Lec 13: Basic law and terminology of flow through granular bed

Hello everybody. Welcome to this massive open online course on solid-fluid operations. So as we are discussing about basic understanding on you know terminal velocity when the flow over a solid body. So what will be the basic characteristics of that that we are discussing in the previous module in three successive lectures. Here we will start again different module which is called that flow through granular beds and in this module there will be another three lectures and in this three lectures we will try to cover about that the flow phenomena through granular beds and in that case we will be having different basic law and terminology of this you know flow phenomena whenever it will be flowing through the granular beds. And there also what will be the hydrodynamics like what will be the void fraction even what will be the frictional pressure drop whenever liquid will be flowing through the bed and also what will be the interfacial area or you can say that surface area whenever the fluid will come in contact with the solid particles in the granular bed.

So in this lecture we will try to have some basic law and also terminology of the flow through granular beds. So in this lecture we will try to learn about the Darcy's law and permeability specific surface and voidage and also what is that Kozeny-Carman equation based on which you will be able to calculate what will be the frictional resistance whenever fluid will be flowing through the granular beds. So here what is that Darcy's law that you have to first understand. So you will see that whenever any single fluid flowing through the bed of solids.

This law will be actually talking about that phenomenology of that flow of a fluid whenever it will be flowing through the porous media or bed of the solid particles. So according to this Darcy's law that is actually given by Darcy in 1856 the flow of water whenever it will be flowing through a packed bed of particle that will be governed by certain relationship. So that flow of water whenever it will be flowing through of course it will have some volumetric flow rate at which that it will be flowing. So that volumetric flow rate will be related with what will be the pressure resistance that is given by that bed of solid. And also we will see that what will be the cross sectional area that is occupied by that solid particles as well as fluid element inside the bed of solids.

And there you will see that whenever it will be flowing that is flow will be flow through that packed bed it may be laminar flow or turbulent flow. But during that flow there you will see that some resistance will be incurred by that solid particles or surface of the particles that is in the bed or conduit. So here in this picture you will see that some solid particles are intact in a conduit or in a you will see that horizontal pipe or you can say it is that tube. So in that case through that tube whenever fluid will be flowing at a certain flow rate that may be volumetric flow rate and it will be passing through that direction here it is given in this slide and this is volumetric flow rate. And during that flow you will see that there will be a pressure drop between two points of length l here as shown in the picture.

The pressure at this point A that will be PA and pressure at this point B it will be PB. So there will be a certain difference of this pressure. So that pressure drop of course that PB will be less than PA. So at the beginning that pressure will be higher whereas that in the downstream there will be a lower pressure. So there would be a difference of that pressure it is called pressure drop.

Now this pressure drop can be evaluated by the manometer. If you fixed two manometers in this upstream and the downstream position that is A and B position there will be a certain difference in the pressure manometer head that meniscus head of that delta S and from which you will be able to calculate what will be the pressure difference by rho g S. That I think you know that is the basic understanding of that how to measure the pressure drop. So you will see that as per this Darcy's law that Darcy told that Q that means volumetric flow rate will be proportional to the cross-sectional area of the bed and inversely proportional to the viscosity of the fluid and also it will be directly proportional to the pressure gradient. So that means here what will be the pressure difference per unit length that will be.

So in that case we can write this equation here that is Q will be is equal to minus KA by mu into PB minus PA divided by L.

Here the Equation

$$Q = \frac{-\kappa A}{\mu}\frac{(P_b - P_a)}{L} = \frac{-\kappa A \rho g}{\mu}\frac{dh}{dL}$$

Here mu is the viscosity, K is called kappa. Kappa is basically you know that permeability coefficient is called that means here this allowing the fluid pass through that bed that is capability of that you know bed at which the fluid will be passing through that. Now that permeability or you can say that allowing capability that is called permeability. So that permeability coefficient can be represented by this K that is basically that coefficient.

So here and A is called cross-sectional area of the tube or pipe or you can say vessel here also and then you will see that pressure difference that is PB minus PA and L is called the length between the two points at which that you are measuring that pressure drop. And then we can represent these here PB minus PA by L that means here it is measured from the manometer head. So manometer head basically the difference is delta S. So we can write here this delta S will be changing with respect to the length. So here we can write here PB minus PA that will be is equal to here rho g dh by dL.

