Lec 17: Degree of mixing and Its Assessment

Hello everybody! Welcome to this massive open online course on solid-fluid operations. So, in this lecture we will discuss about the degree of mixing and its assessment. In the previous lecture we were discussing about the mechanism of mixing as well as what are the different equipments those are being used for getting intense mixing between solids of different types and different sizes. Here we will try to understand what are the mechanism based on which we can assess the degree of mixing that means how much mixing has happened some quantitative analysis to be done and also how to analyze the rate of mixing and there will be a certain parameter based on which that we can analyze the mixing is called mixing index that also will be discussing here. So as that the performance of the industrial mixer that is determined by the degree of mixing that means how much mixing has happened or intensity of the mixing sometimes it is called what is the effectiveness of the mixing and that can be assessed by some mixing index that will be discussing and based on the concept of that random distribution of the material. As we know that whenever materials will be mixed the mechanism is that there will be I think three mechanism that we are discussing about that one is called diffusion by diffusion that mixing happened by shear that diffusion happened by that convection that mixing happened.

So due to that convection or shear or diffusion we can say that this distribution of the particles randomly in a mixer. So because of which we can say that there will be certain degree of mixing and based on which we will assess that what will be the that effectiveness or degree of mixing the mixer and that is determined by analysis of a number of spot samples that will be collected from the mixer. If you are having certain amount of mixer output like that in a mixing equipment you are mixed different types of solid materials and after certain operations if you take out some amount of sample from that mixer outlet or equipment outlet and then if you analyze it and that analysis can be done by that segregating or you can say that making that n number of spot sample on, spot sample of that outlet mixer output. So in that case from that number of spot samples that will be collected from that mixer that will be analyzed.

Now if you consider a solid mixer in which a tracer material is added which does not react with the other materials in the sample. Now question is that some tracer materials to be added there because that based on that analysis of tracer material in the mixer you will be able to analyze how much or what extent of mixing happened there. So that is why some tracer materials which will not be reacting with the other materials will be added before mixing those materials by mixer. So in that case solid mixer in which a tracer material to be added that will not be reacting with the other materials in the sample. Now if you take the number of samples let it be denoted by capital N and it will be taken randomly from the various locations of the mixer here.

See here this is a mixer in the picture it is shown and from this mixer you are taking different spot samples like given like this, this type of different samples we are collecting. So here n number of small samples will be taken out from this mixer. After that you have to determine the fraction of tracer Xi in each samples of i. Suppose if you are taking the samples i from these nth samples you can say in that case you just measure what will be the fraction of that tracer material out of this. So if you are considering this sample now in this case you will see that out of suppose 1, 2, 3, 4, 5, 6, 7 out of 7 2 are tracer materials or you can make it a mass fraction.

You can take it as a number fractions but number fractions sometimes very difficult but you can take it a mass fractions after segregating this tracer material from this spot sample. So determine the fraction of tracer Xi in each small samples i and also you can measure what will be the average concentration of sample of that tracer material that can be denoted by X bar. So average value of the measured concentration is X bar. That means here you will see that what is the average concentration that is X bar that means Xi there are suppose X different spot samples you are getting the different you know concentration of that tracer material. So out of that total mixture what will be the average concentration of that tracer material that will be X bar.

So here let the overall average fraction of tracer in the mixture as mu, mu is what is that total average concentration. Whereas you will see that in spot samples in each spot samples may not be that X bar will be same. So X bar will be different from different n number of spot samples, n number of spot samples. So here if you are considering this spot samples here average concentration is X bar. Here also X bar, but overall concentration of the tracer material will be is equal to mu.

So if n tends to very less suppose if you are taking n number of samples 100 more than 100, 200, 500 if you increase the number of spot samples taking then you will see that that average value of measured concentration that is X bar will be almost equals to the overall average fraction of tracer in the mixture that is mu. That means here if n tends to very large X bar tends to mu otherwise X bar will be different for different samples. So this you have to remember. So here basically we are getting what there are material 1 and material 2 which will be mixed along with that tracer material and then after mixing you are taking that mixture sample here and out of that mixture you are taking n number of spot samples. In each spot samples what will be the average concentration of that tracer material that may be different for different n number of spot samples but if you are considering n number of that means if you increase the number of spot samples taking then your X bar will be tends to mu which is called overall average fraction of the tracer in the mixture which is denoted by mu.

