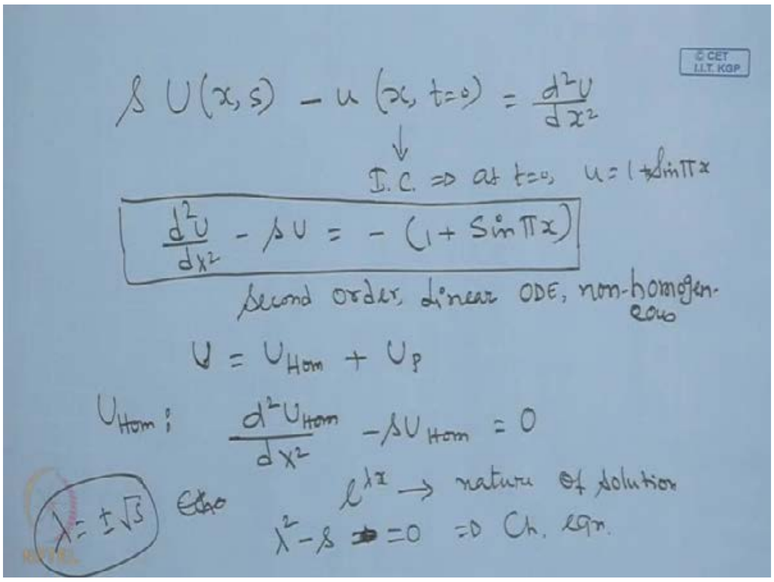

So, we have already seen how this Laplace of del u del t and how Laplace of del square del x square operator looks like in the earlier slide. So, this becomes s U x - s U, capital U basically Laplace transformed variable - u in the s domain minus u x at t is equal to 0; this initial condition is equal to d square u d x square. So, since it is no longer in t domain, this is in s domain; so, the partial derivative becomes total derivative.

Now, we know the u at x is equal to 0, it is given 1 plus sin pi plus x; so, this is the initial condition; and we know at t is equal to 0, u was given as 1 plus sin pi x. So, we rearrange this equation and see what you get, d square u dx square minus s of u is equal to minus 1 plus sin pi x. Now, if you look into this governing equation, this governing equation is second order linear ODE, but non-homogeneous.

Now, if you remember the solution technique of non-homogeneous, second order ordinary differential equation, we break down this problem into two sub problems: one is U homogeneous plus U particular integral - particular solution.

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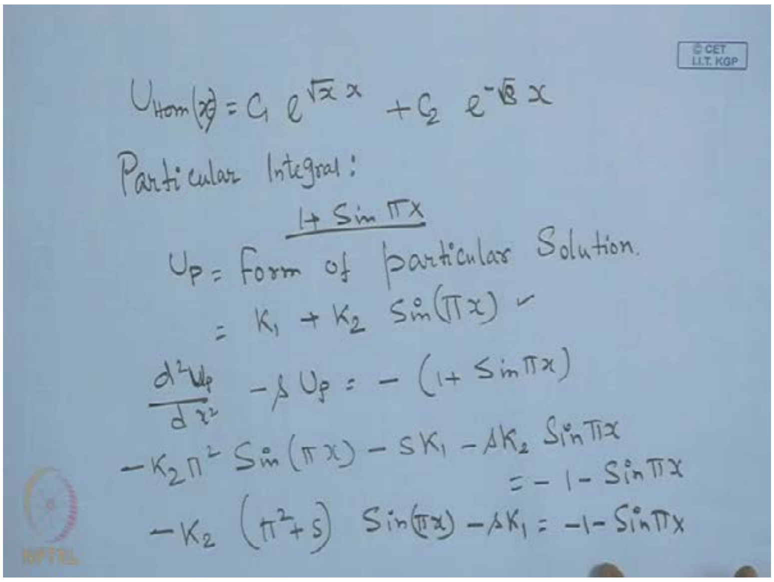

So, let us solve the homogeneous part. The homogeneous solution will be obtained, if you put the non-homogeneous term to be 0 in the governing equation; so, d square U homogeneous divided by dx square minus s of U homogeneous is equal to 0. So, you put Hom subscript to denote that it is homogeneous. So, the solution, the characteristic equation of this, so e to the power lambda x is the nature of the solution. So, lambda square minus s will be equal to 0, is the characteristic equation. So, lambda plus minus root over s is the solution. So, therefore, the homogeneous part has the solution which is composed of two terms: so this one, c 1 e to the power root over s x plus c 2 e to the power minus root over s x. So, that goes for the homogeneous solution.

Now, we are looking for the particular integral. Since the non-homogeneous term is composed of 1 sin pi x, the particular solution will be in the form, form of particular solution and this will be K 1 plus K 2 sin pi x. So, this equation must be satisfying the governing equation, So d square U p dx square minus s of U p should be equal to minus 1 plus sin pi x.

Now, this solution must be satisfying this equation (Refer Slide Time: 14.27); so, if we put it there, this becomes K 2 pi square sin pi x minus s times K 1 minus s K 2 sin pi x is equal to minus 1 minus sin pi x. So, you will be having, by rearranging, you will be having K 2 minus K 2 pi square plus s sin pi x minus s of K 1 is equal to minus 1 minus sin pi x.

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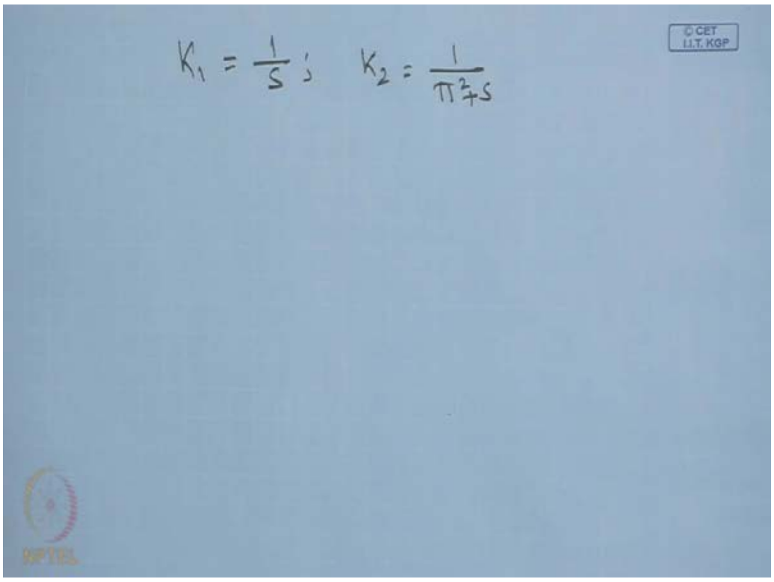

