Advanced Mathematical Techniques in Chemical Engineering
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Module No. # 01
Lecture No. # 24
Solution of Parabolic PDE: Separation of Variables Method

Welcome to this session of this particular class. So, in the last class, we are looking into the solution of the basic problem, in when the boundary condition is a Robin mixed boundary conditions.

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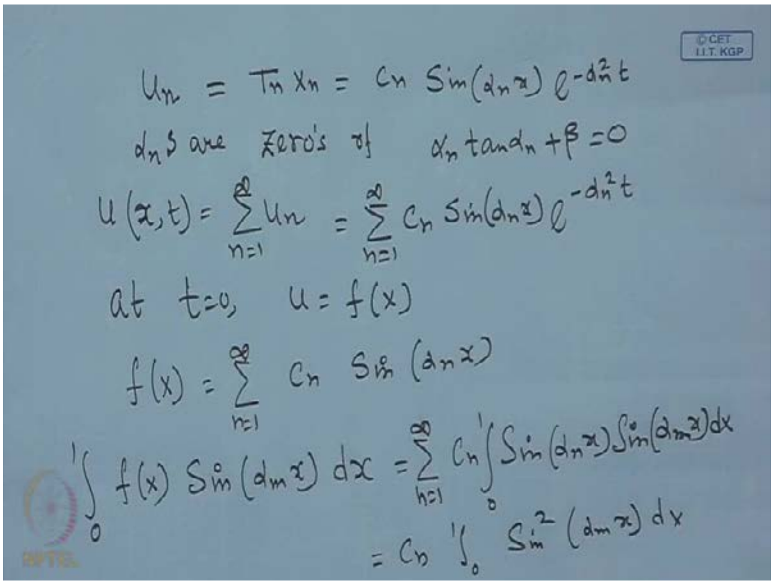

And we have solved, up to the value of u n, so what the solution corresponding to nth eigenvalue. So, this becomes T n times X n is equal to C n sine alpha n x e to the power minus alpha n square t. And if you remember, the alpha n's are the roots or zeros of transcendental equation alpha n tan alpha n plus beta is equal to 0.

Now, in this class, we will complete this problem. So, the complete solution of this problem is a linear superposition of all the eigen solution, so, therefore, u x t is nothing but summation of u n, where n is equal to 1 to infinity, therefore we will be getting summation n is equal to 1 to infinity C n sine alpha n x e to the power minus alpha n square t. Now, we have to evaluate this constant C n and for that we have to utilize the unused initial condition that at t is equal to 0, u is equal to sum function of x.

So, f of x is nothing but n is equal to 1 to infinity C n sine alpha n x. And we have already seen earlier that the eigenfunctions are orthogonal to each other, so to evaluate C n and to break open this summation, we multiply both side by sine alpha m x dx and integrate over the domain of x, that is from 0 to 1 f of x sine alpha m x dx its integration n is equal to 1 to infinity C n sine alpha n x sine alpha m x dx. Therefore, if you open up this summations series, will all the other terms will be 0, except the one term will be, where m is equal to n.

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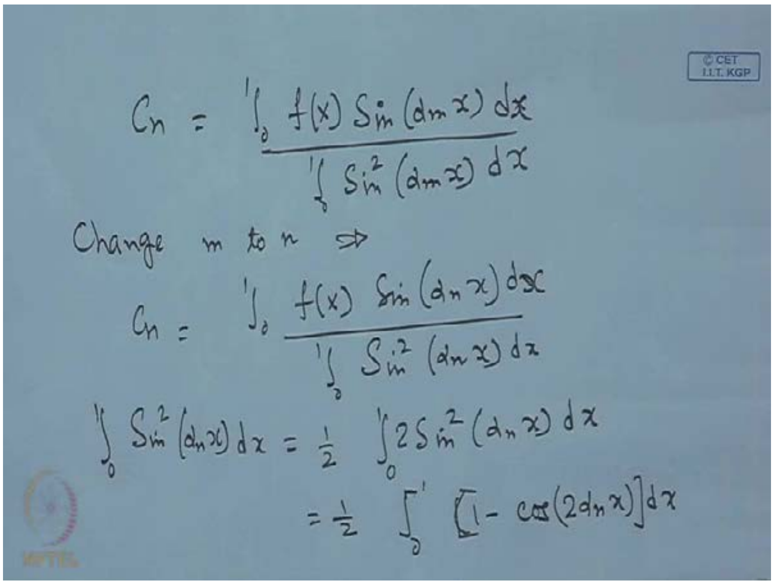

Because of the orthogonal property, the sine functions will be orthogonal to each other for m is not equal to n, 0 to 1 integral sine alpha n x, alpha m x equal to 0. Only one term will survive, that will be is equal to m when n is equal to m, so 0 to 1 sine square alpha m x dx. Now, let us solve this problem, so what is C n? C n is nothing but 0 to 1 f of x sine alpha m x dx divided by 0 to 1 sine square alpha m x dx. So, what we do? We now change the running index m to n, so C n becomes 0 to 1 f of x sine alpha n x dx plus 0 to 1 sine square alpha n x dx.

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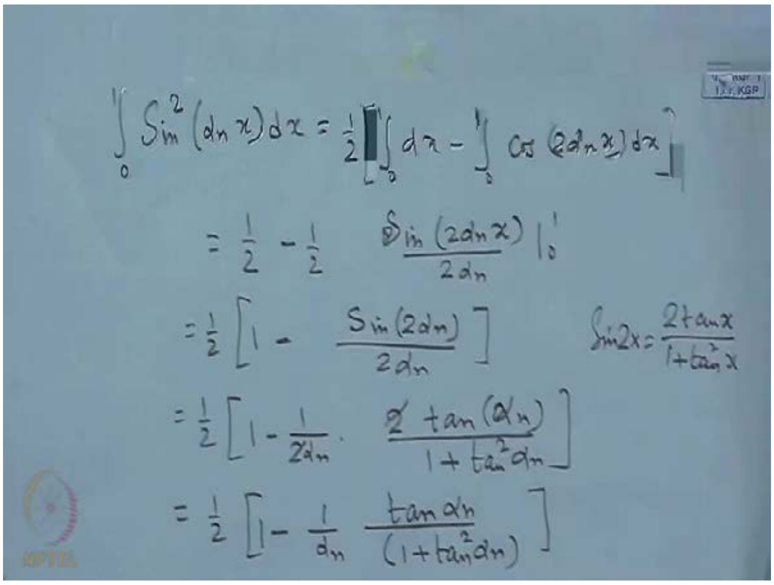

Now, let us look into the new denominator and evaluate this integral 0 to 1 sine square alpha n x dx for this particular problem. We multiply both side by 2, the numerator and denominator this becomes sine square alpha n x dx, so 0 to 1 2 sine square alpha n x dx is nothing but 1 minus cos 2 alpha n x. So, this is nothing but 0 to 1 1 minus cosine 2 alpha n x dx. So, we carry out this integral, so integration 0 to 1 sine square alpha n x dx is nothing but half 0 to 1 dx minus 0 to 1 cosine 2 alpha n x dx, so this becomes half, minus half is there, now it will be 0 to 1 cosine 2 alpha n x dx, will be nothing but sine 2 alpha n x divided by 2 alpha n from 0 to 1.

