

**Thermodynamics of Fluid Phase Equilibria**  
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**Lecture – 07**  
**Mass – Energy analysis of open system**

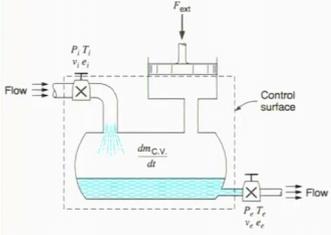
Welcome back. In this lecture we are going to summarize mass energy balance for open system.

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**Conservation of mass principle**

**The conservation of mass principle for a control volume:** The net mass transfer to or from a control volume during a time interval  $\Delta t$  is equal to the net change (increase or decrease) in the total mass within the control volume during  $\Delta t$ .

$$\left( \text{Total mass entering the CV during } \Delta t \right) - \left( \text{Total mass leaving the CV during } \Delta t \right) = \left( \text{Net change in mass within the CV during } \Delta t \right)$$



$$m_{in} - m_{out} = \Delta m_{CV} \quad (\text{kg})$$

$$\dot{m}_{in} - \dot{m}_{out} = \frac{dm_{CV}}{dt} \quad (\text{kg/s})$$

For steady-flow process  $\frac{dm_{CV}}{dt} = 0$

$$\sum \dot{m}_{in} = \sum \dot{m}_{out} \quad (\text{kg/s})$$

So, we start with the conservation of mass principle. For a simple control volume as depicted here the total or the conservation of mass principle says that the net mass transfer to and from a control volume during a interval time delta t is equal to the net change in the total mass within the control volume during delta t or the time change time interval.

So, in a simpler form one can simply write down in expression the total mass entering the control volume during a certain time interval minus total mass leaving the control volume during delta t and that should be same as or that should be equal to net change in mass within the control volume del delta t. Or, in other word  $m_{in} - m_{out}$  is equal to  $\Delta m_{CV}$ , for in the case of rate expression you will be considering dots here and this will be your  $dm$  by  $dt$ .

For a steady-flow process, the mass of the control volume will remain constant and hence this expression this will be 0, this will turned out to be 0 for steady flow process and thus you will have m in minus is equal to m out for a many inlets and outlets this would be a generic expression for the case of steady flow process.

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**Mass balance for steady-flow process**

Many engineering devices such as nozzles, diffusers, turbines, compressors, and pumps involve a single stream (only one inlet and one outlet).

$\dot{m}_1 = 2 \text{ kg/s}$        $\dot{m}_2 = 3 \text{ kg/s}$

CV

$\dot{m}_3 = \dot{m}_1 + \dot{m}_2 = 5 \text{ kg/s}$

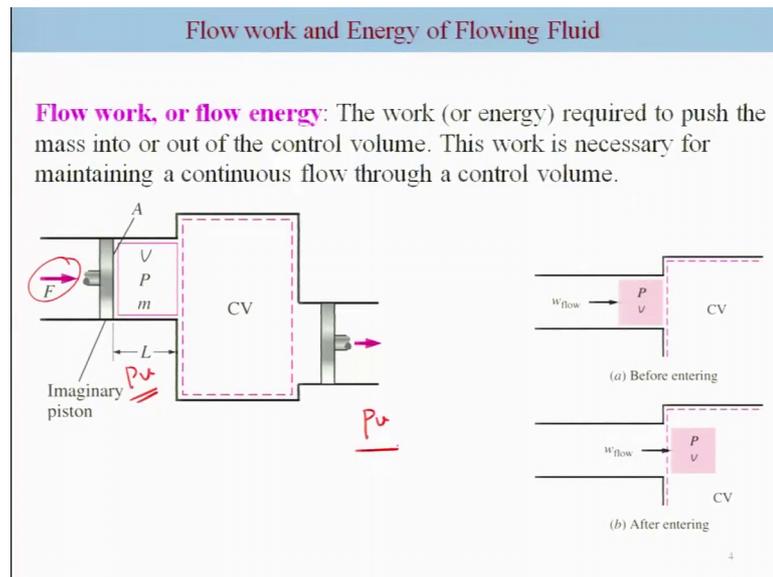
$\sum \dot{m}_{in} = \sum \dot{m}_{out}$   
 $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$

