

**Thermodynamics of Fluid Phase Equilibria**  
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**Lecture - 35**  
**Partition Function for Independent Particles**

Welcome back. In the last lecture we started discussing statistical mechanics we will continue that discussion. So, today we will start with partition functions for independent particles.

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**Partition Functions for Independent Particles**

**Independent Particles**

- We now consider the partition function for independent particles, i.e., particles that do not in any way interact or associate with other molecules
- We consider two cases: distinguishable and indistinguishable

**Distinguishable Particles**

- Particles that can be differentiated from each other
- The particles could be in some way labeled (e.g. red vs. blue) or kept at a fixed position (e.g. particles in a crystal lattice)

**Indistinguishable Particles**

- Particles that cannot be differentiated from each other
- These particles can interchange locations, so you cannot tell which particle is which (e.g. gas particles)

So, what is a first independent particle? That is the particles that do not or in any way interact or associate with each with other molecules ok. We will consider two cases one is distinguishable another is indistinguishable. So, distinguishable particles are the particles that can be differentiated from each other.

Now, the which essentially you can imagine that the particle could be some way labelled that is red of blue or kept at a fixed position example particle in a crystal that is and you can know exactly which particle it is. But in practice molecular system are mostly in distance distinguishable particle if the particles of same type and that is what we consider another case here the particles that cannot be differentiated from each other are called in distribution particles these particles can interchange locations. So, you cannot tell which

particle is which that is typically the gas particles can they move around and hence they can interchange positions.

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**Partition Functions for Independent Particles**

**Model System**

- Consider a system with energy levels  $E_j$
- The system consists of two independent subsystems with energy levels  $\epsilon_i$  and  $\epsilon_m$

**Distinguishable Particles**

- Label the two systems  $A$  and  $B$
- The system energy is

$$E_j = \epsilon_i + \epsilon_m$$

• Where,  $i = 1, 2, \dots, a$  and  $m = 1, 2, \dots, b$

- The partition function of each subsystem is

$$q_A = \sum_{i=1}^a e^{-\epsilon_i/kT} \quad \text{and} \quad q_B = \sum_{m=1}^b e^{-\epsilon_m/kT}$$

So, we will be considering model system to appreciate this understanding ok. So, let us assume that a model system has energy levels  $E_j$  and  $j$  of course,  $k$  will vary. The system consists of two independent subsystems with energy  $\epsilon_i$  and  $\epsilon_m$  and we are considering independent particles and we are also going to consider distinguishable particles let this systems  $A$  and  $B$ , we are going to label the systems  $A$  and  $B$  because  $A$  and  $B$  corresponds to two independent subsystems.

So, imagine that you have a system which you can separate as  $A$  and  $B$  ok; and the in such a way that they are independent and hence you can just focus on the energy levels simply summing up for each energy levels can be sum up as subsystems and energy levels simply  $E_j$  is small  $\epsilon_i$  of system  $A$  and this is of system  $B$  subsystem, where  $i$  can vary and  $M$  can vary from 1 to  $b$ ,  $i$  can vary from 1 to  $a$ .

Now, given this you know that given this you can calculate the partition function for each subsystem. Partition function for  $A$  would be simply this summing out from 1 to  $a$ , because  $i$  varies from 1 to  $a$ , and  $b$  would be simply this where we are summing up the Boltzmann factor from 1 to  $b$  ok.

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Partition Functions for Independent Particles

**Distinguishable Particles**

- The partition function for the entire system is

$$Q = \sum_{j=1}^r e^{-E_j/kT} = \sum_{i=1}^a \sum_{m=1}^b e^{-(\epsilon_i^A + \epsilon_m^B)/kT} = \sum_{i=1}^a \sum_{m=1}^b \underbrace{e^{-\epsilon_i^A/kT}} \underbrace{e^{-\epsilon_m^B/kT}}$$

- Because the subsystems are independent and distinguishable by their labels, the sum over  $i$  levels of  $A$  has nothing to do with the sum over  $m$  levels of  $B$
- Therefore, the partition function can be factored

$$Q = \underbrace{\sum_{i=1}^a e^{-\epsilon_i^A/kT}} \underbrace{\sum_{m=1}^b e^{-\epsilon_m^B/kT}} = \underline{q_A} \underline{q_B}$$

- More generally, when we have  $N$  independent and distinguishable particles the partition function simplifies to

$Q = q^N$ 

 $(q_1) \dots (q_N)$

Now, if you want to find out the entire system partition function of the entire system we will be simply summing up the Boltzmann factor where this single summation will be replaced by a double summation by replacing  $E_j$  as  $E_i^A$  plus  $E_m^B$  and  $i$  will vary from 1 to  $a$ ,  $m$  will vary from 1 to  $b$ .