Here PB means rho g H then delta PB means delta P means here delta H per unit length that means delta L. So here we are having this Q relationship like this and in this case this equation can be represented other way also that from this Q and A that means we can write this Q will be is equal to U into A or U is equal to Q by A. Here the Equation

$$u = K\frac{(-\Delta P)}{L} = Kh\frac{dh}{dL}$$

Q is the volumetric flow rate A is the cross-sectional area. So U is equal to your superficial velocity. So superficial velocity here that means Q divided by this Q divided by A that will be represented by U.

Other terms are here. Here other terms what is that minus K minus kappa by mu this can be represented by capital K. So it will be minus delta P by L and in this case you will see that this minus delta P by L which is basically what that we can write this one. This is basically what this rho g dH by dL and here rho g you will see that this you know K will be is equal to kappa by mu that is K and here another terms that is kH we are considering here. This will be actually kH, kH is in suffix.

So this kH is basically what here this is kappa rho g by mu. Here including that rho g with that kappa by mu it will be called as that you know capital KH. So this kH will be equal to kappa rho g by mu.

Here the Equation

$$Kh = \frac{\kappa\rho g}{\mu} = hydraulic\ conductivity$$

This will be actually called hydraulic conductivity. So this is basically what here what will be the you know resistance by you know that particles that is solid particles whenever fluid will be flowing through the pipe or conduit.

In this case that total you know that constant that is kappa rho g by mu this will be your you know hydraulic conductivity. So how that hydraulic conductivity is defined here that is capital KH which will be kappa rho g by mu. What is kappa? Kappa is basically that you know permeability coefficient. So how to calculate that permeability coefficient or hydraulic conductivity from the experiment. So in this case very simple that you have to allow the fluid at a certain flow rate like you know volumetric flow rate Q.

It is known to you that you can measure it by rotameter. So if you use that rotameter and if you allow that liquid from a storage tank through a rotameter and allow it to pass through this you know bed of solid particles you will see that it will be a certain flow rate Q and at that particular flow rate if you measure the pressure between these two points then you will get you know delta P. So delta P by L you can easily calculate from that dH by dL here from this equation. So once you know this dH by dL and then your velocity is equal to U that means Q by cross sectional area then you can easily calculate what will be the KH value hydraulic conductivity. So that hydraulic conductivity will come here that means U by dH by dL.

So this is your hydraulic conductivity. So once you know that hydraulic conductivity from the experiment you will be able to calculate what will be the permeability coefficient. That permeability coefficient it will come what that means here from that hydraulic conductivity from this equation that means here hydraulic that is permeability coefficient Kappa that will be is equal to here KH mu by rho g this will be your permeability coefficient. So in this way that you can understand what is that Darcy's law. So Darcy's law basically that whenever fluid will be flowing through the pipe of you know bed of solid particles they are you will see that the relationship between that volumetric flow rate and the pressure drop that will be represented by this Darcy's law.

Example here in a cylinder of cross sectional area is 0.002 meter square is packed with particles to make it porous bed. The hydraulic gradient is found to be minus 0.5 during a flow of fluid at 20 meter cube per hour through the porous medium of packed bed. Now find the hydraulic conductivity of the porous medium the same thing that here you have to know what the Q, Q value is given to you that means volumetric flow rate is given 20 meter cube per hour.

So it is basically 20 by 3600 that means meter cube per second and then use that Darcy's law as Q is equal to minus Kappa A by mu into PB minus PA by L that means KH into A into dH by dL. Now you know that dH value what is the dH value it is given I think it is minus 0.5 and dL value that means length of that pipe it is given or vessel it is given if it is unit length you can consider that one. So that is why you can say that dH by dL is equal to what and then A is the cross sectional area it is given to you and also this K is value then after substitution of this value then it will be coming as 5.

56 meter per second. So this unit of that hydraulic conductivity is basically meter per second. And then another term it is called surface area and also what is specific surface area and also voidage. So you will see that the general structure of a bed of particles can often be characterized by the specific surface area of bed that is denoted by SB here capital SB. So be careful here the notation here SB this is basically the specific surface area of the bed and also this general structure of the bed is characterized by the voidage of the bed that is represented by the notation epsilon f. Now in this case you will see that this specific surface area generally we know that the surface area per unit mass but here we will get here this specific surface area based on volume of the bed.