$\dot{m}_1 = \dot{m}_2 \rightarrow \rho_1 V_1 A_1 = \rho_2 V_2 A_2$       Single stream

Now, in many systems of course, there are many inlets and outlets. So, one for simple steady flow process an example would be let us say if you have two inlets you have m 1 and m 2 and if this outlet is 1, then based on simple your m in is equal to m out your m in is m 1 plus m 2 and that should be same as m 3 .

But, in many engineering devices such as nozzles, diffusers, turbine, compressors and pumps they involve only a single stream. In such case you have m 1 is equal to m 2 or the m 1 dot is equal to m 2, and this you can represent in terms of density volumetric flow rate and the cross sectional area as well ok.

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So, since we are talking about energy balance also so and we know that for generic expression the net energy balance would be through heat work as well as from us. So, me just talk about component which we have not discussed earlier.

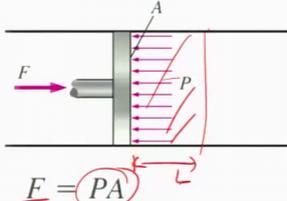
Now, for a flow system and for a steady state system say for a steady-flow system the fluid continuously flows through the inlet and exit through the outlet and in order to maintain the control volume when the steady state there will be work which is needed to continuously flow. Now, there will be work which we need to push the certain mass here before the inlet and similarly, there will be work which will have to be pushed out in order to maintain the continuity.

So, ah, let us consider again ah, so, this is the pressure of volume of the element of the flow which need to be pushed in and this push this work which say is work of or flow work or flow energy. So, be this is before the entering and it is after the entering and so, consider this that this pushing this work is being done by some imaginary piston, ok, there is a certain force which is being applied and this is basically the steady state there is no acceleration and the constant force is being applied and this mass gets in.

So, the question is what is basically the work for such a case, ok. So, me just take a simpler view of it.

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**Flow work**



In the absence of acceleration, the force applied on a fluid by a piston is equal to the force applied on the piston by the fluid.

$$F = PA$$
$$W_{\text{flow}} = FL = PAL = PV \quad (\text{kJ})$$
$$w_{\text{flow}} = PV \quad (\text{kJ/kg})$$

Note: unlike other work quantities, flow work is a product of two properties of a fluid Thus viewed as a flow energy or transport energy!

So, this is again the force and this element here have to be pushed in by a certain distance  $L$  and this force in absence of acceleration is equal to the force applied on the piston. So, this would be equal to the opposite force here assuming this to be a slow process this  $P$  into  $A$  is going to be the force, ok. Now, this work would be the force multiplied by that is an  $L$  and that would be  $PAL$ , that will be  $P$  into volume. Now, in terms of a kilo joules this will be  $P$  into specific volume.

So, this will be the work of flow of course, the  $PV$  value here and  $PV$  value in the outlet may be different, because that will depend also on the pressure of the outlet and the specific volume of the outlet. So, that means, there is a some  $PV$  work here which is associated with it and there is also  $PV$  associated with the outlet.

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**Total Energy of Flowing Fluid**

*Nonflowing fluid*  $e = u + \frac{V^2}{2} + gz$

*Flowing fluid*  $\theta = Pv + u + \frac{V^2}{2} + gz$

The total energy consists of three parts for a nonflowing fluid and four parts for a flowing fluid.

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Now, we can summarize this concept in terms of comparing both the case of non flowing and a flowing fluid for the non flowing fluid you have energy of the fluid as internal energy or energy of the fluid contains internal energy kinetic energy potential energy as shown here. So, this internal energy per unit mass the kinetic energy and the potential energy; for the case of a flowing fluid in addition to that internal energy kinetic energy and the potential energy it also poses a flow energy because it moves inside and certainly outside. So, it contains that energy and we use that energy as a part of the energy of the flowing fluid there is PV or in other word we can also make use of the basic definition this u plus PV comes in the flowing case and this can be written as simply h ok.

