

Petroleum Reservoir Engineering

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Lecture 12: Volumetric Balance in Oil and Gas Reservoir

Hello everyone, I welcome you again to the class of petroleum reservoir engineering. We are in section 2 of our SLAVERS. In this section 2, we are going to discuss several aspects of the reservoir engineering, mostly in the mathematical sense. In last lecture, we discussed in detail about the general volumetric balance, a tank model where the zero-dimension concept of the volumetric balance was applied and the physical process happening within this tank volume with respect to oil, gas, water and the rock formation was set up in the mathematical form to have the relationship between different parameters. So the volumetric performance equation was set up and that was related to a straight line concept. So let us quickly recap what we learned in the last lecture, then we will continue the discussion for the oil and gas reservoir in today's lecture.

So in the last lecture, we started with a reservoir volume that is a tank volume where the possibility of oil, gas, water and the rock formation is there, where the possibility of change in the volume of oil, gas, water and the rock formation was discussed. We also considered the injection wells. This model was developed under certain assumptions that is the temperature is constant, pressure is in equilibrium within the control volume is considered and the reservoir volume is also constant and the reliable production data are obtained when we are counting how much oil, gas and water produced to the surface. So in general what we discussed in the last lecture, a tank model where N denotes for the oil, G for the gas, W for the water and F for the rock formation.

A parameter M was introduced that is the ratio of initial volume of gas cap to the volume of oil initially in place. Using this parameter, we can eliminate one of the requirement either the oil is known initially present in the reservoir or the gas initially present in the reservoir. If we are having this parameter M known to us, it means we are knowing the ratio of these two important properties of the tank model. So this is the zero dimension model where the change in the volume of the oil, gas, water and rock was established with the volumetric concept and we discussed in detail how to account for the change in the volume of the oil where the N is denoting related to oil. For example, NP this is the cumulative oil produced to the surface while N is the amount of the oil that is initially present in the reservoir that is balanced with formation volume factor to account the conditions to the reservoir condition.

Similar has been set up for the gas where the different process those are happening and those are contributing for the change in the volume of the gas are included in this expression. For example, the oil that is saturated in the reservoir condition within the oil

when the oil is getting produced some amount of the gas is also getting liberated out that is also accounted in this expression. Similar for the water, the influx of the water-by-water encroachment and the produce water to the surface is included in the change in the water volume. Similar the rock and the water that is going through the pressure change so the compressibility of the rock and the water is included in this expression that is going to account for the change in the volume of rock formation because of water and the rock compressibility. Using this equation we could reach to the expression where N the initial oil in place can be explicitly expressed in terms of different parameter those are involved in this tank model.

For example, NP cumulative oil production the amount of the gas that is going to produce is included in terms of M we are having the water term we are having the gas injection we are having the water injection and all the terms are related with respect to the formation volume factor for oil, gas and water that is relating the reservoir condition to the surface condition. Another expression we obtained for the same with the consideration of this gas cumulative gas produced to the surface with respect to cumulative oil produced to the surface with the help of RP that is the cumulative ratio for gas to oil. The compressibility factor is included in the expression knowing the definition of the isothermal compressibility coefficient. Similarly the pore volume that we discussed in the last class is included in this expression to account for the change in the volume or the amount of the pore volume that is occupied by the water and the rock expansion during the production process. Another concept that we discussed about two phase volume formation factor that accounts combinedly the oil volume formation factor as well as gas volume formation factor.

The General Material Balance Equation Zero Dimension

Tank model $(N+G+W+F)$

$m = \frac{\text{Initial volume of gas cap}}{\text{Volume of oil initially in place}}$

$$NB_{oi} - (N - N_p)B_o = -W_e + B_w W_p$$

$$\Delta V_p + \Delta V_g = -\Delta V_w - \Delta V_{rock}$$

$$NmB_{oi} = \left[\frac{NmB_{oi}}{B_{gi}} + NR_{si} - N_p R_p - (N - N_p)R_s \right] B_g$$

$$\frac{NB_{oi}(1+m)}{1-S_{wi}} (S_{wi}c_w + c_f) \Delta p$$

$$N = \frac{N_p [B_o + (R_p - R_s)] B_g - (W_e - W_p B_w) - G_{inj} B_{ginj} - W_{inj} B_w}{(B_o - B_{oi}) + (R_{si} - R_s) B_g + m B_{oi} \left[\frac{B_g}{B_{gi}} - 1 \right] + B_{oi} (1+m) \left[\frac{S_{wi} c_w + c_f}{1 - S_{wi}} \right] \Delta p}$$

$B_t = B_o + (R_{si} - R_s) B_g$

$$N = \frac{\{N_p [B_t + (R_p - R_s) B_g]\} - (W_e - W_p B_w) - G_{inj} B_{ginj} - W_{inj} B_w}{(B_t - B_{ti}) + m B_{ti} \left[\frac{B_g}{B_{gi}} - 1 \right] + \frac{[S_{wi} c_w + c_f]}{1 - S_{wi}} \Delta p}$$

the external side

$G_p = R_p N_p$

$c = -\frac{1}{V} \frac{\partial V}{\partial p} |_{T}$

$\Delta V = V c \Delta p$

$P \cdot V = \frac{NB_{oi}(1+m)}{1 - S_{wi}}$

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Combining this two phase volume formation factor we could reach to the expression where the n is represented in terms of the parameters like the cumulative oil production that can be monitored at the surface, cumulative water production, cumulative gas production and if there is any injection of the water and gas that is going inside the reservoir from the external side. So this overall general material balance equation that we reached to obtain the initial oil in place can further be adjusted to understand what are the dry mechanism those are responsible to produce this oil and gas to the surface. By adjusting this equation we could reached to an expression where this complex expression

can further be divided into four part keeping this in the denominator that is make the right hand side equal to 1. So the contribution of each term in this expression is going to make right hand side equal to 1. So if one term is dominating means others are not effective or it might be a situation when all the factors are contributing in a proportionality but the summation of all the dry mechanism or the index should be equal to 1.

The General Material Balance Equation

Zero Dimension

$$N = \frac{\{N_p[B_t + (R_p - R_{si})B_g]\} - (W_e - W_p B_w)}{(B_t - B_{ti}) + mB_{ti} \left[\frac{B_g}{B_{gi}} - 1 \right] + B_{ti}(1 + m) \left[\frac{S_{wi}c_w + c_f}{1 - S_{wi}} \right] \Delta p}$$

Driving mechanisms and their contribution to the production

$$\frac{N(B_t - B_{ti})}{A} + \frac{N m B_{ti} (B_g - B_{gi}) / B_{gi}}{A} + \frac{(W_e - W_p B_w)}{A} + \frac{N B_{oi} (1 + m) \left[\frac{S_{wi} c_w + c_f}{1 - S_{wi}} \right] (p_i - p)}{A} = 1$$

$A = N_p [B_t + (R_p - R_{si}) B_g]$

DDI = depletion-drive index expansion of the original oil volume with all its original dissolved gas

SDI = segregation (gas-cap)-drive index expansion of the original free gas cap

WDI = water-drive index net encroachment of water

EDI = expansion (rock and liquid)-drive index **be ignored in the derivation.**

and fluid expansion to the oil recovery is too small and essentially negligible and can be ignored

What those dry mechanisms are depletion dry mechanism that is TDI this is because of the expansion of the original oil volume with all its original dissolved gas. Second is SDI that is the gas cap dry mechanism WDI is the water dry mechanism and EDI because of the expansion of the rock and the liquid the contribution of this expansion that is going to produce the reservoir fluid to the surface. The contribution of rock and fluid expansion to the recovery is small and essentially can be ignored in the derivation. So what we are going to discuss in today's class using this material balance equation that is based on the volumetric basis can further be adjusted to get the linear relationship and this linear relationship in terms of f , n and the other parameter is expressed by Havelina and Oh in 1963. This straight-line equation giving us lot of flexibility to calculate the unknown parameters those are present in this equation.