So I think you understood this portion and then what you have to do you have to calculate that standard deviation.

Here the Equation(s)

$$s = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N-1}}$$

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

So here again if solids were perfectly mixed every measured value of Xi would be equal to X bar. Now Xi may not be equal to X bar because there is not perfectly mixed solids. If the mixing is not complete the measured value of Xi will differ from X bar and suppose if there is a uniform mixing that means everywhere mixing happened uniformly then you can say that Xi and X bar almost will be same and also mu that is overall concentration also will be same average concentration. The standard deviation about the average value of X bar will be a measure of degree of or quality of mixing.

So what is the deviation standard deviation? Standard deviation that means what will be the deviation from the X bar average value of that concentration with the sample concentration Xi and if you make it square and then summation of for all n number of spot samples and divided by n minus 1 then you will get this standard deviation which is defined by this equation which is denoted by S. The value of S is a relative measure actually of mixing which will be valid only for test of a specific material in a specific mixture and X bar is the average concentration which can be measured or which can be estimated by this taking number of you know Xi that means concentration of that tracer for n number of spot samples and summation after that if you divide it by its number of samples you will get that mean or average value of that concentration of tracer material. So here standard deviation is you will see that diminishes towards 0 as mixing proceeds. So here this standard deviation if suppose the mixing tends to complete mixing then the standard deviation will diminishes towards to 0. And so low value of this standard deviation will give you the good mixing indication.

So this is the main concept of that analysis of this mixing or degree of mixing. There are other way to analyze this mixing that is called mixing index. You will see that various statistical measures that are independent of amount of tracer. In this case, Hernby et al. 1997 they have introduced a mixing index to quantify the degree of mixing.

That mixing index it is generally or they have considered for paste. So the mixing index for paste they have given the definition as Ip will be equal to sigma 0 by S that will be equal to root over n minus 1 into mu into 1 minus mu by summation of I2 n 1 to n that is Xi minus X bar whole square. So here in this case sigma 0 what is that sigma 0? This sigma 0 is basically that when there is a standard deviation at 0 mixing will be calculated that can be considered as a sigma 0. That means standard deviation at 0 mixing and the index is the reciprocal of the ratio of that S. S is what is that? It is not that 0 mixing there will be certain extent of mixing will be there.

So the ratio of these 0 mixing that means standard deviation at the 0 mixing and the standard deviation at the mixing at a certain time you will see that will be regarded as a mixing index for the paste. Where sigma 0 that means that standard deviation at 0 mixing can be calculated by this equation that is called root over mu into 1 minus mu. What is mu? So we have got that overall average fraction of tracer in the mixer. So before mixing has begun the material in the mixer that exists as two layers one of which contains no tracer and one of which contains all tracers. Under this condition it is called 0 mixing.

So at that 0 mixing condition we can have this 0 mixing at its standard condition will be is equal to root over mu into 1 minus mu. So the mixing index for paste will be is equal to IP which can be defined by this equation.

Here the Equation(s)

$$I_P = \frac{\sigma_0}{s} = \sqrt{\frac{(N-1)\mu(1-\mu)}{\sum_{i=1}^{N}(x_i - \bar{x})^2}}$$

$$\sigma_0 = \sqrt{\mu(1 - \mu)}$$

Now in this case you will see that interesting point here that in any batch mixing the mixing index IP is unity at the start. That means when mixing will not happen at the very beginning we can say that IP will be equal to what is that is 1 and higher the value of mixing index will indicate the higher mixing and the mixing index would become infinite at long mixing time. Of course for infinite time if you allow to mix those materials your mixing index will be infinite.

