Advanced Mathematical Techniques in Chemical Engineering
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Module No. # 01
Lecture No. # 11
Eigenvalue Problem in Discrete Domain (Contd.)

Welcome to this session. Now, we will be continuing whatever the example we are talking about in the last lecture, we have taken up one example of a set of non-homogeneous algebraic equation for the solution using the Eigen value Eigen vector method. So, we took up a two-dimensional problem, 2 into 2 matrix for demonstration purpose, the whole procedure can simply be extended for higher dimensional problem numerically, that we have already discussed in the last class.

Now, let us come back to the specific example that we are talking about. So, in this example it is a two-dimensional problem. So, we followed the various steps to be adopted to solve this problem. The first step, one has to get the evaluation of Eigen values. So, we have identified the, we have evaluated the Eigen values for the matrix A in this particular case, and these Eigen values turned out to be minus 4 and minus 1.

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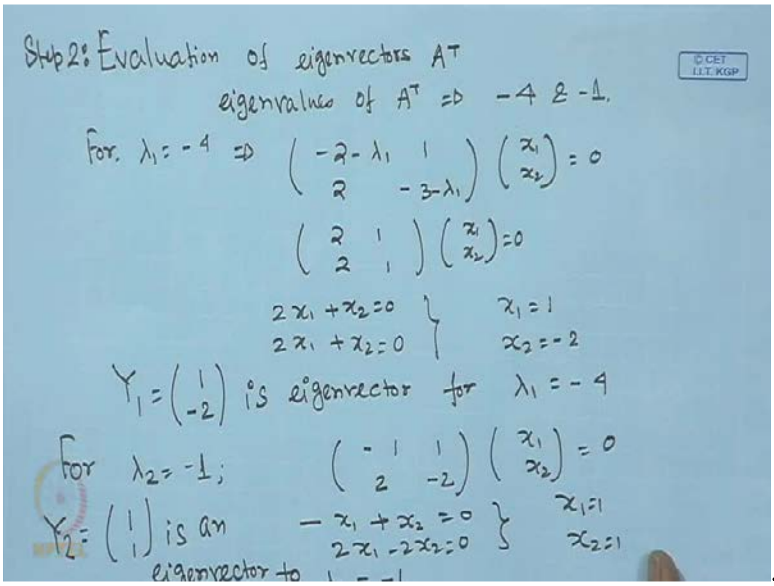

Next step is to evaluate the Eigen vectors of the corresponding Eigen values, and we have already evaluated the Eigen vectors for this matrix A, X 1 and X 2. Next step is evaluation of Eigen values Eigen vectors A transpose. We have already proved the Eigen values will be the same for A and A transpose, that we have already proved earlier. So, this is step number two.

So, evaluation of Eigenvectors of A transpose. So, Eigen values for this A transpose are identical, A transpose are same as minus 4 and minus 1. So, for lambda 1 is equal to minus 4, let us evaluate the Eigen vectors of this A transpose matrix. So, minus 2 minus lambda 1 1 2 minus 3 minus lambda 1, X 1 X 2 are the elements of this vector is equal to 0. So, if we put it as lambda 1 is equal to minus 4 will be getting 2 1 2 1 X 1 X 2 equal to 0.

So, we will be getting 2 x 1 plus x 2 is equal to 0 from this. And 2 x 1 plus x 2 equal to 0. So, they are identical. So, if you select X 1 is equal to 1, then X 2 has to be minus 2. So therefore, Y 1 is equal is 1 minus 2 is an Eigen vector for the Eigen value lambda 1 is equal to minus 4. So, in case of lambda 2 is equal to minus 1, we will be having minus 1 1 2 minus 2, I am just writing from these equations I am just writing this one. So, I am just omitted one more one step here, they are identical, by now it has to be clear to you and you also know how to evaluate Eigen values or Eigen vectors given a matrix.

So, this will be minus X 1 plus X 1 is equal to 0 and 2 X 1 minus 2 X 2 is equal to 0. They are identical. So, if you select X 1 is equal to 1, then X 2 turns out to be 1. So therefore, Y 2 is equal to 1 1 is an Eigen vector to Eigen value lambda 1 is equal to minus 1.

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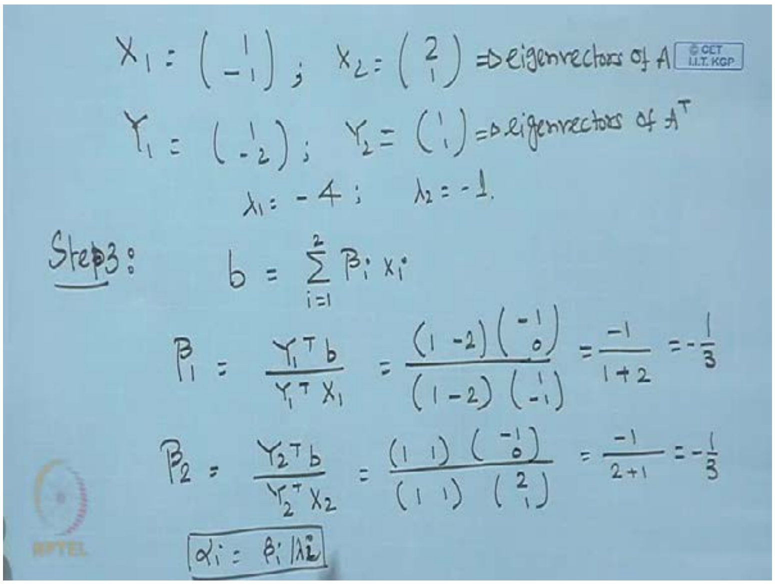

So therefore, this is the same method, this is the same way we have evaluated the Eigen vectors of matrix A, we can evaluate the Eigen vectors of matrix A transpose. Now, let us write down this the solutions in one place. So, we have the Eigen vectors, X 1 is 1 minus 1 and X 2 is 2 1 and Y 1 is 1 minus 2 and Y 2 is 1 1. So, this corresponds to lambda 1 is equal to minus 4, these two corresponds to lambda 1 is equal to minus 1. So, they are Eigen vectors of A and they are Eigen vectors of A transpose.

So, an Eigen values we know, lambda 1 is equal to minus 4 and lambda 2 is equal to minus 1. Next step, that is step three. We have already finished step one and step two, that is evaluation of Eigen vectors of A and evaluation of Eigen vectors A transpose. So, next step is to get the solution of b is equal to summation beta i X i and in this case i is going to 1 to 2. So, beta 1, if you remember beta 1 is nothing but, Y 1 transpose b divided by Y 1 transpose X 1. So, if we write it down it becomes Y 1 transpose, so it will be 1 minus 2, this will be simply 1 minus 2, the column becomes rows. So, it becomes 1 minus 2, rows becomes columns, and b is minus 1 0, and Y 1 transpose is same as 1 minus 2, and X 1 is 1 minus 1.