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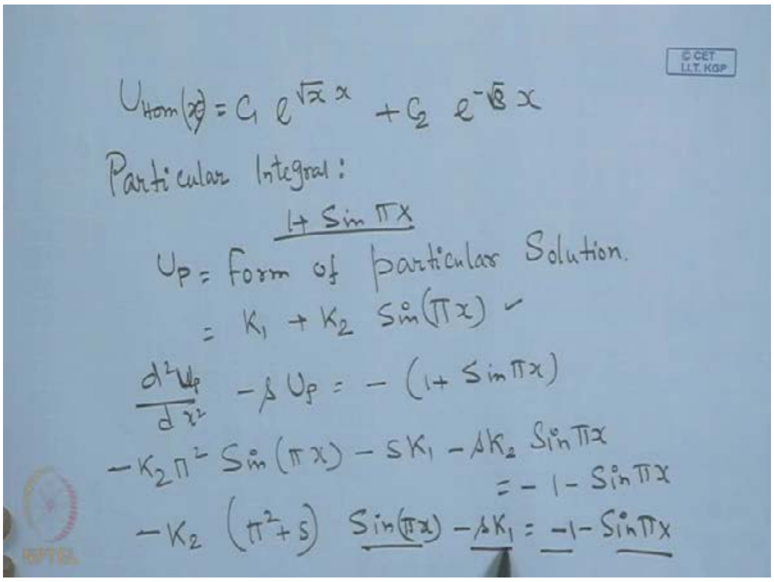

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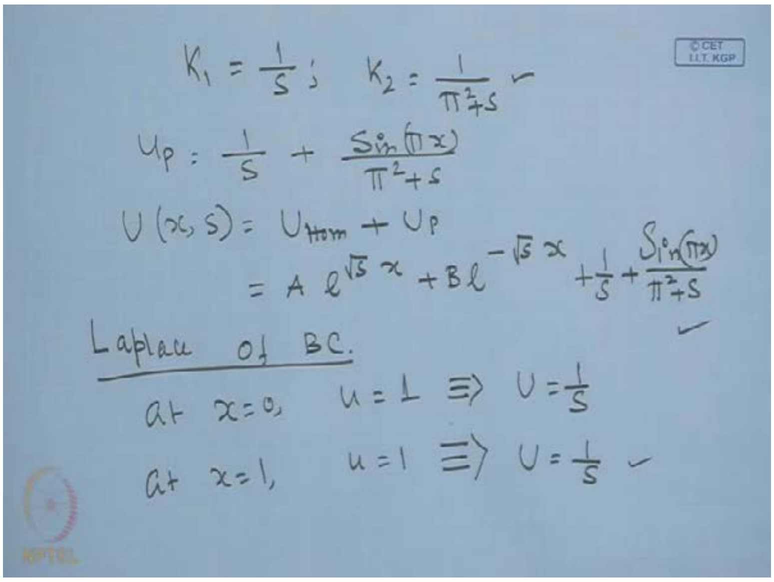

So, therefore, we will be getting K 1 is equal to 1 over s and K 2 is equal to 1 over pi square plus s. So, you just compare the term of the corresponding coefficients. So, in this equation, sin pi x, sin pi x, so K 2 pi square plus s is nothing but 1; so K 2 is equal to 1 over pi square s. And in case of K 1, s is equal to your K 1 is equal to 1 upon s, so compare the other two (Refer Slide Time: 15.56). So, we can get the constants K 1 and K 2. So,therefore, u particular will be nothing but 1 over s plus sin pi x divided by pi square plus s.

So, we obtain the total U x and s, in the s place becomes U homogeneous plus U of p is equal to A e to the power root over s x plus B e to the power minus root over s x plus 1 over s plus sin pi x divided by pi square plus s. So, that is the complete solution of the problem u in x and s space.

Now, let us look into the initial and boundary conditions and take the Laplace transform of them as well. Laplace of boundary condition - so initial condition is already utilized; now, if we look into the boundary condition at x is equal to 0, we have u is equal to 1. So, transform of 1 is nothing but 1 upon s; so, that means at x is equal to 0, this is equivalent to capital U in the s domain, which is nothing but 1 upon s. We have already seen in the beginning of this class that, if f of t is equal to 1, then Laplace transform of f is 1 upon s. At x is equal to 1, we had u is equal to 1; again for this case also, capital U is equal to 1 over s.

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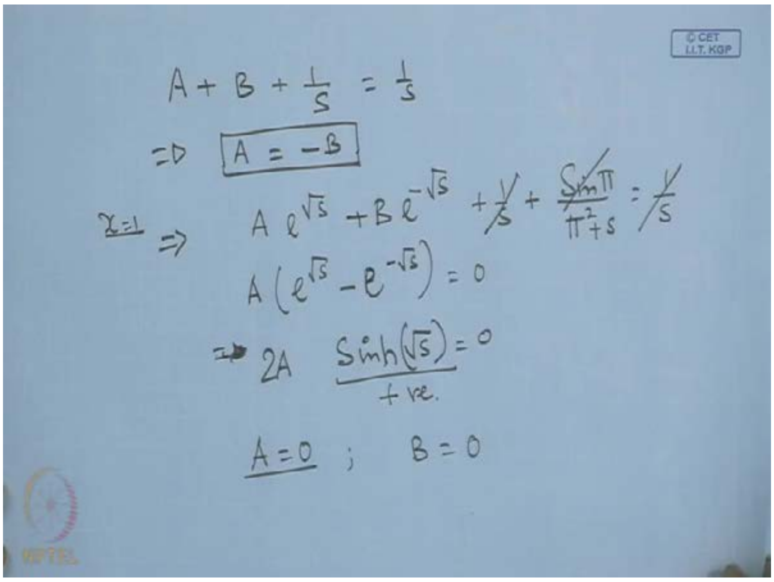

Now, we can use this boundary condition; we can put x is equal to 0 there and we can see how the coefficients of A and B varies. So, put x is equal to 0, we will be getting this; put x is equal to 1 in this equation, we will be getting this. So, if we really do that, then we will be getting A plus B plus 1 upon s - x equal to 0, sin pi x is 0 - so, we will be getting 1 upon s.