That means here mixing of the solid materials will give you the better mixed or you can see intensity of the mixing or degree of mixing will be higher. So it depends on time how much you are allowing it to mix. The mixing index falls within a range of 10 to 150. Generally it is seen that from the experiment that mixing index will be ranging between 10 to 150. The rate of mixing as measured by the rate of change of IP.

Now how then what will be the rate of mixing? Rate of mixing can be calculated based on the change of this IP with respect to time. So at initial condition it will be 1 that means there will be no mixing. Whereas if you increase the time or mixing with respect to time if you increase if you allow the mixing with respect to time more time you are allowing then in that case mixing index will be increasing. So this increasing rate of IP with respect to time will be regarded as a rate of mixing. And of course it depends on the kind of mixture it depends on the nature of materials either this sticky materials or dry materials those characteristics of that materials will change this IP value that means mixing index value with respect to time.

Now let us do an example for this. Suppose a loose soil containing 14% moisture was mixed in a large Mueller mixer we have discussed earlier that what is the Mueller mixer with average 10 weight percent of a tracer consisting of dextrose that means tracer material here dextrose and picric acid. After 3 minutes of mixing 12 random samples were taken from the mix and analyzed calorimetrically for tracer material. The measured concentration in the sample were in weight percent tracer is 10.

24, 9.30, 7.94, 10.24, 11.08, 10.

03, 11.91, 9.72, 9.20, 10.76, 10.97 and 10.55 that is 12 samples will give you that 12 concentrations here and calculate the mixing index and the standard deviation is. So, in this case you see that there is a mixer from which you are taking 12 samples and you have already calculated what will be the average concentration that is 10 weight percent. After 3 minutes of mixing you are taking that 12 random samples and those random samples will give you the concentration here as shown in the problem.

Now in this case what would be the mixing index that you have to calculate. So to calculate that mixing index Ip what is needed? You need how many number of samples you have taken that is n number of samples here 12 capital N is equal to 12, mu is required that means overall average concentration of the tracer it is given 10 weight percent and also you need what will be the concentration for each sample that is Xi and what is the average concentration of that samples it is given X bar. Now in this case what will be the X bar? X bar is summation of Xi by n summation of Xi after summing up all those concentrations 12 concentration and divided by 12 you will get this value 0.101617 and then you have to calculate mu, mu is already given to you that means 0.10 and number of samples is n is equal to 12 and then you have to calculate the deviation for all samples that is summation of Xi minus X bar whole square that will come like this and then you substitute these values here in Ip that is given in equation.

After calculation you will get this value as 28.85 so this is your mixing index so this mixing index will give you that this degree of mixing there it is not that zero mixing this is not that infinite mixing or uniform mixing you can say certain degree of mixing here and then standard deviation is simple that summation of Xi minus X bar whole square divided by n minus 1 then you have to make it square root and after that you are getting 0.0104 so this is your standard deviation. So, I think you understood this problem. Next that let us consider that granular material instead of paste.

This is not a sticky material granular material dry material the individual particles you can count even you can assess also that particle in the granular materials. Now with granular solids the mixing index is based on the standard deviation that would be observed with a completely random fully blended mixture here. In this case according to that paste whatever we have considered based on that zero mixing index here it will not be that zero mixing index. In this case there will be a random mixing and you have to observe that what will be the deviation that would be observed with a completely random fully blended mixture there. And in this case consider again as the same way a completely blended mixture of salt and the sand grains from which n spot samples is containing n number of particles are taken.

That means we are taking here n number of spot samples in each samples there will be a n number of particles n number of particles. The fraction of sand in each spot sample is determined by counting particles of this kind we are counting particles of this kind like n. So n out of total n out of here how many total number of particles here how many total number of particles here how many total number of particles how many total number of particles and for a particular tracer material particles you can say that what will be the number concentration that you can obtain here suppose here in this samples ith sample here n number of particles here the tracer particles will be ni. So it will be your number particle fraction in this way. So suppose the fraction of sand in each spot sample is determined by counting particles of each kind.