For example the f term here is total fluid withdrawal that is getting produced to the surface. So water oil gas that is producing to the surface is accounted by the f . The contributing factor for example expansion of oil and its originally dissolved gas is included in terms of E_o expansion of the gas cap in terms of E_g and the expansion of connate water and rock in terms of E_{fw} in this expression. So what are the unknown in this equation there are 3 unknown in this material balance equation n that is the initial oil in place $\{W_e\}$ how much amount of the water that is getting influx to the control volume and the ratio of the original gas cap to oil m . So initially how much oil is there initially how much gas is there and how much amount of the water is getting into the control volume are the unknown in this equation.

The General Material Balance Equation

Havlena and Odeh (1963)

Zero Dimension

MBE : Equation of a Straight Line

$$N = \frac{N_p[B_o + (R_p - R_s)]B_g - (W_e - W_p B_w) - G_{inj} B_{ginj} - W_{inj} B_w}{(B_o - B_{oi}) + (R_{si} - R_s) B_g + m B_{oi} \left[\frac{B_g}{B_{gi}} - 1 \right] + B_{oi} (1 + m) \left[\frac{S_{wi} C_w + c_f}{1 - S_{wi}} \right] \Delta p}$$

$$N = \frac{\{N_p[B_t + (R_p - R_{si})B_g]\} - (W_e - W_p B_w) - G_{inj} B_{ginj} - W_{inj} B_w}{(B_t - B_{ti}) + m B_{ti} \left[\frac{B_g}{B_{gi}} - 1 \right] + B_{ti} (1 + m) \left[\frac{S_{wi} C_w + c_f}{1 - S_{wi}} \right] \Delta p}$$

Three unknowns in MBE

- The original oil in place N
- The cumulative water influx W_e
- The ratio of original gas cap to oil m

$$F = N[E_o + mE_g + E_{f,w}] + (W_e + W_{inj} B_w + G_{inj} B_{ginj})$$

$$F = N_p[B_o + (R_p - R_s) B_g] + W_p B_w$$

Production

$$E_o = (B_o - B_{oi}) + (R_{si} - R_s) B_g$$

Oil Expansion

$$E_g = B_{oi} \left[\left(\frac{B_g}{B_{gi}} - 1 \right) \right]$$

Expansion of Gas Cap Gas

F = Total fluid (oil, gas and water) withdrawal

E_o = Expansion of oil and its originally dissolved gas

E_g = Expansion of the gas cap

$E_{f,w}$ = Expansion of connate water and rock

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gas cap expansion the driving force or the pressure energy is maintained to produce the

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So this equation can be adjusted for a different cases depending on the types of the reservoir that is the discussion for today's class. Before we go further to discuss the oil and gas reservoir let us recap again what is this F term that is total production or the withdrawal so that is including for the oil for the gas and for the water. E_o because of the oil expansion and the dissolved gas that is oil expansion factor. E_g is including for the gas cap initially present in the reservoir and because of that gas cap expansion the driving force or the pressure energy is maintained to produce the fluid to the surface and E_{fw} that is accounting for the water and the rock formation. The expression here are given in the single phase volume formation factor those can be converted in the form of two phase volume formation factor B_t and B_{ti} this is for the initial condition this is for any condition of the investigation.

The General Material Balance Equation

Havlena and Odeh (1963)

Zero Dimension

MBE : Equation of a Straight Line

$$N = \frac{N_p[B_o + (R_p - R_s)]B_g - (W_e - W_p B_w) - G_{inj} B_{ginj} - W_{inj} B_w}{(B_o - B_{oi}) + (R_{si} - R_s) B_g + m B_{oi} \left[\frac{B_g}{B_{gi}} - 1 \right] + B_{oi} (1 + m) \left[\frac{S_{wi} C_w + c_f}{1 - S_{wi}} \right] \Delta p}$$

$$N = \frac{\{N_p[B_t + (R_p - R_{si})B_g]\} - (W_e - W_p B_w) - G_{inj} B_{ginj} - W_{inj} B_w}{(B_t - B_{ti}) + m B_{ti} \left[\frac{B_g}{B_{gi}} - 1 \right] + B_{ti} (1 + m) \left[\frac{S_{wi} C_w + c_f}{1 - S_{wi}} \right] \Delta p}$$

Three unknowns in MBE

- The original oil in place N
- The cumulative water influx W_e
- The ratio of original gas cap to oil m

$$F = N[E_o + mE_g + E_{f,w}] + (W_e + W_{inj} B_w + G_{inj} B_{ginj})$$

$$F = N_p[B_o + (R_p - R_s) B_g] + W_p B_w$$

Production

$$E_o = (B_o - B_{oi}) + (R_{si} - R_s) B_g$$

Oil Expansion

$$E_g = B_{oi} \left[\left(\frac{B_g}{B_{gi}} - 1 \right) \right]$$

Expansion of Gas Cap Gas

$$F = N_p[B_t + (R_p - R_{si}) B_g] + W_p B_w$$

Production

$$E_o = (B_t - B_{ti})$$

$$E_g = B_{ti} \left[\left(\frac{B_g}{B_{gi}} - 1 \right) \right]$$

Expansion of Gas Cap Gas

F = Total fluid (oil, gas and water) withdrawal

E_o = Expansion of oil and its original dissolved gas

E_g = Expansion of the gas cap

$E_{f,w}$ = Expansion of connate water and rock

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So with this background that we discussed in the last class let us utilize the concept

$$E_{f,w} = \frac{B_{oi}(1+m)}{1-S_{wi}} (S_{wi} C_w + c_f) \Delta p$$

Expansion of water and reduction in pore volume

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So with this background let us go ahead to discuss the different cases of oil and gas reservoir. So with this background that we discussed in the last class let us utilize the concept of straight line proposed by Havelina and Ohr to understand different cases of oil and gas reservoir. So for example this is the linear equation that is having the relationship on the withdrawal contributing factor and any additional term that is going to the control volume either the water encroachment injection of the water from the surface or injection of the gas. There could be the cases when the gas and water are not injected into the reservoir if that is the situation happening this expression will reduce to a further simplified form that is F is equal to n all the contributing factor plus water encroachment.

So these two term injection of the gas and injection of the water will be withdrawn from this expression.

If the contribution of the formation and water compressibility is negligible or other way the values of C_f and C_w are very less or those can be ignored we can get further simplification of this equation by just withdrawing this term. So if we withdraw this E_{fw} we will get another straight line equation that is little simpler than the previous equation. If the reservoir is having no gas cap initially it is just the oil reservoir or the reservoir that is under saturated reservoir in that case the expression will further be simplified by dropping this term. So we will get F is equal to nE_o plus W . So this is having the contribution towards the production because of oil expansion and because of the water encroachment.

