Lec 14: General expressions for flow through packed beds-Ergun Equation

Hello everybody. Welcome to this massive open online course on solid fluid operations. So in this lecture we will try to learn about the general expression for flow through packed beds which is given by Ergun which is called as Ergun equation. Already we have discussed about that what will be the flow phenomena whenever the fluid will be flowing through the bed of packing materials or it is called the granular beds and what are the basic equations for that like we learned what is Darcy's law and what is Kozeny-Carman equation and how that Kozeny-Carman equation can be derived from the basic momentum equation that is given by Hagen-Poiseuille's equation and that concept of that Hagen-Poiseuille's phenomena of flow through that circular pipe that concept was used to derive that Kozeny-Carman equation and based on that Kozeny-Carman equation we have learned how to calculate the frictional pressure drop whenever fluid will be flowing through the porous media or bed of packed materials or granular beds. And also some characteristics factor like surface area per unit volume of the bed as well as surface area per unit you know volume of particles and how those surface areas are interrelated, also how those surface area will be important for analyzing that fluid flow through that porous media. So that we have learned in the previous lecture and in this lecture also we will concern about that flow through that packed beds but here this will be concerned with the general expression just by considering that flow phenomena in a streamline as well as turbulent flow whereas in Kozeny-Carman equation that we have learned about that flow phenomena only at that Reynolds number less than 10 that means very laminar flow regime whereas here we will try to consider that laminar flow regime as well as turbulent flow regime.

So in that case how that frictional pressure drop can be derived? I think it is given by Ergun in his derivation. In this case we will try to learn about that initially what is that application of that packed beds and also derivation of the Ergun equation. How it can be derived based on that laminar and turbulent flow effect or you can say that viscous effect and inertia effect. So before going to that we will try to learn something about where that packed bed or packing bed material which will be used for that chemical engineering processes and where that packed bed reactor can be used for that chemical engineering processes.

You will see that some industry they are using that for separation of naphthene in refinery. like BPCL they are using that packed bed system and also Dewpoint isothermic hydro processing They are using that packed bed where some solid materials as a catalyst particle be intact and through which that fluid will be flowing and then that reaction will be happening for the hydro processing and also you will see that Lurgi mega methanol plant there how methanol can be produced from that methane and from the oxidation of the methane and how that methanol can be produced that is you know Lurgi mega methanol plant where this production is being done. So there also this packed bed is being used there. Some other industrial reactions over heterogeneous catalyst bed in a packed bed reactor can be carried out like here some example here given in the slides like catalyst metals as an example like nickel, palladium, platinum as a powders or on support or a metal oxides like chromium oxide can be used for the reaction of polypene with hydrogen which will give you the paraffin. Similarly other catalyst like nickel that is actually being used for you know production of you know methane that is called methaneation process with that you know synthesis gas that means carbon monoxide and hydrogen gas which will give you that methane in presence of nickel catalyst.

Even you will see that sometimes that iron as a catalyst that supported or you know sometimes promoted with alkali metals that is being used for production of ammonia from the hydrogen and nitrogen. So this hydrogen and nitrogen will be passed through that bed of iron where that ammonia to be produced after reactions. Also copper like supported of zinc oxide with other components like aluminum oxide also is being used as a catalyst particles of the packed bed where synthesis gas to be produced as methanol production. So here this copper or aluminum oxide particles should be as a packing material to be used in the packed bed where that synthesis gas to be converted into methanol. Similarly other you know catalyst particles like rhenium and platinum you know mixture as a catalyst particle which is to be used for dehydrogenation of the paraffin even isomerization and dehydro cyclization for reaction that this catalyst rhenium and platinum mixed catalyst to be used.

Other application like some solid acids of the catalyst you can say silica and aluminum oxides mixture and then you will see that zeolites. So when you are there you will see that sometimes you need to crack that heavy hydrocarbons to produce that lighter hydrocarbons there. So for paraffin cracking this type of catalyst are being used. Even you know that palladium supported on acidic zeolite those are used as a hydro cracking of that paraffin. Even sometimes you will see that metal oxide supported complexes of like chromium, silicon even zirconium those catalyst are being used for polymerization of for production of polyethylene from the ethylene.