So from this, you will be getting A is equal to minus B. So, that will be one condition; then, let us put in to the other condition at x is equal to 1. So, A e to the power root over s plus B e to the power minus root over s plus 1 over s plus sin pi divided by pi square plus s should be equal to 1 upon s. So, that is the boundary condition.

When you put the boundary condition at x equal to 1, will be getting this expression; 1 upon s, 1 upon s will be cancelled out and sin 0 equal to 0, sin pi is equal to 0, so this term will be also going off; and A is equal to minus B, therefore B is equal to minus A. We can take A common, so this will be, e to the power root s minus e to the power minus root s is equal to 0. Therefore, A, this is a sine hyperbolic problem, so 2A is nothing but root over, this will be sin h root s.

Now, 2 is not is equal to 0; sin h root over s is always positive; so, this can be, in order to hold this equation, A has to be equal to 0. If A is equal to 0, then B has to be equal to 0.

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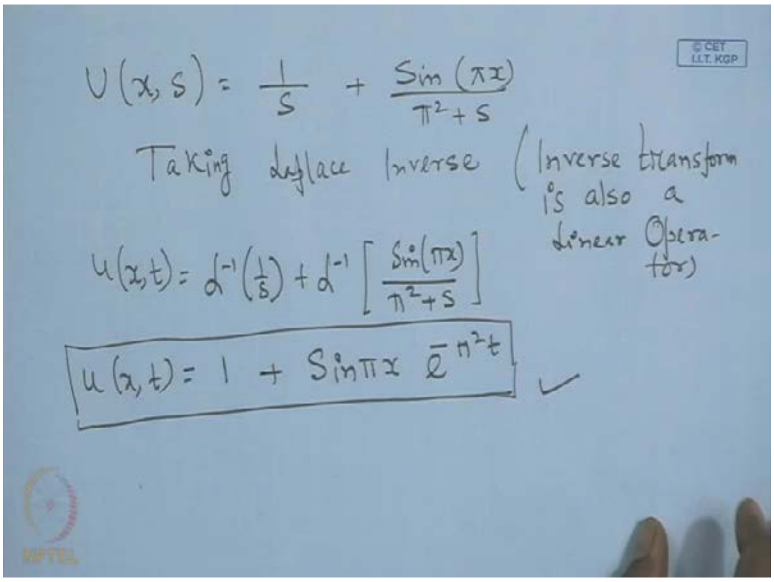

Now, therefore, let us look in to the final form of the solution of U. U as a function of x and s is equal to 1 over s plus sin pi x divided by pi square plus s. Now, we take the Laplace inverse; Laplace transform being a linear operator, the inverse transform is also a linear operator.

So, therefore u of x and t should be Laplace inverse of 1 upon s plus Laplace inverse of sin pi x divided by pi square plus s. So, Laplace inverse of 1 upon s is nothing but 1 and this will be sin pi x e to the power minus pi square t; so, u of x and t, that is the solution of u in the x and t domain;

Laplace inverse is over time domain; so, therefore inverse will be on s domain, so sin pi x will be treated as constant and it will be out of the solution and 1 over pi square plus s will be in the form of 1 over s plus a; and in t domain, this will be e to the power minus a t. So, this will be e to the power minus pi square t. So that gives you the complete solution of u as a function of x and t.

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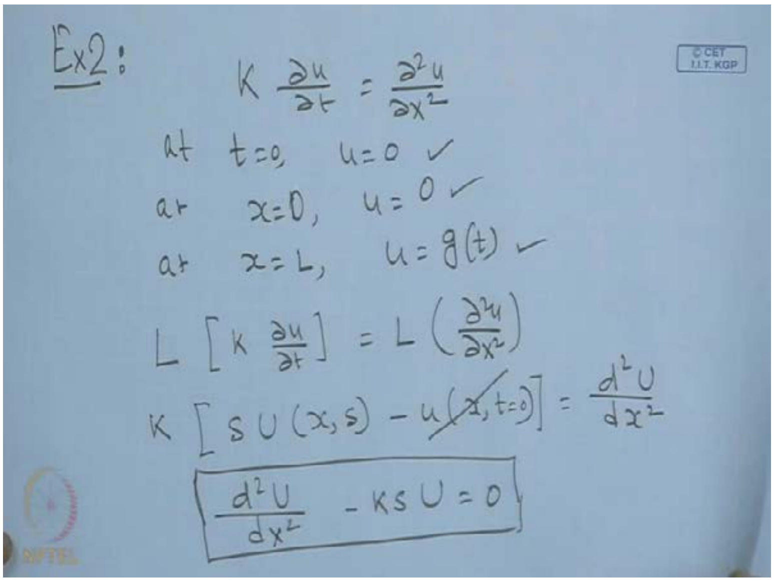

Next, we will look in to the example 2. In this example, K of del u del t is equal to del square u del x square. Again, this is a parabolic problem and at time t is equal to 0, u is equal to 0; at time x is equal to 0, we have u is equal to 0; at x is equal to L, we have u is equal to g of t.

So, this is the problem. We have the 0 initial condition and one of the boundary condition is 0 and at x equal to L, we have u as a function of g t. Now, we take the Laplace transform of this equation. Laplace of K del u del t, where K is a constant, is equal to Laplace of del square u del x square.

So, K is a constant; so then Laplace of del u del t, we have already seen, s u of x and s minus u at x at time t is equal to 0; that means, it is an initial condition. This will be nothing but d square u dx square.