So that is why this SB is the surface area presented to the fluid per unit volume of the bed that is capital SB here. So be careful here to remember this notation SB this is basically the surface area presented to the fluid per unit volume of bed. What is the surface area that means whenever the solid particles will be intact in that vessel you will see that whenever fluid will be pass flowing that is through that bed of course there will be certain gap between the solid particles through which that fluid will be flowing. So that gap will be called as voidage. Now whenever that fluid will be passing through that voidage that fluid will come into contact with the solid surfaces.

Now how much that solid surfaces will come in contact with that fluid that will be represented by the surface area. Now that surface area you cannot measure it absolute value there. So you have to measure what will be that surface area total in total that means out of total volume that you can say. So that is why this specific surface area in this case it is defined as the surface area that means which will be available to get contact with the fluid per unit volume of the bed. So when the particles are packed in the bed in this case this specific surface area will be you know dimension will be as here that means meter square per meter cube.

So here area unit of area is meter square and unit of volume is meter cube so this is basically coming as 1 by meter that means 1 by length unit you can say. And then coming to the another you know characteristic factor it is called void fraction. Void fraction it is simply called as voidage also. So it is basically that how much volume of the void out of total volume of the bed that is called voidage or void fraction. So this void fraction will be represented by epsilon f that means what are the volume can be occupied by the fluid that is voidage that will be represented by a fraction out of total bed volume.

So epsilon f is the fraction of the volume of the bed not occupied by the solid materials that means occupied by the fluid element only. Thus the fractional volume of the bed occupied by the solid material will be what? Remaining portion. So if I get the fractional volume of the liquid as epsilon f then remaining volume fraction it will be 1 minus epsilon f. So this 1 minus epsilon f is basically epsilon of P that means particle volume fraction.

Here the Equatio

$$\varepsilon_p = 1 - \varepsilon_f$$

volume fraction of solid. Now another terms will be known to you that will be called the specific surface area of the particle that will be denoted by only capital S. Here it is not Sb here only capital S. This is basically specific surface area of the particle which is defined as the surface area of the particle per unit volume of particle per unit volume of particle.

Here the Equation

$$S = \frac{\pi d_p^2}{\pi d_p^3/6} = \frac{6}{d_p}$$

So in this case also you can get the dimension 1 by length because here surface area is meter square and volume is meter cube.

So meter square by meter cube it is basically that 1 by meter. So basically 1 by length unit. So for a sphere if we consider that these things the specific surface area what will be that? So for sphere the surface area will be pi dp square and the volume of the sphere it will be as pi dp cube by 6. So it is generally after simplification it will come 6 by dp. So this is called specific surface area of a sphere.

So you have to know that what is the specific surface area of sphere. It is basically 6 by dp. What is dp? Dp is the particle diameter. If suppose the particles are not in regular in shape that means irregular in shape there you have to consider what will be the sphericity that we have discussed in the earlier lectures that what is the sphericity, how to calculate it. If you know that sphericity then you have to multiply the sphericity with that particle diameter.

So then you have to have that 6 by pi dp. Pi is basically that sphericity. And then what will be the relation between that S and Sb. Sb is basically that surface area per unit volume of particle and Sb is basically surface area per unit volume of bed.

Here the Equation

$$S_B = \frac{Surface\ of\ particle}{Volume\ of\ bed} = \frac{Surface\ of\ particle}{Volume\ of\ particle} \times \frac{Volume\ of\ particle}{Volume\ of\ bed} = S \times \varepsilon_p = S \times 1 - \varepsilon_f$$

So what is the relation between these two characteristic factors. Now they are related like this here it is given in the slides.

So Sb will be is equal to what? Here Sb will be equal to surface of particle, surface of particle divided by volume of bed. This is the definition of Sb that is equal to surface of particle by volume of particle into volume of particle by volume of bed. We are having here just you know multiplying by volume of particle and also dividing by that volume of particle here. So it will be cancelled out ultimately surface of particle by volume of bed.

So it will be coming like this. Now surface of particle by volume of particle it is basically S specific surface area of And here volume of particle by volume of bed is basically what is that? Volume fraction of particle in the bed or you can say S into here 1 minus epsilon f. Epsilon p is basically what? 1 minus epsilon f. Epsilon f is basically volume fraction of fluid. So that is why we can simply write that Sb will be is equal to 1 minus epsilon f into S or epsilon p into S. So this is the relationship between these two characteristic factor.