Now, consume this to be independent you can segregate this ok, and then since this is you know because of the independent you can separate it out in this way and this way and this is nothing, but  $Q_A$  and  $Q_B$  because this summation has nothing to do with  $i$  and this one has nothing to do with  $m$  here ok. So, considering the independent you can independent particles you can simply write  $Q$  as a simple partition function multiplication of the partition function of subsystems.

Now, if there are many subsystems or if you are considering  $n$  independent and distinguishable particles then you can extend this exercise and you can write  $Q = q_1 \dots q_n$  here which essentially means since they are independent and distinguishable and have the same type and hence each of them will be same and thus you can write simply  $Q = q^n$  this is what we are going to consider otherwise you can simply write as  $Q = q_1 \dots q_n$  ok.

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**Partition Functions for Independent Particles**

**Indistinguishable Particles**

- There are *no labels A or B* the particles from each other
- The system energy is

$$E_j = \epsilon_i + \epsilon_m$$

- Where,  $i = 1, 2, \dots, t_1$  and  $m = 1, 2, \dots, t_2$
- The system partition function is

$$Q = \sum_{i=1}^t e^{-E_j/kT} = \sum_{i=1}^{t_1} \sum_{m=1}^{t_2} e^{-(\epsilon_i + \epsilon_m)/kT}$$

- The summation can no longer be separated
- As a result of performing the full summation, you overcount by a factor of  $2!$
- Therefore, for  $N$  indistinguishable particles, the partition function evaluates to

$$Q = \frac{q^N}{N!}$$

Now, let us consider the complexity when you cannot segregate as a distinguishable particle ok. So, let us consider in that case. So, when the particles are indistinguishable then you cannot put any labels A and B because the particles are not distant distinguishable. So, though this no other energy of the system will still be like this, but you cannot segregate ok. So, you can write out this e as this and here i can also vary from let us say 1 to  $t_1$  and this 1 to  $t_2$ , but the summation no longer can be separated. In that as a result what you have done is you are going to over count the summation by a factor of two factorial for this two particle system.

In that case you have to divide this by 2 factorial, but for n indistinguishable particle for n indistinguishable particle the partition function in this case can simply be Q to power n divided by n factorial n factorial comes because of the over counting of the summation because you cannot segregate or separate it out the summation. So, this turns out to be the case for indistinguishable particle partition function ok, for n particle system of same type, ok.

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**Thermodynamic Properties and the Partition Function**

**Internal Energy**

- In our original analysis, we postulated that the energy was given by the following summation

$$\langle E \rangle = U = \sum_{j=1}^i p_j E_j = Q^{-1} \sum_{j=1}^i E_j e^{-\beta E_j} \quad \beta = 1/kT$$

- Where, we have introduced the definition  $\beta = 1/kT$
- Now consider the derivative of  $Q$  with respect to  $\beta$

$$\left( \frac{\partial Q}{\partial \beta} \right) = \frac{\partial}{\partial \beta} \sum_{j=1}^i e^{-\beta E_j} = - \sum_{j=1}^i E_j e^{-\beta E_j}$$

- Comparing the previous two expressions gives

$$U = - \frac{1}{Q} \left( \frac{\partial Q}{\partial \beta} \right) = - \left( \frac{\partial \ln Q}{\partial \beta} \right)$$

So, with this we can now extend and try to understand how do we calculate energy internal energy from the Boltzmann distribution ok. So, in our original analysis we proposed we postulated that the energy can be given by simply average of  $E$  and it will be simply the first moment of the probability distribution weighted by first moment of probability distribution which is nothing, but summation of  $p_j E_j$ .

Now, we can replace  $p_j$  by  $E_j$  to power minus beta  $E_j$  divided by  $Q$  which can be written in this way.

Now, let me, so what we have introduced here is beta which is nothing, but  $1/kT$  which is written here. Now, if we take the derivative of this partition function  $d \ln Q$  by  $d\beta$  term it turns out that this is nothing, but if you take it the  $E_j$  comes out here is nothing, but this part ok.

