Lec 20: Minimum Fluidization Velocity

Hello everybody. Welcome to this massive open online course on solid fluid operations. So we are discussing about the fluidization operation under this solid fluid operations as a module. So, in the previous lecture we were discussing about what is fluidization, what is the concept of the fluidization, what are the applications of that fluidization and what are to be learned about that fluidization. In this lecture, we will try to discuss about the minimum fluidization velocity. We generally required to get the particle fluidized at a minimum certain velocity and beyond which you will get that different flow patterns of the fluidization that we have discussed.

So, how to estimate that minimum fluidization velocity to get this particle just suspended in the fluidized bed. Before going to that we again just brush up about that different flow patterns. So, in the fluidized bed we generally have different types of flow patterns like you know that homogeneous flow patterns, some bubbling flow patterns, churn turbulent flow patterns, fast fluidization condition, even pneumatic transport fluidized condition. So, out of those some will be batch operated and some patterns will be considered for the transport operated fluidization operation and whenever that particles will not be suspended in the bed those condition will be called as fixed bed condition or packed bed condition.

So, we observed and also discussed the different flow patterns like in this slide it is shown that we are getting that homogeneous flow pattern where the particles will get just suspended at a minimum gas or liquid velocity and beyond which if you increase the gas velocity you will get different flow patterns like bubbling, churn turbulent, fast fluidization and pneumatic fluidization. And all this flow pattern depends on different factors like particle size, particle size distribution, even gas velocity, even pressure, temperature, even other geometrical condition also of this fluidized bed. Now, we will try to learn about that how to actually estimate that minimum fluidization condition based on some mathematical expression or some from that basic understanding of that energy balance equation or force balance equation. So, before going to that we have to know that what is basically that fluidization minimum fluidization condition. So, this is basically a fluidization velocity at which that all the particles are just suspended by the upward flowing gas or liquid.

At this velocity that frictional force between particle and fluid just counter balances the weight of the particle and also this pressure drop through any section of the bed about equals the weight of the fluid and the particles in that section. The bed is considered to be just fluidized and is referred to as an incipiently fluidized bed or a bed at minimum fluidization. Now, at this minimum fluidization condition, then we can have a force balance like you know what is the drag force by upward moving gas by which that particles will be moving up to get suspended that will be of course equal to the apparent weight of the particles at this minimum fluidization condition. So, we can write here that drag force by upward moving gas is equal to apparent weight of the particles. Now, how to estimate the drag force by upward moving gas, we have already discussed in our earlier lectures also that what is the definition of drag force, how to calculate the drag force, what is the drag force when different shape of the particles will come or whenever flow will be flowing over the solid surface either it is a cylindrical shape or spherical shape that we have learned earlier.

So, please refer to that also lecture. So, in that case the drag force we can calculate from the frictional pressure drop across the bed. So, if we know the frictional pressure drop from the measured value or some other way if we know that frictional pressure drop in the bed and if you multiply it by the cross sectional area of the bed then you will get this total drag force that is given by the upward moving gas. So, we can write the drag force by upward moving gas that will be equal to pressure drop across bed into cross sectional area of the bed. Then it will be equals to the apparent weight of the particles.

Now, how to calculate that apparent weight of the particles? This apparent weight of the particles can be calculated by this equation what that this is basically that weight of the particles net weight of the particles which is going downward that means here you will see the weight of the particles minus its buoyancy effect buoyancy force that will be your net force that is acting on the particles and that will be equal to that pressure drop across the bed there into its cross sectional area. So, that will be that equal to drag force by upward moving gas. So, that net force that can be calculated by this the volume of the bed into fractional or fractional consist of solids into specific weight of solids. So, this will be your apparent weight of particles that means here volume of bed is basically A into LMF,

LMF means L is the length of the bed at its minimum fluidization condition, MF means minimum fluidization condition and 1 minus epsilon f is basically the solid volume fraction in the bed into rho s, rho s means the density of the particles or solid. So, here what will be the total weight of the solid here minus that volume of the solid will be displaced or by this gas that volume of that solid will be displayed by that gas that will be your buoyancy force.