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**Mass and Energy balances for a steady-flow process**

**Energy balance**

$$\dot{E}_{in} - \dot{E}_{out} = \frac{dE_{system}/dt}{\text{Rate of change in internal, kinetic, potential, etc., energies}} \overset{0 \text{ (steady)}}{=} 0$$

Rate of net energy transfer in by heat, work, and mass = Rate of net energy transfer out by heat, work, and mass (kW)

$$\dot{Q}_{in} + \dot{W}_{in} + \sum_{in} \dot{m} \left( h + \frac{V^2}{2} + gz \right) = \dot{Q}_{out} + \dot{W}_{out} + \sum_{out} \dot{m} \left( h + \frac{V^2}{2} + gz \right)$$

Useful when magnitude and directions of heat and work transfers are known!

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So, again we go back to the generic energy balance that is  $e_{in}$  minus  $e_{out}$  is equal to  $e_{dE_{system}}/dt$  and we are considering the rate expression considering is a flow process. Now, in this case the net energy transfer is heat work and due to heat work and mass ok; for the case of a steady state of course, the  $e_{in}$  is equal to  $e_{out}$ , so, if we know the directions of various different interactions we can write this expression where  $Q_{in}$  plus  $Q_{out}$  this part of  $E_{in}$  plus whatever the contribution due to the flow  $ah$ , that is, mass that could be your enthalpy plus kinetic energy plus  $gz$  potential energy and that should be equal to  $Q_{out}$   $W_{out}$  plus the energy contained by the flowing mass. So, that would be  $m$   $h$ .

Now, this is summation is for each inlet and summation here is for each outlet. So, this is for the case where we know the direction here.



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**Nozzles and Diffusers**

$V_1 \rightarrow$  Nozzle  $\rightarrow V_2 \gg V_1$

$V_1 \rightarrow$  Diffuser  $\rightarrow V_2 \ll V_1$

Nozzles and diffusers are shaped so that they cause large changes in fluid velocities and thus kinetic energies.

$\dot{E}_{in} - \dot{E}_{out} = \frac{dE_{system}}{dt} \overset{0 \text{ (steady)}}{=} 0$   
Rate of net energy transfer by heat, work, and mass      Rate of change in internal, kinetic, potential, etc., energies

(since  $\dot{Q} \cong 0$ ,  $\dot{W} = 0$ , and  $\Delta pe \cong 0$ )

$\dot{E}_{in} = \dot{E}_{out}$

$\dot{m} \left( h_1 + \frac{V_1^2}{2} \right) = \dot{m} \left( h_2 + \frac{V_2^2}{2} \right)$

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So, for example, nozzles and diffusers are typically used to change the fluid velocity and kinetic energy where there is no heat interaction and the work there the body does not change. So, that is your Q and W's are zeros, delta p e are going to be also considered to be 0. So, you can apply this basic definition and you get E in minus is equal to E out or in the other word  $m h_1 + \frac{V_1^2}{2}$ ,  $V_2$ ,  $V_1$  square by 2 is equal to  $m h_2 + \frac{V_2^2}{2}$  square by 2. Now, you can do this analysis knowing that inlet and outlet conditions and the ones unknown variables you can find out by using simple algebra ok.

So, the this is the final expression of that and you can do the analysis subsequently.

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### Turbines and Compressors

**Turbine** drives the electric generator in steam, gas, or hydroelectric power plants.  
*- work producing device*

**Compressors**, as well as **pumps** and **fans**, are devices used to increase the pressure of a fluid.  
*Work is supplied to these devices from an external source through a rotating shaft.*

A **fan** increases the pressure of a gas slightly and is mainly used to mobilize a gas.

