Lec 12: Multiple particle Interaction/Sedimentation: Hindered settling velocity

Hello everybody welcome to this massive open online course on solid fluid operations. So, we are discussing about the module of solid whenever it will be immersed in the liquid then how the drag force even what will be the terminal velocity and also what will be the you know lift force and then what are the different regimes of that flow where the drag force will be applied. And also how to calculate the terminal velocity whenever that particle diameter will be known and also the particle diameter to be calculated when terminal velocity will be known at any flow regimes. In this lecture we will try to learn more about this when the particle will be that falling under the gravity and also under the action of interaction with the other particles. In that case this particles will not be free of other particles. So, these particles will be falling downward under the gravity and also there are parallel interaction among the particles as well as fluid and in that case what will be the relative velocity at which that particles will be falling downward and whenever these particles will be falling downward with these interaction there will be a hindered of this particle to settle down or falling downward.

So, that under this hindrance force whenever particles will be falling downward with the relative velocity that velocity will be called as settling velocity. So, here we will try to learn about that settling velocity. So, this present lecture will cover that hindered settling velocity what is that and in a batch settling or sedimentation how that relative velocity of that particles will be falling downward and also there will be some interaction between that solid particles with the other particles and because of which there will be some effective velocity or it is called relative velocity how that relative velocity will be related to that particle concentration in the suspension or slurry. So, there will be a certain relationship relative velocity and the terminal velocity along with that at a certain particle concentration of that slurry that relation will be called as that Richardson-Zaki relationship that will also here we will learn.

So, here we will see that the settling of a suspension of particles in this case a suspension what is that suspension? The suspension is basically a heterogeneous mixture in which you will see that solute like particles very fine particles you can say settled out of a solvent like phases. So, in a solvent like phases that very fine particles will be settled down. So, this is basically a heterogeneous mixture which is called a suspension. As an example you can say the dust in air is a suspension. So, in that case the sand suspended in the air also you will see that some sand particles or other particles you will see that suspended in the river.

So, that is also a you know example of this suspension. So, air dust, dust different types of dust also you will see that in carbon particle even other types of you know very fine you know dry particles also will be suspended in the air. So, this dust in air is a suspension as well as the sand river like Brahmaputra Ganga you will see that the dramatic water that will be the particles will be suspended in the river water. So, these are called suspension. Now, if the particles are too small to you know ever settle they are said to form a colloid.

So, in that case you will see that sometimes in the suspension the very very fine particles will be settling down very slowly and then that type of particles of course will be having a certain range of size may be 1 nanometer to 1 micrometer in range. So, those will be called as colloid. And also in a suspension you will see that some liquid droplet or fine solid particles in a gas it will be you know that suspending. So, that type of droplet liquid droplet or fine solid particles in the air will be called as aerosol. That aerosol you know particle size will be within a range of 10 to the power minus 4 to 10 micrometer.

So, here we are having that particles in presence of other particles who those are actually settling down in a suspension slowly. So, there you will see that there will be a significant interaction between the particles and the frictional forces exerted at a given velocity of the particles relative to the fluid which will result the hindering of the particle for its you know settling. So, it will be called as hindered settling. So, hindered settling basically whenever that hindrance will be coming because of that interaction among the particles and also with the fluid in which that it will be suspending. So, in that case the velocity at which it will be settled down it will be very low or it will be you know that hindrance or it will be you know changing because of that hindrance force.

So, that is why it will be called as hindered settling. So, generally free settling will be always greater than the hindered settling. Free settling velocity I think we have discussed in the previous lecture that what is that free settling velocity which is called as a terminal velocity. Whereas the hindered settling velocity will be actually the velocity of the particles in presence of other particles that will be called hindered settling velocity. So, free settling velocity will be greater than hindered settling velocity.