Now, if you compare this to term here it appears that you can write this expression simply minus  $1/Q$ ,  $dQ/d\beta$  or in other word minus  $d \ln Q$  by  $d\beta$  ok. So, this is a simple way simpler or compact way of this expression which we have calculated.

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**Thermodynamic Properties and the Partition Function**

**Internal Energy**

- In terms of the temperature

$$U = kT^2 \left( \frac{\partial \ln Q}{\partial T} \right)$$

**Internal Energy for Independent Particles**

- For a collection of independent and distinguishable particles

$$Q = q^N$$

- The average energy  $\langle \varepsilon \rangle = \frac{U}{N}$  is given by

$$\langle \varepsilon \rangle = kT^2 \left( \frac{\partial \ln q}{\partial T} \right) = - \left( \frac{\partial \ln q}{\partial \beta} \right)$$

In terms of energy you also in terms of temperature you can write this as you can replace beta by k t 1 by k t and then this, you can show that U is nothing, but k t squared del n and then Q by del t.

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**Thermodynamic Properties and the Partition Function**

**Entropy**

- We previously defined the entropy as

$$S = -k \sum_{j=1}^i p_j \ln p_j$$

- Substituting with  $p_j$  given by the Boltzmann distribution law gives

$$S = -k \sum_{j=1}^i \left( \frac{1}{Q} e^{-E_j/kT} \right) \left( \ln \left( \frac{1}{Q} \right) - \frac{E_j}{kT} \right)$$

- This expression simplifies to

$$S = k \ln Q + \frac{U}{T} = k \ln Q + kT^2 \left( \frac{\partial \ln Q}{\partial T} \right)$$

For the case of an independent particle I can write an independent distinguishable particle, I can write Q is equal to Q by n ok so, for case of indistinguishable particle will be writing Q n by n factorial ok. So, for the case of this which is independent and distinguishable particle this is Q and then I can find out what is the average energy U by

n average energy per particle we can plug in here this expression or in the other case and this can be shown to be in this way ok.

Similarly, I can write entropy you can plug in p here in this way and this can be shown to follow this expression, ok.

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**Thermodynamic Properties and the Partition Function**

**Free Energy**

- Combining  $U$  and  $S$  via the expression

$$F = U - TS$$

- Gives,

$$F = -kT \ln Q$$

- This expression is often referred to as the bridge equation

Similarly, you can extend this exercise you can write U as in this form you can show that F is nothing, but minus kT and then Q which basically relates the free energy to the macroscopic property of the partition function and sometimes called bridge equation.

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**Thermodynamic Properties and the Partition Function**

|                       |   |
|-----------------------|---|
| Internal Energy       | $U = kT^2 \left( \frac{\partial \ln Q}{\partial T} \right)_{V,N}$           |
| Entropy               | $S = k \ln Q + kT^2 \left( \frac{\partial \ln Q}{\partial T} \right)_{V,N}$ |
| Helmholtz Free Energy | $F = -kT \ln Q$   |
| Chemical Potential    | $\mu = -kT \left( \frac{\partial \ln Q}{\partial N} \right)_{T,V}$          |
| Pressure              | $p = kT \left( \frac{\partial \ln Q}{\partial V} \right)_{T,N}$             |

*Handwritten notes:*  $\epsilon \epsilon T = U/N$  (with arrows pointing to U and T),  $1 + e^{-\epsilon_0 / kT}$  (with arrows pointing to T and N), and a red squiggly line on the right side of the table.

Now, without deriving in detail all of it, we can summarize this expression in this way where we have written the macroscopic or thermodynamic properties to the microscopic properties or the partition function ok. So, this relates  $U$  to  $Q$   $S$  to  $Q$   $F$  to  $Q$  chemical potential to  $Q$  pressure to  $Q$ . And this is a very useful relations to connect to the molecular information's to the thermoelectric properties and there is some time also using molecular simulation to evaluate thermal properties ok.

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**Example: The Schottky two-state model**

Consider a system that has  $N$  independent and distinguishable particles with two energy levels for each particle: a ground state with energy zero, and an excited state with energy  $\epsilon = \epsilon_0 > 0$ . Determine the average particle energy  $\langle \epsilon \rangle$ , the heat capacity  $C_V$ , the entropy, and the free energy per particle from the partition function.