So, let us see what we get, half 1 minus sine 2 alpha n x, so it is becomes sine 2 alpha n divided by 2 alpha n minus sine 0, that will be equal to 0. Now, we break down this into tan alpha n, so this becomes half 1 minus 1 by 2 alpha n, sine 2 x is nothing but 2 tan x divided by 1 plus tan square x, using this identity, you put it here, so this becomes 2 tan alpha n divided by 1 plus tan square alpha n, so 2 2 will be cancelling out. So, if you remember, so this becomes half 1 minus 1 over alpha n divided by 1 plus tan square alpha n and in the numerator will be having tan alpha n.

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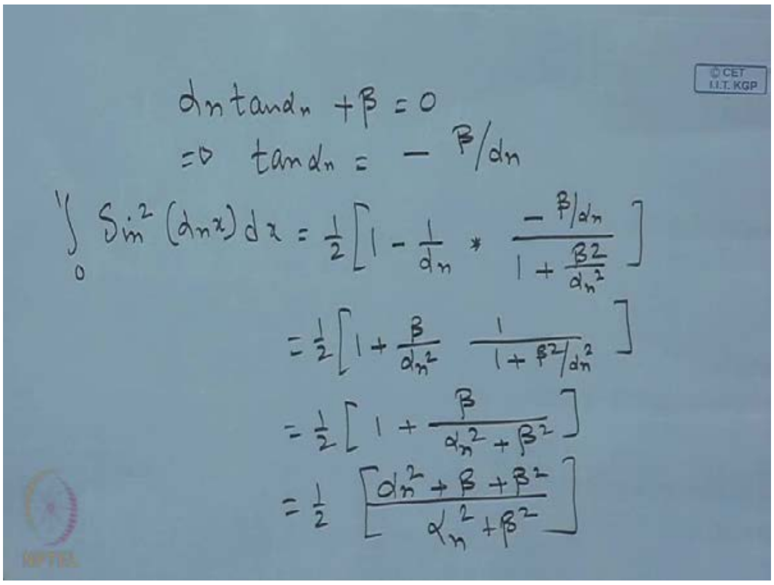

Now, if you remember the transcendental equation, we were getting was alpha n tan alpha n plus beta should be equal to 0. So, we can get expression of tan alpha n from here, beta minus beta by alpha n, so we are going to substitute it there, so integral 0 to 1 sine square alpha n x dx is equal to half 1 minus 1 over alpha n, tan alpha n is minus beta by alpha n divided by 1 plus tan square alpha n is beta square by alpha n square.

So, what we get is that half 1, minus minus plus, so this becomes beta by alpha n square and this becomes 1 plus beta square by alpha n square. So, multiply by alpha n square in the denominator, so 1 plus beta plus alpha n square plus this becomes beta square, so will be having half alpha n square plus beta square is equal to alpha n square plus beta plus beta square, so that is the evaluation of the integral sine square alpha n x.

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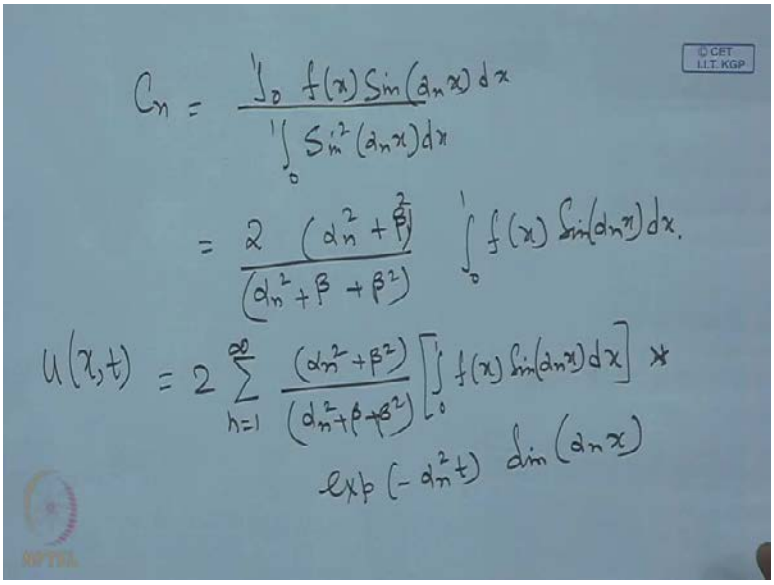

So, therefore, if we put into the so, now, we will be in a position to calculate the value of C n, C n is nothing but integral 0 to 1 sine square alpha n x dx and in numerator we had 0 to 1 f of x sine alpha n x dx. Therefore, we just evaluated the sine square alpha n x and so we just put it there, so 2 half will becomes 2 alpha n square plus beta square divided by alpha n square plus beta plus beta square and this will be 0 to 1 f of x sine alpha n x dx.

So, therefore, will be in a position to construct the complete solution, the complete solution will be u as a function of x and t is summation of C n, so C n we just put it these, so this becomes 2, summation n is equal to 1 to infinity, so this will be alpha n square plus beta square divided by alpha n square plus beta plus beta square integral 0 to 1 f of x sine alpha n x dx, this is within the third bracket multiplied by exponential minus alpha n square t sine alpha n x. So, that gives the complete solution and alpha n's are the root of the transcendental equation alpha n tan alpha n plus beta is equal to 0.

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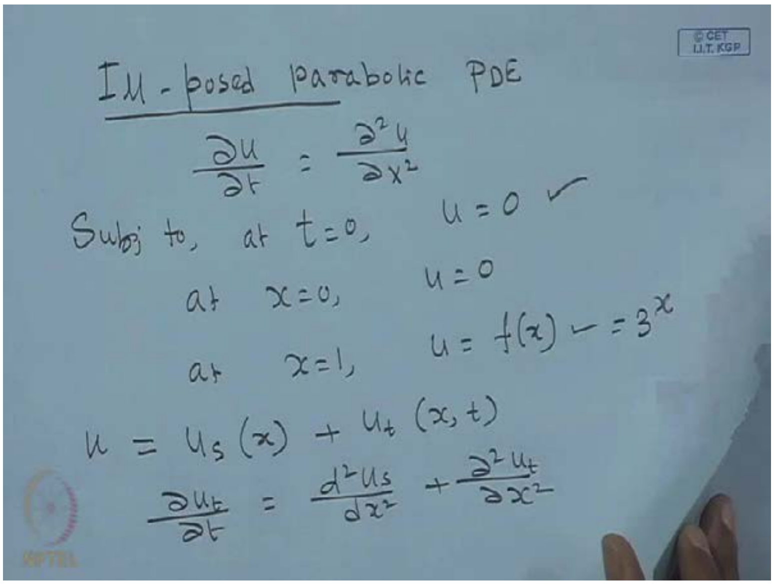

So, this gives the complete solution for the basic parabolic partial differential equation with a Robin mixed boundary condition. Next, will be looking into the ill-posed problem, ill-posed parabolic PDE and that becomes very important when we will be looking into the solution of an actual problem. So, the ill-posed problem will be something like this, let us look into the so in the in the Dirichlet boundary condition. One of the boundary condition is non-homogeneous and the initial condition is homogeneous. So, this becomes del u del t is equal to del square u del x square, subject to these set of boundary conditions and initial conditions at t is equal to 0, u was equal to 0 and at x is equal to 0, we had u is equal to 0 and at x is equal to 1, we had u is equal to f of x.

So, the initial condition is homogeneous and one of the boundary condition is non-homogeneous. So, if you remember, we looked into this problem earlier as well, in a generic case we considered and this is specific case we are taking. And for the in a time being, you just take a value of f x, then that will demonstrate the problem more appropriately let say f x is 3x.