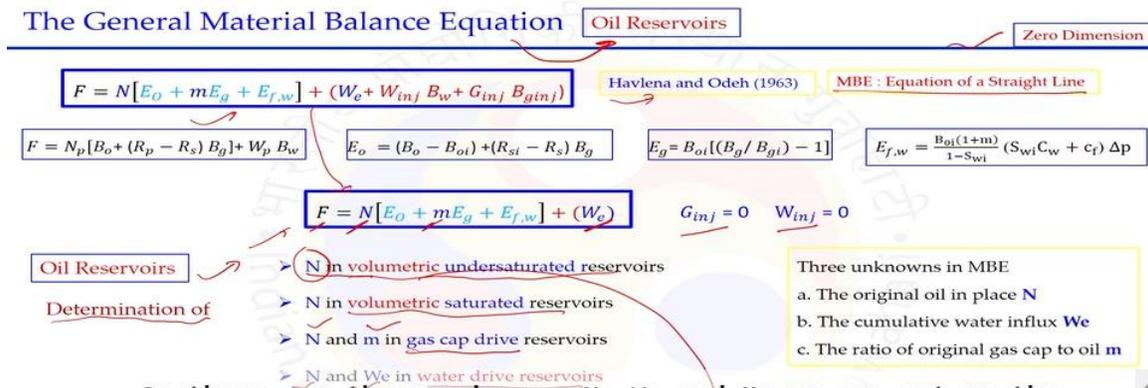
If there is a situation when there is no water influx there is no source of the water that kind of the reservoir is called the volumetric reservoir this term will also further be dropped out and we will get very simple expression F is equal to nE_o . So the volumetric reservoir definition is a situation when neither the water is getting encroached into the control volume or the tank volume or the reservoir volume under consideration and no water is getting produced from the reservoir. In that situation the reservoir is called the volumetric reservoir. Now we ignore this formation and water compressibility coefficient or the contribution because of the water and the rock formation expansion if we include that that should also be included in the expression if it cannot be ignored and then the expression will be changed to this situation. Similarly, this expression could also be included here when no water encroachment is there.

So depending on the situation we can adjust this linear equation to a form that can be utilized to understand the reservoir drive mechanism as well as to estimate the unknown parameter those are not known to us in this straight line equation. Under this situation when the reservoir is volumetric under saturated condition the solubility of the gas in the reservoir as well as the r_p value that is the cumulative gas oil ratio all will be equal and remain same because all produced gas is dissolved in the oil because reservoir is under saturated. So let us recap what we learned about different types of the oil reservoir. So, this is the classification for the oil reservoir. The similar thing under saturated oil reservoir P_i the initial pressure of the reservoir is greater than the bubble point pressure P_b under the saturated condition this P_i is equal to bubble point pressure P_b and the gas cap reservoir that is where the reservoir pressure is below the bubble point pressure and it is in the two phase region and the reservoir is called the two phase reservoir then.

So we discuss the PVT behavior or the thermodynamics behavior of the oil and gas reservoir in detail previously similarly we also discuss in detail about the different drive mechanism those are responsible for the oil and gas production. We discuss those qualitatively in today's discussion using this straight line concept of the volumetric balance. We can discuss them in the mathematical sense and as well as knowing the unknown parameters in the expression. For example the general material balance equation can be reduced to the oil reservoir the equation is same which is including everything this is the straight line equation of the material balance equation this is again emphasizing zero dimension equation we are not counting the fluid flow is happening

any particular direction. Where F, Eo, Eg and Efw are having their usual notation in the comprehensive manner so we did not drop any term this is very general material balance equation.

If we further classified a reservoir where there is no injection of the water and no injection of the gas is happening this expression is simply reduced to this form. So in case of the oil reservoir where there is no gas injection there is no water injection we are having just a reservoir where the pressure is not maintained by injecting the gas and water the oil reservoir could be classified in different form to determine the unknown parameter in this expression. So if we see here the unknown could be N could be M could be We in this case. So if we are having the classification of reservoir under different cases we can get this straight line for to an appropriate situation where we can plot x parameter versus y parameter and the slope and intercept of any linear equation can give us the unknown parameter in that form. So let us see the cases of the oil reservoir the performance equation or the straight line equation can be set up for the determination of N in case of when the reservoir is volumetric under saturated reservoir.

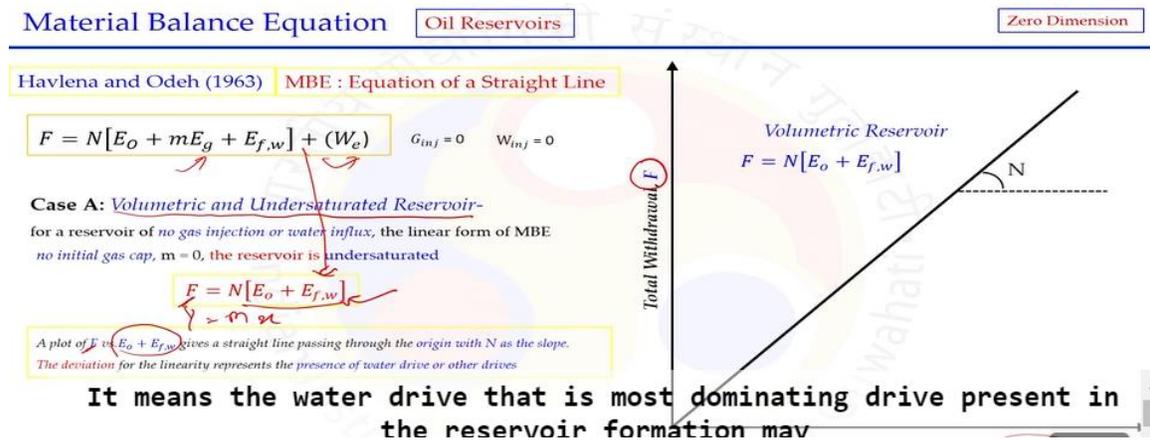


Here expression can be established for the volumetric saturated reservoir when we are having the gas cap drive two terms N and M are unknown when we are having the water drive reservoir no gas cap reservoir then we are having unknown N and We and when the combination of drive mechanisms are responsible to produce the oil we are having three unknown N, M and We. So what is volumetric reservoir? Volumetric reservoir is the reservoir when neither water is encroaching into the reservoir nor the water production is happening. So, We and Me we can set up the equation to estimate this unknown. So the oil reservoir simple equation as the straight line there could be a case when the reservoir is volumetric and under saturated reservoir. It means we are considering the reservoir to be volumetric means no water is getting produced no water is getting encroachment under saturated reservoir means no gas is getting produced.

So the value of M can also be dropped out because the initial gas cap is not present in the reservoir conditions the reservoir pressure is above the bubble point pressure and the reservoir is under saturated. So this simple expression can further be simplified by dropping this term by dropping this We. We means no water encroachment because of the volumetric nature of the reservoir. So why we are doing all these things we can reduce this equation of a straight line to a simplified form and can see the plot of one parameter versus the set of the other parameter if it is resulting straight line or not. If it

results the straight line it means the assumptions that we made to simplify this equation are valid and we can consider the types of the reservoir is following the assumptions that we made and we can classify the reservoir accordingly.

So in case A when we consider the volumetric under saturated reservoir the equation is simplified to this form where we did not ignore the contribution of the rock expansion and the water expansion. So by seeing this equation we can see the plot of F that is on this side versus combination of E_o and $E_{f,w}$ that is here. So this is y is equal to Mx. So if we plot y versus x the straight line should result if this kind of the reservoir is there that is volumetric and under saturated and we should be getting the straight line of this resultant equation. So let us see when we are having F on the y axis combination of E_o and $E_{f,w}$ on the x axis and plotting the data for the volumetric reservoir we are supposed to get the straight line and the slope of that straight line should give us the value of n that is the initial oil present in the reservoir.