So, if you compute, this is 1 into minus 1 minus 1 plus minus 2 into 0, that is 0 divided by 1 into 1 plus minus 2 into minus 1, so this will be plus 2, so this will be minus one-third. So, beta 1 is minus one-third. Similarly, we calculate beta 2, beta 2 will be Y 2 transpose b divided by Y 2 transpose X 2. And Y 2 transpose will be 1 1 minus 1 0 1 1 and X 2 will be 2 1, X 2 will be 2 1. And we get this, this will be minus 1 plus 0 into 1, that will be 0 2 into 1 plus 1 into 1. So, this will be minus 1 upon 3.

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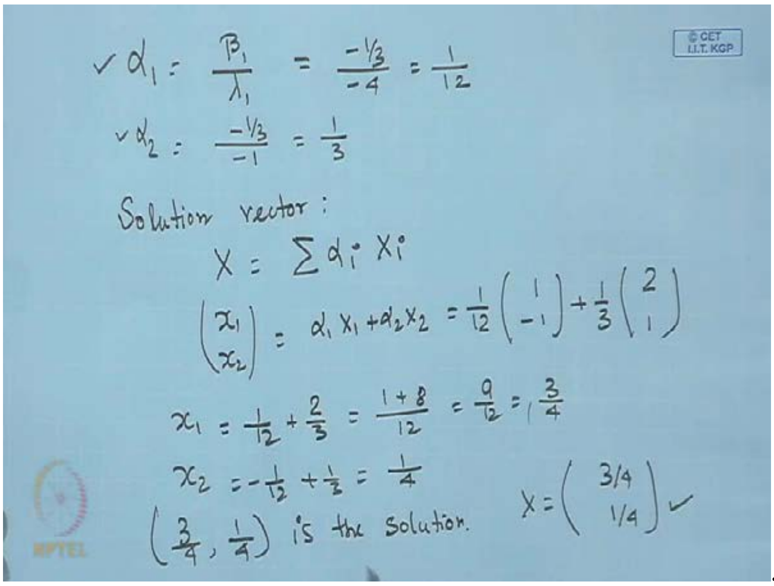

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Next, we evaluate alpha 1 and alpha 2. If you remember that alpha i was equal to beta i by lambda i. So, once you get the beta's, then you will be able to and you already know the lambdas, so you can get the alpha i's. So, alpha 1 will be nothing but, beta 1 by lambda 1. So, beta 1 will be minus 1 upon 3 and lambda 1 is minus 4. So, this will be plus 1 by 12 and alpha 2 is minus 1 upon 3 divided by minus 1. So, this will be plus 1 upon 3.

So, now since the alpha's are known and x i's ,the Eigen vectors of A are already determined, we will be in a position to get the solution matrix. So, the solution matrix is or solution vector is given as, vector is nothing but, a special matrix, so solution vector is given as X is equal to summation alpha i X i. So, you will be having two elements of the solution vector, X 1 and X 2. So, this will be alpha 1 X 1 plus alpha 2 X 2. Alpha 1 is 1 by 12 and X 1 we have already found out 1 minus 1 plus, alpha 2 is 1 upon 3, so this will be 2 into 1, 2 and 1. So, this will be equal to.

So, we will be getting the solution matrix, solution of this problem, solution elements X 1 is equal to 1 by 12 into 1 plus 1 by 3 into 2, so it will be 2 by 3. So therefore, 12 1 8, so this will be 9 by 12, so it will be three by four. Similarly, the other solution is 1 by 12 with a minus sign plus 1 upon 3. So, this will be 1 upon 4. So, three by 4 and 1 over 4 is the solution of this particular problem.

So, the solution vector x will be comprised of two elements, 3 by 4 and 1 by 4, and that gives the complete solution to this problem. So, that is how this problem clearly demonstrates the solution using the Eigen value problem, how to get the solution using the Eigen value and Eigen vector method, without taking a request to the matrix inversion that is necessary for utilizing Gauss Seidel algorithm or Gauss Elimination method.

So, we will be taking up one more example in this type of problem, simply because the Eigen values in that particular problem becomes imaginary, it becomes imaginary. So, you need not to worry about that, depending on the situation, depending on the definition of the problem, one can get real valued Eigen values or imaginary Eigen values. But the final solution will be a real solution, if it is a real valued problem, because all chemical engineering systems are real valued problem. So, finally, you will be getting a real solution even if the Eigen values are imaginary. That is the reason I will be taking up this particular example, where the Eigen values you will be getting as imaginary.

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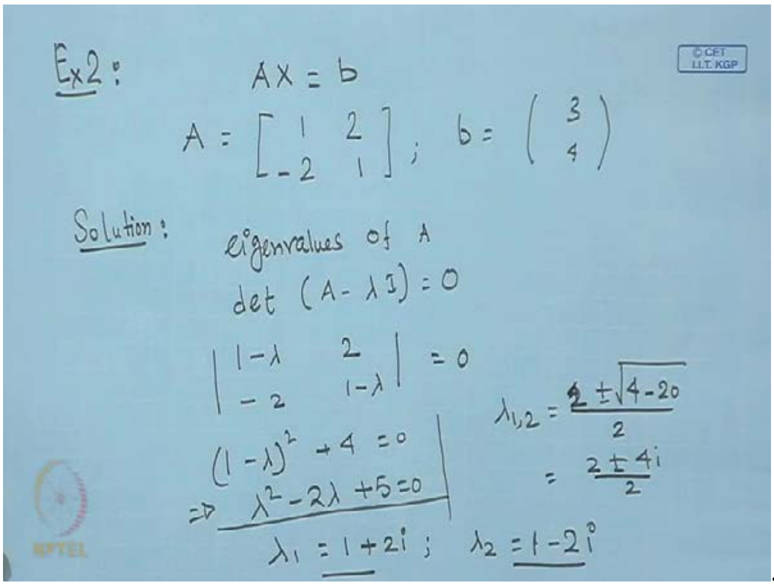

So, the next problem is example two. And we are going to solve this equation non-homogenous set of algebraic equation, A X is equal to b, where the matrix A is 1 2 minus 2 1 and the vector b is 3 and 4. So, let us look into the solution. Step number one is Eigen values of A, that is nothing but, determinant of A minus lambda i is equal to 0. So, we compute that 1 minus lambda 2 minus 2 1 minus lambda, that will be equal to 0. Open up this determinant. If you open up this determinant, this becomes 1 minus lambda square plus 4 is equal to 0. So, you will be getting lambda square minus 2 lambda plus 5 is equal to 0. So, it is a quadratic, the took solutions correspond to the two Eigen values. So, lambda 12 will be minus b, so it will be plus 2 plus minus under root b square 4 minus 4 a c. So, 20 divided by 2. So, it will be 2 plus minus 4 i divided by 2. So, you will be having imaginary Eigen values in this particular problem. So, lambda 1 becomes 1 plus 2 i and lambda 2 is equal to 1 minus 2 i. And this roots, they occur as complex conjugate.