So, what we did in the earlier classes if you remember, that we broke down this problem into two sub problems, one is the steady state part, another is the transient part. So, if you really do that, so this problem has been divided into two sub problem, one is the steady state part, which is a function of x alone and another is the transient part, which is a function of x and t both. Now, next what we do? We formulate the governing equation of u s and u t and set the appropriate initial and boundary condition.

So, first we formulate the governing equation of u s and u t, for that what we do? We substitute this in the mother problem, that is del u del t is equal to del square u del x square. If you do that, so this will be giving you, if you put u is equal to u s plus u t, you will be getting del u t del t, there will be no u s, because u s is a sole function of space, so this becomes d square u s dx square plus del square u t del x square. If you put u is equal to u s plus u t, the u s is space varying part only, so therefore the partial derivative becomes are total derivative, on the other hand, in case of u s, in case of u t, it remains a partial derivate, then what we do, we collect the similar terms and get the governing equation of u s and u t, this is possible because the operator is a linear operator.

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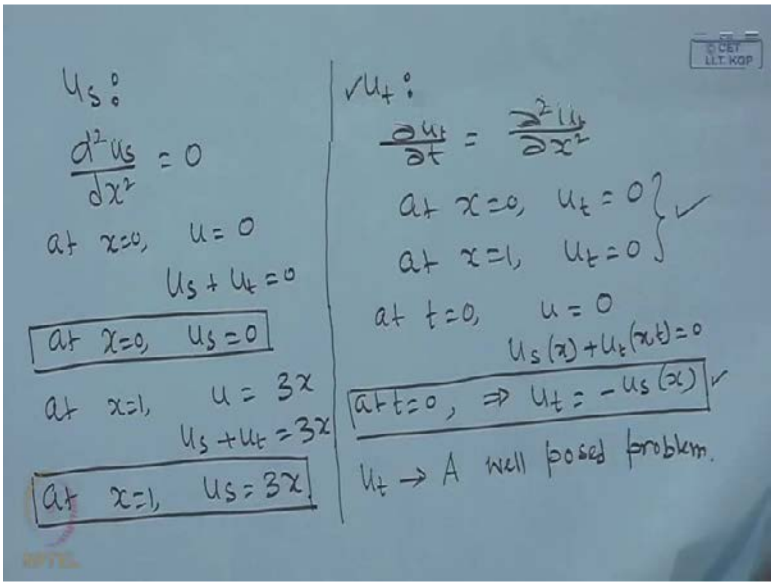

So, therefore, what we do next is that, we collect the similar terms and get the governing equation of u s and u t. So, we formulate the governing equation of u s and formulate the governing equation of u t, so d square u s dx square is equal to 0 and del u t del t is equal to del square u t del x square that was the governing equation of u t.

So, now, we set the boundary conditions, if you look into the boundary condition of the original problem, do not require an initial condition in the space varying part, because we require to have the boundary condition only. So, if you look into the boundary condition, the original problem that is at x is equal to 0, u was is equal to 0, if you put u is equal to u s plus u t, then each of them will be individually 0, therefore at x is equal to 0, u s is equal to 0 and at x is equal to 0, you can get u t is equal 0.

Now, we put the other boundary at x is equal to 1, u is equal to 3x, so you put u s plus u t is equal to 3x. Now, a fundamental comes here, we associate the non-homogeneous part with the steady state part, so that we will enforce the transient part to be homogeneous. If you do that see what you get? At x is equal to 1 will be getting u s is equal to 3x and at x is equal to 1 will be getting u t is equal to 0. So, what we did here, we associated the non-homogeneous term with the steady state solution and we associated so the u t becomes a homogeneous. Why we did that? We did that intentionally, because the we would like to have a standard eigenvalue problem or a well posed problem in the u t form.

So, therefore, we in this processes, we made the boundary conditions of u t to be homogeneous, so the boundary condition u s is, this is the first boundary condition and boundary condition on u s, this is the second boundary condition. And in case of u t, we have fixed up the boundary conditions and they turned out to be homogenous in this case. Now, in the next, we find out what is the initial condition of this problem. So, let us go back to the initial condition of the mother problem, the mother problem had u is equal to 0.

So, we put u is equal to u s, which is a function of x and u t, which is a function of x and t both, therefore at t is equal to 0, u t becomes minus of u s x, therefore this is the initial condition for the time varying part. And see what we have obtained in this case, we completely defined the steady state part, so will be able to get a complete solution of the steady state part u s and we are going to get it within a couple of minutes. And then after by doing this transformation by doing this modification or mathematical manipulation, what we have done that, we have defined u t and this u t is having a non-homogeneous initial condition and homogeneous boundary condition, so we converted u t as a well posed problem.

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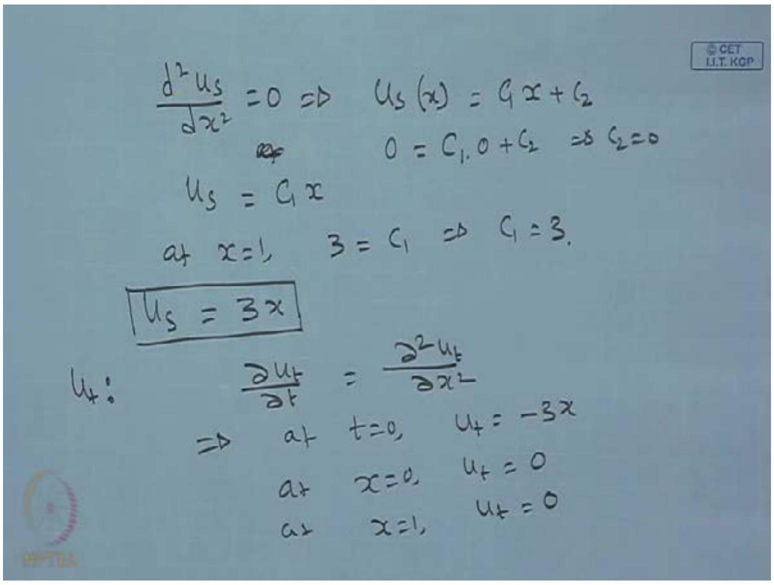

So, this u t has now become a well posed problem. And we know the solution of the well posed problem the earlier case, now let us try to solve the steady state part at least. The solution of the steady state part is d square u s dx square is equal to 0, solution is u s, it is a function of x, is nothing but 1 x plus c 2 and at x is equal to 0, u s is equal to 0, so u s is equal to 0 is equal to c 1 times 0 plus c 2, so c 2 is equal to 0.

So, u s is nothing but c 1 x, let us put the other boundary condition, at x is equal to 1, u s was equal to 3 of x, so at x is equal to 1, actually we should not have put u is equal to at the boundary condition, it should be x is equal to 1, so u s is equal to 3, so 3 is equal to c 1, so your c 1 will be is equal to 3. So u s, the solution of u s is nothing but 3 times x, this is the solution of u s steady state part, so u t becomes, if you look into the solution, whether different the condition on u t, it will be del u t del t is equal to del square u t del x square, so this becomes x is equal to at time t is equal to 0, your u t is equal to minus 3x, at x is equal to 0, we had u t is equal to 0, at x is equal to 1, we had u t is equal to 0.