So, that minus will be equal to then a LMF into 1 minus epsilon f into rho g into g.

Here the Equation

$$\Delta P_b A = W_a = AL_{mf}(1 - \varepsilon_{mf})(\rho_s - \rho_g)g$$

So, here you will get this net force which will be acting on the solid particles and the downward whereas the drag force will be acting upward direction that is basically that delta PB into A. So, we are having this drag force that will be is equal to your apparent weight of the particles. So, from this equation you will be able to calculate what will be the minimum fluidization velocity. Now, in this case what are the terms condition here it is given that L should be the height of the bed, A is the cross sectional area of the bed, epsilon is the void fraction, rho is the density of the particle or solid, rho f is the density of fluid either liquid or gas, wa is basically apparent weight which is basically a LMF into 1 minus epsilon f into rho s minus rho g and suffix mf will represent the minimum fluidization.

Here the Equation

$$\frac{\Delta P_b}{L_{mf}} = (1 - \varepsilon_{mf})(\rho_s - \rho_g)g$$

So, we are getting this equation. Now, in this case you have to find out what will be the bed pressure drop. Now, why that minimum fluidization condition that to be found to obtain that minimum velocity. Now, you will see that at its minimum fluidization condition the particles is considered to be as a maximum fixed condition at the maximum condition at its fixed. So, here this dotted line is represented that this is the minimum gas velocity at which you can get this fixed condition.

Up to this velocity you will get this particles will be stagnant, it will not be moving, but beyond these particles will start to just fluidized. So, up to these here if you increase the gas velocity the pressure drop will linearly increase that is in packed condition that is in packed bed condition. In packed bed condition you will see that the pressure drop will be increasing with respect to liquid or gas velocity. So, up to these here we are getting this fictional pressure drop which will be increasing with respect to gas velocity. After that, whenever it will be coming at its minimum fluidization condition, the pressure drop will remain almost constant here and beyond that if you increase gas velocity that flow pattern will be changing either bubbling or unturbulent or fast fluidization condition.

So, here in this case, you will see that at this minimum fluidization condition, there will be no change of frictional pressure drop. So, here it will be called as at this minimum condition it will be called as minimum fluidization condition and what will be the velocity that will give you that you know driving force to get this particle suspended just suspended from its rest position. So, here we are having this minimum fluidization condition and what will be the pressure drop from this pressure drop we will be able to calculate what the gas velocity. Now let us see here. So, we obtain that governing equation just by balancing this drag force with the apparent weight.

Now from this equation, if you rearrange it, then you will get delta PB by LMF that will be equal to 1 minus epsilon MF into rho s minus rho g into g. So, this is your bed pressure drop per unit length of that minimum fluidization condition. Now at this onset of fluidization that means at this minimum fluidization condition, the voidage is a little larger than in a packed bed because at that time the particles will get just suspended. So, that is why the minimum voidage will be little larger than the packed bed condition. So, Wane and EO found for many systems that this void fraction depends on the shape of the particle and accordingly from his observation, he has suggested this equality or equation to find out that minimum void fraction at its minimum condition of fluidization.

So that is phi s epsilon MF cube that will be is equal to 1 by 14.

Here the Equation

$$\Phi_s \varepsilon_{mf}^3 \cong \frac{1}{14}$$

So this epsilon MF is the minimum fluidization void fraction that depends on shape of the particles. For spherical particles this epsilon MF at this minimum condition varies from 0.

4 to 0.45. Now this frictional pressure drop or bed pressure drop that you can measure experimentally by manometer here as shown in the picture from this manometer you can measure this what is the frictional pressure drop. This measured frictional pressure drop that will come from the two components. One is frictional pressure drop another is one of that hydrostatic pressure drop. So if you are using liquid instead of gas there will be frictional pressure drop, there will be effective frictional pressure drop that will be measured pressure drop minus hydrostatic pressure drop. Whereas if you are using gas this hydrostatic pressure drop will be negligible compared to this frictional pressure drop.