So, let us talk about the first root and we will be evaluating the corresponding Eigen vectors. So, corresponding to Eigen values, next we evaluate the Eigen vectors of the matrix A.

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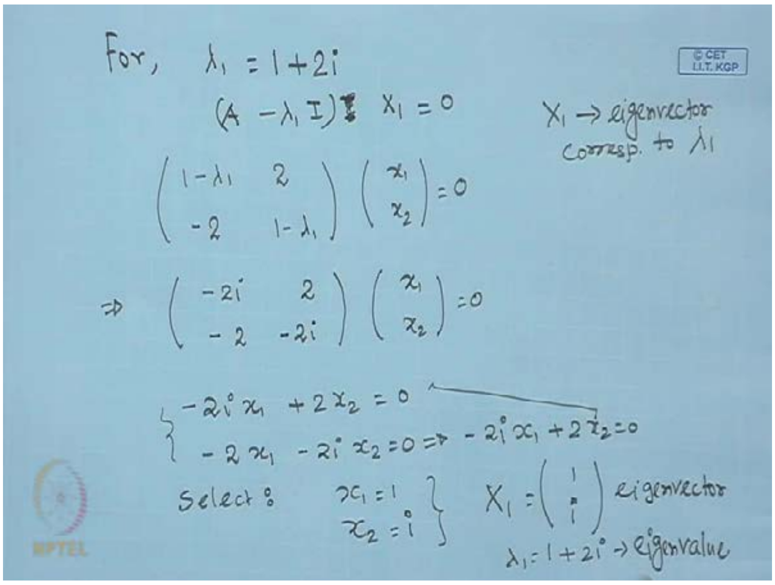

So, for Eigen value lambda 1 is equal to 1 plus 2 i, we will be having A minus lambda 1 i X 1 is equal to 0. So, X 1 is the Eigen vector corresponding to Eigen value lambda 1. So therefore, we will be getting this as 1 minus lambda 1 2 minus 2 1 minus lambda 1, X 1 X 2, these are the elements of the Eigen vector X 1. So, you put the value of lambda 1 as 1 plus 2 i. So, this becomes minus 2 i 2 minus 2 plus and 1 minus lambda 1, sgain it is minus 2 i. So, you formulate the two equations. So, minus 2 i X 1 plus 2 X 2 is equal to

0. Minus 2 X 1 minus 2 i X 2 is equal to 0. These two equations are identical because if we multiplied these equation by i, what we will be getting is that, minus 2 i X 1 i square is minus 1, so minus minus plus, 2 x 2 is equal to 0. So, these two equations are identical and therefore, if we select X 1 is equal to 1, then X 2 will be i. So therefore, X 1 is 1 i is the Eigen vector corresponding to Eigen value lambda 1 is equal to 1 plus 2 i. So, X 1 is the Eigen vector, 1 plus 2 i is the corresponding Eigen value.

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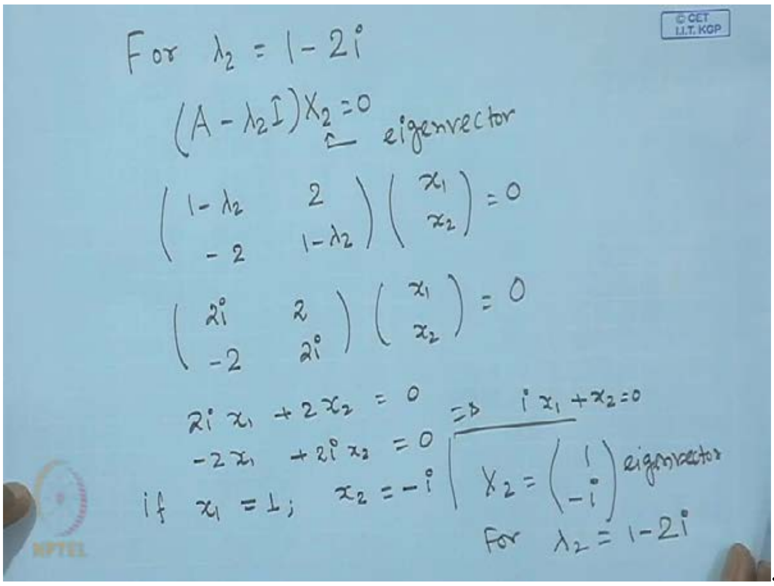

So, next we compute the Eigen vector corresponding to the Eigen value lambda 2. So, for lambda 2 is equal to 1 minus 2 I, again we get the corresponding Eigen vector by solving this equation A minus lambda 2 i multiplied by X 2 is equal to 0, where X 2 is the corresponding Eigen vector. So, therefore, this becomes 1 minus lambda 2 2 minus 2 1 minus lambda 2. And X 2 will be having two elements, X 1 and X 2 that will be equal to 0. So therefore, if you put lambda 2 as 1 minus 2 i here, so you will be getting 2 i 2 minus 2 plus 2 i and X 1 and X 2 is equal to 0. So, the first equation we will be getting 2 i X 1 plus 2 X 2 is equal to 0 and minus 2 X 1 plus 2 i X 2 is equal to 0. Again both are identical because you multiply this equation by i, you will be getting back the other one. So, you will be getting finally i X 1 plus X 2 is equal to 0.

So therefore, if we select X 1 is equal to 1, then X 2 will be is equal to minus i. So, if X 1 is equal to 1, then X 2 will be minus i. Therefore, X 2 is equal to 1 minus i is the Eigen vector corresponding to lambda 2 is equal to 1 minus 2 i. So, we evaluated the Eigen vectors for the corresponding Eigen values. Next step is to evaluate the Eigen vectors for A transpose.