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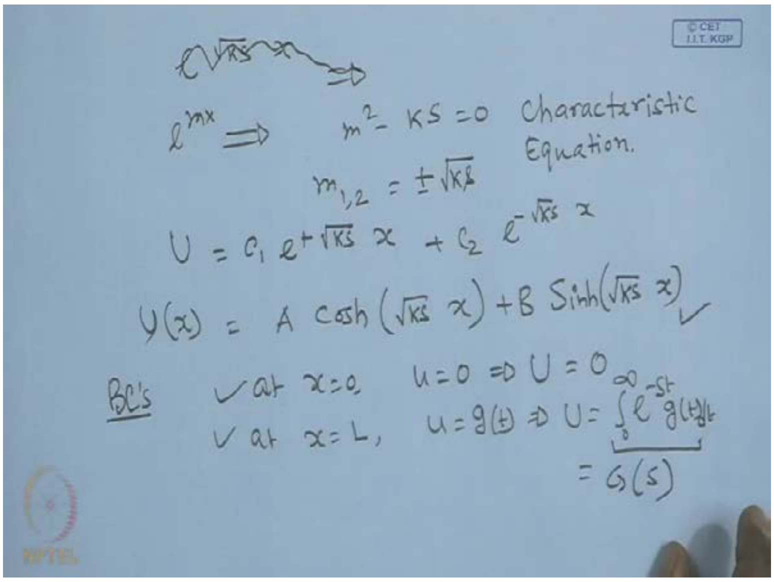

Now, u at x, in it x at time t is equal to 0, we had this initial condition u is equal to 0; this has to be satisfied. So, d square U dx square minus KS U is equal to 0. In this particular problem, this is the governing equation. And if you look this equation is a homogeneous second order ordinary differential equation and we know the solution. The solution will be composed of, the form of the solution will be root over KS x because the characteristic and then we will be e the power so form of the solution will be e to the power mx and then we will be getting m square minus KS is equal to 0 as the characteristic equation.

So, m 1, 2 are the two roots plus minus root over KS; therefore the roots, the solution of this problem will be composed of c 1 e to the power root over KS x plus c 2 e to the power minus root over KS x. The solution will be composed of sine hyperbolic and cosine hyperbolic function.

So therefore, U will be, the final form of solution will be A cos hyperbolic root over KS x plus B sine hyperbolic root over KS times x. The boundary conditions: let us have a transform of boundary condition: at x is equal to 0, my u was equal to 0, therefore capital U is equal to 0; at x is equal to L, u is equal to g of t, so therefore your capital U should be 0 to infinity e to the power minus St g of t dt. So, therefore, we write it in the form of G of s; so, let us write u is equal to G of s.

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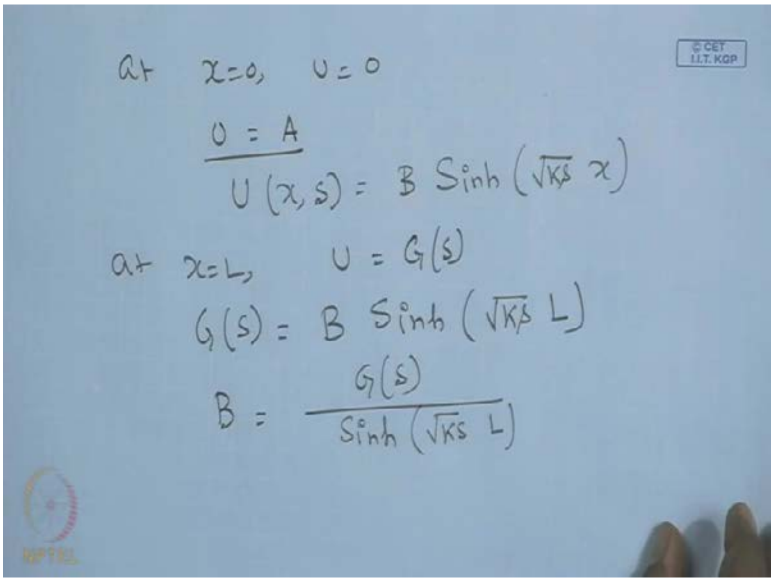

Now, using these two boundary conditions, we can solve the coefficients A and B in the solution. So, if you put these two boundary conditions, we put the first boundary: at x is equal to 0, u is equal to 0; therefore 0 is equal to A. So, that is the solution. So, U as a function of x and s now becomes B times sine hyperbolic root over Ks times x.

Now, we put the other boundary condition. The other boundary condition is at x is equal to L, u is equal to G of S. Therefore, G of s is nothing but B sine hyperbolic root over Ks times L. So, therefore, you will be getting B as G of s divided by sin hyperbolic root over KS times L.

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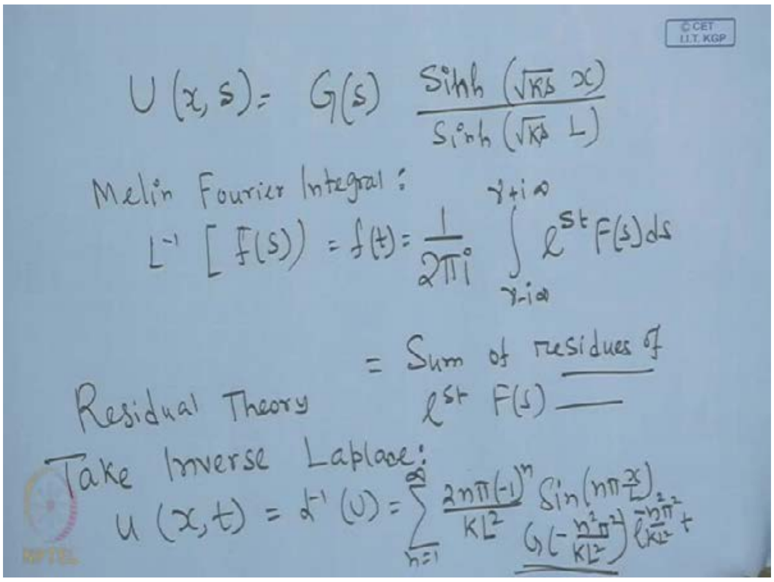

Now, let us look into the complete solution of U. The complete solution of U will be capital U as a function of x and s will be nothing but G of s sine hyperbolic root over Ks times x divided by sin hyperbolic root over Ks times L; then, we can use the Melin's Melin Fourier integral like Laplace inverse of, f of s is nothing but f of t and this f of 2 will be nothing but 2 over 1 over 2 pi i gamma i minus i infinity gamma plus i infinity e to the power St F of s ds and this will be nothing but sum of residues of e to the power St F of s.

So, use the residual theory, one can find out the roots and find out the residues at the singular points. So, you can take the Laplace, you can take the inverse Laplace and we can get u as a function of x and t as Laplace inverse of U. And that will be summation of n is equal to 1 to infinity e to the power 2 n pi minus 1 raised to the power n divided by KL square sin n pi x by L G of minus n square pi square over KL square e to the power minus n square pi square over KL square times t.

So, this gives the complete solution in the actual u and x t by taking the Laplace inverse; only thing is the functional form of G, if this would have been known, then we could have got the complete expression of G.