Here in the slides you will see that some picture here you will see that if you are considering a batch settling operation suppose in a small test tube if you are having some you know that muddy water you will see that the muddy water or slurry or suspension you will see that particles will be gradually you know settling down at the bottom of this you know test tube or column or you will see that in you know real life in a vessel if there is you know that slurry after a certain time or certain days you will see that that muddy particles will be settled down at the bottom. So, there in this case in a bass some you know slurry will be you know allowed to settle down and after certain time you will see that solid particles will be you know settle down at a certain velocity okay that is called settling velocity and in that case you will see that solids are suspended in a column or vessel. Like you will see that generally for wastewater treatment you will see that initially that muddy or you know that particulate water that will be allowed in a you know vessel or clarifier you can say where that particulate solid materials will be suspending downward at the bottom. And then you know that clarified liquid that will be again process further for removing other materials. So, this here you will see that this type of that clarifier are there where that suspended particles you know will be settled down at the bottom with respect to time.

So, upon settling you will see that solids move downward and settle on the bottom of the column due to gravity force. So, here also you will see that in a small test tube you will see that some slurry or suspended solid particles are allowed to settle down and after certain times you will see that at the top there will be a clarified liquid that means here clear liquid will be coming out where that particles would be you know suspending at the bottom with respect to time. But whenever it will be suspending you will see there will be a formation of different concentrated zone at the bottom you will see very concentrated zone and also at that upper position at different you know concentrated slurry will be shown and with a certain demarcation you know line will be there. So, it will be called as interface between the concentrated layer of that you know slurry. So, in that case with respect to time you will see that after a certain time all the particles will be settled down here at the bottom whereas it will leave that clear liquid above its you know settled amount of solid.

So, in this way here some you will see that if suppose that if you are allowing that suppose a slurry of zinc oxide solution here zinc oxide basically white you will see that if you make that slurry of that zinc oxide what will happen after certain of that zinc oxide will be settled down at the bottom whereas it will be leaving that you know clear liquid at the top. So, basically these are called the batch settling or sedimentation. So, in case of batch settling you will see that the solids are suspended in a column or vessel and during that settling you will see that the solid will move downward and settle on the bottom of the column due to gravity force. Now, whenever we are going to assess this hindered settling velocity or that efficiency of the process for the settlement of the particles you have to know that what would be the you know viscosity of that slurry and also density of the slurry because this slurry viscosity and slurry density would not be the same as that pure liquid you know viscosity or viscosity or density of the liquid. So, in that case you have to know what will be the effective viscosity of the slurry.

You will see that whenever you will add some powder in a you know pure liquid what you will see that there will be a formation of slurry and if you keep on increasing the concentration of the you know slurry by adding more powder in the solution you will see that you know that viscosity that means that the solution or slurry will become more sticky as you increase the particle concentration there. So, that stickiness increase of stickiness will actually give you that indication of that increasing the viscosity of the you know slurry. So, in that case if you keep on increasing that you know particle concentration the slurry you will get that the change of viscosity will be there or density of the slurry will be changing there. So, how to you know calculate or how to estimate that viscosity or effective viscosity or you can say that apparent viscosity or you can say actual viscosity of that slurry there. So, it depends on particle concentration that concentration will be volumetric concentration out of total slurry what will be the volume of particles in that slurry.

So, that will actually change that viscosity of the you know slurry. Now, if you are considering that for a suspension of particles in a fluid the viscosity and density will be represented by the effective suspension viscosity and density will be represented by the effective or average suspension density. Now, in the case of particle concentration in a slurry if you are making a slurry and if you make that slurry concentration volumetric by volume you can say that volume of particles out of total volume of that slurry. If you make this you will see that if your you know slurry concentration will be less than 20 percent and low shear stress will be there less than suppose 1 kilo Pascal then you can calculate what will be the you know effective viscosity of that you know slurry. So, that condition that is within a slurry concentration less than 20 percent and low shear stress that is 1 kilo Pascal in that case Einstein 1906 you know he has given or he has suggested an equation for calculating that effective viscosity of the slurry.

So, it is given here in the slide as equation number 1 that is mu E, mu E is basically what is that effective viscosity of the slurry and this will be is equal to mu F into 1 plus 2.5 epsilon P. Here mu F is basically the viscosity of the of your fluid and 1 plus 2.5 epsilon P, epsilon P is basically the volumetric concentration of the particles in the slurry. So, here we can write here simply then mu F into function of epsilon P this is basically 1 plus 2 point epsilon P is a function of epsilon P, epsilon P is basically what that particulate concentration in the slurry by volume.