$Q = \mathcal{V}^N$

$q = 1 + e^{-\frac{\epsilon_0}{kT}}$

$\langle \epsilon \rangle = \frac{\epsilon_0}{2}$

(a) Average Energy

(b) Heat Capacity

Now, having this information you can also apply to many different cases or many different examples Schottky two state model is a classical one. So, here I will not go through the how to solve this problem, but this is a typical problem which you can try to attempt. So, here you consider a system that has a  $n$  independent and distinguishable particle; that means, you do not have to worry about dividing by the over counting. So, it will be simply  $q$  to the power  $n$  the partition function and it has two energy level for each particle a ground energy state with 0 particle and an excited state with  $\epsilon_0$  which means  $q$  is nothing, but  $1 + e^{-\epsilon_0/kT}$ .

Now, determine the average particle energy of course, you can calculate this which is  $U$  by  $N$  and this is something which we have done it earlier we have this is what we have we can use  $q$  we plug in  $2 + e^{-\epsilon_0/kT}$  and  $Q$  is here  $1 + e^{-\epsilon_0/kT}$  and from here you can obtain the expression  $U$  by  $N$  which is going to be average  $\epsilon$  ok.

Similarly, you can try to get a heat capacity entropy free energy per particle from all this exercise and using the expressions here ok. And this is a typical expressions which turns out to be as a function the temperature which you can show that this indeed is directly from these calculations ok. So, as I said without going into details you are welcome to do this exercise this will help you to further appreciate the molecular information or pictures or how to evaluate the thermodynamic properties from statistical mechanics principles.

So, now I am going to extend this exercise and summarize what is an ensemble.

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What is an Ensemble?

**Ensemble**

- The collection of all possible microstates, or snapshots, of the arrangements of a system
- 'Ensemble' is also used to describe the constraints placed on a system
- Canonical ensemble: constant  $(T, V, N)$
- Isobaric-isothermal ensemble: constant  $(T, p, N)$
- Grand canonical ensemble: constant  $(T, \mu, N)$
- Microcanonical ensemble: constant  $(E, V, N)$

Now, this is something which we commonly use to connect to the macroscopic properties. So, let me just first define an ensemble. Ensemble is the collection of all possible microstates or snapshots of the arrangement of a system ensemble is also used to describe the constraint placed on a system. So, what are the typical common ensembles? Canonical ensemble where we keep it T V N constant. Isobaric isothermal, ensemble where we keep T p N constant and grand canonical ensemble where we keep and where we keep T V mu; that means, chemical potential constant micro canonical ensemble which is a common the ensemble where basically energy volume N, N is called as far as the expressions are concerned.

But from the experimental point of view these two are the most commonly used in experimental conditions and that is why we need to know how to evaluate the probability distribution in these two ensemble, ok. So, as I said we if you want to find out the

probability distribution we have to minimize the relevant thermodynamic property as we have done for almost free energy for the case of N V T we found the Boltzmann distribution as a probability distribution for that e to the power minus beta e divided by Q. For the case of N p T we have to also consider other constraints and use Lagrange multipliers to solve it.

Let me first try to explain the microcanonical ensemble and then we can try other ensemble.

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Microcanonical Ensemble

**An Isolated System**

- The microcanonical ensemble corresponds to an isolated system. i.e., one that is not connected to a thermal, pressure, or particle bath
- The microcanonical ensemble is qualitatively different than other ensembles it involves no fluctuations of any extensive property across the boundary
- The equilibrium probability distribution is obtained by finding the probabilities that maximize the entropy with the constraint that the probabilities sum to one
- Using the Lagrange multiplier method, we get

$$dS = -k \sum_{j=1}^{\Omega} \left[ (1 + \ln p_j^*) + \alpha \right] dp_j^* = 0$$

- This expression is satisfied when

$$p_j^* = \frac{1}{\Omega} = \frac{1}{W(U, V, N)}$$

So, micro canonical ensemble is nothing, but an isolated ensemble because energy is fixed and the important thing is not to be connected to any thermal pressure or particle bath.

So, micro canonical ensemble is qualitatively different from other ensembles because it involves no fluctuation of any extensive property across the boundary. So, the equilibrium property distribution is one that is obtained by finding the probability that maximizes the entropy with the constraint that the probability sum to one that is something which we have already done in the last lecture ok.

I remember that we have tried to use the Lagrange multiplier method where we have used dS is equal to this expression we have used the maximization of entropy with just one constraint which is this ok, where summation of p\_i is equal to 1 and this expression satisfies the fact that this is nothing 1 by t the number of energy levels or the number of

degeneracy  $W$  here, the maximum way of arranging the particle having a constant energy  $E$ .