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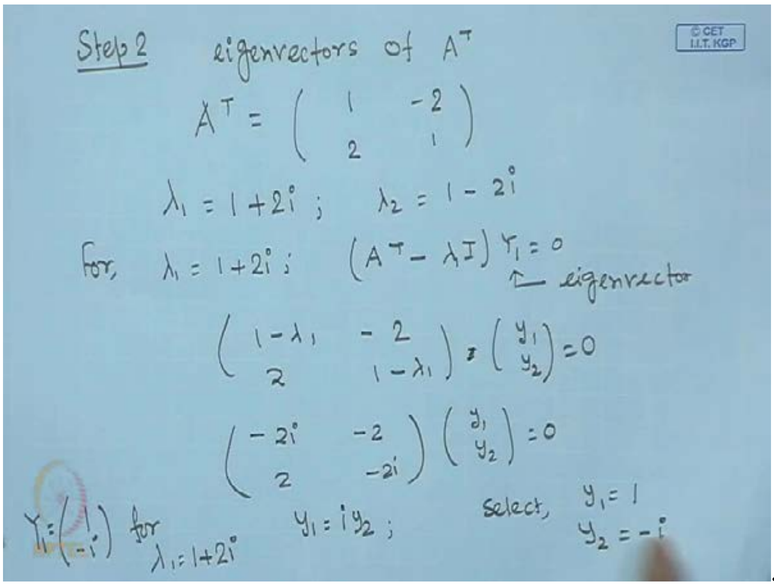

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So, that is step two, Eigen vectors of A transpose. So, if you look into the A transpose, this becomes, this is a transpose matrix 1 2 minus 2 and 1. So, lambda 1 and lambda 2 remain the same because you have already proved the Eigen values of A and Eigen value of A transpose are identical. So, they remain same, only the transpose matrix elements will be changed, inter changed, rows becomes columns, lambda 2 becomes 1 minus 2 i, they are identical. So therefore, for lambda 1 is equal to 1 plus 2 i, we have A transpose minus lambda 1 i Y 1. So, Y 1 is the corresponding Eigen vector of A transpose for the Eigen value 1 plus 2 i. Therefore, it becomes just put the values here. So, it becomes 1 minus lambda 1 A transpose 1 minus lambda 1 minus 2, then 2 and 1 minus lambda 1, Y 1 Y 2 are the elements of the Eigen vector Y 1. So therefore, minus 2 i minus 2 2 minus 2 i Y 1 Y 2 is equal to 0. Now, if we construct the equations, these equations becomes Y 1 is equal to i Y 2. So therefore, if we select Y 1 is equal to 1, then Y 2 has to be is equal to minus i.

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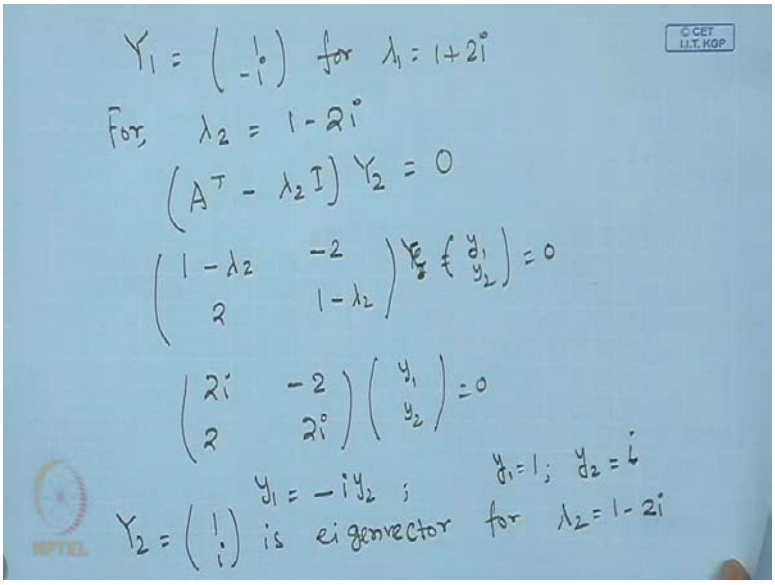

So, 1 minus i is the first Eigen vector for lambda 1 is equal to 1 plus 2 i. Similarly, 1 can evaluate the corresponding Eigen vectors for lambda 2. So, Y 1 is equal to 1 minus i for lambda 1 is equal to 1 plus 2 i. Next, we evaluate the Eigen vector for next lambda values, the Eigen value 1 minus 2 i, we formulate the same thing A transpose minus lambda 2 i multiplied by Y 2 should be equal to 0. So, this becomes 1 minus lambda 2 minus 2 2 1 minus lambda 2 Y , Y 2 will be having the elements, let us say Y 1 and Y 2 is equal to 0. So, it becomes put the value of lambda there, lambda 2, it becomes 2 i minus 2 2 plus 2 i Y 1 Y 2 will be equal to 0. So, we will be having two identical equations and this equations in this case will be Y 1 is equal to minus i Y 2. So therefore, if we select Y 1 is equal to 1, then Y 2 will be plus i.

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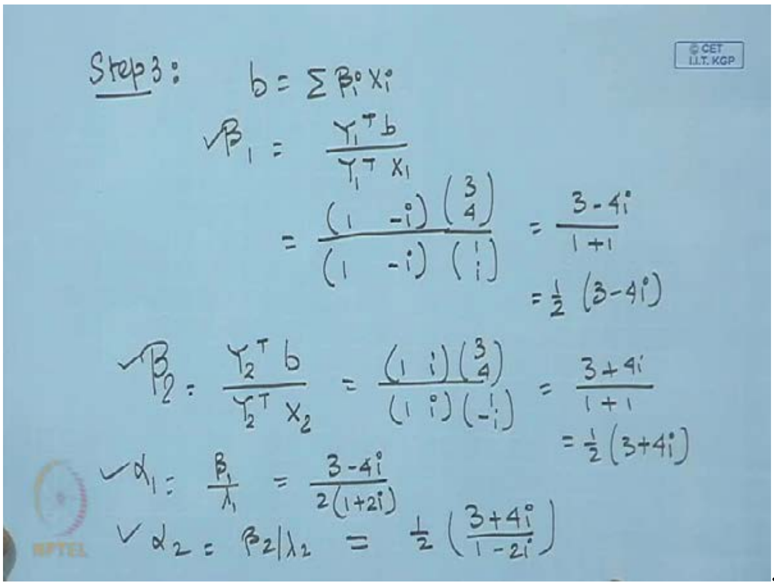

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So, 1 plus i Y 2 is equal to1 and i is Eigen vector for lambda 2 is equal to 1 minus 2 i. So therefore, we have evaluated the Eigen vectors for A, we have evaluated the Eigen vectors of A transpose. Next step is step 3. For step 3, we evaluate the coefficients beta in the equation b is equal to summation beta i X i. So, if you remember beta 1 is equal to Y 1 transpose b divided by Y 1 transpose X 1. So, this becomes 1 minus i and b is nothing but, 3 4 1 minus i and X 1 is nothing but, 1 i. So, you will be having 3 minus 4 i divided by 1 plus, i square is minus, minus minus plus 1, so it becomes 1 by 2 3 minus 4 i. So, that is beta 1. Similarly, we can evaluate beta 2, beta 2 is Y 2 transpose b divided by Y 2 transpose X 2. So therefore, this becomes 1 i 3 4 and it is 1 i 1 minus i. So therefore, you will be getting 3, minus minus plus, 3 plus 4 i divided by 1, minus i square, so minus minus plus 1, so this becomes half 3 plus 4 i. So, you will be getting alpha 1 as beta 1 by lambda 1. So, alpha 1 becomes 3 minus 4 i divided by 2 and lambda 1 becomes 1 plus 2 i. So, lambda 1, you just put the value of lambda 1 there. And similarly, we can get alpha 2 as beta 2 by lambda 2 and this becomes 1 by 2 3 plus 4 i divided by 1 minus 2 i.