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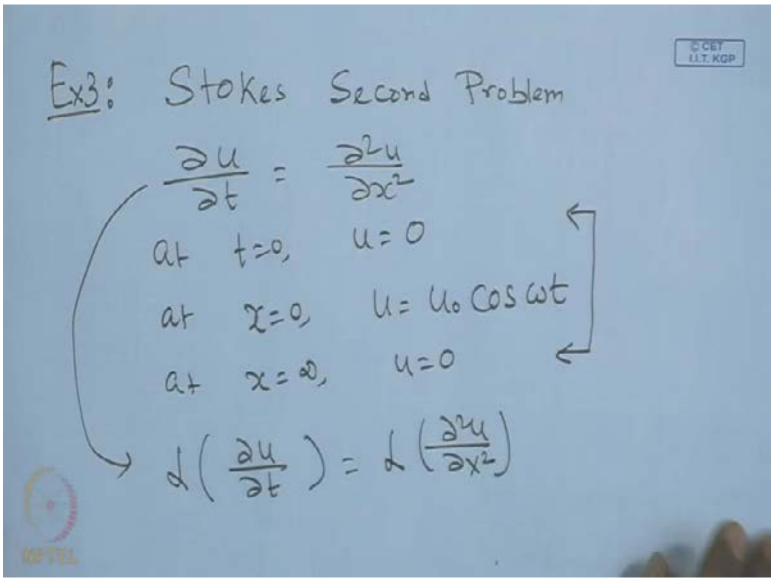

Next, we look in to, we take up one more example in the Laplace transform; that is, exercise 3, example 3; this problem is Stokes second problem; if we remember that we have already talked about Stokes first problem; the Stokes first problem was if there was a stationary fluid and we put a plate, then at time t is equal to 0, this plate starts moving in the forward x direction with a uniform velocity u naught; then the fluid particle adjacent to this plate will be having the velocity u naught, as you go up in the y direction the fluid, because of the viscous effects, the fluid particle, the velocity of the fluid particles will keep on reducing and beyond a particular point in s, in y direction, the fluid particle does not experience the presence of the moving plate at y is equal to 0.

Therefore, the velocity will be equal to 0 at y is equal to infinity, that will be a typical boundary condition and we would like to find out the velocity profile within the boundary layer and typically this profile is a function of y and time.

We have solved this problem completely by using the similarity transformation method. Now, in this part, we will be taking up the Stokes second problem. What is Stokes second problem? Stokes second problem is that the same; the problem statement remains the same; we have a stationary liquid and then at we place a plate at time t is equal to 0 at t is equal to 0 plus, this plate starts executing a simply harmonic motion a an oscillatory motion given by u is equal to u naught sine omega t or u naught cosine omega t.

So, what is the difference between the first problem and second problem? In the Stokes first problem, the plates starts moving with uniform velocity in the forward x direction with a velocity u naught, but in the second problem it is starts executing a simple harmonic motion.

So, we would like to find out what will be the velocity of the velocity profile within the boundary layer. So, the governing equation is del u del t is equal to del square u del x square; that is the problem, that the governing equation at t is equal to 0 the u was is equal to 0 at x is equal to 0, the plates starts executing a simple harmonic motion, let us say oscillatory motion; let us say u is equal to u 0 cosine omega t and at x is equal to infinity u was equal to 0.

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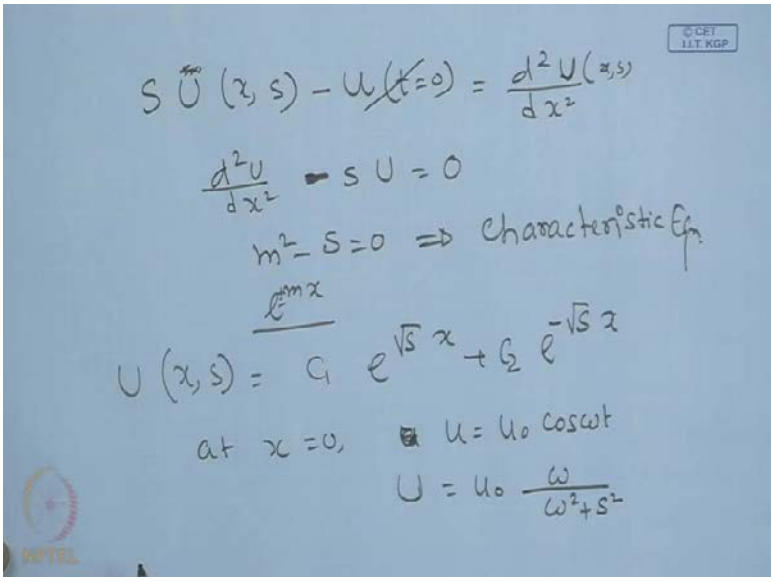

So, this formulates the initial and boundary condition of this governing equation; then, what we do, we take the Laplace transform of the governing equation; so Laplace of del u del t is equal to Laplace of del square u del x square; so what we will be u getting is that S u bar x s minus u at t is equal to 0; this is equal to the on the right hand side d square u d x square, where u is nothing but x and s, so you need to write bar; either you write bar or you write capital U to represent the transform variable; so and we have already seen, that at time t is equal to 0 u is equal to 0 so d square u dx square becomes s U so or this minus this thing will be equal to 0.

So, we have already seen the solution of this the m square minus s will be the characteristic equation and e to the power plus minus mx will be constituting the solution; therefore, the solution will be constituted by U as a function of x and s will be nothing but c 1 e to the power root over s x plus c 2 e to the power minus root over s x.