Even silver like on inert support promoted by alkali metals like vanadium pentoxide or platinum all those are being used for conversion of ethylene oxide or sulphur trioxide there. And bismuth molybdate also as a catalyst in a packed bed to be used for you know oxidation of propylene to produce that you know acroline. So that case we are using that different catalyst particles which is to be intact in a packed condition or sometimes as a fluidized condition to be used for you know production of different hydrocarbons even different inorganic chemicals also. So that is why it is called that packed bed and this packed bed reactor to be used since there is a reaction happens in presence of this catalyst particle. So here we can say that this packed bed is very very important for the production of different carbons, hydrocarbons even you will see that different organic and inorganic chemicals there.

So in that case you have to know some hydrodynamics of that packed bed reactor as this you know usually label at least you have to know what the frictional resistance of that packed bed and how that packed bed reactor can be designed based on that frictional resistance or frictional pressure drop. So you have to know what is the frictional pressure drop whenever fluid will be flowing through that packed bed and what is the governing equation, what is the general expression that you have to know. So let us learn that you know general expression for estimation of the frictional pressure drop whenever fluid will be flowing through the bed of packed materials. So in that case we know that whenever fluid will be flowing through that you know packed bed, so practically that packed bed will be you know packed with a randomly arranged packing material that may be irregular size of that particles who have some sphericity value. So in that case what should be the frictional pressure drop in that packed bed that may be occurs that frictional resistance that may be occurs due to some viscous effects and also that inertia effects.

Here the Equation

$$\Delta P_{f,total} = \Delta P_{f,viscous} + \Delta P_{f,inertial}$$

be flowing through that packed bed you will see that there will be an effect of viscous effect that means fluid will be come in contact with the you know solid surface and in that surface there will be some viscous effect because of that you know viscosity of the fluid and that is called frictional forces and also whenever fluid will be flowing at a certain velocity there will be some inertia effects also. So both the you know effects, viscous effects and inertia effects will contribute the frictional pressure drop in the packed bed. At high Reynolds number you will see that inertia effects will be dominating compared to that you know viscous effects whereas that viscous effects will also be important at low Reynolds number. So in general we can say that this total you know frictional pressure drop will be contributed by pressure drop due to viscosity and pressure drop due to the inertia. So these two components will give you that total you know frictional pressure drop in the packed bed.

Here you will see that in the suffix is represented as you know frictional. So in 1952 actually Ergun given and you know general expression to you know estimate the frictional pressure drop and he has derived this frictional pressure drop whenever fluid will be flowing through the bed of packing materials that is in packed bed and from this contribution of the viscous and inertia effect. So let us learn those things first. So if we consider that for a packed bed it is proposed that you know this drag force per unit total you know surface area of the particle you will see that which will be defined as drag over the channel wall consisting of packed bed particles divided by total surface area of the particle. Now here channel wall we are considering whenever that fluid will be flowing through that you know packed bed you will see that the fluid will be flowing through a voids.

So that voids will be itself called as a channel, small channel there. So that is why whenever it will be passing through that voids or channels there will be some drag force acting over the channel wall and of that you know packed bed particles. So that drag force per unit surface area of the particles will be represented by this FD by AS that will be actually called as wall shear stress, wall shear stress in laminar turbulent flow in both the cases. So whenever fluid will be flowing through that you know void fractions or void region through the void you can say that the wall shear stress will be based on that laminar as well as turbulent flow. So we can say that wall shear stress in laminar flow plus wall shear stress in turbulent flow both will be contributing to that you know drag force per unit surface area of that particle.

Here the Equation

$$\frac{F_D}{A_s} = \frac{Drag\ over\ the\ channel-wall\ consisting\ of\ packed\ bed\ particles}{Total\ surface\ area\ of\ particles}$$

the laminar flow? That will be actually proportional to the viscosity of the fluid, also proportional to the velocity of the actual velocity of the fluid and also it will be inversely proportional to the hydraulic radius of the channel. So in that case we can represent by this mathematics here. So FD by AS will be proportional to the mu F UA by RH that will be contributed by wall shear stress. Similarly, that FD by AS that means drag force will be you know proportional to that density of the fluid and also that square of that velocity of the fluid through which fluid that is through the void. So here we are having these two contribution as a proportionality factor.