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### Grand Canonical Ensemble

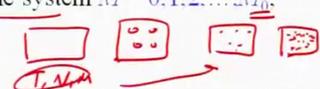
**Constant  $(T, \mu)$**

- The grand canonical ensemble describes a system with diathermal, rigid, and permeable walls, i.e., a system at constant volume connected to a heat and particle bath
- To obtain the appropriate potential function at constant  $(T, \mu)$ , we can Legendre transform from the Helmholtz free energy  $U, V, N$
- A system at constant  $(T, \mu)$  reaches equilibrium when the following function is minimized

$$F - \mu N = U - TS - \mu N = -pV$$

$F \leftarrow U, V, N$   
 $G \leftarrow T, P, N$   
 $T, \mu$

- The macroscopic energy  $U = \langle E \rangle$  is the ensemble average of the energy over all microstates
- The particle number  $N$  is the ensemble average over the microscopic number of particles in the system  $M = 0, 1, 2, \dots, M_{\text{max}}$ , with  $N = \langle M \rangle$



Now you can extend these exercises for grand canonical ensemble. So, as I said I am not going to go into details. So, grand canonical ensemble describe a system with a diathermal rigid and permeable walls. So, it allows the fluctuation the. So, basically there is a therma there is a heat bath across the system the and as well as the particle bath. So, the system is at constant volume connected to the heat and particle bath.

To obtain the appropriate potential function at  $T, V, \mu$  we need Legendre transformation from the Helmholtz free energy Remember that we have done this exercise earlier ok. So, from  $U, V, N$ , we have got  $T, V, N$  right and from or  $U$  or  $S$  and from there we got function a similarly we have got  $T, P, N$  this is function  $H$  and from here we got sorry similarly we got  $T, P, N$  we got a function  $G$ . So, from here we got a function  $G$  ok. And now the question is that if you want to find out from here what would be  $T, V, \mu$ , what would be the appropriate function.

So, again we use the same Legendre transformation and this is going to be  $A$  or sometimes we also write  $F$ . So,  $F$  is Helmholtz free energy minus  $\mu$  times  $N$  because we are changing  $n$  2  $\mu$ . So, minus of  $\mu$  times  $N$  and this is the function which we are going to minimize. So, if you replace  $F$  by  $U$  minus  $TS$  then this is becomes  $U$  minus  $TS$  minus  $\mu N$  this is nothing, but minus  $P, V$ , ok.

So, we can try to minimize this by considering us average  $E$  as we know the expression and  $N$  is of course, fluctuating all the time. So, we have to worry about that as well. So, what kind of a constraints we are going to put? Let us assume that we are going to put  $U$  as average  $E$  and the particle number  $N$  is the ensemble average over microscopic number of particle in the system which can vary from 0 to  $M_0$ , which can be, which can vary which is arbitrary because  $M_0$  could be extremely large. So, the  $N$  is nothing, but average of  $M$

So, remember that the particles can change because you are not fixing the particles you are fixing only the chemical potential, so it can vary. You can have also almost 0 there is no particle inside and you can consider extremely fill system and this will have different probabilities, ok. So, all of them will have  $T V \mu$  as external conditions, ok.

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Grand Canonical Ensemble

Constant  $(T, V, \mu)$

- The free energy of a system  $F - \mu N$  is now given by
 
$$F - \mu N = \langle E \rangle - TS - \mu \langle M \rangle$$
- The differential at constant  $(T, V, \mu)$  is
 
$$d(F - \mu N) = d\langle E \rangle - TdS - \mu d\langle M \rangle$$
- The constraints are:
  - The probabilities must sum to one
  - The energies, summed over energy levels and particle numbers, must give the appropriate average  $\langle E \rangle$
  - The number of particles, summed over all the states of the system, must equal the appropriate average particle number  $\langle M \rangle$

Now, with this we can consider now this function as this and if you want to differentiate this in order to get the minima then we are going to consider  $d$  of  $F$  or in general  $d$  average  $E$   $d$   $TdS$  minus  $\mu$   $d$  average  $M$ . And the constraints are the probability must sums to 0 the energy summed over energy levels and particle number must give the appropriate energy average of  $E$ . The number of particles summed over all the states of the system must equal the appropriate particle number average of  $M$ .