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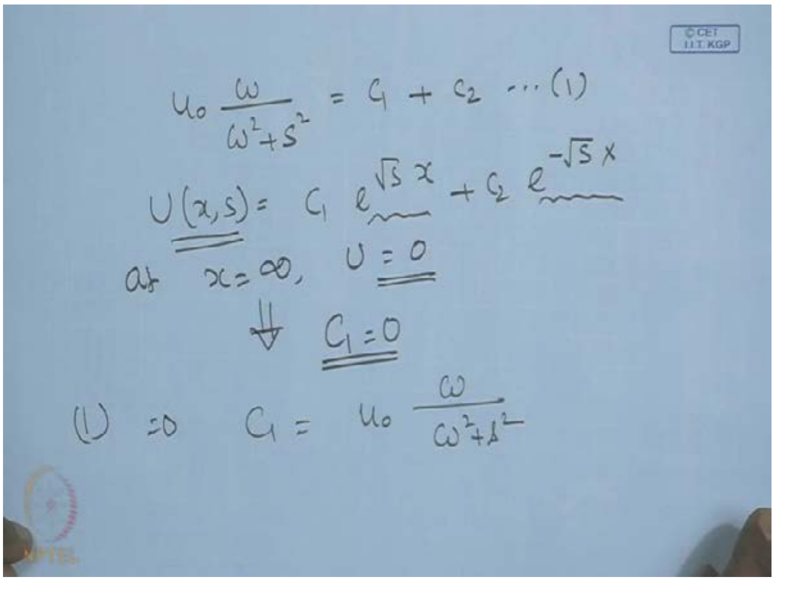

Now, at x is equal to 0, we have u is equal to u naught cosine omega t; therefore, at in the Laplace transform domain capital U becomes u naught omega divided by omega square plus s square; so this will be the Laplace transform domain; so by if you put this equation here, then, put x is equal to 0; so this becomes u naught omega divided by omega square plus s square is equal to c 1 e to the power root over s x will be c 1 is equal to c 1 plus c 2 e to the power c 1 plus c 2; then, we can have c 1 is equal to 0. Now, if we put the other boundary condition that are x is equal to infinity, so this will be equation number 1; so, when you put x is equal 1, so just look into the other boundary condition; the solution is u x and s is c 1 e to the power root over s x plus c 2 e to the power minus root over s x.

Now, if we put the other boundary condition, that is, at x is equal to infinity, U is equal to 0; that means, we the U becomes 0 it is finite, but if you put infinity, then, this term becomes 0, but this term blows off, it goes to infinity; so in that case, because of the presence of the first term on the right hand side, the problem becomes instead of 0 it becomes infinitely large; so, that means, this is a point where the solution will blow off at x is equal to infinity. In order to avoid that, the solution will be constituting must with the solution, that will be constituting the final form must be having c 1 is equal to 0 if c 1 is equal to 0; then, the solution will be having a finite solution, when we write the boundary condition x is equal to infinity U becomes finite or 0; so, therefore, c 1 becomes 0 from the other boundary condition; so from 1, we will be getting c 1 is nothing but u 0 omega omega square plus s square.

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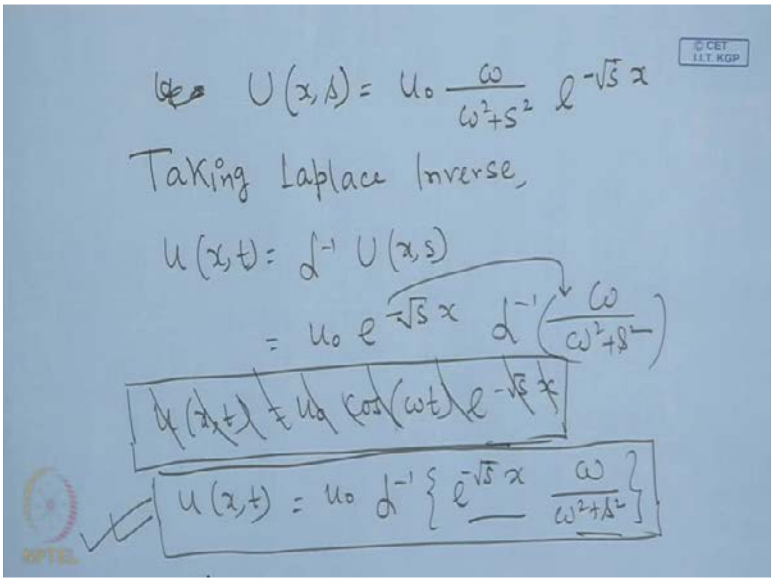

So, we write down the Laplace transform. So the now, the solution becomes U as x and s domain, this becomes u naught omega omega square plus s square e to the power minus root over s x, because e to the power plus root over s x becomes infinitely large.

So, therefore, you take the Laplace transform, taking Laplace inverse you take inverse Laplace. So, this becomes u x and t becomes Laplace inverse of U function of x and s; so the inverse is on t domain so u 0 e to the power minus root s x Laplace inverse of omega omega square plus s square, we know the solution the solution is cosine a t.

So, u 0 cosine omega t e to the power minus root s x is the solution; no, this is a, there is s also; so this is a combined inverse of the 2; so u of x t should be u naught Laplace inverse; this is also incorrect; this should be getting into it; so Laplace inverse of e to the power of minus root s x times omega omega square plus s square.

Now, if you know a proper Laplace inverse of this function, one can get the u as a function of x and t, so that gives the complete solution. I am just putting, keeping this Laplace inverse, to evaluate the Laplace inverse on the readers, are the students to evaluate this inverse transform of Laplace inverse transform of this function e to the power minus root s x omega divided by omega square plus s square that will be giving you the complete form of u as a function of x and t. So, Laplace transform.

So, this completes the demonstration of Laplace transform and application of Laplace transform in the solution of partial differential equations in various chemical engineering applications. Now, since the time domain becomes very important when we are talking about the transient chemical engineering problem, for example, transient heat conduction, for example, transient mass transport equations or transient momentum transfer equations, those equations becomes very important some times, because all these chemical engineering systems, there exists a transients before they reach a steady state, as I said earlier. We have discussed earlier problem, in the beginning of this course, that steady state of any chemical engineering process is extremely important, because that dictates the product quality of a plant.

So, if we would like to maintain a constant product quality of a plant, then the transients, the maintenance of steady state becomes important, but whenever you start up a problem the steady state cannot be attend instantaneously; depending on the kinetics dynamics of the system, dynamics of the system and the system parameters, the transients may be existing for few seconds, for up to few minutes or up to few hours. So, if you are talking about a slow filtration process, the transients may be existing up to several hours or days before reaching to the steady state.

If we talk about a quick response method in the system, the transients may be existing for few seconds. So, transient analysis becomes very important in most of the chemical engineering application, but looking into the system variables and doing a some order of magnitude analysis, one can really identify and one can readily identify, whether the transients will be existing for few seconds or minutes or few hours or few days.

If transient analysis becomes very important, few hours or few days, then the transient analysis of extremely important for the fresher's engineers, then you have to do the mathematical modeling using all the known techniques.