Then some important points that you have to remember here. It is said that for a given shape of particle S increases as the particle size reduced. Of course it will be there as per the definition that we are having here S will be equal to 6 by dp. So if dp is increases that means surface area will be you know reducing. If dp decreases that means smaller particles will give you the more surface area.

Also as epsilon f that means volume fraction of fluid if it is increased flow through the bed becomes easier. And so the permeability coefficient Kappa will increases that you have to remember. This is also important point. Also if the particles are randomly packed then volume fraction of the fluid should be approximately constant throughout the bed and the resistance to flow of the same in all directions will be you know constant. So this is the things that important three points that you have to remember whenever fluid will be flowing through the pipe.

Now let us do one example. In a porous medium it is seen that the fluid is moving at a fluid volume fraction of 50%. If the specific surface area of the particles is equal to 100 meter square per meter cube of bed what should be the surface area per unit volume of the bed when the particles are packed in a bed. Here this you will see that specific surface area of the particles is given that is 100 meter square per meter cube of that particle not that bed. Then you have to find out what will be the surface area per unit volume of bed when the particles are packed in the bed. So we know that what is the relationship between that you know surface area of the bed and the surface area of the particle.

So in this case surface area of the bed is equal to SB and surface area of particle is S and the 1 minus epsilon f is basically the volume fraction of the particle. So if you know this S value, S value is given to you here 100 and 1 minus epsilon f also volume fraction of the fluid is given to you. So after calculation you will get this 50. What is the unit will be there? So this is the unit will be that meter square per meter cube of bed here. Here meter square surface area of the particle per volume of the bed.

So this you can have. Next we will discuss about that Kozeny-Kuhrmann equation. This is very important whenever fluid will be flowing through that you know bed of the solid particles and whenever flow will be in streamline flow that means the Stokes flow regime that means very laminar flow we can say. In that case what should be the pressure drop during the flow of a fluid at a certain velocity through the porous media at that laminar condition. In this case we will derive this Kozeny-Kuhrmann equation based on that the concept of flow through a tube and flow through a pores in a bed of particles. In that case we know that in fluid mechanics we have learned something about that Haugen-Poiseuille equation whenever any single fluid will be flowing through the pipe in absence of any solid particles that means there will be no resistance only thing wall resistance will be there.

So whenever fluid will be flowing through the pipe that pipe is not filled with the solid particles. In that case what will be the frictional resistance? What will be the frictional pressure drop? How is it related to the you know velocity of the fluid or volumetric flow rate of the fluid? I think you know that that is actually I think taught in your fluid mechanics subject that that equation that relationship between that volumetric flow rate and the pressure drop that is represented by the Haugen-Poiseuille equation. So that Haugen-Poiseuille equation as per that we know that u will be is equal to that means velocity will be is equal to dt square by 32 mu into minus delta p by L. Here in this case dt is called tube diameter or pipe diameter and mu is called viscosity and delta p by L is basically pressure drop per unit length of the pipe. So in that case the equation for flow through a circular tube which will be represented or which will be suggested or which can be derived from the momentum balance and that is given by Haugen-Poiseuille and which is known as Haugen-Poiseuille equation for laminar flow which can be represented by this equation number 1.

Here the Equation

$$u = \frac{d_t^2}{32\mu} \frac{(-\Delta P)}{L}$$

Again we have learned that what is Darcy's law? In the case of Darcy's law we know that whenever fluid will be flowing through the pipe that velocity of the fluid through that bed of solid particles or porous media that also will be related to the pressure drop. So there itself u will be is equal to k into minus delta p by L.

Here the Equation

$$u = K \frac{(-\Delta P)}{L}$$

So this is basically the Darcy's law. So equation 1 is Haugen-Poiseuille equation in absence of particle and equation 2 is the Darcy's law equation which is in presence of solid particle.

Both are related with the velocity and pressure drop. Now if we use this concept of Haugen-Poiseuille equation in porous media how then it will be there? Let us consider this. If the free space in the bed is assumed to consist of a series of tortoises in equation 1 we can then manipulate or that equation 1 can be written for flow through a particle bed as equation number 3.