A **compressor** is capable of compressing the gas to very high pressures.

**Pumps** work very much like compressors except that they handle liquids instead of gases.

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Now, the similar kind of exercises you can do for turbine which is a work producing device for compressor fans and pumps again where the work is needed in order to change the pressures. So, you can do this exercises again based on very simple logics ok.

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### Example

$\dot{E}_{in} - \dot{E}_{out} = \frac{dE_{system}}{dt} \overset{0 \text{ (steady)}}{=} 0$   
 Rate of net energy transfer by heat, work, and mass = Rate of change in internal, kinetic, potential, etc., energies

$\dot{E}_{in} = \dot{E}_{out}$   
 $\dot{W}_{in} + \dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2 \quad (\text{since } \Delta ke = \Delta pe \cong 0)$   
 $\dot{W}_{in} = \dot{m}q_{out} + \dot{m}(h_2 - h_1)$

Applicable to various devices such as  
 - Throttling valves, mixing chamber, heat exchanger...

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For example, this is the case for compressor where air is being compressed from 100 kilopascal to 600 kilo Pascal whereas, the flow rate of mass is given here 0.02 kg per second and what we need to find out is the work which is required the rate of work which is required and in this process the heat also lost.

So, you can start with again the basic definition of energy balance here, where is it this is a steady state then you can consider this and then you can write this  $E_{m,i}$  is equal to  $E_{out}$ . Now, knowing the direction here already we can simply write  $W_{in}$  plus the flow contribution  $m h_1$ , this is going to be equal to  $Q_{out}$  plus  $m h_2$  ok. So,  $Q_{out}$  is here this part plus  $m h_2$  is due to this. So, you can rewrite this expression in this form.

Now, this you can find it from the tables or you can use  $C_p dT$  this if you know the expression in terms of  $C_p$  in terms of temperature and  $Q_{out}$  also of course, we know that. Now, this kind of analysis is of course, applicable to other cases throttling valves, mixing, heat exchanger and so forth. So, I am not again going to in details of that you can apply and revisit some of the example to hone your skills again.

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**Energy Analysis of Unsteady-Flow Process**

Many processes of interest, however, involve *changes within the control volume with time*. Such processes are called *unsteady-flow*, or *transient-flow*, processes.

- Discharge of a fluid from pressurized vessel.
- Inflating tires or balloons

Charging of a rigid tank from a supply line is an unsteady-flow process since it involves changes within the control volume.

The shape and size of a control volume may change during an unsteady-flow process.

Now, I will just talk about the case where we have unsteady flow process which is also very common. These are the processes where the changes in the within the control volume with time such processes are called a transient flow process or as I said unsteady flow, ok. The examples are discharge of the fluid from pressurized vessel, inflating tires or the balloon. So, for example, here in this case during this process of the filling of the control volume this piston can also move ok.

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**Energy Analysis of Unsteady-Flow Process**

Most unsteady-flow processes can be represented reasonably well by the *uniform-flow process*.

**Uniform-flow process:** The fluid flow at any inlet or exit is uniform and steady, and thus the fluid properties do not change with time or position over the cross section of an inlet or exit. If they do, they are averaged and treated as constants for the entire process.

Mass balance  $m_{in} - m_{out} = \Delta m_{system} \quad (\text{kg})$

$$\Delta m_{system} = m_{final} - m_{initial}$$
$$m_i - m_e = (m_2 - m_1)_{CV}$$

$i = \text{inlet}, e = \text{exit}, 1 = \text{initial state}, \text{ and } 2 = \text{final state}$

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So, let me just quickly explain how to work with this kind of system. So, most steady flow processes can be represented by uniform flow process where we assume that the fluid flow at any inlet or exit is uniform and steady and thus the fluid properties do not change with time or position over the cross section of an int inlet or exit and if they do they are averaged and treated as constant for the entire process.