Here the Equation(s)

$$\mu_e = \mu_f(1 + 2.5\varepsilon_p) = \mu_f f(\varepsilon_p)$$

So, here a function of epsilon P that will be equal to 1 plus 2.5 epsilon P

Here the equation

$$f(\varepsilon_p) = (1 + 2.5\varepsilon_p)$$

as convenient we are considering it is a function of epsilon P whereas this epsilon F is equal to 1 minus epsilon P.

Here the Equation

$$\varepsilon_f = 1 - \varepsilon_p$$

So, epsilon P in terms of you know volumetric concentration of fluid that will be you know 1 minus epsilon F and in case of you know particle concentration greater than 20 percent and also higher shear stress if you are considering if it is greater than 1 kilo Pascal the relative viscosity or effective viscosity of that slurry given by Kitano et al from which you know you can calculate it. So, Kitano et al suggested a correlation based on which you can calculate what will be the effective viscosity if your suspension concentration or slurry concentration is greater than 20 percent and shear stress is greater than 1 kilo Pascal. So, in that case we can represent that mu U will be equal to mu F into 1 minus epsilon P by 0. 68 whole to the power minus 2.

Here the Equation

$$\mu_e = \mu_f\left(1 - \frac{\varepsilon_p}{0.68}\right)^{-2} = \mu_f f^{'}(\varepsilon_p)$$

So, this is the correlation that is developed by Kitano et al 1981. So, as per their suggested correlation you can easily calculate what will be the effective viscosity for the you know slurry whose concentration will be greater than 20 percent. So, here again we can you know express this you know as a function like mu F into F dash into epsilon P here F dash is basically 1 minus epsilon P by 0.68 whole to the power minus 2.

So, by this equation number 2 you can easily calculate what will be the slurry viscosity and then average suspension density or effective suspension density that will be again depending on that you know particle concentration. So, average density can be you know calculated by this equation here this is epsilon F rho F plus 1 minus epsilon F into rho P

Here the Equation

$$\rho_{ave} = \varepsilon_f\rho_f + (1 - \varepsilon_f)\rho_p = (1 - \varepsilon_p)\rho_f + \varepsilon_p\rho_p$$

here rho F is basically the fluid density and rho P is the particle density or you can express here 1 minus epsilon P into rho F plus epsilon P into rho P because here 1 minus epsilon P that will be is equal to epsilon F. So, this is the equation based on which you can easily calculate suspension viscosity. Then we are coming to the point that whenever the particles will be falling downward or settling downward you will see there will be a drag force acting on the particles. Now for that what will be the you know drag force.

In this case you cannot consider the pure liquid here instead of liquid you have to consider the slurry whose viscosity and average density will be different from the pure viscosity and density. So, in that case what will be that drag force. So, in that case drag force will be is equal to as per definition whatever earlier definition was there. So, FD will be is equal to what CD into you know projectional area into rho U square by you know 2. So, in the in the case of you know slurry in that case you have to consider that rho to be considered as a rho average and you will see that U velocity U that means you particle at which that it will be settling down.

So, it will be relative velocity instead of that absolute velocity in absence of particles. So, here U relative will be there instead of U. And also other things that will be there even if mu will be mu effective there. So, in that case drag force will be is equal to this you know CD into 1 by 4 pi Dp square into half rho average into U relative square.

Here the Equation

$$C_D = \frac{F_D}{\left(\frac{1}{4}\pi d_p^2\right)\left(\frac{1}{2}\rho_{ave}U_{rel}^2\right)}$$

So, from this equation number 5 you can calculate what will be the drag force.

So, to calculate the drag force you need CD value. CD is what? CD is the drag coefficient. So, drag coefficient again you can calculate based on that flow regimes that already we have discussed in the previous lecture. So, in that case in the Stokes flow regime or Stokes law regime you can say that. So, here in that case CD will be is equal to 24 by Rep.