Here the Equation(s)

$$\propto \frac{\mu_f u_a}{r_h} + \propto \rho_f u_a^{\,2} = K_1 \frac{\mu_f u_a}{r_h} + K_2 \rho_f u_a^{\,2}$$

So what is that proportionality constant here? K1 is the proportionality constant for that you know wall shear stress in laminar flow. So we can write here that K1 into mu F UA by RS this will be your you know drag force at this wall per unit area that is called wall shear stress in laminar flow and then K2, K2 is here is called that again constant, proportionality constant that will be coming from that wall shear stress in the turbulent flow. So total wall shear stress in the turbulent flow will be K2 into rho F into UA square. So here we can say that this portion will be dominating that means wall shear stress in laminar flow will be dominating if the Reynolds number of the particle is less than 10 and wall shear stress in turbulent flow will be dominating if Reynolds number will be greater than 10. So here these two components will be contributing to the total drag force here and also this Reynolds number here will be defined by this equation.
Here the Equation

$$= \frac{\rho_f u_a \phi_p d_p}{(1-\varepsilon_f)\mu_f}$$

In this case you will see here Reynolds number of the particle as a star we are considering here it is defined as rho F UA phi P dP by 1 minus epsilon F into mu P. So here epsilon F is the void fraction that we have discussed in the previous lecture also and UA is the velocity of the fluid and phi P is the sphericity of the particle, dP is the particle diameter, mu F is the viscosity of the fluid and rho F is the density of the fluid. And UA is the actual velocity of the fluid it is basically the superficial velocity divided by the void fraction. And then you have to know what is that hydraulic radius because this hydraulic radius is actually will be giving the contribution to that wall shear stress there. And this hydraulic radius that is the total cross section of the conduits divided by weighted perimeter then we can write here will be equals to total cross section of the conduits divided by weighted perimeter into L by L.

Here the Equation

$$r_h = hydraulic\ radius = \frac{Total\ cross-section\ of\ conduits}{Wetted\ paremeter} = \frac{Total\ cross-section\ of\ conduits}{Wetted\ paremeter} \times \frac{L}{L} = \frac{Total\ volume\ of\ voids}{Total\ surface\ area\ of\ partic}$$

volume of the void shear and also here L we are considering the total surface area of the particle. So we are having this value here that hydraulic radius will be h0L into epsilon F by As. Whereas As is called this is basically that total surface area of the particle. Now how can you find out that total surface area of the particles? Now you do not know how many particles are there in the packed bed. So if you know that number of particles let it be considered here that Np as a total number of particles there.
Here the Equation

$$A_s = N_p \times S_p Total\ no.\ of\ particles \times Surface\ area\ of\ one\ particle = \frac{S_o L(1-\varepsilon_f)}{V_p} \times S_p = S_o L(1-\varepsilon_f)\frac{6}{\phi_p d_p}$$

And if you know the surface area of individual particles or one particles which is represented by Sp then we can have the total surface area as Np into Sp. So total number of particles into surface area of one particle will give you the total surface area of the particles. Now Np how will you calculate that Np? Np that is number of particles. Number of particles how will you calculate? For that you have to know what will be the volume of that particle that total volume of the particle in the packed bed. The total volume of the particle in the packed bed it will be basically that if you consider h0 is the cross sectional area of the packed bed and L is the length of the packed bed.

So it will be your volume total volume of the packed bed. Out of which some volume will be for particles. So that will be the 1 minus epsilon f. 1 minus epsilon f is basically the particle volume fraction. Epsilon f is the volume fraction of fluid.