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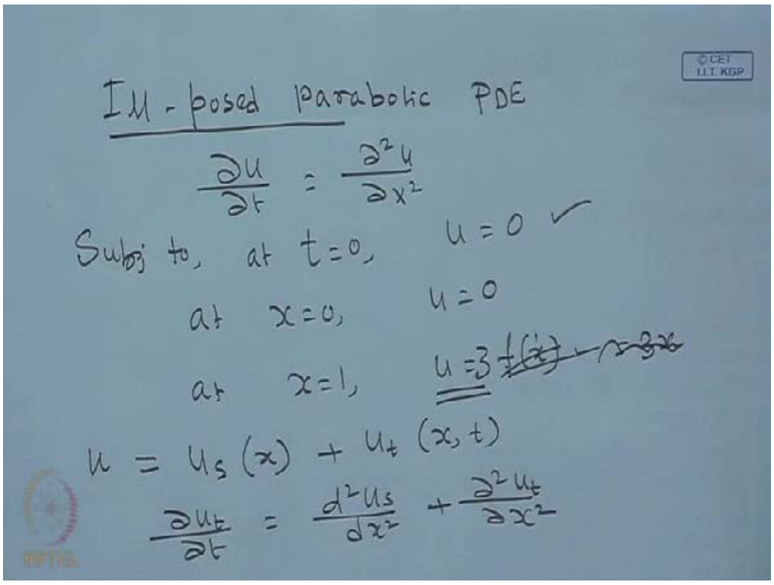

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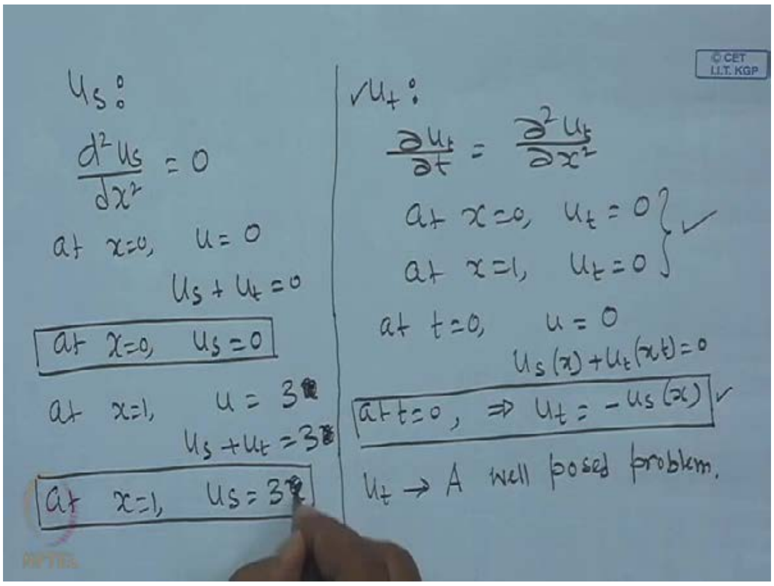

So, just only one correction, the correction is that whenever we are putting this non-homogeneity in the boundary condition, we should not put u is equal to some function of x, it will be some non-homogeneous term and in general, this should be a function of, it must be a constant term. Because, if we put x is equal to 1, then it becomes independent of x, so this will be some function, may be a constant 3. So, this is the non-homogeneous term that we are talking about, so the next case, whenever we are solving, formulating the governing equation of u s and u t at x is equal to 1, u should be 3. And this should be is equal to 3 and whenever we are getting the solution of a space varying part, so u s is equal to 3, at x is equal to 1.

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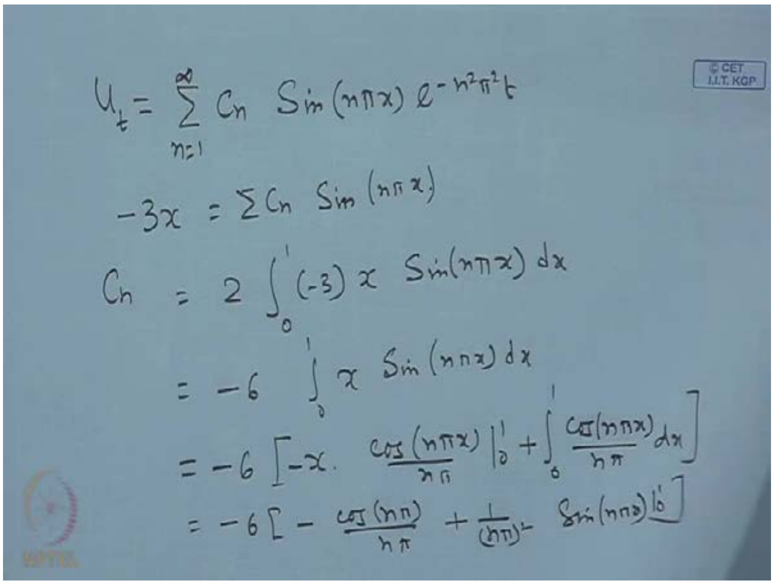

So, this is fine, so rest of the solution is all right. So, we have already seen the solution to this problem that the eigenvalues will be constituted by n pi and eigenfunctions will be constituted by sine n pi x. So, will be getting a complete solution here, that u t is nothing but C n sine n pi x e to the power minus n square pi square t, n is equal to 1 to infinity and we use the initial condition that is minus 3x should be is equal to C n sine n pi x.

So, we can get the C n as 2 integral minus 3 times x sine n pi x dx from 0 to 1, so it will be minus 6 0 to 1 x sine n pi x dx. And we can integrate this out by using separation of variable and can get the solution of, expression of C n. Let me try to finish it of, so first function integral of, with a minus sign in a second function divided by n pi, so integration of sine is minus cosine n pi, so that is why this minus comes, 0 to 1 minus minus plus differential of the first function integral of second function n pi x divided by n pi dx from 0 to 1. So, when I put 0, the whole thing vanishes, this will be minus 6 minus cosine n pi over n pi and this becomes 1 by n pi square sine n pi x from 0 to 1 and sine n pi is 0, sine 0 is there 0, so C n in this particular case is it turns out to be 6 cosine n pi by n pi.

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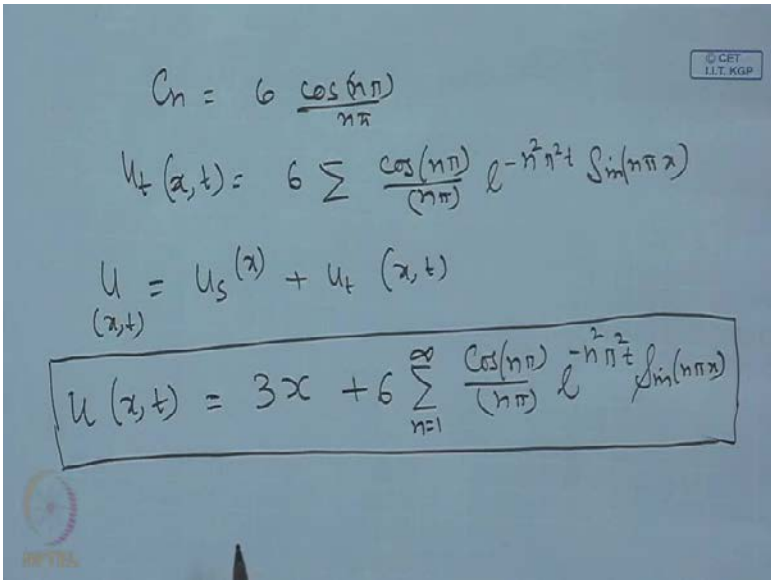

So, C n turns out to be 6 cosine n pi over n pi, therefore u t as a function of x and t becomes 6 summation cosine n pi divided by n pi e to the power minus n square pi square t sine n pi x. So, the complete solution can be constructed of this problem is linear superposing of both the solution, u s as a function of x and u t as a function of x and t.