Here the Equation

$$u_a = \frac{u}{\varepsilon_f} = \frac{d_m'^2}{K'\mu} \left(\frac{-\Delta P}{L'} \right)$$

So in this case be careful that we are considering here that bed of solid particles will have the voidage in such a way that the fluid will be flowing through that voidage that means fluid will be flowing through an imaginary channel very tortoise channel or you can say that very you know that narrow channel. That means here the fluid will be passing through the gap between particles here like this suppose particles are you know arranged like this you will see that. So fluid will be flowing like this through that you know gap that the channel as a channel here like this here channel like this through the gap here again like this here it will be flowing like this.

So there will be that you can say that n number of you know channels n number of tortoise channels zigzag channels that is basically the gap through which that fluid will be flowing so as a channel you can say. So that channel will be represented as that one circular pipe of narrow dimension of narrow you know diameter range. So that is why if the free space in the bed is assumed to consist of a series of tortoise channels here equation 1 can be rewritten for flow through a particulate bed as like this. So what is that ua, ua is the actual velocity of the fluid here is equal to u by epsilon f u is the superficial velocity and epsilon f is the volume fraction of the fluid.

Here the Equation

$$u_a = \frac{u}{\varepsilon_f}$$

So superficial velocity divided by the volume fraction of the fluid it will be your actual velocity I think we have discussed about that superficial velocity and actual velocity in the previous lecture itself also.

So how that actual velocity will be defined that will be superficial velocity by volume fraction of the fluid. So this actual velocity can be then as per Hagen Poiseuille equation represented by this d dash m square by k dashed into mu into minus delta p by L dash.

Here the Equation

$$d'_m = \frac{\varepsilon_f}{S_B} = \frac{\varepsilon_f}{S(1-\varepsilon_f)}$$

What is the Hagen Poiseuille equation here? Here this is basically what u will be is equal to Hagen Poiseuille equation is basically u is equal to dt square by 32 mu into minus delta p by L this is your Hagen Poiseuille equation where there will be no particle present but in presence of particles we are considering this equation again but here dt will not be absolute dt here. So dt will be the channel diameter.

There may be n number of channels. So for that how many channels will be that we do not know. So in that case total you know cross sectional area if we add all those cross sectional area of the channels that will be you know that dm dash we are representing. So in that case we are considering dm dash then it will be square. Now this instead of 32 here we do not know what will be the coefficient here. So there we are considering here k dash and mu of course will be there viscosity and remaining part minus delta p by L.

So this equation 3 is basically this. Now in this case also one important point L dash. This L dash may not be the same as what is the length of that circular pipe what is considered for that Hagen Poiseuille equation. So this L dash will not be exactly the same. Here why it will not be same because this channel will be having that zigzag path. So actual straight distance and its zigzag path distance will not be same but this distance may be you know proportional to this L.

So L dash here channel length will be proportional to the L. We are considering that it will be proportional to the L. So here in this case L to be represented by L dash here.

So see as per Hagen Poiseuille equation and as per concept of that Hagen Poiseuille concept we can write this equation here in porous bed by this equation number 3 where epsilon f is the void fraction, ua is the actual velocity of the fluid, dm dash would be equivalent diameter of the pore channel, L dash is the length of the channel and k dash is the constant depends on the you know structure of the bed.

Here the Equation

$$u_a = u/\varepsilon_f and L^{'} \propto L = kL$$

Now question is that why ua? ua is basically actual velocity. So whenever fluid will be flowing through that porous bed the cross-sectional area which will be occupied by the fluid will not be the same as total cross-sectional area of the you know circular pipe.

So in that case since the cross-sectional area will be reducing that fluid element will have more velocity whenever it will be flowing through the void fraction. So that is why actual velocity here to be considered but this actual velocity will be related with the superficial velocity where there will be no you know solid particles only the empty vessel cross-sectional area will be considered. In that case the superficial velocity will be related to the actual velocity by volume fraction of the fluid. So which is defined as ua will be is equal to u by epsilon f.

So this is your actual velocity. So ua and L dash are not the same as u and L that is as per Hougen-Poiseuille equation. The actual velocity of fluid through the void of the packed bed and is related to the super velocity by u by this equation. Then Kozeny 1927 he proposed that this what will be that dm dash value that is channel diameter. So dm dash you know will be related to that epsilon f volume fraction of the fluid as well as you know the surface area per unit volume of the bed. So this will be is equal to epsilon f by S into 1 minus epsilon f this is given in equation number 4.