So that is why their measured pressure drop will be exactly equals to the frictional pressure drop. So once this frictional pressure drop and then equate it with the afferent weight of that particles at this minimum fluidization condition you will be able to find out what will be the minimum fluidization velocity. Now if you do not have that experimental data but that other operating variables like minimum fluidization void fraction, viscosity of the fluid, particle diameter and also sphericity of the particle then you can get this frictional pressure drop from the Ergun equation because at this minimum fluidization condition you are considering the bed is still a packed bed condition. That is why at this onset condition we can consider that frictional pressure drop can be equated or can be predicted by that Ergun equation. So what is that Ergun equation? Already we have learned in the previous module that flow through that porous media there we have learned that Darcy's equation, Darcy's law, Kozeny-Carman equation, Ergun equation there.

So from that Ergun equation we can calculate what will be the frictional pressure drop per unit length of minimum fluidization condition. So here from this Ergun equation you will be able to calculate what will be the minimum fluidization pressure drop. So here by this equation you can calculate. In this case only u is unknown to you, this unknown u value. Other parameters are known to you, particle diameter, even void fraction and sphericity.

Now if you equate this frictional pressure drop with that apparent weight, this is your apparent weight that we have calculated earlier and if you equate this apparent weight with that frictional pressure drop what has been calculated by the Ergun equation and then rearranging just by dividing it by 1 minus epsilon f of this equation 5 and then multiply it by dp cube rho f by mu square and simplifying you will get this equation number 6 that is 150 into 1 minus epsilon mf by epsilon mf cube into phi s square into rho f u mf dp by mu. Here u you are taking as a u mf because at its minimum fluidization velocity then plus 1.75 by epsilon mf cube phi s into rho f square u mf square into x s v not this x s v it is d p square divided by mu square that will be is equal to dp cube rho f into rho p minus rho f by mu square into g.

Here the Equation(s)

$$\frac{(-\Delta P_{fr})}{L_{mf}} = 150\frac{(1-\varepsilon_{mf})^2}{\varepsilon_{mf}^3}\frac{\mu u}{\Phi_s^2 d_p^2} + 1.75\frac{(1-\varepsilon_{mf})}{\varepsilon_{mf}^3}\frac{\rho_g u^2}{\Phi_s d_p}$$

$$(1-\varepsilon_{mf})(\rho_p-\rho_f)g = 150\frac{(1-\varepsilon_{mf})^2}{\varepsilon_{mf}^3}\frac{\mu u_{mf}}{\Phi_s^2 d_p^2} + 1.75\frac{(1-\varepsilon_{mf})}{\varepsilon_{mf}^3}\frac{\rho_f u_{mf}^2}{\Phi_s d_p}$$

$$150\frac{(1-\varepsilon_{mf})}{\varepsilon_{mf}^3\Phi_s^2}\left(\frac{\rho_f u_{mf} d_p}{\mu}\right) + \frac{1.75}{\varepsilon_{mf}^3\Phi_s}\left(\frac{\rho_f^2 u_{mf}^2 x_{sv}^2}{\mu^2}\right) = \frac{d_p^3\rho_f(\rho_p-\rho_f)g}{\mu^2}$$

So, this equation number 6 from this equation number 6 then you will be able to calculate what will be the u mf this is a quadratic equation you have to solve this quadratic equation for u mf. So, we can rearrange or we can express in different way this equation number 6 like this here.

In this case we are considering here R e mf defining R e mf as like this and also argon equation or it is called Archimedes number or it is also called as Galileo's number. So, Archimedes number and Galileo's number you can say which is defined by this equation. Now equation 6 can be expressed by this equation number 7 in terms of this Archimedes number and Reynolds number at this minimum fluidization condition. Now you will see that this equation 7 is coming as a quadratic equation for minimum fluidization condition Reynolds number. In that case you have to solve this quadratic equation of this Reynolds number at its minimum condition after solving this quadratic equation you will get the value in terms of Reynolds number.
Here the Equation