So, you will be getting u as a function x and t, so u as a function of x and t will be getting once will be obtained as 3x, there is the steady state solution plus 6 summation n is equal to 1 to infinity cosine n pi divided by n pi e to the power minus n square pi square t sine n pi x. So, that gives the complete solution for an ill-posed problem. Then will be also taking up another example of ill-posed problem with a Neumann boundary condition and see what we get.

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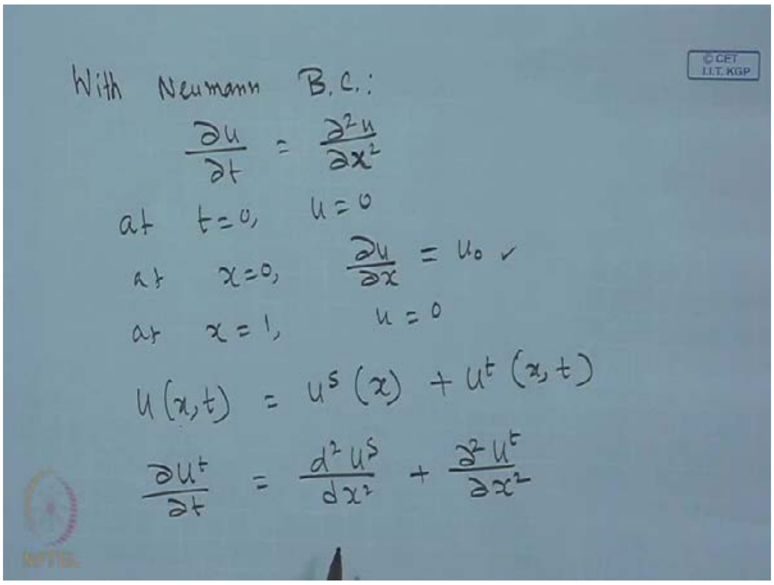

With Neumann boundary condition, if we do that, we will be getting del u del t is equal to del square u del x square. At t is equal to 0, we have u is equal to 0, at x is equal to 0, we have del u del x is equal to 0 or let say a constant heat flux, so that may be a constant u 0, at x is equal to 1, we have u is equal to 0.

So, we have a non-homogeneous term u naught in the formulation of in the boundary condition. So, this as a homogeneous initial condition and a non-homogeneous boundary condition and other boundary condition is homogeneous, so this problem is again it should be divided into two sub problems, one is taking the time dependent part, another is the time independent part. If you do that so u x t is composed of u s, we put the s and t as a superscript, I think that will be quite logical, because u subscript t sometimes define to take to denote that as del u del t. So, in order to avoid confusion, let us put s and steady state part and time varying part as the superscript, so u x of t is u s function of x only and u t as a function of space and time, both.

So, next, we will be putting this equation in the governing equation and substitute the and get the governing equation on the boundary condition are both u s and u t. If you do that will be getting del u t del t is equal to d square u s dx square plus del square u t del x square, then we collect the similar order terms and similar terms and formulate the governing equation of u s and u t.

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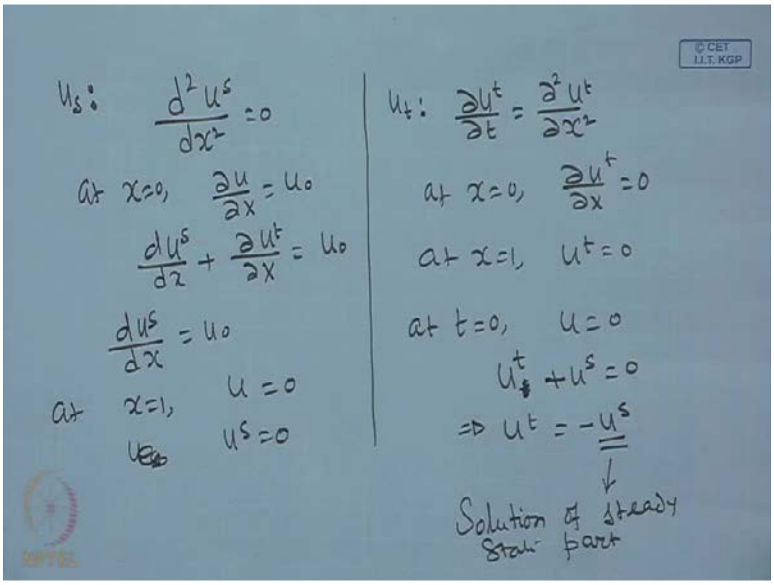

So, u s the governing equation is d square u s dx square is equal to 0 and for governing equation of u t, will be having del u t del t is equal to del square u t del x square. Now, we a get the boundary conditions on x, at x is equal to 0, you have del u del x is equal to u naught that was the boundary condition on the of the mother problem, then we substitute u is equal to u s plus u t, what you will be getting is that you will be getting du s dx plus del u t del t is equal to u 0.

So, we associate, we do a judicial selection of association of u naught to the steady state part, so we associate, we collect the similar terms and associate the non-homogeneous part to the steady state solution, so it becomes du s dx is equal to u naught at x is equal to 0 and at x is equal to 0, this becomes del u t del x is equal to 0. So, by associating the non-homogeneous term with the space varying part makes me the boundary condition of the at the same boundary of the time varying part to be homogeneous. So, at x is equal to 1, we had u is equal to 0, so u s, so at x is equal to 1, u s is equal to 0 and at x is equal to 1, we have u t is equal to 0, so there is no problem in that. And the it let us fix at the initial condition of the time varying part, at t is equal to 0, we had u is equal to 0, there is a condition on the mother problem.

So, therefore, we have u t plus u s is equal to 0, therefore u t is nothing but minus u of s, which is nothing but the solution of the steady state part, so this is the solution of steady state part and that will be typically a function of x.

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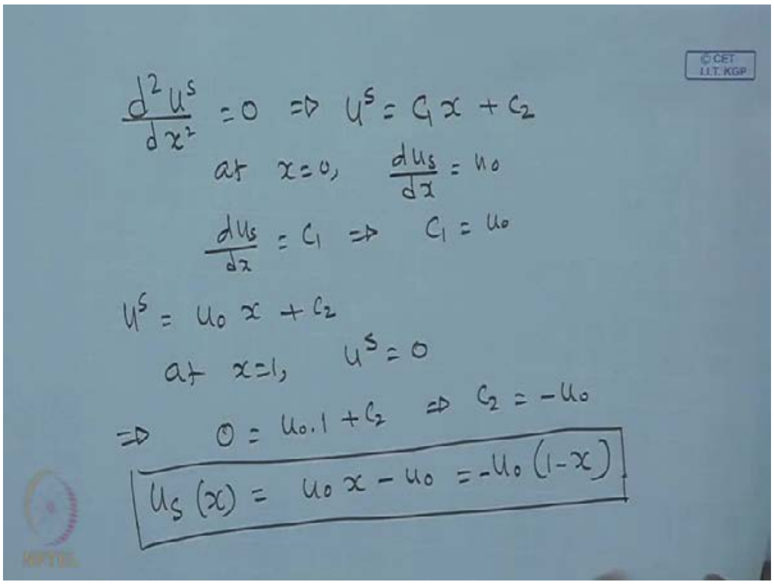

Now, let us look into the complete solution to this problem. So, first will be solving the space varying part, so it will be d square u s dx square is equal to 0, so the solution is u of s is equal to c 1 x plus c 2. So, the first boundary condition is that at x is equal to 0 du s dx is equal to u naught, so we have du s dx is equal to c 1, so that means c 1 is equal to u naught.