Now, most of the transient chemical engineering processes will be multidimensional chemical engineering processes, will be transient in our time dependent and there it will be also special dependent, but in under normalized and simplified conditions, it is generally boil down to a two-dimensional problem - transients in time and a special variation in space in one dimension. So, you talk about a two- dimensional or at most a three-dimensional problem; in such cases, we are landing with the parabolic equations; on the parabolic equations we will be having the form L u is equal to f or L u is equal to 0, for if the system is non-homogeneous, then you will be getting L u is equal to f, if the system is homogeneous, you will be getting L u is equal to 0, the operator L becomes del del t minus del square del x square; so, it is also called a parabolic operator.

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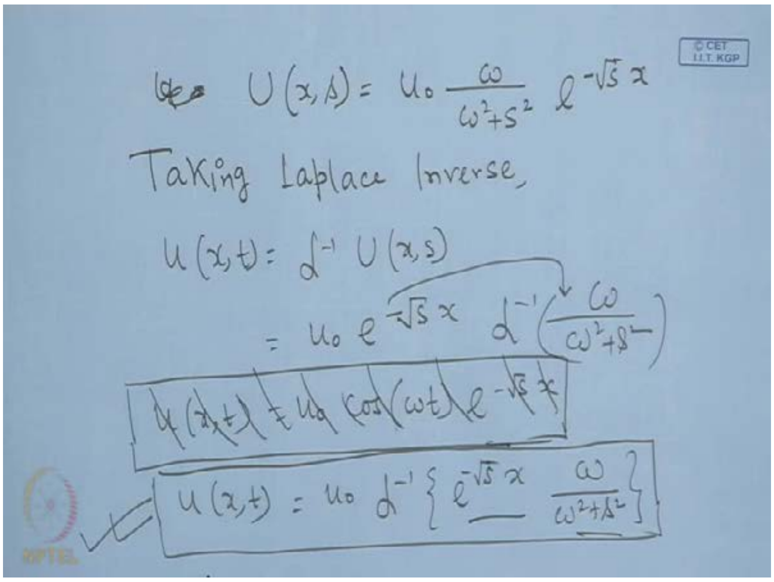

Now, in case of parabolic operator, one can safely use the Laplace transform, because Laplace transform is generally in t domain and the integration of the domain of validity is from 0 to infinity and the kernel is equal to e to the power minus St; that you have already seen earlier and by doing the Laplace transform, one can get if they governing equation is linear, one can use the Laplace transform and by doing a Laplace applying Laplace transform, one can guess, one can reduce the order of the problem; from partial differential equation, it will be reduce to ordinary differential equation.

The ordinary differential equation being easier to solve, it can be solved almost analytically if it is a linear problem and then constants of integration will be evaluated by the boundary conditions; the to apply the boundary condition, we have to use, the take, the Laplace transform on the boundary conditions as well and then convert the boundary conditions in the transformed s domain.

So, using those boundary conditions, one can evaluate the coefficients of integration A and B; once you get the coefficient of integration A and B, then you can constitute the complete solution in x and s domain, then we have to take the Laplace inverse and the whole problem will be then converted in by taking the inverse transformation; it can be converted into x and t domain that will give you the complete solution for Laplace transform.

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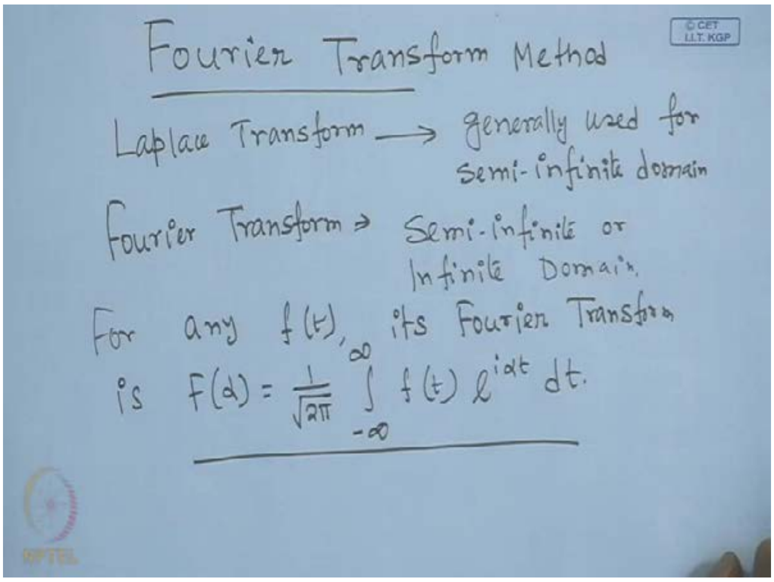

Then, we will be looking into quite common a transform - mathematics in applied to chemical engineering methods; these are called Fourier transformation methods; Fourier transform method, generally, they are used for the semi-infinite or infinite domains. Laplace transform is generally used for semi-infinite domain; this Fourier transform is utilized for semi-infinite or infinite domain; so for any transformed f t for any function f of t it is Fourier transform is defined as is f of alpha is equal to 1 over root over 2 pi minus infinity to plus infinity f of t e to the power i alpha t dt.

So, this is the form of Fourier transform, given any function f of t; so it will be multiplied by e to the power i alpha t dt; it will be multiplied by 1 over root over 2 pi e to the power i alpha t dt and domain of integration is from minus infinity to plus infinity.

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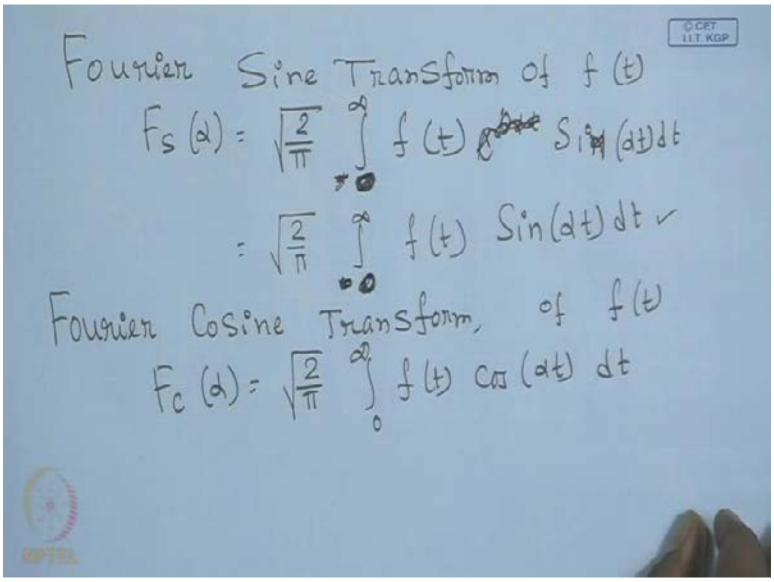

Then, we define Fourier sine transformation and Fourier cosine transform; Fourier sine transform of f t, this becomes F s, that in domain alpha, this becomes root over 2 over pi minus infinity to plus infinity f of t e to the power i alpha t; no, it will I, alpha t will be replaced by the sin part; so this will be f t sin alpha t dt. So, this is called root over.