Now, without going how it is done what we are going to do is write down the expressions of the thermodynamic properties and connections to the partition function of this particular ensemble, ok.

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### Grand Canonical Ensemble

Constant  $(T, \mu, \mu)$

- For each microscopic particle number  $M$ , there are  $t_M$  energy states with energies  $E_{Mj}$
- The probability that the system has  $M$  particles and is in energy level  $j$  is given by  $p_{Mj}$
- The constraints can now be written as

$$\sum_{M=0}^{M_0} \sum_{j=1}^{t_M} p_{Mj} = 1$$

$$\sum_{M=0}^{M_0} \sum_{j=1}^{t_M} E_{Mj} p_{Mj} = \langle E \rangle$$

$$\sum_{M=0}^{M_0} \sum_{j=1}^{t_M} M p_{Mj} = \langle M \rangle$$

So, you can write down this constraint in this way again, ok. The double summation is basically for simple reason that for each  $M$  state; that means, for each particle state because the particle state let us say in this case it may have different energy levels ok. So, it may have different energy levels similarly for others may have energy different energy levels and hence you are summing it up the probability of  $M j$  which is the system has  $M$  particle and his energy level  $j$  and you are summing up from  $j$  is equal to 1 to  $t_M$  correspond to energy levels  $M$  and then you are summing up from  $M$  equal to 0 to  $M_0$ , ok.

Similarly, you are getting the average  $E$  by multiplying  $E_{Mj}$  and multiplying by the  $p_{Mj}$  and then you have double summation and similarly for  $\Delta M$  also. So, these are the constraint retain in this form ok. So, you need to solve that.

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### Grand Canonical Ensemble

**Equilibrium Probability distribution**

- We now eliminate the Lagrange multiplier by using the condition that the probabilities must sum to one
- The grand canonical probability distribution is

$$P_{Mj}^* = \frac{e^{-E_{Mj}/kT} e^{M\mu/kT}}{\Xi}$$

- With the grand partition function  $\Xi$  defined as

$$\Xi = \sum_{M=0}^{M_0} \sum_{j=1}^{j_M} e^{-E_{Mj}/kT} e^{M\mu/kT}$$

And after solving this the for the using the Lagrange multiplier we get the grand canonical probability distribution as simply e to power minus E M j divided by kT multiplied by E M mu by kT divided by this partition function the partition function is double summation of this multiplication exponential terms So, this is nothing, but Boltzmann factor and this is the term which connects the particle and the chemical potential E M mu divided by kT, ok.

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### Thermodynamic Properties and the Partition Function

|                 |   |
|-----------------|---|
| Internal Energy | $U = kT^2 \left( \frac{\partial \ln \Xi}{\partial T} \right)_{V, \mu}$                      |
| Entropy         | $S = k \ln \Xi + kT^2 \left( \frac{\partial \ln \Xi}{\partial T} \right)_{V, \mu}$          |
| Particle Number | $N = kT \left( \frac{\partial \ln \Xi}{\partial \mu} \right)_{T, V}$                        |
| Pressure        | $p = kT \left( \frac{\partial \ln \Xi}{\partial V} \right)_{T, \mu} = kT \frac{\ln \Xi}{V}$ |
| Free Energy     | $F - \mu N = -pV = -kT \ln \Xi$   |

Using these expressions we can connect in a similar way as we have done for canonical ensemble this expressions which connects  $U$  to grand canonical partition function. Remember that earlier the  $U$  was connected to the canonical partition function which was  $Q$ , ok and similarly  $S$  similarly  $N$  because  $N$  is a variable and  $N$  is basically fluctuating. So, you can find a  $N$  with the partition function  $p$  and  $N$ . So,  $N$  is basically the variable here ok.

So, with this we can connect the thermodynamic property and the partition function. Now, as I said I have not gone into details how to do that now just because it is not a statistical mechanical course, but it is important to understand how this has been derived or and how we can make use of it by doing a very simple example ok. So, that was the intent for my side to give a exposure of statistical mechanic briefly.

Now, hopefully you understand the complicity as well as the you can appreciate the beauty of this expect of the you know the thermodynamics where we have we connect statistical mechanics to the macroscopic property. In order to further understand this without going too much in details what we are going to do is, we are going to quantify the different models which we are use it in thermodynamic to express the properties the fluids ok.

So, that means we must understand the intermolecular forces and the models. So, that would be the subject of this course in the next few lectures. So, we will see you next time with a new topic here ok.