Here the Equation

$$C_D = \frac{24}{} = \frac{24\mu_e}{U_{rel}\rho_{ave}d_p}$$

be is equal to 24 by you know rho average Dp U relative divided by mu E. So, here instead of your liquid viscosity and density you have to consider that average or effective viscosity and density of the liquid here. So, here instead of liquid it will be considered as a slurry. So, here by this you can easily calculate what will be the drag coefficient and drag force. Now, under terminal velocity conditions where there will be no you know acceleration force for a particle falling under gravity in a suspension that means in presence of other particles and interacting with the other particles the force balance how it will be there.

The same way that we have discussed for the terminal velocity in absence of other particles that will be is equal to gravity force minus buoyancy force minus drag force that will be is equal to acceleration force.

Here the Equation

$$Gravity\ Force\ -\ Buoyancy\ Force\ -\ Drag\ Force\ =\ Acceleration\ Force \frac{\pi d_p^3}{6}\rho_p g - \frac{\pi d_p^3}{6}\rho_{ave} g - C_D\frac{\pi d_p^2}{4}\frac{1}{2}\rho$$

So, in this case drag force will be is equal to pi by 6 Dp cube into rho p into g and buoyancy force will be pi Dp cube by 6 that means your this is volume into you know that density of the here slurry not pure liquid density. So, average density to be considered into g and then drag force will be Cd into pi Dp square by 4 and half into you know rho average U relative square. Here instead of you know terminal velocity Ut we are considering here U real U real means relative velocity of that particles. So, after simplification from this equation it becomes Cd into pi Dp square by 4 into half rho average U real square and then pi Dp cube by 6 into you know that rho p minus rho average into g which results after substitution of Cd is equal to 24 by Re from equation number 4 earlier.

Here the Equation

$$C_D\frac{\pi d_p^2}{4}\frac{1}{2}\rho_{ave}U_{rel}^2 = \frac{\pi d_p^3}{6}\left(\rho_p - \rho_{ave}\right)g$$

So, finally that equation number 6 will becomes that U relative that means relative velocity will be equal to rho p minus rho average into Dp square into g divided by 18 mu

Here the Equation

$$U_{rel} = \left(\rho_p - \rho_{ave}\right)\frac{d_p^2 g}{18\mu_e}$$

here mu will be mu e and rho liquid will be rho average but particle density will be same. So, this is the equation based on which you can calculate what will be the relative velocity of the particles in presence of other particles or velocity of the particles when it will be flowing downward under gravity in a suspension. Then substituting for average density here we will see that we are getting here average density rho average. Now what will be the rho average value that we have already given the equation for that rho average value here, rho average value by equation number 3. So, if you substitute this rho average value and the effective viscosity that is given earlier also that either 1 or 2 if you substitute that you know viscosity.

So, here rho average and mu e that means effective viscosity of the suspension. We obtain the following expression here for terminal falling velocity for a particle in a suspension. So, it will be here U relative but comma t we are we are giving t here this is the terminal velocity of that particle in presence of other particles that means in a suspension not in a pure liquid or in an absence of particles it is not there only it is a relative velocity or you can say that terminal falling velocity for a particle in a suspension. So, it will be rho P minus rho F to dP square G by 18 mu into function of epsilon P into epsilon F.

Here the Equation

$$U_{rel,t} = \left(\rho_p - \rho_f\right)\frac{d_p^2 g}{18\mu_f f(\varepsilon_p)}\varepsilon_f$$

So, this function of epsilon P that would be you know that based on that either that concentration will be greater than 20 percent or less than 20 percent according to that function of epsilon P will be here.

Now, the velocity U t is here U real t is called particle settling velocity in the presence of other particles or the hindered settling velocity. So, this is your basically hindered settling velocity. Now, comparing this with the expression for the terminal velocity of a single particle in a fluid we can get here equation number 9. Here from this equation number 8 we are just going to substitute the value of U t then we can express from this equation number 8 then we are having this expression here in terms of terminal velocity of a single particle. So, U real t that will be is equal to U t into epsilon F divided by function of epsilon P.