If suppose here tracer is not being used here only two number of particles two types of particles are there. One is sand another is suppose other particles suppose coal. So sand and coal both will be mixed. Now in that case if you take the spot sample after mixing in that spot samples how many particles of coal how many particles of sand. So what will be the fraction that means here n sand if you are considering sand particle concentration or fraction say n sand divided by n sand plus n coal.

So it will be your fraction n fraction. This is your concentration that is Xn you can say or some Xi is equal to say so nS by nS plus nC in this way. So suppose the fraction of sand in each spot sample is determined by counting particles of each kind let the overall fraction by number of particles of sand here we are considering sand which is mixed with other type of material in the total mixture be mu. So overall fraction is mu as per earlier concept and then if n is a small that is say about 100 the measured fraction Xi of sand in each sample will not always be same even when the mix is completely and perfectly blended. Thus for any given size of spot sample there is a theoretical standard deviation for a completely random mixture this standard deviation can be represented by sigma R. So in this case you see that for any given size of spot sample there should be a certain theoretical standard deviation if you have a completely random mixture there so that standard deviation can be denoted by sigma R.
Here the Equation

$$\sigma_R = \sqrt{\frac{\mu(1-\mu)}{n}}$$

So sigma R can be defined by this sigma R is equal to root over mu into 1 minus mu by n where mu is equal to overall mixture fraction of particular solid in the mixture here particular solids like sand out of sand and coal mixture and n is the average number of particles in each spot sample N is the number of spot samples. So for granular solids the mixing index is defined as here Igs, gs means granular solids so Igs mixing index of granular solids that will be equal to sigma R by s instead of paste they are sigma 0 by s here sigma R by s

Here the Equation(s)

$$I_{GS} = \frac{\sigma_R}{s} = \sqrt{\frac{\mu(1-\mu)(N-1)}{n\sum_{i=1}^{N}\left(x_i - \bar{x}\right)^2}}$$

$$s = \sqrt{\frac{\sum_{i=1}^{N}\left(x_i - \bar{x}\right)^2}{(N-1)}}$$

$$\bar{x} = \sum x_i / N$$

here random sample here not 0 mixing here at a particular time what will be that sigma R by s. So this will be is equal to mu into 1 minus mu into n minus 1 divided by n small n summation of i is equal to 1 to n into Xi minus X bar whole square. So in this case you can calculate what will be the mixing index for granular solids where s is defined by this equation and average value of X that will be defined by this summation of Xi by n. So here suffix gs is basically the granular solids.

And then mixing index at 0 time what is that mixing index at 0 time the equilibrium standard deviation for complete mixing sigma R is used as a reference with granular solids with paste the reference the standard deviation at 0 mixing sigma 0 before mixing begins the mixing index at 0 mixing that is called mixing index at 0 time which is defined by this equation. So it will be for the granular solids what will be the mixing index at 0 time that is Igs0 that will be is equal to sigma R by sigma here 0 that will be is equal to what 1 by root over n.

Here the Equation

$$I_{GS,0} = \frac{\sigma_R}{\sigma_0} = \frac{1}{\sqrt{n}}$$

What is the rate of mixing? You will see that whenever you are getting some mixing index that mixing index will be changing with respect to time. So here in mixing as in other rate processes the rate is proportional to the driving force in that case that mixing index Igs is a measure of how far mixing has proceeded toward equilibrium that is with respect to time it will come for short mixing times the rate of change of Igs is directly proportional to 1 minus Igs or it can be written as like this. So it is for a short period of time what will be the rate of change of Igs.

So this Igs which is changing with respect to time that will be proportional to the 1 minus Igs that means 1 minus Igs

Here the Equation

$$\frac{dI_{GS}}{dt} = k(1 - I_{GS})$$

how much materials will not be mixed in that mixture that is proportional. So here D Igs by Dt that will be is equal to K into 1 minus Igs where K is constant here the equilibrium value of Igs is 1. So you will see that the Igs will be changing with respect to time once upon a time you will see that this mixing will be uniform in that case the mixing index will not be changed with respect to time. So that is your equilibrium condition that means here certain time you will see that this mixing index will be like this here with respect to time there will be no change of Igs with respect to time.