So, u s is equal to u naught x plus c 2, we put the other boundary condition that is at x is equal to 1, u s is equal to 0, therefore u s is equal to 0, u 0 into 1 plus c 2, so c 2 equal to minus u 0. Therefore we get u s, solution of u s as a function of x is nothing but u 0 x minus u 0, so this becomes u 0, you can take u minus u 0 common, this becomes 1 minus x, so this is the solution of the steady state part.

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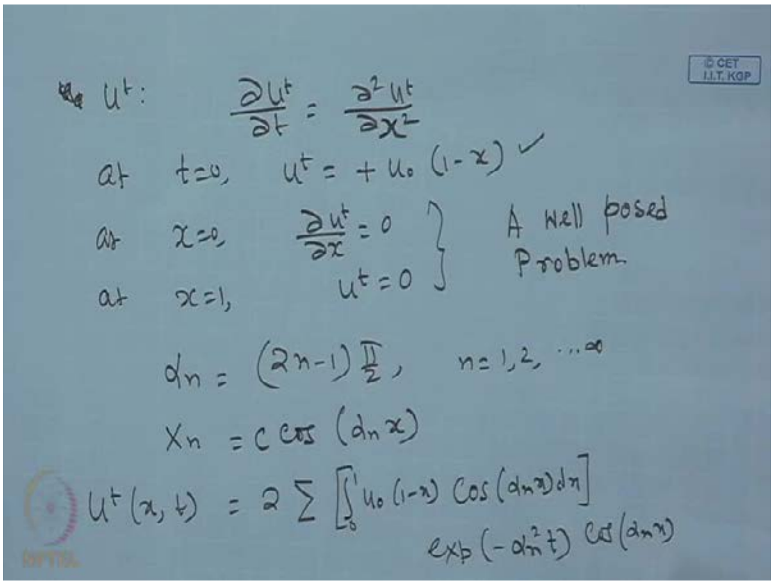

Now, let us look into the solution of the transient part. The transient part, the governing equation is del u t del t is equal to del square u t del x square. So, at t is equal to 0, u t is equal to minus u s x, so minus u s, so will be minus minus plus u 0 into 1 minus x, that is the solution, that is the initial condition. At x is equal to 0, we have del u t, del x is equal to 0 and at x is equal to 1, we had u t is equal to 0.

So, we have the homogeneous boundary condition and a non-homogeneous initial condition, so this is a well posed problem. We know the solution, we have already seen the solution of this problem, the eigenfunctions, the eigenvalues are cosine values 2n minus 1 pi by 2, but n are 1, 2, infinity and eigenfunctions are cosine functions, so this will be cosine of, some constant cosine of alpha n x.

Now, if you look into the complete solution to this problem, we have already solved this problem in the last class, so I am just writing it down the solution, u t as a function of x and t is 2 summation integration of u 0 1 minus x cosine alpha n x dx 0 to 1 multiplied by exponential minus alpha n square t times cosine alpha n x.

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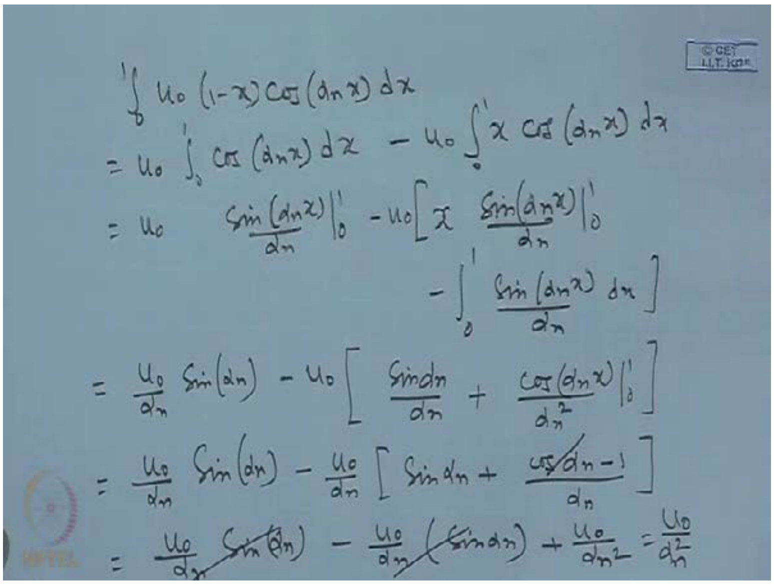

Now, let us evaluate this term, this integral and see what we get. If we evaluate this integral 0 to 1 u 0 one minus x cosine alpha n x dx, this becomes u 0 is constant out, so this will be 0 to 1 cosine alpha n x dx minus u 0 x cosine alpha n x dx from 0 to 1.

So, this becomes u 0 sine alpha n x divided by alpha n from 0 to 1 minus u 0, this has to be evaluated by integration parts, first function integral of second function sine alpha n x divided by alpha n from 0 to 1 minus differential of first function is 1, integral of second function sine alpha n x divided by alpha n dx from 0 to 1.

So, this becomes u 0 by alpha n sine alpha n, because sine 0 is 0, minus u 0 1, so sine alpha n by alpha n, if it is multiply by 0 the whole thing turns out to be 0, minus alpha n will be outside, this alpha n will be outside and integral of sine alpha n is nothing but cosine alpha n, so this becomes minus of that, so minus minus plus 1 by so cosine alpha n x by alpha n square from 0 to 1.

So, you will be having u 0 by alpha n sine alpha n minus u 0 by alpha n, so this becomes sine alpha n plus cosine alpha n minus 1, cos 0 is 1, so this will be alpha n 1, alpha n is taken out. But if you remember the what are the eigenvalues to this problem, the eigenvalues to the problem where 2n minus 1 pi by 2, so cosine 2n minus 1 pi by 2 will be always 0. Therefore this term will vanish, so what will be having u 0 by alpha n sine alpha n minus u 0 by alpha n sine alpha n minus minus plus u 0 by alpha n square. Again this term we also cancelled out, so this integral is nothing but u 0 by alpha n square.

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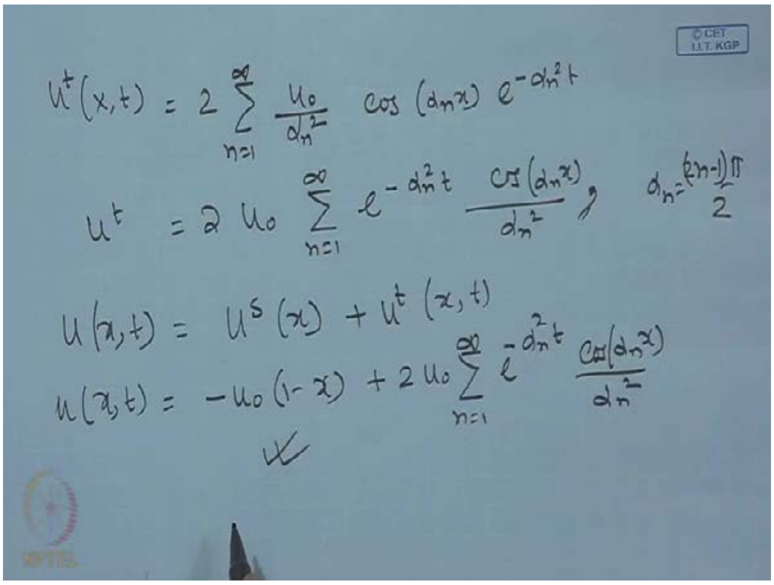

So, if we write down the complete solution, the complete solution takes this form, the complete solution will be u as a function of u t as a function of x and t, will be 2 summation n is equal to 1 to infinity, this whole integral becomes u 0 by alpha n square cosine alpha n x e to the power minus alpha n square t. So, this becomes 2 u 0 summation of n is equal to 1 to infinity e to the power minus alpha n square t cosine alpha n x divided by alpha n square, where alpha n are 2n minus 1 pi by 2.