Here the Equation

$$U_{rel,t} = U_t \frac{\varepsilon_f}{f(\varepsilon_p)}$$

will be the relation between that hindered settling velocity and the terminal velocity of the single particle. So, this equation number will give you the answer. So, the settling velocity of a particles tend to decrease steadily as the concentration of the suspension increased. If your particle concentration increase what will happen that settling velocity of the particles will decrease. This is simple that can be understood from this equation number 9.

So, here whereas this U t value already we have derived in previous lecture there. So, U t will be is equal to rho P minus rho F into dP square G by 18 mu.

Here the Equation

$$U_t = \left(\rho_p - \rho_f\right)\frac{d_p^2 g}{18\mu_f}$$

Now, coming to the point here best settling or sedimentation. So, before going to that sometimes we have to know here what is superficial fluid velocity, what is superficial particle velocity, flow area occupied by the fluid, flow area occupied by the particles, what is the actual velocity of the fluid, what about the actual velocity of the particles, how it will be defined.

So, all those things will be known to you. So, let us consider the superficial fluid velocity, what do you mean by superficial fluid velocity, whenever fluid will be flowing through a conduit only single fluid whenever it will be flowing. So, in that case the volumetric flow rate divided by the total cross sectional area of the conduit it will be called as superficial fluid velocity. So, U f s that means superficial fluid velocity here f of fluid s for you know superficial. So, U f s that means superficial fluid velocity that will be is equal to what will be the volumetric flow rate of fluid divided by cross sectional area. Cross sectional area what is that cross sectional area through which that fluid will be flowing.

Here the Equation(s)

$$U_{fs} = \frac{Q_f}{A}$$

If it is flowing through the column then cross sectional area of the column, if it is flowing through the pipe then cross sectional area of the pipe. In this case you will see that what will be the cross sectional area will be occupied by the fluid that will be considered. So, that is why that superficial fluid velocity basically that the superficial velocity based on that you know empty vessel cross sectional area. So, here so whenever any fluid flowing through the you know conduit or pipe or column in that case the what will be the volumetric flow rate divided by that you know cross sectional area of that column or conduit that will be represented as superficial velocity. Similarly, superficial particle velocity only particle is flowing at a volumetric flow rate of Q p then if you divided by that cross sectional area it will be regarded as superficial particle velocity.

Here the Equation

$$U_{ps} = \frac{Q_p}{A}$$

conduit simultaneously in that case each of the phase like fluid and particle do not have the same cross sectional area. It may be changed the cross sectional area that will be occupied by the fluid and cross sectional area occupied by the you know solid particles will not be same there will be a different cross sectional area that they will occupy. Of course, there because total cross sectional area will be remain same. So, whenever any pipe you are using and through that pipe suppose liquid and solid both will be flowing then some cross sectional area will be occupied by the fluid some cross sectional area will be occupied by the solid. So, in that case what fraction of the cross sectional area will be occupied by the fluid.

So, that will be actually what will be the volume fraction of the fluid that will give you what will be the fractional area occupied by the fluid. So, that can be written by equation number 13 here A f will be is equal to epsilon f into A.

Here the Equation

$$A_f = \varepsilon_f A$$

A f is what that fractional cross sectional area occupied by the fluid that will be is equal to epsilon f into A. Capital A is the total cross sectional area and then flow area that is occupied by the particles that will be A A p that will be epsilon p into A. So, again since epsilon p is equal to 1 minus epsilon f.

Here the Equation

$$A_p = \varepsilon_p A = (1 - \varepsilon_f)A$$

So, we can write A p will be is equal to 1 minus epsilon f into A. Now in this case what will be the actual velocity of the fluid. Now this actual velocity of the fluid will be based on the actual cross sectional area is occupied by that fluid. So, here we can say that this U f actual that means actual fluid velocity U f A, A for actual that will be is equal to what that Q f divided by A means what here actual flow area of the fluid will be is equal to A f.

So, it will be Q f by A f. So, it will be here actual velocity of fluid. So, this is basically Q f divided by A f means what epsilon f into A. Now if we have that Q f by A then into epsilon f. So, this Q f by A will be called as U f s that is superficial velocity of the fluid divided by then epsilon f. So, actual velocity we are having here is equal to superficial velocity divided by volume fraction of that phases.