$$\frac{1.75}{\varepsilon_{mf}^3\Phi_s} + 150\frac{(1-\varepsilon_{mf})}{\varepsilon_{mf}^3\Phi_s^2} = Ar$$

$$K_1 + K_2 = Ar$$

$$= \frac{-K_2 \pm \sqrt{K_2^2 + 4K_1(Ar)}}{2K_1}$$

And for this you need only that minimum void fraction which can be calculated from this when and you correlation here or if you do not know this you can directly take this value within this range here for a different shape of this particle as shown in the table. For spherical particles it will be considered as 0.4 to 0.45 and then what you have to do that quadratic equation of equation number 7 can be rewrite like this. Here these terms of this coefficient of Reynolds number square that will be regarded as or defined as K1 and this Reynolds number for this other terms to be defined as K2 here.
Here the Equation(s)

$$K_1 = \frac{1.75}{\varepsilon_{mf}^3\Phi_s}$$

$$K_2 = 150\frac{(1-\varepsilon_{mf})}{\varepsilon_{mf}^3\Phi_s^2}$$

So it will be coming as K1 into R emf square plus K2 into R emf is equal to Archimedes number where K1 is equal to 1.75 by epsilon emf cube into phi s whereas K2 will be is equal to 150 into 1 minus epsilon f by epsilon emf cube into phi s square. So from these if you are solving this quadratic equation you can get this equation for Reynolds number at its minimum fluidization condition. So once you can calculate the K1 and K2 from the parameters given like our variables given here unknown variables like epsilon emf epsilon emf and phi s and dp value if it then you will be able to calculate what will be the K1 and K2 and substitute that K1 and K2 value here along with that Archimedes number then it will be easier to calculate what will be the Reynolds number at its minimum fluidization condition. So once this minimum fluidization Reynolds number then you will be able to calculate U emf value because R emf that will be is equal to rho f U emf into dp by mu f.

So from this U emf will be is equal to mu f into Reynolds number of particles at its minimum fluidization condition divided by rho f and dp. So from this equation you will be able to calculate what will be the minimum fluidization velocity. So we can calculate what will be the minimum fluidization velocity once we know that particle diameter once we know that minimum void fraction once we know that other physical properties. So in this case we can assess that minimum fluidization velocity depends on what basically three types of variables will be affecting on this minimum fluidization velocity. One is called geometric variables another is called variables as a physical properties and third one is thermodynamic conditions.

You will see that minimum fluidization velocity will be depending on bed diameter, bed height, particle size and distributor hole diameter. That means through which that gas will be distributed that hole diameter will be you know affecting on that minimum fluidization condition. And also physical properties like fluid density, fluid viscosity, fluid surface tension and slurry concentration. Also other thermodynamic condition like pressure and temperature all those variables will affect the minimum fluidization velocity. Now let us do an example to how to calculate that minimum fluidization velocity.

Let us consider an example here like you have to calculate the minimum fluidization velocity for the fluidized bed operating with a sharp sand particles and with air under following properties. Where fluidizing gas is ambient air density is 1.2 kg per meter cube and viscosity given 1.8 into 10 to the power minus 5 kg per meter second and the solid here a solid whose density is 2600 kg per meter cube and diameter of the particle is given 75 micrometer and its sphere is 0.

67. So here we are considering that sand particle of density this and in the fluidized bed from the bottom the gas that means ambient air will be allowed to get this minimum fluidization condition. So you have to find out what will be that minimum fluidization velocity. So to calculate that you need minimum void fraction which is given here 0.45 and

also sphere is it is given 0.67 then you have to calculate what will be the K1 parameter here that means 1.

75 by epsilon mf cube into phi s which will be coming as 28.66 and also K2 value after substitution of those values it will be coming as 2016.81 and then after substitution of K1 and K2 in that Reynolds number then you will get this value as 0.0197 where Archimedes number is calculated as 39.83 since all other variables are given here and once this Reynolds number at this minimum fluidization condition as 0.

0197 from this you will be able to calculate what will be the minimum fluidization velocity here. So it will be coming as 0.00395 meter per second. So this is a simple method to calculate how that minimum fluidization to be obtained. Then if you are considering that liquid instead of gas what will be the minimum fluidization velocity.