Any deviation from this straight line simply says the assumption that we made about the reservoir or the way we simplified the equation is not valid. There is some other dry mechanism also responsible to contribute for the oil production. It means the water drive that is most dominating drive present in the reservoir formation may be contributing significantly towards the production and hence the deviation from this straight line is the indication of not following the assumptions that we made. So this straight line is going to give us the slope n that was the unknown in this expression. Similar thing can be done for the second case.

In second case we consider the volumetric saturated oil reservoir without water influx. So when we say volumetric reservoir means there is no involvement of the water neither water getting engrossed nor it is getting produced. So the expression that is already we simplified for no gas injection no water injection. The case of m is equal to 0 because we are at still at the saturated condition no gas is liberated out from the reservoir oil and in that case m can be considered 0. If we further ignore or not consider the contribution of the rock and the fluid expansion towards the oil recovery in that case the expression will be simplified to f is equal to n eo.

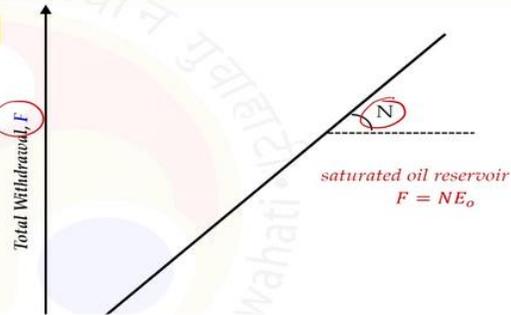
Havlena and Odeh (1963) MBE : Equation of a Straight Line

$$F = N[E_o + mE_g + E_{f,w}] + (W_e) \quad G_{inj} = 0 \quad W_{inj} = 0$$

Case B - Volumetric saturated oil reservoir (without water influx)

$$F = NE_o$$

A plot of F vs E_o gives a straight line passing through the origin with N as the slope.



It means the plot of f versus e_o gives us the straight line that is passing through the origin with n as the slope. So if we plot f versus e_o you remember here in the previous case we plotted e_o plus e_{fw} while here this is just only the e_o and in that case for the saturated oil reservoir the resultant straight line will give us the slope that is initial oil in place. The difference between the previous and this one in this case we also not considering e_{fw} that is we completely ignored or the contribution of rock and water formation expansion is ignored. If we are counting that that is the situation of the under saturated reservoir. Why we can ignore the expression related to rock and fluid expansion because of the pressure change that is happening within the study domain is not significantly and the expression for this rock and water formation factor is including the effect of ΔP .

The third case that we can discuss for this material balance equation for the oil reservoir that is the gas cap drive reservoir means the initial gas cap is there in the reservoir and that is contributing significantly as a drive mechanism to produce the oil. So the case C when has only the gas drive mechanism for the oil production and the size of the gas cap is known. It means the value of M could be known to us or if it is unknown to us we can adjust the equation accordingly to get the value of M. So what could be the situation in case 3 when this is gas cap drive reservoir no water influx is happening and again we are considering the e_{fw} contribution is negligible. In that case what will happen this term will drop out and this term will drop out.

Havlena and Odeh (1963) MBE : Equation of a Straight Line

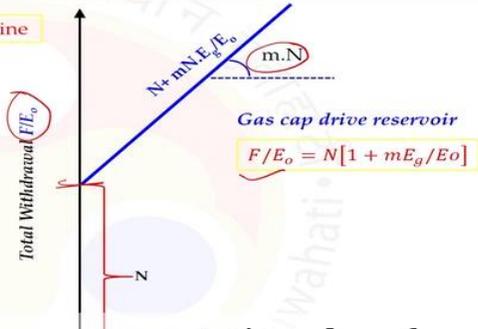
$$F = N[E_o + mE_g + E_{f,w}] + (W_e) \quad G_{inj} = 0 \quad W_{inj} = 0$$

Case C- Gas cap drive reservoir-

has only gas cap drive mechanism for the oil production and the size of the gas cap is known,

$$F = N[E_o + mE_g]$$

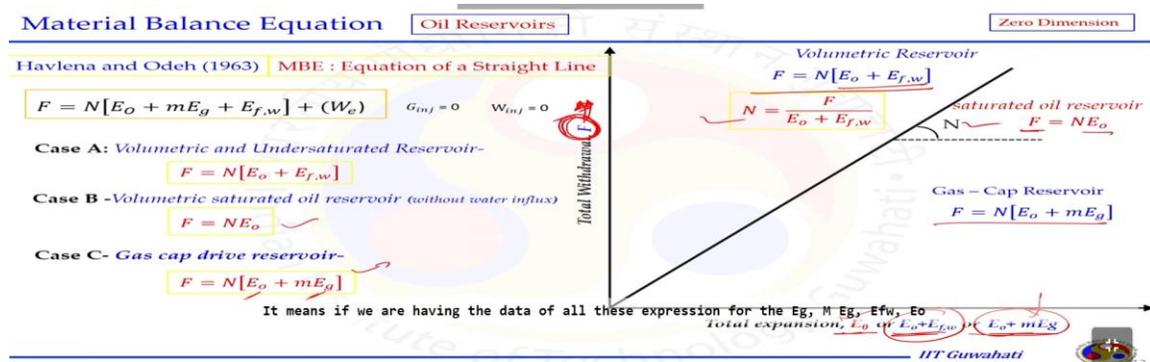
$$F/E_o = N[1 + mE_g/E_o]$$



We will be having the simple expression f is equal to n e_o and m e_g . If we adjust this equation by dividing this expression by e_o on both the side left hand side and the right hand side we will get f divided by e_o is equal to n 1 plus m e_g by e_o . So now if we plot f

by e_o on the y axis and e_g by e_o on the x axis we are supposed to get the expression that is like this one and when we plot this we are going to get the slope of this equation $m \cdot n$ and this equation is also having some intercept that is equal to n initial oil in place. So with this initial oil in place we can get the value of n with the help of this slope we can get the value of m if n is known to us we can replace n here and we are going to get the value of m . So this is the way we can adjust the equation either in this form or in this form to get the unknown.

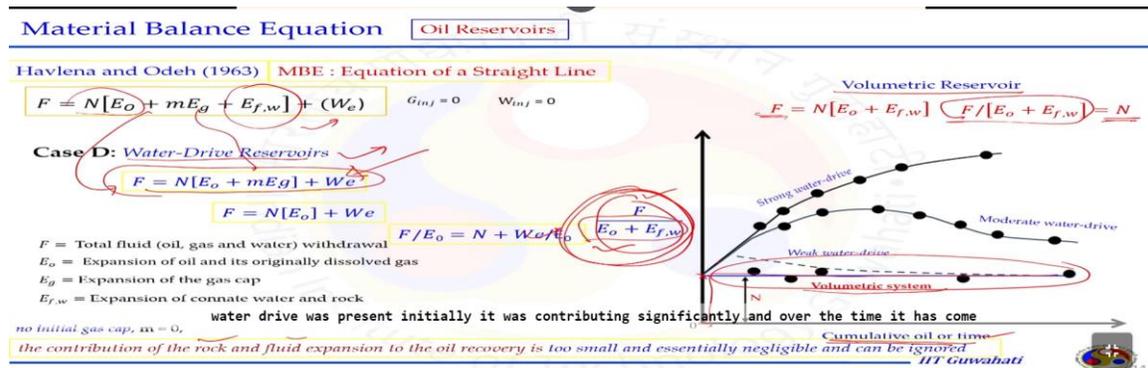
If we are plotting f versus this expression this should give us the value of n just by the slope if m value is known then we can plot this entire things on the x axis. Let us see what happens in that case. So the volumetric reservoir under saturated condition is having simple expression like this in case of the saturated oil reservoir we are going to get the expression very simple one and in case of the only gas cap drive reservoir mechanism we are going to get this expression where the oil expansion and the initial gas cap present in the reservoir is contributing towards the production. So as I was mentioning in the last slide on the x axis we can have the different parameter to plot depending on the situations we are having. On the y axis we can get the f that is the total withdrawal of oil, gas and water to the surface.