So 1 minus epsilon f is the particle volume fraction. So total volume of packed bed into particle volume fraction will give you the total volume of the particles in the packed bed. Now if you divide it by individual particle volume then you will get how many numbers of particles will be in the bed. So that is why we have divided it by Vp. Vp is basically the volume of individual particles.
Here the Equation

$$\frac{S_p}{V_p} = \frac{\pi d_p'^2}{\pi d_p'^3/6} = \frac{6}{d_p'} = \frac{6}{\phi_p d_p}$$

So you are having this and then you have to multiply by the surface area of one particle. So finally we are getting here h0 L into 1 minus epsilon f. Vp is basically 1 by 6 into pi Dp cube and after substitution you will see that of this Sp or Vp. Here you will see that Sp by Vp already we have calculate here Sp by Vp is equal to Sp is basically pi Dp square surface area and volume of this particles then it will come 6 by Dp dash and it will be basically 6 by pi P Dp. So after substitution of this value here we are getting that As will be equal to h0 L into 1 minus epsilon f into 6 by pi P Dp.

Here the Equation

$$A_s = N_p \times S_p \text{Total no. of particles} \times \text{Surface area of one particle} = \frac{S_o L(1-\varepsilon_f)}{V_p} \times S_p = S_o L(1-\varepsilon_f)\frac{6}{\phi_p d_p}$$

So in this way we can calculate the total surface area of the particles. Once we know the total surface area of the particle we can calculate what would be the hydraulic radius from this equation. Next after finding out that total surface area and hydraulic radius then we will be substituting those values here in the drag force per unit surface area of the particle here as FD, FD into As value we are just substituting here this As value and then after this we will be equal to what that will be contributed by the laminar shear stress and the turbulent shear stress. So these two shear stress will be contributing to this total drag force. So after substitution of this Ra value here again then also Ua will be equal to U by epsilon f after substitution of U value here Ua into epsilon f then we are having here all in terms of you know that U value.

So we are having this you know drag force here will be is equal to finally after simplification we are getting this equation that is rho f into U square divided by epsilon f square into 6 K1 mu f into 1 minus epsilon f by U pi P Dp rho f plus K2 this one we are getting.

Here the Equation(s)

$$\frac{F_D}{A_s} = \frac{F_D \phi_p d_p}{S_o L(1-\varepsilon_f)\times 6} = K_1 \frac{\mu_f u}{r_h} + K_2 \rho_f u^2 = K_1 \frac{\mu_f u}{\varepsilon_f^2 S_o L} \times \frac{S_o L(1-\varepsilon_f)6}{\phi_p d_p} + K_2 \rho_f \frac{u^2}{\varepsilon_f^2} = \rho_f \frac{u^2}{\varepsilon_f^2}\left[6K_1 \frac{\mu_f(1-\varepsilon_f)}{u\phi_p d_p \rho_f} + K_2\right]$$

So this is basically that drag force per unit you know surface area of particles. Then you have to substitute the value of FD what is that FD? FD will be in terms of frictional pressure drop. So FD that is drag force can be defined as by this equation.

Here the Equation

$$F_D = \text{Drag force} = \Delta P_{\text{f,total}} \times S_o \varepsilon_f$$

FD will be equal to drag force that will be delta PF total frictional pressure drop into what is that cross sectional area that will be occupied by the fluid because that fluid will give you the frictional pressure drop.

So that total surface area that is occupied by the fluid that would be h0 into epsilon f. So the delta PF frictional pressure drop into surface area occupied by fluid that will give you the drag force here. Similar substitution of this FD in the earlier equation here in this case we can say and simplification we are getting this equation finally. So delta PF total by L that into epsilon f cube by 1 minus epsilon f into 5 P Dp by rho f u square that will be your 36 k1 into mu f into 1 minus epsilon f by u 5 P Dp rho f plus 6 k2.

Here the Equations

$$\frac{\Delta P_{f,total}}{L}\left(\frac{\varepsilon_f^3}{1-\varepsilon_f}\right)\frac{\phi_p d_p}{\rho_f u^2} = 36K_1\frac{\mu_f(1-\varepsilon_f)}{u\phi_p d_p \rho_f} + 6K_2$$

So this you will get. So simply here we are substituting FD value in terms of frictional pressure drop and h0 into epsilon f and after that we are simplifying like this. And then based on that experimental observation by Ergun there will be some constant value this you know k1 and k2 here this k1 value and k2 value. We do not know this value. So based on experimental observation it is found that this k1 value is coming 1 by 50 by 36 and k2 is 1.