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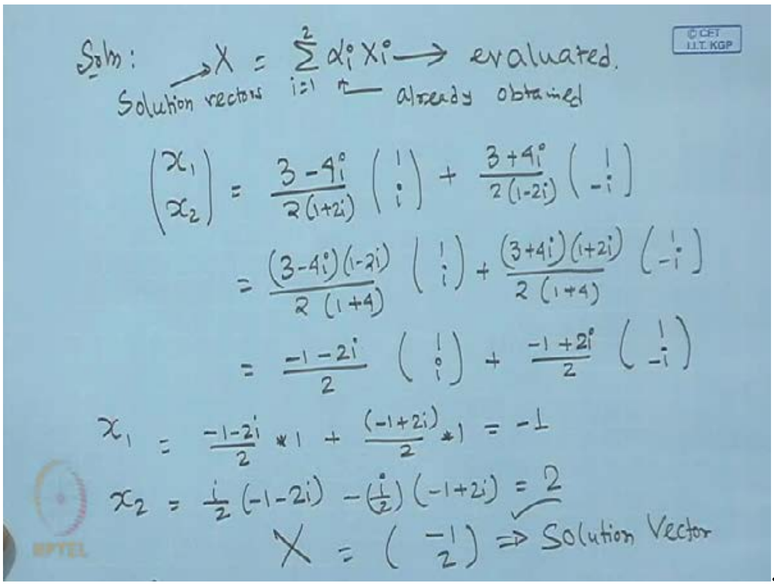

So, next, so we have evaluated the betas, we have evaluated the alphas. Next and final step is the solution X is equal to summation alpha i X i. So, the solution, this becomes a solution vector. Already determined the Eigen vectors, already evaluated, so we just put the solution vector, these are the elements of the solution, vector X 1 and X 2. So, you just open up this summation, this summation runs from index 1 to 2. So, 3 minus 4 i divided by 2 into 1 plus 2 i, that is the alpha 1. Next is X 1, X 1 is 1 and i plus 3 plus 4 i divided by 2 into 1 minus t2 i, that is the alpha 2 and X 2, X 2 is 1 minus i.

So, you will be getting, just simplify the whole thing, just make it, you just multiply both side, multiply both numerator and denominator by complex conjugate of 1 plus 2 i. So, multiply by 1 minus 2 i on top and the bottom. So, it becomes 3 minus 4 i multiplied by 1 minus 2 i divided by 2 into 1, minus minus plus, 4 1 and i plus 3 plus 4 i 1 plus 2 i divided by 2 1 plus 4 1 minus i.

So, this becomes, you just keep on simplifying, this becomes minus 1 minus 2 i, I just omitted one more step, divided by 2 1 and i. Next, we will be getting minus 1 plus 2 i divided by 2 1 minus i. So, X 1 we will be getting as minus 1 minus 2 i divided by 2 multiplied by 1 plus minus 1 plus 2 i divided by 2 multiplied by 1, and if you simplify this whole thing this becomes minus 1. So, you will be getting a real valued solution. Similarly, the X 2 will be i by 2 minus 1 minus 2 i plus minus i by 2 minus 1 plus 2 i. So, this will become 2.

So, minus 1 and 2 is the solution vector for this particular problem. And if you see that, although the Eigen values are not real, they are imaginary but, the solution is a real solution. So, this example demonstrates that even the Eigen values become imaginary, one we will get a real valued solution for this particular problem.

So, this gives a demonstration of how the Eigen values and Eigen vector methods can be implemented to solve the set of algebraic equations, which are typically occurring for the steady state problem in any chemical engineering application, and how one will get an elegant solution using the Eigen value and Eigen vector method. Next, I will talk about solution of, so we have looked into the solution of set of algebraic equation. Next, I will be talking about solution of set of ordinary differential equation.

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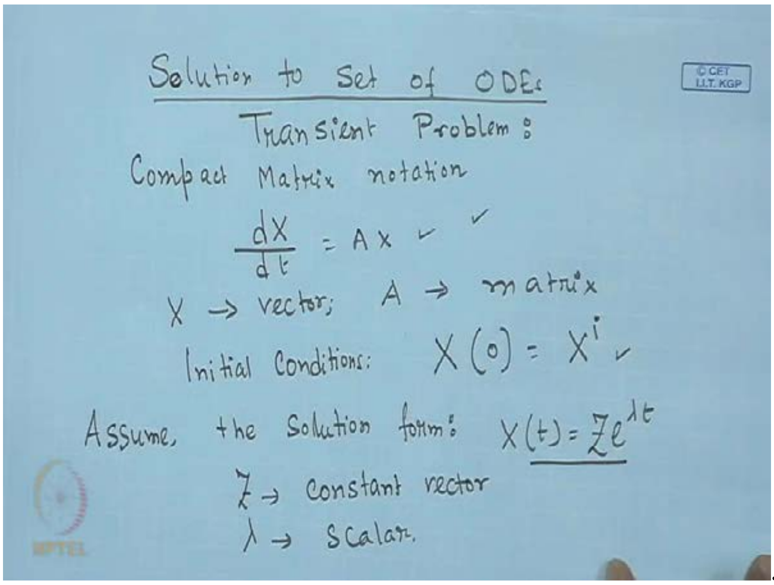

Solution to set of ODEs. Typically, this will be occurring, ODEs will be occurring if we will be talking about a transient problem. If we have a transient problem, then the ordinary differential equations may be the governing equations of the chemical engineering process.

So, if there is a set of ODEs, they can be written in a compact matrix notation. One can use a compact matrix notation to detect this, to write it down d x d t is equal to A X, in this case X is a vector and A is a matrix and the initial conditions are given. This is vector at t is equal to 0 is X i. So, that will be the initial condition specified for this set of governing equations, which are in the form of ordinary differential equation.

Now, in order to utilize the Eigen value and Eigen vector method, we have to assume the solution form as X is Z e to the power lambda t. So, just lambda is a scalar, Z is a constant vector and lambda being scalar, t is the time. So therefore, we just assume the solution in this form. Now, what do we do? Next, we substitute this into this equation and see what we get.