So, for such a case we can simply write mass balance as  $m_{in} - m_{out}$  is equal to  $\Delta m_{system}$ , where  $\Delta m_{system}$  is  $m_{final} - m_{initial}$  sometimes also we write  $m_i - m_e$  is equal to  $m_2 - m_1$  of the control volume ok. So, this is our mass balance.

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**Energy Analysis of Unsteady-Flow Process**

**Energy balance**

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{system}}_{\text{Change in internal, kinetic, potential, etc., energies}} \quad (\text{kJ})$$

$$\left( \underline{Q}_{in} + \underline{W}_{in} + \sum_{in} \underline{m\theta} \right) - \left( \underline{Q}_{out} + \underline{W}_{out} + \sum_{out} \underline{m\theta} \right) = (m_2 e_2 - m_1 e_1)_{system}$$

$$\theta = \underline{h} + \underline{ke} + \underline{pe}$$

$$\underline{e} = \underline{u} + \underline{ke} + \underline{pe}$$

Assuming ke, pe changes of fluid streams and CV are negligible:

$$\underline{Q} - \underline{W} = \sum_{out} \underline{mh} - \sum_{in} \underline{mh} + (m_2 u_2 - m_1 u_1)_{system}$$

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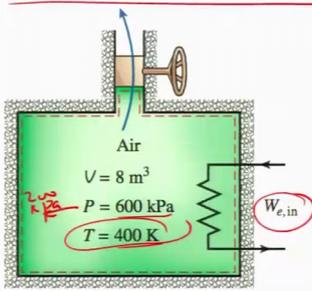
What about the energy balance? so, we will write again here energy balance  $E_{in}$  minus  $E_{out}$  is equal to  $\Delta E_{system}$ . Now,  $\Delta E_{system}$  is not going to be 0, in this case  $\Delta E_{system}$  is  $m_2 e_2$  minus  $m_1 e_1$ , where this contains only the internal energy, potential energy, kinetic energy as shown here whereas,  $E_{in}$  is  $Q_{in}$  plus  $W_{in}$  plus summation of  $m\theta$  where  $\theta$  contains  $h$  because it is a flowing fluid and  $ke$  plus  $pe$ , whereas for the case of outlet is  $Q_{out}$  plus  $W_{out}$  plus summation  $m\theta$  for the outlet.

Now, assuming  $pe$  and  $ke$  changes are negligible, you can write down this generic expression in this form, ok,  $Q - W$  where  $Q$  is net heat minus  $W$  is net work out and this equal to summation  $m_h$  minus summation  $m_h$ , this equal to the contribution due to the changes in the enthalpy from the outlet and inlet and as well as the changes due to the internal energy of the system.

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**Example**

An insulated 8-m<sup>3</sup> rigid tank contains air at 600 kPa and 400 K. A valve connected to the tank is now opened, and air is allowed to escape until the pressure inside drops to 200 kPa. The air temperature during the process is maintained constant by an electric resistance heater placed in the tank. Determine the electrical energy supplied to air during this process.



$$m_2^o - m_e = (m_2 - m_1)_{cv}$$

$$m_e = (m_1 - m_2)_{cv}$$

$$\underline{E_{in}} - \underline{E_{out}} = \Delta E_{sys} \quad \begin{matrix} Q = \\ \Delta KE \\ \Delta PE \\ = 0 \end{matrix}$$

$$W_{e,in} - m_e h_e = m_2 u_2 - m_1 u_1$$

$$v_1 = v_2 = \phi m^3$$

Now, the question is how do you apply this? So, we will take up one example and this we will end the lecture with this example. This is an example where you have an insulated 8 meter cube rigid tank which contains air at 600 kilo Pascal, 400 at 4 400 Kelvin and the wall is connected to the tank which is now opened. Air is allowed to escape until the pressure inside the draw inside the drops to 200 kilo Pascal.

The air temperature during the process is maintained constant. So, this is maintained and by how it is maintained by providing this electrical resistance. So, there is a work electrical work is applied to the system. What we need to find is basically the electrical energy supplied to the air during this process ok. During the process, where the pressure is changed from 600 to 200 Kelvin, keeping the temperature constant.