75 by 6. So after substitution of this k1 and k2 value here in this equation again then we are getting this equation here delta PF total by L here as this or after simplification we are having this equation delta PF by L that is 150 into 1 minus epsilon f square mu f u by epsilon f cube 5 P square Dp square plus 1.75 into 1 minus epsilon f rho f u square by epsilon f cube 5 P Dp.

Here the Equation

$$\frac{\Delta P_{f,total}}{L} = \frac{150(1-\varepsilon_f)^2\mu_f u}{\varepsilon_f^3\phi_p^2 d_p^2} + 1.75\frac{(1-\varepsilon_f)\rho_f u^2}{\varepsilon_f^3\phi_p d_p}$$

So this equation is called that Ergun equation and it will have 2 components here these components this is coming from the viscous effect and this component coming from the inertia effect. So this equation you have to remember.

So this is called Ergun equation. You have to remember throughout your life as a chemical engineer you have to remember this Ergun equation. Whereas this equation will be dominating or this frictional pressure drop will be dominating as viscous loss where this Reynolds number will be less than 10 whereas Reynolds number if it is greater than 10 then kinetic energy loss will be there that means your inertia effect will be dominating. So in that case you have to remember these 2 components of this Ergun equation. Also total Ergun equation that you have to remember. Now defining the friction factor for the back bed.

Now in terms of frictional pressure drop this friction factor can be defined by this equation.

And then after substitution of this frictional pressure drop from that Ergun equation and substituting that we can express this equation from this part. And then we are having that fp will be is equal to here 150 by Rep star plus 1.

75. So this is basically the Ergun equation.

Here the Equation

$$\frac{\Delta P_{f,total}}{L}\left(\frac{\varepsilon_f^3}{1-\varepsilon_f}\right)\frac{\phi_p d_p}{\rho_f u^2} = \frac{150(1-\varepsilon_f)\mu_f}{\phi_p d_p \rho_f u} + 1.75$$

Now from this Ergun equation part we can have this as fp here. So fp will be is equal to what 150 by Rep this is basically what Rep plus 1.75. So we are having this friction factor fp will be is equal to 150 by Rep plus 1.

75. So Rep star is defined as earlier given. So in this way we can define what will be the friction factor for the back bed.

Here the Equation

$$f_p = \frac{150}{} + 1.75$$

Similar friction factor for the circular pipe whenever fluid will be flowing through that circular pipe there again Poiseuille's equation also there it is given that what is the friction factor. That friction factor depends on that flow regime whether it is laminar flow or turbulent flow. In the laminar flow that friction factor generally 16 by Re Reynolds number and for turbulent flow that friction factor is equal to 0.

079 divided by Re to the power 0.25. So that is in absence of particles but in presence of particles here as a packed bed that friction factor will be defined by this. This is also related to the Reynolds number. But this Reynolds number will be defined by this since there are two phases solid and fluid. So Reynolds number will be defined by this and here this friction factor again it will be inversely proportional to the Reynolds number. In earlier cases also in absence of particles that friction factor is inversely proportional to the Reynolds number.

And then we are having from this friction factor in the packed bed that is 150 by Rep plus 1.75 we can say that if Rep is less than less than is equal to 1 V square effect dominates in that case that it will be Fe will be equal to 150 by Rep star which will be called as Kozeny-Carman equation. If Rep is greater than 1000 that is inertia effect will be dominates so in that case Fe will be equal to 1.75 this equation is called Black Plummer equation. Now Ergun equation is used to calculate the pressure drop across packed bed of particle which will be less than 25 millimeter that you have to remember.

Now let us do some example here. In this case you will see that a packed bed of total packed volume of 5 meter cube in which the volume of packing material is 2 meter cube. If the packed bed is operated with a superficial velocity of 2 meter per second then what will be the actual velocity of the fluid in the packed bed. So in this case very interesting that you have to find out what will be the actual velocity. So for that you have to know what will be the volume fraction of the particle first. Here it is given volume of particle is 2 meter cube and the volume of the packed bed is 5 meter cube so volume fraction of the particle it will be 2 by 5 that means 0.

4. So volume fraction of particle it will be 0.4. So remaining fraction it will be of course void fraction. So void fraction of packed bed it will be 1 minus 0.4 that will be 0.6. So the total you can say the actual velocity of the fluid that actual velocity of the fluid will be superficial velocity upon void fraction.