Here the Equation

$$d^{'}_m = \frac{\varepsilon_f}{S_B} = \frac{\varepsilon_f}{S(1-\varepsilon_f)}$$

And then what will be epsilon f by Sb that means volume of voids filled with the fluid divided by weighted surface area of the bed. So that will be cross-sectional area normal to the flow by weighted perimeter. This is basically what 1 by 4 into hydraulic mean diameter. Where hydraulic mean diameter is basically defined as 4 into cross-sectional area normal to flow by weighted perimeter. So that is why we can then you know have the value of dm dash that is effective you know channel diameter of that you know channel through which that fluid will be flowing through the porous media.

So that will be is equal to epsilon f by S into 1 minus epsilon f where epsilon f by Sb will be is equal to 1 by 4 into hydraulic mean diameter. Now then taking that UA will be is equal to U minus f U by epsilon f and L dashed is proportional to L that means KL, K is called proportionality constant. Then from equation number 3 with that equation number 4 we can have this UA will be is equal to U by epsilon f that will be is equal to dm dash square by K dash mu into minus delta P by L dash.

Here the Equation

$$u_a = \frac{u}{\varepsilon_f} = \frac{d_m'^2}{K'\mu}\left(\frac{-\Delta P}{L'}\right) = \frac{(\varepsilon_f/S_B)^2}{K'k\mu}\left(\frac{-\Delta P}{L}\right) = \frac{\varepsilon_f^2}{K''S^2(1-\varepsilon_f)^2}\frac{1}{\mu}\left(\frac{-\Delta P}{L}\right)$$

Now substituting this dm dash here we are having epsilon f by Sb whole square by it will be K and then K dash into mu into minus delta P by L. Then epsilon f square by K double dashed here K double dashed is basically K dash into K we are assuming here K double dashed into S square into 1 minus epsilon whole square where Sb is equal to S square by Sb is equal to S into 1 minus epsilon f we have substituted here and then 1 by mu here into minus delta P by L.

Here the Equation

$$u = \frac{\varepsilon_f^3}{5(1-\varepsilon_f)^2S^2}\frac{1}{\mu}\left(\frac{-\Delta P}{L}\right)$$

So after substitution of dm dash here in equation number 4 in equation number 3 then we are having this equation of axial velocity of fluid which is flowing through the porous media. So in this case K double dashed here K double dashed is generally known as Kozeny is constant that will be is equal to minus K into K dash and a commonly accepted value for this K double dashed is 5 as per Kozeny. Therefore we can write U will be is equal to from this equation number U will be is equal to epsilon f cube by 5 into 1 minus epsilon f whole square into S square into 1 by mu into minus delta P by L. So after simplification we are having this. So this is the equation which is related you know with the pressure drop per unit length of that 5 or packed bed you can say and also it is related to the surface area of the particle volume fraction of the fluid and also viscosity of the fluid and here this U is basically the superficial velocity.

So once we are able to calculate what will be the superficial velocities that is why this equation representation based on the superficial velocity instead of axial velocity. Now if we consider that sphere any spherical particle as a packing material which is to be placed in the packed bed then allowing that fluid through the pipe. So in that case what will be the surface area of that particle that will be the pi Dp square by pi Dp cube by 6 that will be 6 by Dp.

Here the Equation

$$S = \frac{\pi d_p^2}{\pi d_p^3/6} = \frac{6}{d_p}$$

Now after substitution of this S value here in this equation again instead of S here and finally you can get this equation minus delta P by L that will be equal to 180 to mu U into 1 minus epsilon f whole square divided by Dp square into epsilon f cube.

Here the Equation

$$\frac{-\Delta P}{L} = \frac{180\mu u(1-\varepsilon_f)^2}{d_p^2 \varepsilon_f^3}$$

So this equation is called Kozeny-Carman equation and this equation valid only for laminar flow.

Non-porous particle. So in this case you have to remember. So these things that Kozeny-Carman equation has a limitation only that it will be applied for only laminar flow as well as non-porous particle. Also you have to note down that for non-spherical particle in that case a particle size distribution a shorter mean diameter of dps to be considered instead of only that single particle size because there are n number of particles will be in the packed bed and there you will see that what will be the mean particle size. That mean particle size to be considered as a volume to surface area mean particle diameter you have to consider that is called shorter mean diameter that is dps that we have discussed in the earlier lecture. And also sphericity that you have to consider.