In most of the cases so far we discussed the water is not producing to the surface so we are having only the oil and gas that is going to produce at the surface. So in case of the oil reservoir we are considering only the oil is getting produced if initial gas is present that will also get produced to the surface. So the volumetric reservoir simple expression when we are plotting this versus f we are going to get the slope n and then n is equal to f divided by this expression. In case of the saturated reservoir we are going to get f versus u that is here and in case of the gas cap reservoir we are going to get the expression in the x axis as E_o plus $m E_g$. It means if we are having the data of all these expression for the $E_g, M E_g, E_{f,w}, E_o$ with respect to the withdrawal or the production data we can plot and whatever the parameter is giving us the straight line we can consider that is the dry mechanism that is responsible for the oil production or that is the significant contribution from the dry mechanism towards the production.

So all these parameters if plotted should result the straight line depending on the situation. So if it is just the saturated f versus E_o should give us the straight line if it is under saturated f versus E_o plus E_{fw} should give us the straight line and if it is gas cap reservoir f versus E_o plus $m E_o$ should give us the straight line and if none of them are

If the straight line is fitted to this data we can say the reservoir is having the volumetric system from this expression also we can say this should give the constant value of n over the time and if it is happening the value of n can be calculated from here whatever the point we are going to get on the graph and if there is a water drive reservoir means the water drive that is getting enclosed from the nearby is contributing significantly or contributing not significantly can be assessed by getting the same set of the data this versus this over the time it means any deviation from this horizontal line will be the indication of there is a water drive present in the reservoir. So let us see in case of the water drive reservoir this equation can be simplified f is equal to n e o m e z and this part can be ignored because of the water drive cases the rock expansion and the water expansion is really very insignificant because the pressure in the reservoir is maintained by the water encroachment and the expression for EFW that is having the del P that is very less. Another reason for ignoring this because the water drive is significantly contributing towards the production and we can ignore the rock and fluid expansion factor into the expression. So for the water drive reservoir we are going to get very simplified form here that is including now the W e term this W e term is including how much water is getting into the system and that is unknown we do not know exactly how much water is getting into the reservoir but by adjusting this linear equation we can get the value of W e by plotting a set of the parameter with respect to other set of the parameter. So for example here in this case just to understand is any deviation is happening from the volumetric system to the water drive system we are plotting this set of the parameter with respect to time or the cumulative oil production if it is giving us the straight line or the horizontal line this is the volumetric system and if the data those are deviating from that one and they are far away from this straight line we can say there is a strong water drive.



If the data are not in the straight line but near the straight line we can consider this as a weak water drive is present in the reservoir and when somewhere intermediate the parameter on the y axis is increasing and then decreasing over the time it means there is a moderate water drive was present initially it was contributing significantly and over the time it has come to a lower value and in that case we can consider any deviation from this horizontal line is the indication of water drive that can be classified as weak water drive, strong water drive and moderate water drive. If water drive is present by understanding the set of the data those plotted for the volumetric reservoir we can modify our equation considering this W e. In case of no initial gas cap present M this equation can be reduced to F is equal to N e o plus W e and if we adjust this equation divided both the side by e o F

by e_o is equal to N plus W_e by e_o . So now if we plot the data F by e_o versus 1 by e_o we are going to get the intercept N and W_e as a slope. So the unknown here are N and W_e while in the previous case here when we are also having the initial gas cap it means it is a combined drive mechanism where the gas cap is also contributing along with the water encroachment in that case we will be having 3 unknown N , M and W_e .

So if we further simplified this as we did here we just left with 2 unknown N and W_e because we consider initial gas cap contribution is not significant but if we are not getting the straight line it means we have to count for the initial gas cap is also present that is also contributing. So this is the way when we plot a set of the parameter with respect to either cumulative production or time or with another set of the parameter that is contributing for example oil expansion, gas expansion, rock and water expansion and we are seeing the deviation from the straight line that is the indication of presence of the other drive mechanism. So when we are seeing any deviation in that case the combination of the drive mechanism is present and we have to set up the equation in this form where all 3 unknown N , M and W_e are present and we are solving the equation in the same manner fixing some parameter on the y axis those are known to us and on the x axis getting the slope and intercept to calculate the unknown. Let us move next to the gas reservoir so we know the gas reservoir could be classified as wet, dry and the condensate reservoir and the dry mechanism those are responsible for the gas reservoir or the gas expansion and the depletion drive supported by the aquifer that is the water drive or the combination of the drive. So similar analysis as we did for the oil reservoir for different cases can be done for the gas reservoir.

The recovery efficiency however in case of the gas reservoir the gas expansion drive gives much better recovery compared to the water drive. While in the case of the oil reservoir it was the water that was maintaining the pressure in the system and if water encroachment is there that is most favorable condition that is contributing significantly for the oil production while in case of the gas because the influx of the water in the reservoir domain will reduce the critical saturation of the gas and the gas is trapped within the reservoir it is not crossing the critical gas saturation conditions hence the water drive recovery mechanism is less favorable in case of the gas reservoir and this is opposite to the case of the oil reservoir. So the gas expansion can give us the recovery up to 80 to 90 percent of the gas that is initially present in the reservoir while the water expansion may result only up to 50 to 60 percent. So the method can be implemented in the same manner as done for the oil reservoir. We can do the volumetric balance so this is our kind of a tank model.

In this tank model we are considering only the gas is getting produced from this reservoir so initially the gas is there we are not counting for the oil and then there is a possibility where the water is getting encroached into the reservoir domain and we are getting the cumulative gas produced and cumulative water produced. So this is the net water that is the connate water in the reservoir and then the influx of the water is also there. So we can do the similar volumetric balance to know the initial gas in place, gas reserves and the recovery of the gas from this reservoir. Same set of the equation can be

established the linearity or the straight line concept can be implemented to know the unknown. In case of the gas reservoir we are having this BG that is the gas volume formation factor that can be expressed in terms of the temperature, pressure and the compressibility factor.

Material Balance Equation Gas Reservoir

Only Free gas: Dry/Wet/Condensate Volumetric method

- Natural Drive mechanisms operative in Gas Reservoir are:
 - Gas expansion and depletion drive
 - Aquifer water drive
 - Combination drive

➢ Recovery Efficiency: Gas expansion drive gives much more recovery than water drive (which is opposite in case of Oil Reservoir)

- Gas Expansion: 80 - 90%
- Water Expansion: 50 - 60%

✓ Initial gas in place G
✓ Gas reserves
✓ Gas recovery

$B_g = \frac{p_{sc} zT}{T_{sc} p} = 0.02827 \frac{zT}{p}$

So the compressibility factor comes into picture in case of the gas reservoir.