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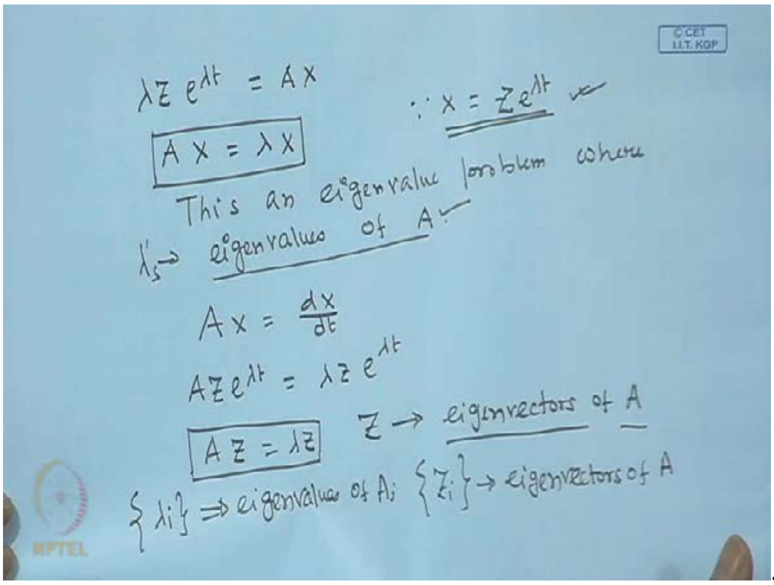

So, if we do that we will be getting d x d t. So, lambda Z e to the power lambda t should be is equal to A X. So, what A X is? A X is nothing but, lambda X because X is nothing but, Z e to the power lambda t. So, if lambda is this scalar given in this form, you will be finally, getting A X is equal to lambda X. Now, I think you will be able to get the, identify the form of this equation, this is a standard Eigen value problem. So, this is an Eigen value problem, where lambda's are nothing but, Eigen values of matrix A.

So, again we write A X is equal to d x d t. So, X we write Z e to the power lambda t. So, it becomes A Z e to the power lambda t and d x d t we write as lambda Z e to the power lambda t. So, A Z is equal to lambda Z. So, what is Z? So, Z is nothing but, Eigen vectors of A. So, if we now see that, what we have assumed the form of the solution, we have assumed as X is equal to Z e to the power lambda t. By doing these algebraic manipulation, we have identified what these lambdas are. These lambdas are nothing but, the Eigen values of A. And what this Z is? These Z's are nothing but, the Eigen vectors of A. So, we write it, lambda i’s are the Eigen values of A, the corresponding set of zeta’s Z i’s are nothing but, the Eigen vectors of A.

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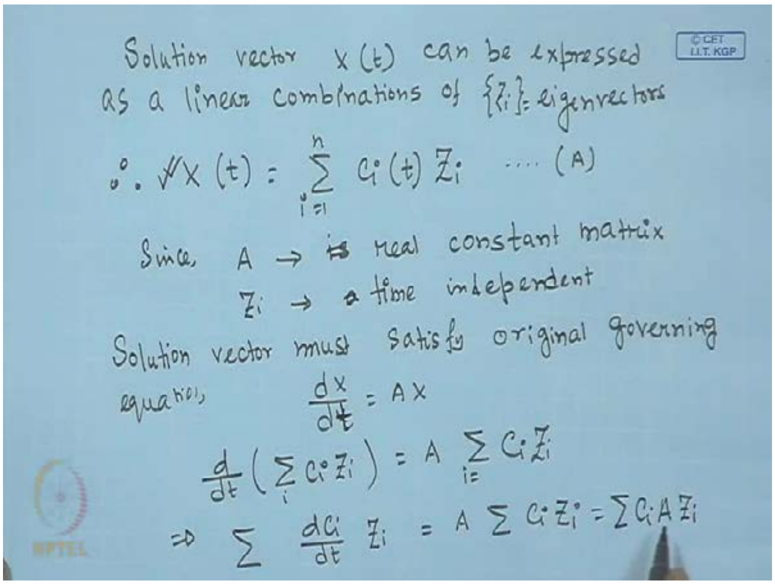

Therefore, the solution form whatever we are assuming, we identified this unknowns, lambda’s are the Eigen values of A, Z’s are the corresponding Eigen vectors. So therefore, the solution vector X t can be expressed as a linear combinations of Z i’s the Eigen vectors. Because we have already proved earlier that Eigen vectors are always independent vectors. So, Eigen vectors form a basis set vector, that is the reason the solution vector, that will be belonging to the same space, can be expressed as a linear combination of the Eigen vectors,

So, therefore, we will be able to write X as a function of time as summation i is equal to 1 to n c i some constant multiplier, which may be a function of t, multiplied by Z i, this let us say equation number A. Now, in this case since A is a real constant matrix, then Z i are time independent because they are the Eigen vectors, they do not depend on the time. Because the matrix itself is a constant matrix, time independent constant matrix. So therefore, the Eigen vectors will also be time independent.

So therefore, X in the equation A must also, this is the solution vector, the solution vector must satisfy the original equation, must satisfy original governing equation. And what was that? That was d x d t should be is equal to A X. So, we write d d t of x, x is summation C i Z i. So, it will be summation C i Z I, this summation is over i, A times X, X is nothing but, summation C i Z i.

So therefore, since Z is independent of time, so this will be nothing but, summation d C i by d t Z i is equal to A summation C i Z i. So, this will be just C i is a scalar multiplier, it may be time dependent. So, C i will be out. So, A Z i. So, we have already seen that A Z i is nothing but, lambda Z i. We have already seen in a few minutes back that A Z i is nothing but, lambda i Z i. So, we substitute that here. So, what we will be getting is that, d C i d t Z i is minus summation C i lambda i Z i is equal to 0. So, we can take the summation series out, i is equal to 1 to n d C i d t minus lambda i C i Z i is equal to 0.