So in that case phi s dps should be used in place of only particle diameter Dp. Now let us do an example here based on this Kozeny-Carman equation. Here it is said that a packed bed of solid particles of density 2500 kg per meter cube occupies a length of 1 meter in a pipe of cross sectional area of 0.04 meter square. The mass of solids in the bed is 50 kg and the surface volume mean diameter that means outer mean diameter of the particles is 1 millimeter.

A liquid of density 800 kg per meter cube and viscosity 0.002 Pascal second flows through the bed. So, under this condition you have to calculate what should be the voidage that means volume fraction occupied by the liquid of the bed. And then also you have to calculate what will the pressure across the bed when the volume flow rate of liquid is 1.44 meter cube per hour. So in this case consider this case A here what is given and what is not given here very simple that particle density is given to you length of the pipe is given to you cross sectional area is given to you even mass of the solid in the bed is given to you particle mean diameter is given to you.

And also their liquid density and viscosity are given to you. Now from the mass of the particle, mass of the particle can be represented by this equation m will be equal to A into L into 1 minus epsilon f into rho p what is that A is cross sectional area L is the length of the pipe. So, A L is volume of the pipe into 1 minus epsilon f that means volume fraction of the particle there into density of the particle then it will be having that mass of the particle. So this mass of the particle is equal to 50 kg whereas cross sectional area is given L is given epsilon f to be found rho p is given to you. So from this equation if you solve this equation after substitution of other values you will be able to find out what will be the volume fraction of fluid.

So it is coming as epsilon f is equal to 0.5. And the case B in that case what is the liquid flow rate is given it is given 1.44 meter cube per hour. So, it will be coming as you have to convert it to meter cube per second.

So you have to divide it by 3600 to be coming as 1.44 by 3600. So it will be your volumetric flow rate. So this volumetric flow rate divided by cross sectional area you will get that velocity of the fluid.

So, it will be 0.01 meter per second. So we know now velocity of the fluid. Now what will be the pressure drop across the bed? Now pressure drop as per that Kozeny-Carman equation it will be coming as 180 into mu u into 1 minus epsilon f whole square by dp square epsilon f cube what will be the value then. So, before coming to that you have to check whether this Kozeny-Carman equation to be valid here or not. So Kozeny-Carman equation valid only for laminar flow when Reynolds number will be less than 10. So, in that case what will be the Reynolds number here as per that given problem after substitution of rho p u dp by 1 minus epsilon f into mu as per definition of this Reynolds number here as per this then it will be coming as 8 which is less than 10. So this Kozeny-Carman equation will be applied only if Reynolds number of particle which is defined by this equation if it is coming less than 10 then only you can apply this Kozeny-Carman equation to calculate the pressure drop.

So applying this Kozeny-Carman equation here we are having this pressure drop as 7200 Pascal. Let us do an another example here. Now in this case 1.28 gram of powder of particle density 2500 kg per meter cube are charged into the shell of an apparatus for measurement of particle size and specific surface area by permeametry. The cylindrical shell has a diameter of 1.14 centimeter and the powder forms a bed of depth 1 centimeter, dry air of density is given to you viscosity is given to you and flows at a rate of 36 centimeter cube per minute through the powder bed and producing a pressure differences of 100 millimeter of water across the bed.

So in this case what will be the surface to volume mean diameter and also specific surface area of the powder sample that you have to find out. So here to calculate the cross sectional area of the shell you have to first know what will be that 1.027 into 10 to the power minus 4 meter square it is given I think because diameter is given so cross sectional area you can easily calculate. Then shell volume what will be the shell volume area into depth that will be equal to this after calculation and then what will be the mass of the powder, mass of the powder it is given I think 1.28 gram you have to convert it to kg this is actually m into 1 minus epsilon f into rho p into shell volume.

So that will be is equal to this kg. So therefore from this equation you can have epsilon f will be is equal to this and then superficial velocity of the air that is flow rate of air divided by cross sectional area that you can calculate. After that you have to calculate the pressure drop equivalent to 100 millimeter water. So this basically H rho g that will be is equal to 981 Pascal after substitution of this value and then substituting into that Kozeny-Carman equation here this value is given to you and other parameters are given except that dp value particle diameter and then after solving you can get dp value is equal to 20.08 micrometer.