So here it will be your equilibrium condition. So here this is your equilibrium time. So K is a constant here the equilibrium value of Igs is 1 here it will be 1 and therefore the driving force for mixing at any time can be considered to be a 1 minus Igs. Here you will see that with rearranging and integrating this equation we can get within it time limits 0 to t then here it will be coming as the mixing index at 0 time to the mixing index at a time t. So after rearranging we can get this equation and then integrating with this time limit we are having t will be is equal to what 1 by K ln 1 minus Igs 0 by 1 minus Igs or you can substitute the value of Igs 0 here. What is the Igs 0? Igs 0 is equal to sigma R by sigma 0 that will be equal to 1 by root over N.

Here the Equation(s)

$$\int_0^t dt = \frac{1}{k}\int_{I_{GS,0}}^{I_{GS}} \frac{dI_{GS}}{1-I_{GS}}$$

$$t = \frac{1}{k}\ln ln \frac{1-I_{GS,0}}{1-I_{GS}}$$

$$I_{GS,0} = \frac{\sigma_R}{\sigma_0} = \frac{1}{\sqrt{n}}$$

after substitution of this Igs 0 here you can get this equation after simplification 1 by K into ln into 1 minus 1 by root over N divided by 1 minus Igs.

Here the Equation

$$t = \frac{1}{k}\ln ln \frac{1-1/\sqrt{n}}{1-I_{GS}}$$

So this is your rate of mixing. So once what will be the Igs at a particular time then you can easily calculate what will be the rate constant and also if that rate constant and the mixing index at a particular time and then number of particles in a granular samples then you will be able to calculate what will be the time required to get that extent of degree of mixing. Now let us do an example here. An experimental data on the rate of mixing of sand and salt particles.

Here sand and salt particles in an air fluidized bed is given below that means in a fluidized bed there mixing happened between salt and sand particles. In each run the initial charge to the reactor was 254.4 gram of salt on top of 300 gram of sand that means here you will see that mixing happens initially 254.4 gram of salt with the 300 gram of sand. The average number of particles in each spot sample was 500 and the mixing time was 45 second that means after 45 second of mixing the average number of particles in each spot sample was 500.

Now you have to find out how long will it take for the mixing index to reach 0.95. In the table it is given that number of fraction of sand in spot samples are like this here. So how many here samples 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20.

Here 20 samples are taken. So out of this 20 sample after completion of 45 second of mixing you are getting the average number of particles in a spot will be 500. After mixing time of 45 you are getting this average. So how long it will take to get this mixing index to reach 0.

95. Here it is not 0.95. Here nowhere it is reached to 0.95 of that mixing index. So to get that mixing index how long it will take. Whereas some samples will give you this data like 45 seconds after that you are getting that average number of particles is 500. So in this case. Now in this case you have to find out time that is t is equal to what t is equal to 1 by k is equal to ln 1 minus 1 by root over n by 1 minus Igs and then mu is equal to overall average fraction of salt in the mixture is 254.

4 by 254.4 plus 300 that will be equal to 0.46 mu value and then x bar will be equal to summation of xi by n that is 0.505 and Igs is equal to sigma R by s that means root over mu into 1 minus mu into n minus 1 divided by n to summation of i is equal to 1 to n to xi minus x bar whole square. This is your definition of Igs. So Igs value is 0.0710 and here 45 it is given to you this time value is given 45 and then from this 45 seconds value you have to calculate what will be the value of k others are given to you.

So k will be is equal to this then you have to find out what will be the time this k is constant is not varying. So for this k value if you again substitute this k value here and Ig value as 0.95 and n is equal to 500 then you will get the time of mixing to get the mixing index of 0.