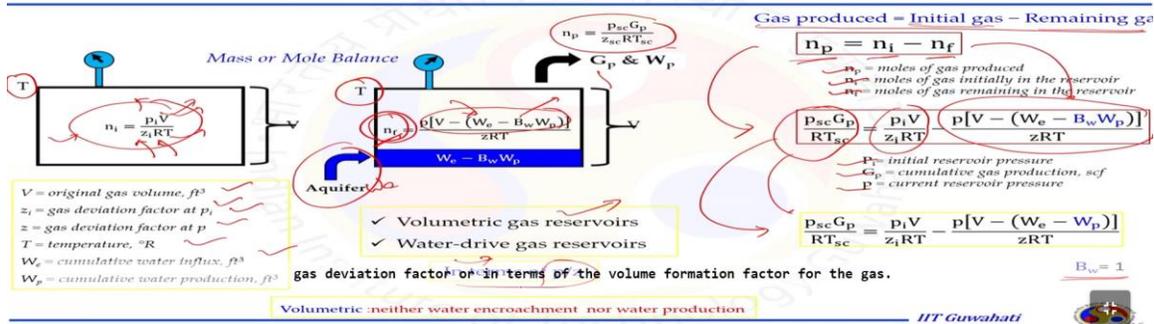
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So the compressibility factor comes into picture in case of the gas reservoir. So let us do the mass or mole balance system for the gas reservoir. Initially we are having this control volume or the tank volume that is having the initial number of the moles of the gas that is present in the reservoir. We can count that initial number of the moles by knowing the compressibility factor ZI initial pressure of the reservoir volume of the tank R is the gas constant and T is the temperature. So we are considering this as an isothermal case so the same temperature is here and after certain time we are seeing the same temperature means the time when we are investigating the tank model.

There is a possibility of water getting encroached into the system and in that case we are also producing the water at the surface GP is the cumulative gas produced, WP cumulative water produced. In this case we can have a simple balance equation that says gas produced is equal to initial gas minus the remaining gas. It means NP is equal to NI minus NF. NF is the mole of the gas those are present in the volume. Now in this case the volume is reduced because the water is encroached and some water got produced.

So the difference of these two should be subtracted from the initial volume to calculate the number of moles of the gas that remains in the control volume. So the NP is number of the moles that got produced, NI initial moles, NF is the moles remains in the reservoir at the time of the investigation. Other terminology is same V is the original gas volume in feed queue, ZI is the gas deviation factor and at initial pressure PI, Z is at any condition, T is temperature, W is cumulative water that is getting influx into the reservoir and WP cumulative water produced to the surface. So when we are doing the balance for the number of moles in the reservoir we can see the NP can be expressed in terms of the temperature, pressure and the cumulative gas that produced to the surface. It can be related to the initial condition and put to the volume, pressure, temperature condition at time of the investigation.

Material Balance Equation Gas Reservoir



So NP, NI and NF can be expressed in this form where PI is the initial pressure, GP is the cumulative gas produced and P is the current reservoir pressure. The expression can further be simplified just considering the volume formation factor for the water is equal to 1 that we discuss in detail in the properties of the water. So the expression can be written in form of the pressure and temperature. Now the reservoir as similar to the oil reservoir, the gas reservoir could be the volumetric gas reservoir or water dry gas reservoir. We can investigate the case for both the things in terms of P, Y, Z pressure divided by the gas deviation factor or in terms of the volume formation factor for the gas.

So the material balance equation for the gas reservoir that is shown in the previous case is simple because we consider the PW is equal to 1. Now if we are having the volumetric gas reservoir assuming no water producing and no water is getting in into the reservoir means there is no water influx, the equation will simplify it just dropping this term, the equation will simplify to this form where we are having this volume and the T. So this volume is actually the original volume in the tank. If we express this expression in terms of P, Y, Z, we can do this just keeping P, Y, Z here remaining P and V can be placed to the other side. So we will get this expression here P I by Z I here and the others are going to the other side and we are going to get this expression.

The initial reservoir gas volume P can be related to the initial gas present at the reservoir conditions and with the help of the volume formation factor. So G is original gas in place at the standard cubic feet condition and V is the original gas in place at the feet Q condition. So the volume formation factor that can relate the volume P to G with the help of this volume formation factor we can relate this equation in the form of other parameters. So what those other parameters? By the definition of BG we can say this expression that is appearing here in the expression here this one can be represented by P I divided by G Z I. If we do so this expression should be resulted in the form of P, Y, Z on the one side P I by Z I as it is and this part we can write P I by Z I multiplied by 1 by G this part with the help of the expression of BG.

If we see this expression we can write this entire thing as M and this again result in the straight line Y is equal to C plus MX. So if we plot the data P Y Z with respect to the cumulative gas production we will get the slope and that slope is going to give us the value of M. So let us say on the Y axis we are having the pressure divided by compressibility on the X axis we are having the cumulative gas rate on the X axis we are having the cumulative gas GP the resultant equation is like this and when we are

collecting the data field data or the tank model data at different P I by Z I we are going to get some initial values. These initial values should fall in the straight line if it is a volumetric gas reservoir we can extend or extrapolate this data with the concept of extrapolation we are going to get the point at the GP axis that is the initial gas that is present in the reservoir. So OGIP original gas in place while on the Y axis this is the initial condition.

Material Balance Equation Gas Reservoir

$$\frac{p_{sc} G_p}{RT_{sc}} = \frac{p_i V}{z_i RT} - \frac{p[V - (W_e - W_p)]}{zRT}$$

Volumetric Gas Reservoirs Assuming no water production and no We

$$\frac{p_{sc} G_p}{T_{sc}} = \left(\frac{p_i}{z_i T}\right) V - \left(\frac{p}{z T}\right) V$$

In terms of p/z

$$\frac{p}{z} = \frac{p_i}{z_i} - \left(\frac{p_{sc} T}{T_{sc} V}\right) G_p$$

The initial reservoir gas volume $V = B_g G = \left(\frac{p_{sc} z_i T}{T_{sc} p_i}\right) G$

G - Original gas in place, scf
 V - Original gas in place, ft³

$$\frac{p}{z} = \frac{p_i}{z_i} - \left[\left(\frac{p_i}{z_i}\right) \frac{1}{G}\right] G_p$$

There could be a situation when the slope of this equation will give us the value of M

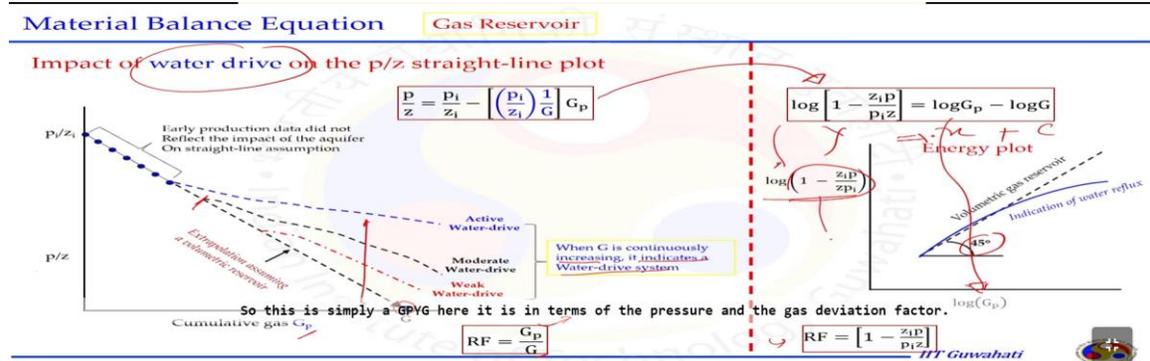
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So this is when the P I by Z I is obtained. There could be a situation when the slope of this equation will give us the value of M and the situation when the reservoir is left as abundant we can calculate the cumulative gas production at that condition and what is the ratio of P I by Z at that condition. And when we are here in this situation the GP value is 0 means we are not producing any gas. So the impact of the water drive on the volumetric reservoir can be assessed by modifying this equation. In this equation what we did P I by Z is plotted with respect to GP we are getting this initial data. This initial data are not reflecting any impact of the water drive they are in the straight line.