So what will be the superficial velocity? So superficial velocity can be obtained from that you know volumetric flow rate divided by the cross sectional area. But here it is given that superficial velocity is 2 meter per second. So you can get the actual velocity of the fluid will be is equal to superficial velocity divided by void fraction that will be is equal to 2 by 0.

6 that will be coming as 3.33 meter per second. So I think you understood this example. Now coming to the another example how to actually calculate that you know void fraction of the packed bed. In that case you will see that in a packed bed of cross sectional area given as 0.10 meter square and length of 1 meter and there it is found that the total surface area of all particles is 10 meter square. Under this condition if the hydraulic radius of the packed channel is found to be 0.

002 meter then what should be the void fraction of the packed bed that you have to find out. Now you know that hydraulic radius is defined as hydraulic radius that we have already discussed there the total cross sectional area of the conduits divided by weighted perimeter. And you know total cross sectional area of conduits by total perimeter that will be is equal to you know that into L by L that you have to multiply by L by L then you will get the total volume of voids by total surface area of the particles which will be defined as by this H0L into epsilon 8 by As. Whereas H0 is the cross sectional area of the bed and L is the length of the bed epsilon f is the volume fraction of the fluid and As is the total surface area of the particles. Now which will imply that epsilon from this equation we can write then epsilon f will be equal to RSAs by H0L.

Now after substitution of this you know value of RS, RS is given to you that is hydraulic radius 0.002, As value is also given to you and H0 that means cross sectional area is given to you and L is given to you. So after substitution of this value you will be finding out what will be the void fraction. Now coming to the another example where we will be able to find out what will be the pressure across the bed when the volume flow rate will be given to you. And also there what will be the you know particle density and other parameter.

Now let us have this problem here. The problem is that a packed bed of solid particle of density 2500 kg per meter cube that occupies a depth of 1 meter in a vessel of cross sectional area is 0.04 meter square. The mass of solids in the bed is 50 kg and the surface volume mean diameter of the particles is 1 millimeter. Now a liquid of density 800 kg per meter cube and viscosity 0.002 Pascal second flows upward through the bed which is restrained at its upper surface.

Now in this case you have to calculate what will be the voidage that means volume fraction occupied by the liquid of the bed and also calculate the pressure across the bed when the volume flow rate of the liquid is given as 1.44 meter cube per hour. So in this case first of all you have to know what will be the voidage here. So how can you calculate the voidage? To calculate the voidage you know if you have the value of mass, mass of the solid and also cross sectional area of the bed and also length of the bed and also particle density I think everything is known to you. So the mass of the solid will be related as Al into 1 minus epsilon f into rho p.

This is the equation. What is that Aa is the cross sectional area into Al that means total volume of the bed into 1 minus epsilon f that is volume fraction of the particle into density of the particle that will give you the mass of the solid. So that mass of the solid is given to you as 50 kg. So upon substitution of this 50 kg here and cross sectional length and density you will be able to calculate what will be the void fraction.

So it is coming as 0.5. You will see that after that if you use that Ergun equation to estimate the pressure drop across the bed at this flow rate. Now flow rate is given 1.44 meter cube per hour. You have to calculate what will be the superficial velocity. So superficial velocity it will be very simple that volumetric flow rate by cross sectional area.

Volumetric flow rate is 1.44 meter cube per hour. You have to convert it to meter cube per second and then you have to divide it by cross sectional area then you will get velocity that means that velocity you can get it. Then epsilon f is known to you. Viscosity is known to you.

Epsilon f is known to you. Phi p also is known to you. That means sphericity is known to you. If it is spherical particle then phi p will be is equal to 1. And then you will see that dp particle diameter is given to you and also here rho p density of the particle is given to you. So all the parameters variables value is given to you. So after substitution of those values you can get what will be the frictional pressure drop per unit length that will be is equal to 6560 Pascal per meter.