So, actual fluid velocity will be is equal to superficial fluid velocity divided by volume fraction of the fluid. Similarly, actual velocity of the particle will be is equal to superficial particle velocity divided by its volume fraction. So, in this case one important thing that you have to remember that actual velocity always will be greater than superficial velocity because here the cross sectional area what is occupied by the fluid or solid will be less than the total or you know that total cross sectional area. So, in that case since the cross sectional area will be you know less than that total cross sectional area its velocity of course will be increased. So, in that case we are having that actual velocity of the phase will be greater than superficial velocity of the phase.

Now as per continuity equation we will see that then Q f can be written as U f s into A that will be is equal to U f A into A into epsilon f

Here the Equations

$$for\ the\ fluid: Q_f = U_{fs}A = U_{f,a}A\varepsilon_f for\ the\ particles: Q_p = U_{ps}A = U_{p,a}A(1 - \varepsilon_f)$$

and for the particle similarly you know Q p will be is equal to U p s A is equal to U p A into 1 minus epsilon f.

So, this will be your you know simple what will be the volumetric flow rate of the fluid and what will be the actual velocity of the fluid this is the relationship. Similarly, for the volumetric flow rate of the particle with the actual velocity of the particle how it will be related. Next we are coming here that settling flux as a function of suspension concentration. Here you will see that whenever particles will be falling downward that will be certain rate at a certain rate it will be falling downward.

Now if we consider that a base of solid in suspension are allowed to settle say in measuring cylinder as shown in the picture in the laboratory there is no net flow through the vessel there is no net flow through the vessel because here the slurry will not be coming out from the vessel all the slurry will remain there whereas at the same time the particles will be settling downward. So, here liquid will not be flowing whereas particles will be flowing. So, in that case we are having here total flow will be zero. So, that is why you can say that there will be no net flow through the vessel that is why you can write here equation number 18 as Q f plus Q p will be is equal to 0.

Here the Equation

$$Q_f + Q_p = 0$$

Now what would be the Q f? Q f is basically that here this U f epsilon f into A so this is your Q f similarly what is Q p that will be is equal to U p A into 1 minus epsilon f into A.

Now A will be cancelled out so we can simply write here U f A epsilon f plus U p A into 1 minus epsilon f that will be equal to 0.

Here the Equation

$$U_{f,a}\varepsilon_f + U_{p,a}(1 - \varepsilon_f) = 0$$

So, from which we can have after simplification U f A will be is equal to minus U p A into 1 minus epsilon f by epsilon f.

Here the Equation

$$U_{f,a} = -U_{p,a}\frac{(1-\varepsilon_f)}{\varepsilon_f}$$

So, in this way we can you know have the relationship between actual velocity of the fluid in terms of you know that actual velocity of the particle and also volume fraction of the particles or fluid. Here you will see that in the picture it is shown that in a test tube the slurry or suspension is allowed to settle down. You will say with respect to time the particles will be settled down and after a certain time you will see that the all the particles will be settled here okay and you know giving the clear liquid above it.

Now you will see that during that you know settlement with respect to time here with respect to time it will be happened and here you will see that the concentration level of that particles will be changing or you can say that actual velocity of that particles will be changing that depends on that you know particle concentration. So, here initially you will see that all the slurry can be represented by B at this stage and with respect to time whenever it will come after some time you will see that that slurry will be suspending and the solid particles will be suspending and at the bottom the small amount of solid will be depositing whereas above it there will be a you know concentrated layer of that slurry and above which there will be a clear liquid. Now you will see that at this you know zone B there will be a again division of that layer of this you know zone B as per the particle concentration and after certain time again this you know zone will be reducing just you know allowing or keeping that clarified zone above it whereas the bottom zone that settled zone will be increasing because more the particles will be settled down at the bottom here with respect to time and at the end you will see that when all the particles will be settled down only this you know that settling zone will be forming just leaving that you know clarified zone above it. Okay, so it is happening with respect to time. So, during that settlement there will be a you know particle flux you can say that particle flux means here the particle will be falling down with respect to certain velocity with respect to time and also as per that cross-sectional area.