But when we are extrapolating this data for the assumptions of volumetric reservoir this should give us the straight line and we are supposed to get the value of G here. But if there is any deviation in this extrapolation when we are collecting the more data over the time with respect to cumulative gas if the deviation is happening not much we can consider this as a water drive is present but it is weak. When deviation start very early stage and deviating more we can say moderate water drive and when early stage itself we start getting the deviation from this straight line we can say active water drive is present and the deviation is also more in that case. So when G is continuously increasing it indicates water drive system is present in the gas reservoir. The recovery factor can be calculated total gas produced cumulative gas produced to the surface divided by the original gas that is present.

The unit must match with the help of the formation volume factor. Further another way of plotting the data with the same equation can be taken on the log scale in that case we are having this Y is equal to X plus C. In that case when we are having this part on the Y axis log of cumulative production on the X axis we should get the equation that is having the slope 1 because 1 dot X here or the 45 degree angle. If it is a volumetric gas reservoir we should get the straight line in that case if there is any deviation is happening from this straight line it means we are having the water influx in the reservoir. This is the indication of the water influx and we should consider the impact of the water drive on the

reservoir performance that is initially was the volumetric gas reservoir under the assumption. The deviation is indicating the volumetric gas reservoir is also having the contribution from the water influx.



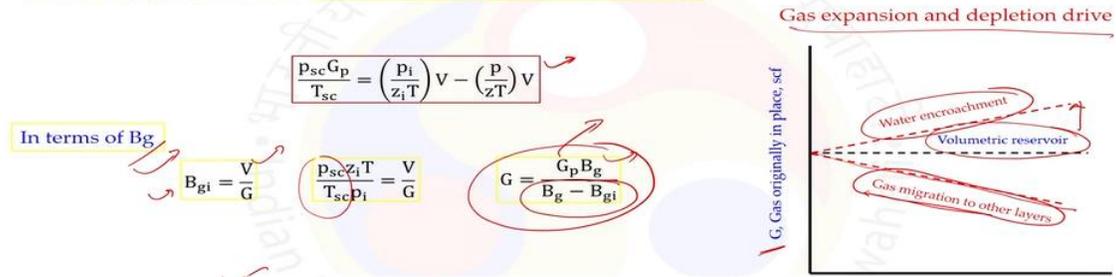
In this case the recovery factor can be obtained by using this expression here. So this is simply a GPYG here it is in terms of the pressure and the gas deviation factor. So the volumetric gas reservoir assuming no water production and no WE we are going to get this expression. This expression can be arranged in the form of BG that is the gas volume formation factor. We can also express in terms of the expansion gas factor that is the reverse of BG. But let us see in terms of BG that is initial BGi is V divided by G if we express those in terms of the pressure and temperature with respect to standard condition and the condition initially present in the reservoir.

We can adjust this expression in form of G is equal to cumulative gas produced to the surface multiplied by the gas volume formation factor divided by change in the volume formation factor happens during this cumulative production. So here V is the original gas volume in feed cube, G is the volume of the gas originally but measured at a standard condition STP that is where we are getting the standard temperature and pressure in the expression, Z is the deviation factor and at the standard condition Z SC is considered as 1. So that is why it is not appearing the expression. So if the gas expansion and depletion drive is present and we are plotting the gas versus the cumulative gas production at the standard conditions if we are getting the straight line it means it is the volumetric reservoir and the gas expansion depletion drives are significantly contributing towards the production. If we are getting the tabulation on the upper side it means it is supported by the water, water is getting encroached in the reservoir domain.

Material Balance Equation

Gas Reservoir

Volumetric Gas Reservoirs *Assuming no water production and no We*

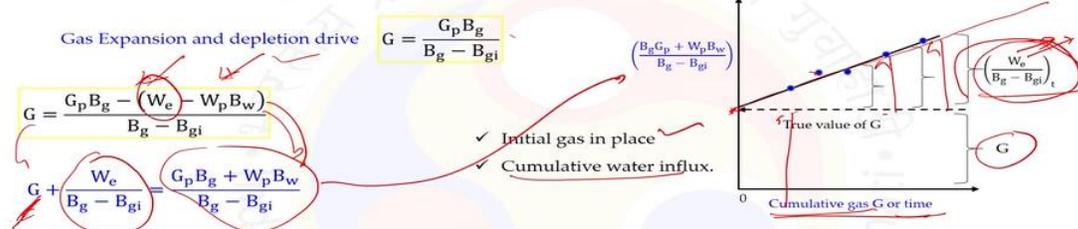


On the other side if the value is decreasing as the cumulative gas is getting produced we can suspect the gas is migrating from the reservoir domain to some other domain in the reservoir itself or from one layer to the other layer it is not just getting contributed towards the production side it is also having the migration to the other side of the production well. So the water drive is present in the gas reservoir the same gas expansion depletion drive equation can be modified including the water encroachment and the water production. So the G on one side on the other side and bringing this term that is accounting for the water influx on the left hand side we can adjust this equation in this form where G plus this water encroachment is kept on the left hand side and the production data for the gas and the water are kept on the other side. So if we plot this versus the cumulative gas or the time that is G the equation should give us the value of G it means it is starting from somewhere that is the intercept here it is starting here and as we move with respect to cumulative gas or time we are collecting the data point and when we are plotting this would result in the straight line that is the indication the water drive is present and at any particular time we can calculate how much deviation is happening with respect to this horizontal line and with the help of that we can calculate the value of W_e .

Material Balance Equation

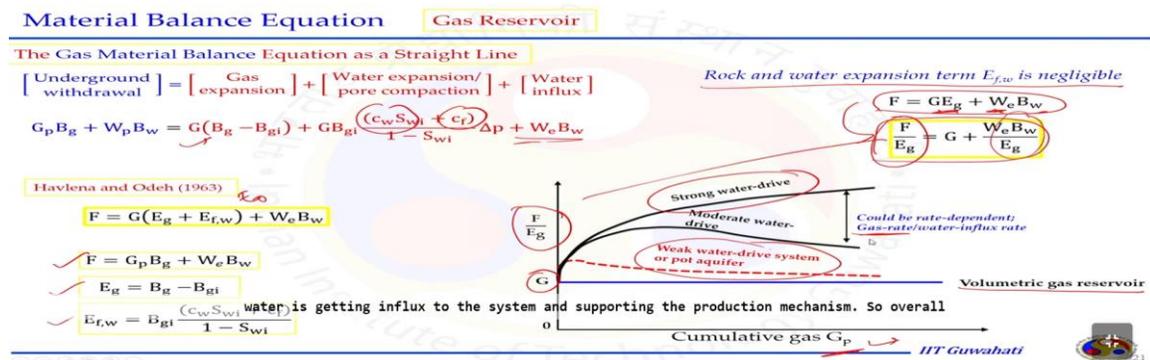
Gas Reservoir

Water-Drive Gas Reservoirs



So in that case the water encroachment value for the gas reservoir can be estimated. So this will give us the initial gas in place that is here and the cumulative water influx with this parameter. So the ultimate recovery of 80 to 90 percent are common in the volumetric gas reservoir while the value typically decrease in the gas reservoir when it is supported by the water drive. So the material balance equation for the gas reservoir with the same concept of the tank volume can be written in a more generalized form