I think you understood this problem how to calculate the frictional pressure drop whenever fluid will be flowing through the packed bed. Let us do another example here to calculate the friction factor due to that frictional pressure drop whenever fluid will be flowing through the packed bed. So let us have this problem of like this a packed bed of cross sectional area 0.0036 meter cube of materials of sphericity 0.68 and diameter 3 millimeter is operated for a catalytic cracking of heavy hydrocarbon whose density is 860 meter cube per second and viscosity 0.

18 Pascal second at its flow rate of 0.018 meter cube per second. What is the friction factor of the flow through the bed of porosity 0.4? Here porosity 0.4 means void fraction is given 0.

4. Now we know that friction factor the packed bed is as 150 by Rep star plus 1.75. So Rep star what is that Rep star that is you know rho f u 5P dP by 1 minus epsilon f into mu f. Rho f is given to you u is the velocity, velocity also is given to you okay velocity I think given as a flow rate so you have to divide it by cross sectional area, cross sectional area is given to you so you will get that, that is velocity of the fluid. So this is known to you and also 5P I think sphericity is given to you 0.

68 dP diameter of the particles is 3 millimeter that means 0.003 meter and then epsilon f that means void fraction is given 0.4 and mu f viscosity of the fluid is given 0.18 Pascal second. Now after substitution of all those values here we can calculate what will be the Reynolds number.

Now this Reynolds number is coming as 81.22. Now after substitution of this Reynolds number here in this equation you will be getting the value of friction factor of the packed bed which is coming as 3.60 okay. I think you understood this example problem.

Now let us have another example problem. This problem is given in GATT 2011. Here it is said that for a liquid which will be flowing through a packed bed the pressure drop or unit length of the bed is expressed by Ergun equation. Given that the particle diameter is 1 into 10 to the power minus 3 meter, sphericity is 0.8, density of the fluid is given 1000 kg per meter cube, viscosity of fluid is 1 into 10 to the power minus 3 kg per meter second, particle density is given 2500 kg per meter cube and acceleration due to gravity is given 9.

8 meter per second square. When the velocity of the fluid is 0.005 meter per second and porosity is 0.5, what is the ratio of the viscous loss to the kinetic energy loss? Now as per that Ergun equation that we have derived there we got that Ergun equation of that final form. So this is your final form of the Ergun equation. It has 2 components of its right hand side. This components will give you the viscous loss and this components will give you the kinetic energy loss.

Now you have to find out what will be the ratio of this viscous loss to the kinetic energy loss. So viscous loss by kinetic energy loss just by dividing these 2 factors you will get this value viscous loss by kinetic loss. Now for calculating this ratio you need viscosity, you need density, you need void fraction of the fluid okay. This is void fraction of the fluid and density of the fluid, particle diameter and sphericity everything is known to you.

So we are substituting here 150 by 1.75 into 1 into 10 to the power minus 3 the mu f, rho f is the 1000 and then 1 minus epsilon f is 0.5 and then what is that 5 is given 0.80 dP 1 into 10 to the power minus 3 and then u is 0.

005 meter per second. After substitution you can get it as 10.71. So the ratio of viscous loss to the kinetic loss will be is equal to 10.71. That means viscous loss will be 10.

71 times of this kinetic loss okay. I think you understood this problem here okay. So we learned here in this lecture what is the Ergun equation, how that Ergun equation can be derived, what are the different components of that Ergun equation, where those components will be dominating from each other. If Reynolds number is greater than 10, then we can say that inertia effect will be dominating. If Reynolds number which is defined which is given here is less than 10, then viscous effect will be dominating. And also what will be the viscous loss and what will be the kinetic energy loss that we understood. And also we can say how to calculate the drag force that is coming from the frictional pressure drop and also we understood that how to calculate the friction factor in the packed bed okay.

So also we obtained or we understood how to calculate the frictional pressure drop from the Ergun equation just by solving different example. So I think you enjoyed this lecture and I think came to know different information or various information about this fluid flow through the packed bed. So in the next lecture, we will also try more about this flow through the packed bed. There we will consider two phase flow instead of single phase flow. Till now we have discussed about the single phase flow and the laws are Darcy's law, Mosisson-Coin- carbony equation and Ergun equation.

But in the next lecture, we will try to learn about the flow phenomenon whenever two phase flow will be flowing through that and also their application. Until next time, have a nice day.