So in that case instead of gas simply you consider all other properties of the liquid then you will be able to in that case liquid and solid it will be a slurry and the physical properties of the slurry will be changing in that case mass of that or volume of that physical properties of that systems will be different. There will be slurry properties will be considered that is mixed here in this case you will see that this will be your apparent weight that is rho p into 1 minus epsilon mf plus rho l into epsilon mf into g that will be equating with the Ergun equation where that liquid will be flowing through the packed bed.

Here the Equation

$$150\frac{(1-\varepsilon_{mf})^2}{\varepsilon_{mf}^3\Phi_s^2}\frac{\mu_l u_{mf}}{d_p^2} + 1.75\frac{(1-\varepsilon_{mf})}{\varepsilon_{mf}^3\Phi_s}\frac{\rho_l u_{mf}^2}{d_p} = \left[\rho_p(1-\varepsilon_{mf}) + \rho_l\varepsilon_{mf}\right]g$$

$$\varepsilon_{mf}\cong\left(\frac{1}{14\Phi_s}\right)^{1/3}$$

So once you equate this equation as per this Lee et al 2016 they have considered this and for this minimum void fraction also you can calculate as per that when and you model here. So once you substitute those values except that u mf you will be able to calculate what will be the minimum fluidization velocity for liquid solid system. Now let us do some examples which given in GATE examination also based on this theory here one problem it is given in GATE 2010.

In this case the height of fluidized bed at incipient fluidization is 0.075 meter and the corresponding voidage is given 0.38. If the voidage of the bed increases to 0.5 then what would be the height of the bed very simple.

In this case you will see that though the voidage will be changed the amount of solids will remain same this is the case here. The height of the fluidized bed at its minimum condition is 0.075 and corresponding voidage is 0.38. Now you are increasing the voidage but the amount of solid will not be changed that means here mass of the solids will remain same.

So in that case only voidage will be changed amount of solid will not be changed. So we can write in that way initially what will be the amount of solid that will be is equal to what is that what will be the volume of that bed that will be cross sectional area of the bed into height of that fluidized bed at its initial condition and then 1 minus epsilon 1 is basically void fraction at its initial condition. So 1 minus epsilon 1 is basically the volume fraction of solid at its initial condition. So from this you will be able to calculate what will be the amount of solid this amount of solid will be exactly equals to the same amount of solid even if your height will be increased and void fraction will be changed. So in this case pi by 4 dB square that means your cross sectional area the same and height here since void fraction will change height also will be changed so it will be h2 and then 1 minus epsilon 2 this is your volume fraction of the solids.

So here total amount of solids will be like this. So we can say that considering that amount of solid remains same only by changing that height and epsilon A epsilon that means void fraction of that bed from this relation we can then say that h2 will be is equal to h1 into 1 minus epsilon 1 by 1 minus epsilon 2. So from this equation you will be able to calculate what will be the height at its second condition when the void fraction will be changed. So that will be is equal to 0.093 after substitution of this epsilon 1 and epsilon 2 which is given to you and also initial height of the bed. So I think you understood this problem here how to calculate that height of the bed when it will be changed based on this void fraction.

Then another problem that is given in a GATE 2015 here it is said that a cylindrical packed bed of height 1 meter is filled with equal sized spherical particles. The particles are non-porous and have a density of 1500 kg per meter cube. The void fraction of the bed is 0.45. The bed is fluidized using air whose density is given 1 kg per meter cube.

If the acceleration due to gravity is 9.8 meter per second square what is the pressure drop in Pascal across the bed of incipient fluidization that means at incipient fluidization condition you have to find out what will be the pressure drop. So we know that at its incipient fluidization condition or minimum fluidization condition the pressure drop across the bed that will be is equal to your apparent weight. Here in this case from this relationship you will get that is given earlier. So here it will be delta PB that will 1 minus epsilon mf into rho s minus rho g into g into l mf. Everything is given to you here epsilon mf is given to you rho s is given to you rho g is given to you and g and lmf is also known to you.