considering the more contribution from the water expansion from the rock expansion or the gas is contributing the water influx is there just not counting the injection of the gas and water from the outside. The same straight line concept as we did for the oil reservoir this is withdrawal rate this is contribution of the gas this is the contribution of formation water and the rock towards the gas production and in that case we are not considering the oil is getting produced this is the gas reservoir that is why E_o is not appearing here in this expression. So we can have different cases when this is negligible $E_{f,w}$ then we are going to get this simple expression F is equal to G plus E_g plus W_e that is the water influx multiplied by the volume formation factor of the water.



We can adjust this equation by dividing by E_g and again the plotting of this parameter with respect to the non parameter should result the straight line and we should catch the resultant equation if it is the volumetric gas reservoir we should get the straight line when we are plotting F by E_g that is here with respect to the cumulative gas production and this is going to give us the intercept that is the initial gas present in the reservoir and if any deviation from this that is happening if the deviation is too high then it is a strong water drive if it is not too high just near to this horizontal line we can say it is a weak water drive and somewhere in between we can say it is a moderate water drive. So similar to the oil reservoir we can also see the impact of the water drive with respect to the data F by E_g and the cumulative gas production data on y axis and x axis respectively we can see the impact of the water drive mechanism on the reservoir. How much deviation is happening between this moderate water drive and the strong water drive that will depend on at what rate the gas is getting produced and at what rate the water is getting influx to the system and supporting the production mechanism. So overall the situation could be like this in fact over the time if the value is decreasing we can say the gas is migrating to some other zone that might be the situation when the deviation from this volumetric gas reservoir on the other side it is decreasing. So far we discuss about the volumetric valence of the gas reservoir where we consider the tank volume we are not worry about the dimension of the reservoir the porous volume available to hold this thing but the gas reservoir material valence or the volumetric valence can also be done in this way when we are knowing some of the parameter like A area H is the Poisson thickness ϕ is the porosity of the reservoir domain and the conic water saturation or initial water saturation this can be balanced with the volume formation factor the

numerical coefficient that is appearing because of the unit valence area is measured in acre while the height is measured in feet.

So this is the way we can get the initial gas present in the reservoir if the parameter on right hand side appearing in this expression are known or other way if we are getting the value of G from the previous study that we did with the straight line concept we can get some of the information of the reservoir using this equation. So the G value is known from the straight line concept now whatever is unknown here can be estimated with the help of this expression. So this concept is similar gas produced is called to initial gas minus remaining gas and the cumulative gas production can also be expressed with the same expression where B_{gi} is the initial gas volume formation factor B_{ga} is the gas volume formation factor at the abundant conditions. In this case we consider the B_g , B_g can be calculated at particular temperature and pressure and knowing the deviation factor for the gas at that conditions. Other terms are having the usual meaning like G_e is gas in place area of the reservoir important is it is in acre H is the average reservoir thickness or the page on thickness, ϕ is the porosity water saturation at the initial condition gas volume formation factor at the initial conditions.

So the calculation of the unit recovery from this volumetric gas reservoir can also be done before going to do we can also discuss like the connate water saturation with respect to this unit recovery. Unit recovery we are considering the bulk volume A multiplied by H of the reservoir domain if we multiply that with the ϕ it is the volume available to be occupied by the oil, gas and water in the reservoir. But the unit recovery concept can be applied to know the connate water with respect to unit volume. So that is why you are not seeing A and H in this expression. So this is with respect to A and H we can say connate water numerical factor porosity multiplied by the saturation of the water at that condition.

Similarly the gas volume we are considering only gas and water is present in the reservoir. Hence the 1 minus initial water saturation that is equal to the gas saturation we can get the expression or the value of the reservoir gas volume. Similarly, the pore volume this is numerical factor just multiplied by the porosity of the reservoir we can get the reservoir pore volume. So the unit recovery is the difference between the initial gas in place and that remaining at the abundant pressure conditions. So the unit recovery can be expressed in this form per unit area here you can say per acre feet that is why A and H are not appearing here.

Material Balance Equation Gas Reservoir

Gas Reservoirs $G = \frac{43560Ah\phi(1-S_{wi})}{B_{gi}}$

Gas produced = Initial gas - Remaining gas

$$G_p = 43560Ah\phi(1-S_{wi})\left(\frac{1}{B_{gi}} - \frac{1}{B_{ga}}\right)$$

Calculation of unit recovery from volumetric gas reservoirs

Unit recovery is the difference between the initial gas in place and that remaining at abandonment pressure

$$\text{Unit recovery} = 43560\phi(1-S_{wi})\left(\frac{1}{B_{gi}} - \frac{1}{B_{ga}}\right) \text{ SCF(acft)}$$

So in this balance it is assumed the pore volume occupied by the gas is constant, so $\frac{100(B_{gi} - B_{ga})}{B_{gi}} = 100\left(\frac{B_{gi}}{B_{ga}} - 1\right)$

Volumetric balance

$$B_g = \frac{p_{sc} zT}{T_{sc} p} = 0.02827 \frac{zT}{p}$$

G = gas in place, scf
 A = area of reservoir, acres
 h = average reservoir thickness, ft
 ϕ = porosity
 S_{wi} = water saturation
 B_{gi} = gas formation volume factor ft^3/scf

- ✓ Connate water: $43560\phi(S_{wi}) \text{ ft}^3$
- ✓ Reservoir gas volume: $43560\phi(1-S_{wi}) \text{ ft}^3$
- ✓ Reservoir pore volume: $43560\phi \text{ ft}^3$

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This is the expression can be used to get the unit recovery from the gas reservoir. Recovery factor that is the gas produced divided by the initial gas multiplied by 100 to get the recovery factor on the scale of 100 in terms of the percent. So we can get this is the initial gas this is the gas at the abundant condition the difference of this is gas produced or we can say cumulative gas produced GP divided by G and we can express this in terms of the volume formation factor Bgi at the initial condition Bga at the abundant condition and finally we can get the expression to get the recovery factor that is equal to 100 multiplied by 1 minus Bgi divided by Bga so this is minus sign here 1 minus. So in this balance it is assumed the pore volume occupied by the gas is constant. So in summary we discuss about the volumetric performance equation for oil and gas reservoir different cases of oil and gas reservoir are discussed. We discuss the volumetric oil and gas reservoir we discuss water dry supported oil and gas reservoir.

We also discuss the recovery factor for the gas production. So in the next lecture we are going to continue our discussion about the oil and gas reservoir. We will start with the fundamental of oil and gas flow in porous media then set up the general equation for the radial flow of oil and gas reservoir. In this case as this one was the zero dimension model this is one dimension model that is we are going to discuss in the next week. With this I would like to thank you for watching the video we will meet in the next lecture. Thank you.