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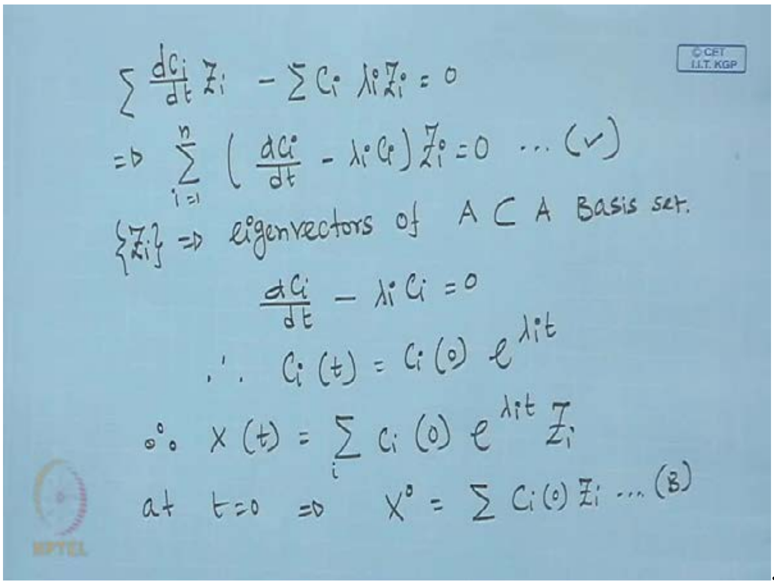

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Now, you remember Z i's are the Eigen vectors of the matrix A, that means they always form a, they belong to a basis set. And what the basis set is? The basis set is a set of independent vectors. So therefore, in order to satisfy this equation, all the corresponding coefficients should be equal to 0, we have already proved this theorem earlier.

So, therefore, to satisfy this equation, all the corresponding coefficients should be is equal to 0. So, d C i d t minus lambda i C i should be equal to 0. So, what is the solution to this? The solution is very simple. So, C i as a function of t will be C i at time t is equal to 0, that is C i at t time t is equal to 0 initial condition e to the power lambda i t. And therefore, you can write down the complete solution X as a function of t is nothing but, summation C i e to the power lambda i t times Z i, where Z i at the Eigen vectors at time t is equal to 0. We have X 0 that is known to us, this initial condition set is known to us. This is nothing but, summation C i 0 Z i. So, let us say this is equation number B. Now, let us consider that Y i are the Eigen vectors of A transpose. So, these are all talking about the Eigen vectors of A.

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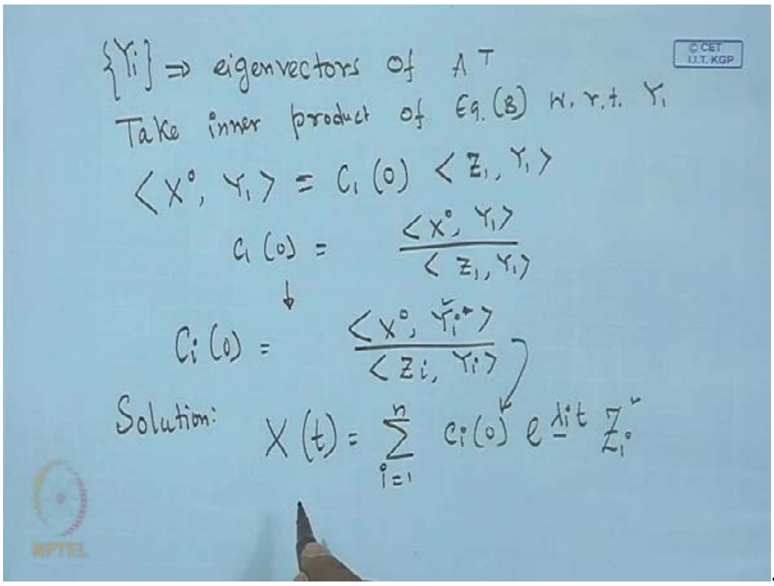

Now, let us say Y i forms a set of Eigen vectors of the matrix A transpose, then what do you do? You take inner product of equation B with respect to Y 1 and let us see what we get. So, inner product of equation B with respect to Y 1 will give you inner product of X naught Y 1 should be is equal to C 1 naught inner product of Z 1 and Y 1. So, C 1 naught we can get as inner product of X naught Y 1 divided by inner product of Z 1 Y 1. So, likewise we can generically you can write as C i 0 is inner product of X naught Y i 0 divided by Z i Y i.

So, solution becomes the complete solution now becomes X t, the time dependent solution vector is nothing but, summation i is equal to 1 to n C i naught e to the power lambda i t Z I, where lambda i's are the Eigen values of A known to us, Z i's are the corresponding Eigen vectors of A matrix, they are already known to us. And C i naught will be obtained from this. And if you look into the expression of C i naught, this is inner product of X naught and Y i. So, X naught is known to us, that is the initial value vector and Y i is the Eigen vector of A transpose. So, since we know, we have already found out the Eigen vectors of A transpose. So, this is known to us. Z i are the Eigen vectors of

A, Y i are the Eigen vectors of A transpose. So, they are known to us. So, we can compute completely what C naught C i naught is, that will be put here, lambda is unknown, Z i unknown. So, one can we get the complete solution matrix. That is how one can obtain the solution of set of ordinary differential equation in using Eigen value Eigen vector method.

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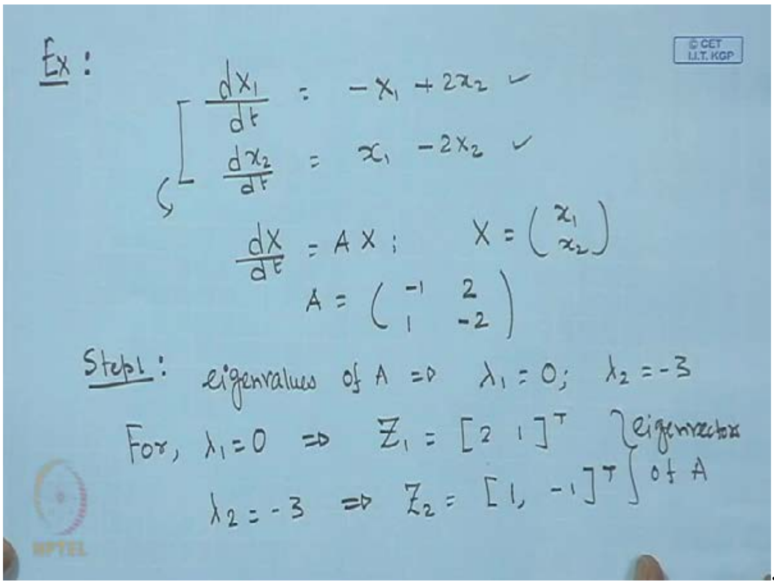

Next, I will take up an example to demonstrate this method. Again I will be taking up a two-dimensional problem. So, that this can be solved in the class here but, one can do the similar thing for a multivariable, multi skilled problem, multi dimensional problem using numerical techniques.

So, d X one d t is equal to minus X 1 plus 2 X 2, d x 2 d t is equal to X 1 minus 2 X 2. Suppose these two are the corresponding, the governing equation of a transient chemical engineering problem, so we write these two equation into a compact matrix notation d x d t is equal to A X, where X is the solution vector, X 1, X 2 and A is the coefficient matrix, that is minus 1 2 1 minus 2.