So after substitution of these values you will be able to calculate what will be the that delta PB. So it is coming 8087.86 Pascal. Another problem the same type of problem it is given in 2007.

Here a fluidized bed whose diameter is given 0.5 meter and height is given 0.5 meter of spherical particles diameter 2000 micrometer specific gravity is given 2.5 that means density you can calculate uses water as medium here instead of gas.

That minimum porosity of the bed is given 0.4. The argand equation for the system at its minimum fluidization condition is given like this. So at this minimum fluidization condition what is the pressure drop per unit length and what is the minimum fluidization velocity in millimeter per second. So what you have to do here you have to calculate pressure drop at its minimum fluidization condition. So pressure drop at its minimum fluidization condition is given by this equation here.

That is actually given earlier how it has come. After substitution of these values epsilon, rho s and rho g then you will get this 829 value. Now this value will be equating with this equation that is given Ergun equation as per problem here. So if you substitute this value here and solve this equation for this VMF for minimum fluidization velocity you will be able to calculate what will be the minimum fluidization velocity.

So it is coming after calculation it is 15.8 you check it once. Then another problem is given to calculate that frictional pressure drop from which you have to calculate minimum fluidization velocity. Here it is said that a bed of spherical glass beads density 3000 kg per meter cube diameter is 1 millimeter bed porosity is 0.5 is to be fluidized by a liquid of density 1000 kg per meter cube and viscosity of 0.1 Pascal second. In this case assume that the Reynolds number based on particle diameter is small compared to 1.

If g is equal to 10 then what is the minimum velocity required to fluidize the bed. So interesting here that liquid is here not gas it will be liquid as water here. And in this case diameter of this particle is I think it is given 1 millimeter bed porosity is 0.5 and also viscosity is given to you. So Reynolds number is the main important point here that Reynolds number is close to 1.

That means here you can say that it is only Kozeny-Carman equation you can use to calculate the frictional pressure drop. So for Reynolds number based on particle diameter is a small compared to 1. One can write this Kozeny-Carman equation here. So Kozeny-Carman equation is given by this that you have already learnt earlier in earlier lecture that we have discussed. So if you substitute this delta P by L here at its minimum fluidization condition and equating with this Kozeny-Carman equation then you will get this equation and after rearranging you will be able to find out what will be the minimum fluidization velocity.

So it is coming like this and then after substitution of this other parameters other variables like here Phi P that means sphericity and then dP and rho s, rho f, g, mu f all are given to you even epsilon f also is given to you. So after substitution and calculation you will get this 3.33 into 10 to the power minus 4 meter per second. I think you understood this problem. Then another problem is given 2020 very recently a vertically held packed bed has a height of 1 meter and void fraction of 0.

1 meter when there is not flow through the bed in this case the incipient fluidization is set in by injection of fluid of density 1 kg per meter cube. The particle density of the solid is 3000 kg per meter cube. Acceleration due to gravity is given 9.81 meter per second square. What is the pressure drop across the height of the bed? So in this case again you have to calculate the pressure drop at its minimum fluidization condition.

So for minimum fluidization we know this pressure drop will be is equal to this. So here epsilon is void fraction is given to you rho s is given to you that means solid density and gas density also is given to you length of the bed is 1 meter it is also given then what will be the friction of pressure drop of this bed. So after substitution of those values and calculation we can get 26478.

17 Pascal. I think this problems you understood. So based on this theory we are having concept of that minimum fluidization how to calculate that minimum fluidization velocity and also we have done some examples to calculate the minimum fluidization velocity and frictional pressure drop and also height of the bed if your void fraction will be changing accordingly. So I think you understood this concept of that minimum fluidization velocity how to calculate based on that Ergun equation concept at its minimum fluidization condition or incipient condition from the maximum packed bed condition. So thank you for giving attention in the next lecture we will try to understand more about this fluidization of solid particles in gas liquid mixer and there is a special case it is called flotation we will try to understand the basic fundamental of that flotation process. So thank you have a nice day.