So, we get the complete solution of u as a function of t x and t, is nothing but u s as a function of x, plus u t as a function of x and t. So, we get complete solution as minus u naught into 1 minus x plus 2 u naught summation n is equal to 1 to infinity e to the power minus alpha n square t cosine alpha n x divided by alpha n square, but alpha n is equal to 2n minus 1 pi by 2. So, that gives the complete solution of an ill-posed problem when the boundary condition is a Neumann boundary condition.

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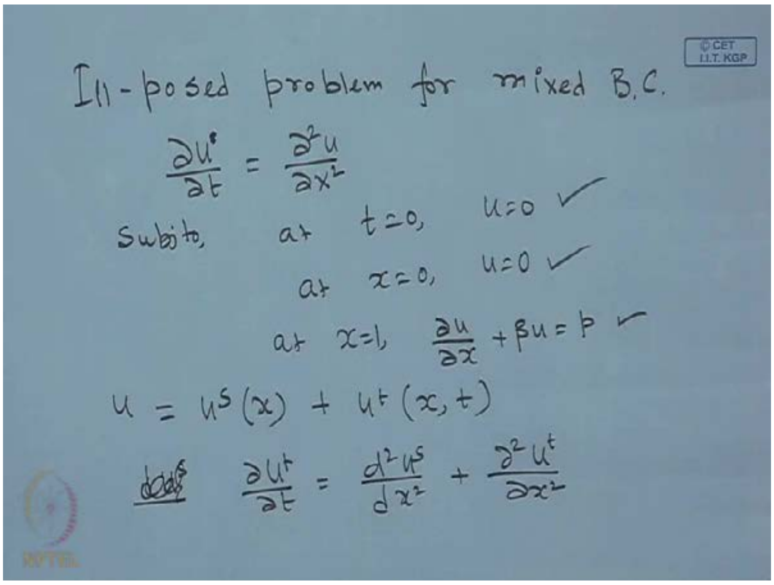

Next, we take up and ill-posed problem, where the boundary condition is a Robin mixed boundary condition. So, if we do that ill-posed problem for mixed boundary condition, so this will be del u del t is equal to del square u del x square subject to at t is equal to 0, u is equal to 0, at x is equal to 0, let us have a Dirichlet boundary condition, they are u is equal to 0 and at x is equal to 1, we have let say del u del x plus beta u is equal to some constant let say p.

So, if you now see this set of equation, the initial condition is homogeneous, the boundary condition is, one of the boundary condition is homogeneous, the other boundary condition is non-homogeneous. Therefore, this problem is ill-posed problem, we have to as we have discussed earlier, this problem has to divide into two sub parts, one is the time dependent part on and another is the time independent part and the steady state part.

If you do that, will be getting u s, which is the steady state part and it is a function of x alone and the other part is the time varying part, which is the function of time and space both.

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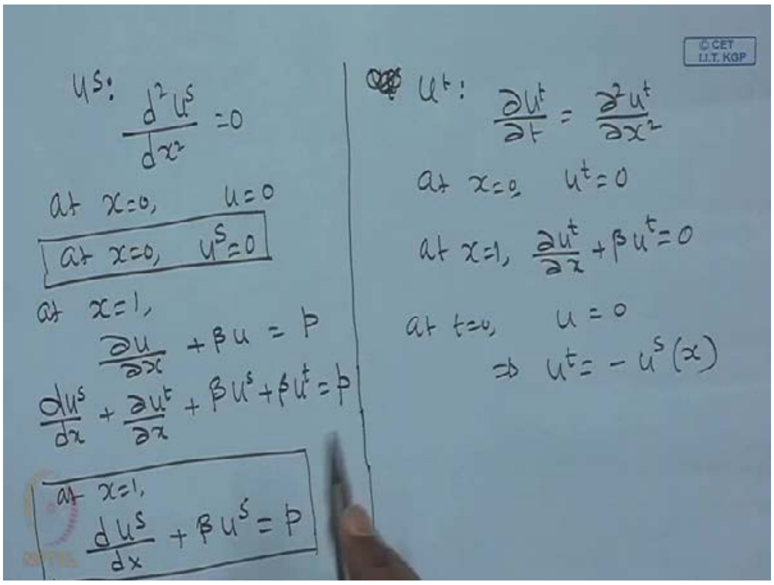

Now, you substitute there in governing equation, this becomes del u t del t, because u s is function of space alone, this becomes d square u s dx square plus del square u t del x square. Now, we collect the similar term and get the governing equation of u s and u t, so u s and u t, the governing equation of u s becomes d square u s dx square is equal to 0 and governing equation u t becomes del u t del t is equal to del square u t del x square.

Now, we set the boundary conditions of u s and u t, so at x is equal to 0, the boundary condition of the mother problem was u is equal to 0, therefore at x equal to 0, u s equal to 0 and at x is equal to 0, u t is equal to 0, both are equal to 0. And on other hand, the non-homogeneous boundary condition at x is equal to 1, the boundary condition of the mother problem was del u del x plus beta u is equal to p.

So, what we can have, du s dx plus del u t del x plus beta u s plus beta u t is equal to p, therefore we correct the similar term and associate the non-homogeneous term with the steady state solution, so that the transient problem will be having a homogeneous boundary condition. That means at x is equal to 1, will be having del u t del x plus beta u t should be is equal to 0 and here will be having at x is equal to 1, du s dx plus beta u s is equal to p, that is the other boundary condition. So, this two are the boundary conditions of the steady state part and about about the initial condition of the transient solution, is that at t equal to 0, we had u is equal to 0 of the original problems, therefore u t will be having minus u s, which is a function of x.

So, therefore, the solution of the steady state part has become an initial condition of the transient part with a minus sign associated with it.

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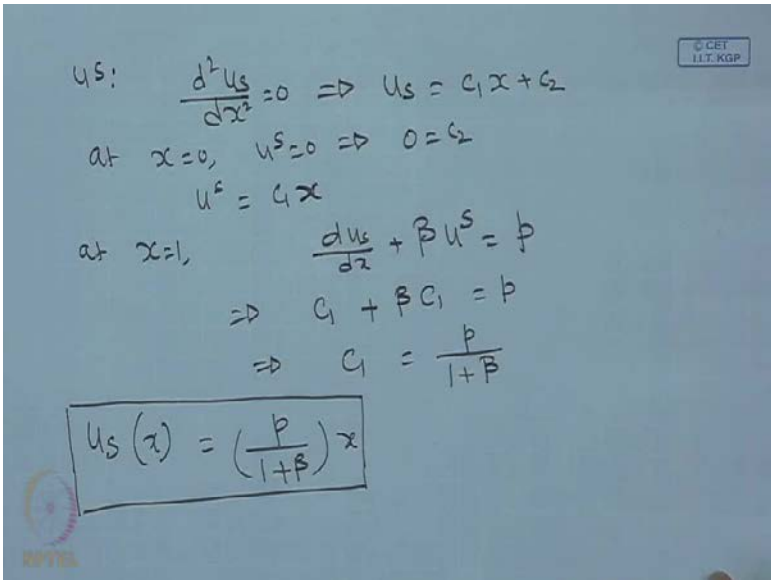

Now, let us look into the solution to this problem. So, if you look into the steady solution of the steady state part, so this becomes d square u s dx square is equal to 0, so the solution will be in the form of c 1 x plus c 2, it is a linear profile.