Now, let us go step by step. Step one is evaluation of Eigen values. Eigen values of A will be, I am not, we have already seen how to evaluate Eigen values. So, the Eigen values are in this case is lambda 1 equal to 0 and lambda 2 is equal to minus 3. Now, you can evaluate the corresponding Eigen vectors X 1 capital X 1 and capital X 2, which are basically Eigen vectors of A. So, we have already seen how to do that in detail in the earlier examples, I am just writing the solution.

So, for lambda 1 is equal to 0, the corresponding Eigen vector is 2 1 transpose, specifically 2 1. And corresponding to lambda 2 is equal to minus 3, we will be getting Z 2 is 1 minus 1 transpose. So, these are Eigen vectors of matrix A.

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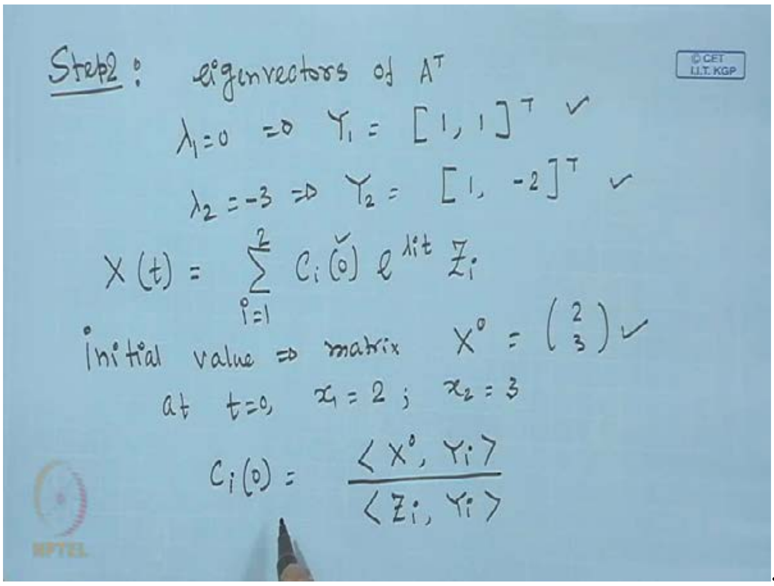

Next, we evaluate that is step number 2. Next, we evaluate in step 2 the Eigen values Eigen vectors of A transpose. And the corresponding Eigen vectors are for lambda 1 is equal to 0, the Eigen vector is 1 1 transpose and for lambda 1, for lambda 2 is equal to minus 3 Y 2 is 1 minus 2 transpose. So therefore, one can construct the solution vector as X t is summation of C i 0 e to the power lambda i t Z i, where i goes from 1 to 2. So, once we get that, then next step is how to obtain the C i's

Now, in this particular problem, let us say the initial value matrix is given as X naught 2 and 3, that means at time t is equal to 0, X 1 is equal to 2 and at the time t is equal to 0, my X 2 was 3. So, initial value matrix is given by 2 and 3, then you will be in a position to compute C i naught's. So, this C i naught's, because now Z i's are all known, you have already determined, lambdas we have already determined, so next is C i naught. C i naught can be determined by inner product of initial value matrix 2 2 and 3, inner product between X naught and Y I, that is the Eigen vector of A transpose and divided by inner product between Z i and Y I, Z i is basically the Eigen vector of A and Eigen vector of A transpose. So, we can and all these evaluated Y 1 and Y 2. So, we will be in a position to evaluate C 1 naught and C 2 naught.

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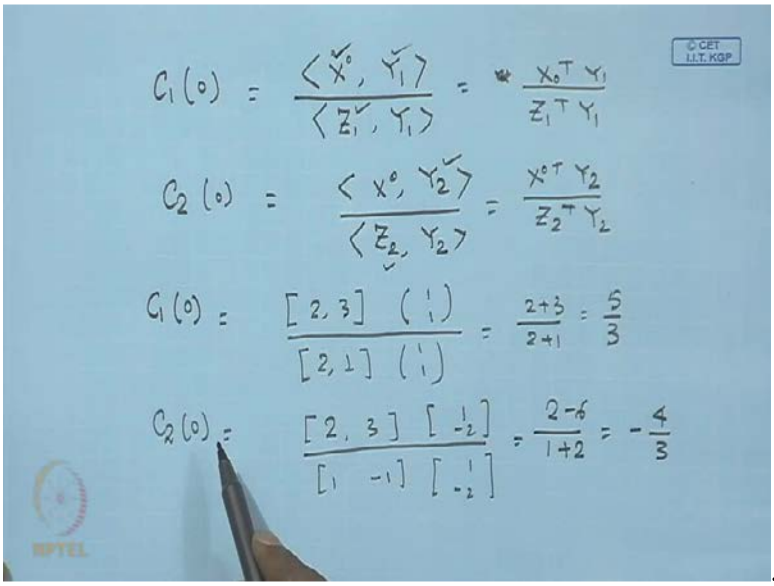

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So, if you remember just put the values of C 1 naught is nothing but, inner product of X naught and Y 1 divided by inner product of Z 1 and Y 1. And C 2 naught is nothing but, the inner product of X naught and Y 2 and divided by Z 2 and Y 2. So, we know the initial value matrix, we know the Eigen vector initial value vector, we know the Eigen vectors of A transpose, we know the Eigen vectors of A, so we will be in a position to compute C 1 and C 2. So, C 1 turns out to be 2 3 1 1. So, basically you can write it as X naught t Y 1 Z 1 t multiplied by Y 1, this will be X naught t Y 2 Z 2 t Y 2.

So, C naught C 1 naught can be written as 2 3 1 1, this will be 2 1 1 1. So, if you compute this thing, this will be 2 into 1 plus 3 into 1, so this will be 2 into 1 plus 1 into 1, this becomes 5 by 3. And C 2 naught will be simply 2 3 1 minus 2 divided by 1 minus 1 1 minus 2. So, this becomes 2 minus 6 divided by 1 plus 2. So, this will be minus 4 by 3.

So, once we have computed C 1 naught and C 2 naught, then we will be able to construct the complete solution. And in the next class we will be looking into the complete solution and finish of this problem, and that will give you a clear demonstration of how Eigen value Eigen vector method can be utilized in order to solve the homogeneous ordinary differential equation, set of ordinary differential equation, by using the using this particular method quite elegantly. And we also, in the next class we will be looking into what is the solution of set of non-homogeneous ordinary differential equation by using the Eigen value Eigen vector method. So, I stop this class at this point and continue in the next class from this point onwards. Thank you very much for your kind attention.