95 it is coming 4759 seconds. I think you understood this problem. Next let us consider some other analysis of that mixing which is some limits based on that variance. So based on the theoretical limits of variance we can analyze the different that way of analysis of

mixing phenomena of that solid materials. Now if you consider that for a two component system the theoretical upper and lower limits of a mixture variances upper limit of that variance is sigma 0 square that will be equal to mu into 1 minus mu

Here the Equation(s)

$$(a)\ Upper\ limit\ (completely\ segregated)\sigma_0^2 = \mu(1 - \mu)(b)\ Lower\ limit\ (randomely\ mixed)\sigma_R^2 = \frac{\mu(1-\mu)}{n}$$

where the completely segregated samples that means there will be no mixing whereas lower limit the randomly mixed that is sigma r square that means randomly mixed you can say so it will be mu into 1 minus mu by n where mu and 1 minus mu are the fractions of two components that will be determined from samples and here n is the number of particles in each sample. So actual values of mixture variance will be lying between these two extreme values here. So this is the upper limit and lower limit of mixing that means where completely segregated that means there will be no mixing in that case sigma 0 square and for lower limit where randomly mixed it will be sigma r square.

Now another mixing index which will be depending on this upper limit and lower limit value that is defined by or that is given by Lacey 1954. In that case Lacey defined that mixing index that is called Lacey mixing index. So Lacey mixing index is defined as IL that will be equal to sigma 0 square minus s square by sigma 0 square minus sigma r square. So here this is the lower limit and this is the upper limit.

Here the Equation (s)

$$Lacey\ mixing\ index\ (I_L) = \frac{\sigma_0^2 - s^2}{\sigma_0^2 - \sigma_R^2}$$

$$\sigma_0^2 = \mu(1 - \mu)$$

$$\sigma_R^2 = \frac{\mu(1-\mu)}{n}$$

$$s = \sqrt{\frac{\sum_{i=1}^{N}\left(x_i - \bar{x}\right)^2}{(N-1)}}$$

some mixing so that is represented by this you know Lacey mixing index where sigma 0 square is equal to mu into 1 minus mu and sigma r square is equal to mu into 1 minus mu by n which is defined earlier and s is the standard deviation at a time when that mix will be happen up to a certain degree. So here this s, now a Lacey mixing index if it comes 0 that will represent that complete segregation that means there will be no mixing and a value of unity will give you that completely random mixture. So your objective is to find that completely random mixing and which will be that around 1. So from this Lacey mixing index you are getting the value between 0 and 1. So if it is coming near about 0 then you can say that the mixing will be good okay or you can say completely random mixing there.

Practical values of this mixing index however are found to lie in the range of 0.75 to 1. So another way to assess that mixing that is called Poulet et al mixing index okay. So further mixing index as suggested by Poulet et al which is defined as Poole mixing index P double OLE, Poulet mixing index that is IpO that is s by sigma r.

Here the Equation

$$Poole\ index\ (I_{PO}) = \frac{s}{\sigma_R}$$

So this index gives better discrimination for practical mixtures and a process unity for completely random mixtures.

So here s is defined by earlier already you have given this s is defined by this equation and sigma r is also defined by this equation okay. So from this you have to substitute this value and get the IpO. So IpO value if it comes near about 1 then you can get that completely random mixing okay. So I think you understood the basic understanding how to assess the mixing of solids okay by theoretical analysis based on that statistical value whether it is the random distribution or not whether this standard deviation will give you that lower value or not. If standard deviation give you lower value then you get that better mixing and other parameters for assessing that mixing it is called Poulet mixing index, Lassie mixing index and also other mixing index it is called here we have got that granular mixing index that is called IGS and also we have defined or we have learned about that mixing index like Ip for paste.

So these are the different mixing index value based on which you can say whether the solid mixing will be intense or not. So I think it will be helpful for assessing that mixing index and if you go through that examples you will better understand this. So thank you for your concentration and the next lecture we will try to discuss more about this mixing and agitation of fluids and slurries and they are also what are the different mechanism of that mixing of fluid and slurries there. Thank you.