So, let me write it more clearly, minus infinity to plus infinity f of t sin alpha t dt and Fourier cosine transform of some function f of t becomes F c alpha is equal to root over 2 over pi 0 2. So, this will be again, this is a problem, this will be the integration is from 0 to infinity; so, this will 1 domain, it is not from minus infinity to plus infinity, this from 0 to infinity; so, again, here it is from 0 to infinity f of t cosine alpha t dt.

So, these are definition of Fourier transform, Fourier sine transform, Fourier cosine transform. If you just look into the definition, the Fourier transform, the domain is minus infinity to plus infinity kernel is e to the power i alpha t 1 over root over 2 pi; the kernel is, 1 over root over 2 pi e to the power i alpha t; in case of Fourier sine transformation, the limit is from 0 to infinity and the kernel is root over 2 by pi sin alpha t. And in case of cosine transformation, the limit is from 0 to infinity and the kernel is root over 2 divided by pi.

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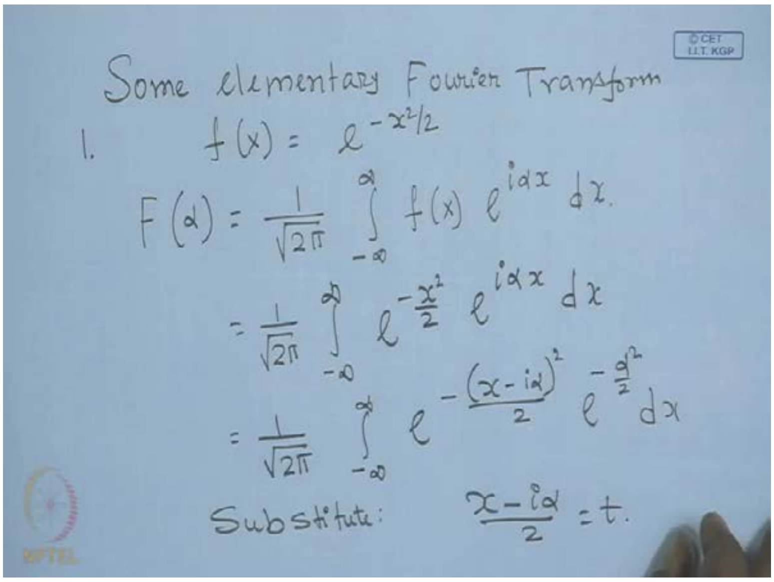

So, some elementary Fourier transform may be of interest to us and we should to know some elementary forms; so number 1 is if f of x is e to the power minus x square by 2. Then, what will be the corresponding form of elementary Fourier transform? This will be 1 over root over 2 pi minus infinity to plus infinity f of x e to the power i alpha x dx; so, this is the form of if the Fourier transform in alpha domain.

So, this becomes 1 over root over 2 pi minus infinity to plus infinity f of x is e to the power minus x square by 2 e to the power i alpha x dx. So, you will be having 1 over root over 2 pi minus infinity to plus infinity e to the power minus x minus i alpha square by 2 e to the power minus alpha square by 2 x. We can write these two terms in the form of this; so, then, we substitute x minus i alpha by 2 is equal to t.

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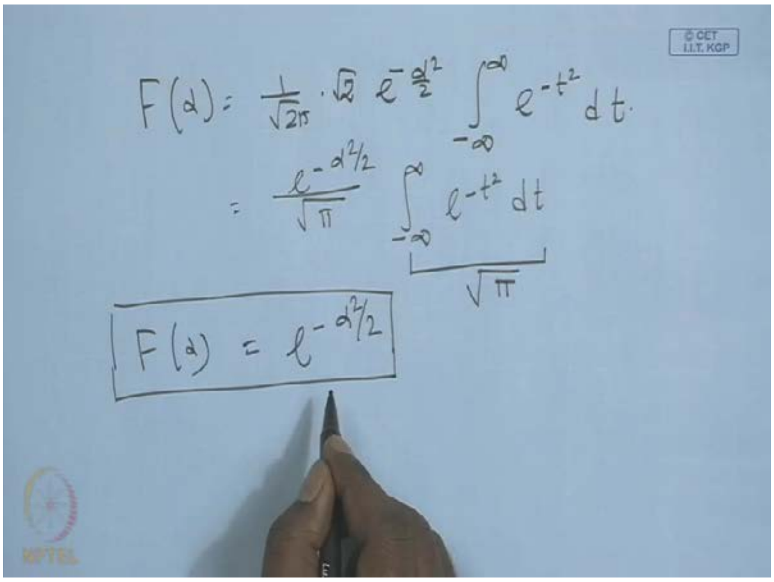

So, F of alpha, now becomes 1 over root over 2 pi root 2 e to the power minus alpha square by 2 minus infinity to plus infinity e to the power minus t square dt; so this becomes, e to the power minus alpha square by 2 divided by root over pi minus infinity to plus infinity e to the power minus t square dt and what is minus infinity to plus infinity e to the power minus t square dt? This becomes root over pi; therefore, root over pi root over pi will be cancelled out; so it will be nothing but F of alpha is nothing but e to the power minus alpha square by 2.

So that is the Fourier transform of e the power minus x square by 2. So we will look into some of the common functions and see how the Fourier transforms of these functions are evaluated.

So I stop in this class here, at this point. I will take up in the next class, the Fourier transformation method and see how the Fourier transform can be utilized for solving the various chemical engineering problem. And then, we will wind up the course and we will do a summary of whatever we have done, and how things can be solved for chemical engineering problems and how the mathematics can be used as a tool for solving actual chemical engineering problem in real life.

Thank you, very much.