Whereas Reynolds number also to be calculated whether this Kozeny-Carman equation to be valid or not. So in that case Reynolds number is found to be 0.0153 which is less than 10 and then you have to calculate what will be the surface area of the particle. This is pi dp square by pi dp cube by 6 that means 6 by dp, dp already you have found here 20.

08 micrometer. So 6 by 20.08 into 10 to the power minus 6 that is converted to meter then finally you are getting 2.998 into 10 to the power 5 meter square per meter cube. And also what will be the surface area per unit volume of that particle that you can calculate what will be the surface area that you got here. So in this case after that you have to divide it by you know that density of that particle that will be 2500 then it will be coming as 119.

5 meter square per kg. So surface area per unit mass of the particle that will be is equal to 119.5 meter square per kg. So I think you understood this problem here. Next problem here you will see that water flows through 4 kg of solid particle of density 2500 kg per meter cube forming a packed bed of depth of 0.

475 meter and diameter 0.0757 meter. The variation of friction of pressure drop across the bed with water flow rate in the range of 200 to 1200 centimeter cube per minute is shown in columns 1 and 2 in table here. In this case viscosity of water is given here. Then in this case you have to find out what will be the mean surface volume diameter of the particles and also calculate that relevant Reynolds number. So here the pressure drop against that flow is given to you and then what is the volume surface mean diameter that you have to find out and also you have to calculate what will be the relevant Reynolds number. So in this case you have to apply that Kozeny-Carman equation where that the pressure drop is related to the simple that velocity.

So in this case if we plot those values in a graphical form here in this case in X axis U and in Y axis that is delta P by L then we are getting this equation and from this you know profile or equation you can say that what will be the that slope of this you know graph and it will be coming as 2 into 10 to the power 6. So here we are getting this slope as 2 into 10 to the power 6 from the graph and from the Kozeny-Carman equation we can write here this delta P by L that will be 180 mu into 1 minus epsilon f whole square dP square by epsilon f cube into U. Now this except this U that this one only we can say this portion will be the coefficient. This coefficient will be equals to the slope of this you know graph.

So that is why you can write this value will be equals to that you know slope value it is 2 into 10 to the power 6. So from this we can have you know that Reynolds number it will be as what is that 2.664. You know in this case very interesting that epsilon f is equal to 0.25 after substitution of dP value, mu value and epsilon f to be found out.

So from this slope you will be found out what will be the epsilon f value. So from this equation again then you have to calculate what will be the mass of that then from the above equation you can find the particle diameter dP as 0.0018. So once you know that epsilon f value from this you know mass of that value given whatever it is in the problem and from this slope if you are not having that dP value as per problem you have to find it out from the slope value what will be the dP value. So once you know that dP value and epsilon f value and then what will be the Reynolds number that you can calculate from this equation. So here interesting that from this you know pressure and velocity relationship you have to first make a graph and what will be the slope from that slope you can calculate what will be the you know parameter which is not known to you.

If epsilon f is not given to you dP is given then you can easily calculate what will be epsilon f. If epsilon f is given to you but dP is not given to you then you can easily calculate what will be the dP value. Other way also if suppose that epsilon f is given dP is given but viscosity is not given then you can calculate viscosity also from this slope. So once you know those values you have to calculate Reynolds number that you have to find out as per problem. So this is the way to calculate that Reynolds number as well as other parameters from the pressure velocity relationship. I think you understood the problem and also in the lecture what is Darcy's law, what is Kozeny-Carman equation, what is that Hagen-Poiseuille equation, how that Hagen-Poiseuille concept can be utilized to you know derive that Kozeny-Carman equation and what is the validity of that Kozeny-Carman equation to calculate or to estimate the pressure drop.

I think you understood these things and in the next lecture we will also try to discuss more about this flow phenomena, fluid whenever it will be you know flowing through the packed bed. There also we will discuss about more about that flow phenomena under the laminar as well as inertia condition. So there we will also derive another important very important equation that is called Ergun equation based on which you can calculate the frictional pressure drop through a packed bed where laminar and you know turbulent condition to be considered. So thank you for giving attention. Have a nice day. Thank you.