So, the first thing is since the valve is opened certain mass has escaped. So, we will be applying the mass energy mass balance first followed by energy balance. So,  $m_{in}$  or  $m_{in}$  minus  $m_{exit}$  is equal to  $m_2$  minus  $m_1$  of control volume. Now, this there is no inlet, so, this  $m_{exit}$  is nothing, but  $m_1$  minus  $m_2$ , of control volume.

Now,  $E_{in}$  minus  $E_{out}$  is  $\Delta E_{system}$  ok. What is  $E_{in}$   $W_{e,in}$  here and what is  $E_{out}$  because only the mass flow then we can simply write  $m_e h_e$  ignoring the kinetic energy and potential energy. So, and as well as since it is insulated, so,  $q$   $\Delta E_{KE}$  and  $\Delta PE$  are going to be 0, and this is  $m_2 u_2$  minus  $m_1 u_1$ .

Now,  $V_1$  is constant ok.  $P_1$  we know  $P_2$  we know, now the thing is it is air ok.

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Handwritten calculations:

$$m_1 = \frac{M P_1 V_1}{R T_1} = 41.81 \text{ kg}$$

(Note:  $R = 0.287 \text{ kJ/kg}\cdot\text{K}$ )

$$m_2 = \frac{M P_2 V_2}{R T_2} = 13.94 \text{ kg}$$

$$m_e = m_1 - m_2 = 27.87 \text{ kg}$$

Energy balance equation:

$$\textcircled{2} \Rightarrow W_{e,in} = m_e h_e + m_2 u_2 - m_1 u_1$$

(Note:  $u(400\text{K}) = 286.16 \text{ kJ/kg}$ )

$$W_{e,in} = 400.90 \text{ kJ}$$

Final result:

$$W_{e,in} = 3200 \text{ kJ}$$

So, we can consider air to be ideal gas and thus we can make use of the ideal gas equation to obtain  $m_1$  which will be your  $m = \frac{P_1 V_1}{R T_1}$  and this itself is  $0.287 \text{ kJ/kg}\cdot\text{K}$  and if you plug in the  $P_1$  and  $V_1$ ,  $P_1$  is  $600 \text{ kPa}$ ,  $T_1$  is and  $T_1$  and  $T_2$  is constant,  $V_1$  and  $V_2$  is also constant is  $41.81 \text{ kg}$  ok.

Similarly, you can get  $m_2 = \frac{M P_2 V_2}{R T_2}$  you get  $13.94 \text{ kg}$ . Thus, you can obtain  $m_e$  which is  $m_1 - m_2 = 27.87 \text{ kg}$ . So, now knowing this  $m_e$  we can now plug in in this equation ok. So, so, you can plug in this information  $m_e$  and what about  $h_e$ ?

So, you can plug in  $27.87$ ;  $27.87$  will become  $W_{e,in}$  and you can rearrange here this is  $m_e h_e + m_2 u_2 - m_1 u_1$  ok. The reason where you have taken again this is because this is the  $\Delta E$  of the system which is non flowing is only of the control volume and this  $u$  since this ideal gas is your only depends on the temperature which is  $400 \text{ Kelvin}$  you can use the table given in any book and this comes out to be  $286.16 \text{ kJ/kg}$ .

Similarly,  $h_e$  which  $h_e$  is basically the enthalpy, which can be approximate because this can be enthalpy of the air at  $400 \text{ Kelvin}$ , ok, because enthalpy is the temperatures is a fixed over there and this from the table also we can get there. So, with this information

and with the value  $m_e$  which we have calculated we know  $m_2 - m_1$  as well this value  $W_{in}$  comes out to be 3200 kilojoules ok.

So, with this we complete this lecture and we have just covered the energy balance for the open system. So, we will continue our reviewing process of basic engineering thermodynamics first. So, with that I will end this lecture and I will see you in the next lecture.