Now, at x equal to 0, we have u s is equal to 0, therefore 0 is equal to c 2, so u s is nothing but a linear profile c 1 x. Now, let us put the other boundary condition, at x is equal to 1, du s dx plus beta u s is equal to p, so what is du s dx is nothing but c 1 plus beta u s evaluated at x equal to 1, so that is also c 1 is equal to p, so c 1 turns out to be p divided by 1 plus beta.

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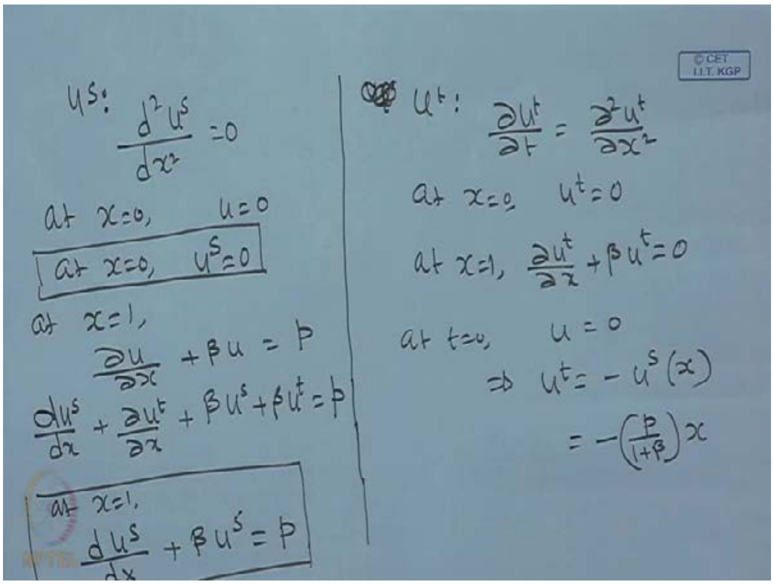

So, therefore, the steady state solution becomes p divided by 1 plus beta x, so this is the steady state solution. Now, if we are interested, we can look into the transient solution as well. So, if you look into the transient solution, so if you look into the initial condition of the transient solution, we can put u t is equal to minus u s x, so it will be minus, the solution you already obtained, p divided by 1 plus beta times x.

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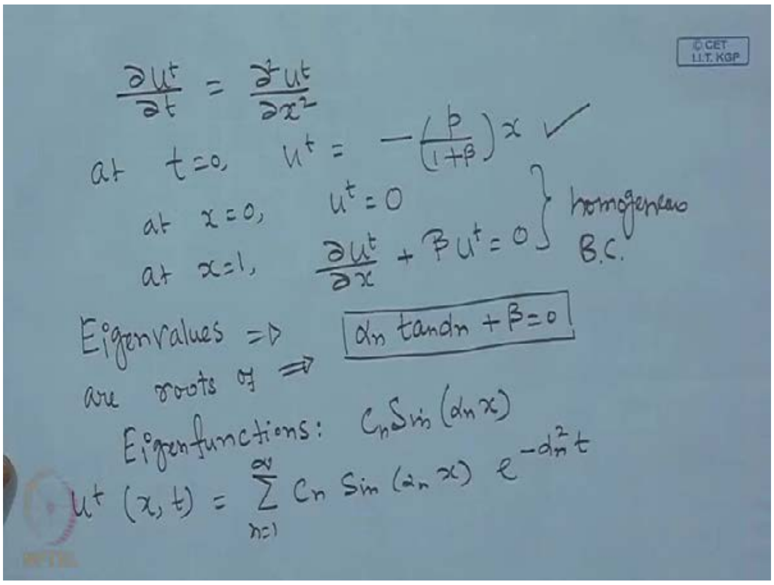

So, if you now look into the transient solution, the transient solution you have already did earlier, the similar type of thing, that del u t del t is equal to del square u t del x square. At time t is equal to 0, we had u of t is equal to minus p divided by 1 plus beta times x and at x is equal to 0, we have u t is equal to 0 and at x is equal to 1, we had del u t del x plus beta u t is equal to 0.

So, therefore, this problem has a non-homogeneous initial conditions and homogeneous boundary condition. So, again by using separation of variable, this can be solved and if you remember the eigenvalues to this problem, they became the roots of transcendental equation alpha n tan alpha n plus beta equal to 0. So, eigenvalues are roots of this equation and eigenfunctions are sine functions, some constant multiplied by sine alpha n x.

So, if you look into the complete solution to this problem, the solution is given as u t as a function of x and t, is nothing but summation n is equal to 1 to infinity C n sine alpha n x e to the power minus alpha n square t.

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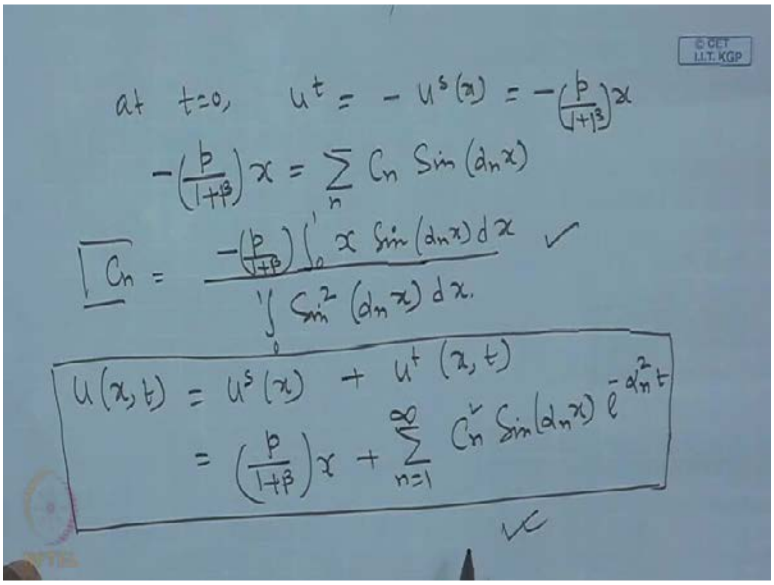

So, that gives the complete solution to this problem. Now, let us evaluate the C n from the initial condition, if you remember the initial condition of this time varying part, that is at t is equal to 0, my u t was nothing but minus of steady state solution, so it was minus of p divided by 1 plus beta times x. So, we use that, so this becomes minus p divided by 1 plus beta into x is equal to summation of C n sine alpha n x.

So, we can evaluate the value of C n as integral minus p into 1 plus beta integral 0 to 1 x sine alpha n x dx and 0 to 1 sine square alpha n x dx. So, we know how to solve this equation, we have taken care of this equation earlier. And I am not going to rewrite this equation once again, so this can be solved. And we will be getting the complete solution as u as a function of x and t, should be u s, which is a function of x alone plus u t, which is a function of x and t both. So, this becomes p into 1 plus beta times x plus u t will be nothing but summation n is equal to 1 to infinity C n sine alpha n x e to the power minus alpha n square t, where C n will be evaluated from this expression.

So, that gives the complete solution of the ill-posed problem. So, I stop here, in the next class, I will be taking up a complete actual chemical engineering problem, where we will be applying the separation of variable type of solution to get the complete formulation and the solution of partial differential equation in its parabolic form; thank you very much.