So, that is why it will be called as a flux. Then in hindered settling under gravity the relative velocity between the particles and the fluid that we know that whenever that particles will be flowing downward what would be the relative velocity then it will be UpA minus UpUfa this is the actual velocity difference between actual velocity of particles and fluid. So, that will be regarded as U relative T that means hindered settling velocity here in this case Ufa may be you know that 0 but in some cases you will see the slurry also will be moving upward whereas the particles will be going downward. So, in that case that Ufa to be considered there. Okay, so here we can say that it is basically the UT into epsilon f by function of epsilon p. So, after substitution of Ufa value here Ufa value that will be in terms of you know by equation number 20 in terms of UpA we are having this equation number you know 22 here after substitution of Ufa here and simplifying we can get that UpA that means actual particle velocity in the vessel it will be UT into epsilon f square by function of epsilon p and now this is this you can say that equation number 22 this is basically that UpA is the actual you know that settling velocity or hindered settling velocity in a vessel and which is again a function of you know that fluid volume fraction or particle volume fraction you can say and also the terminal velocity of the of the single particles in the absence of other particles.

Here the Equations

$$U_{p,a} - U_{f,a} = U_{rel,t} = U_t \frac{\varepsilon_f}{f(\varepsilon_p)}$$

$$U_{p,a} = U_t \frac{\varepsilon_f^2}{f(\varepsilon_p)}$$

So, here from this equation number 22 you can easily calculate what would be the you know hindered settling velocity and how is it related to the terminal velocity of the single particle in a particle free liquid. Then based on this observation research and Jackie they have made one relationship for the hindered settling velocity as a function of fluid volume fraction and also they have you know suggested this relationship that will be nonlinear relationship like this equation number 23 that means UpA would be is equal to UT into epsilon f to the power n

Here the Equation

$$U_{p,a} = U_t \varepsilon_f^n$$

where n is called the research on Jackie hindered settling index. Now for uniform sphere that is forming a suspension of a particle volume fraction less than 10 percent by volume n will be is equal to 2.5. So, as per their observation from their experimental observation they actually made this relationship here they have you know correlated these things with the you know free terminal velocity of the particles.

So, UpA that will be is equal to UT into epsilon f to the power n whereas from the derivation we obtain these that UpA will be is equal to UT here UpA will be is equal to UT into epsilon f square by function of epsilon p. So, this is basically a this is a function of epsilon p. So, that is why we can write here this equation UpA is equal to UT into epsilon f to the power n or you can say that UT into 1 minus epsilon f whole to the power n like this. So, this is your UpA hindered settling. So, n is called you know research on Jackie hindered settling index and this n will be is equal to 2.

5 when the particle concentration will be less than 10 percent by volume. And for any flow regime Khan and Ryszardson 1989 recommend the use of the following correlation here as shown in the equation 24 to calculate the exponent n that is Jackie Ryszardson index n over the entire range of Reynolds number. So, they have given this correlation 24 from which you can easily calculate what will be the you know hindered settling index n this n value. So, this is basically 4.

8 minus n divided by n minus 2.4 that will be equal to 0.043 Archimedes number to the power 0.57 into 1 minus 2.4 into dp by dc whole to the power 0.27 where Archimedes number is defined as dp cube rho f into rho p minus rho f into g by mu square.

Here the Equations

$$\frac{4.8-n}{n-2.4} = 0.043Ar^{0.57}\left[1 - 2.4\left(\frac{d_p}{d_c}\right)^{0.27}\right]$$

$$Ar = \frac{d_p^3\rho_f(\rho_p-\rho_f)g}{\mu^2}$$

Here dp is the particle diameter and dc is the vessel of tube diameter vessel or tube diameter.

So, in this case we can you know calculate what will be the value of n in the general case for the entire range of Reynolds number. Next we can say that if we are having this you know Ups will be is equal to Up into 1 minus epsilon f that relationships that already we have derived here that what will be the superficial particle velocity how it is related to the actual particle velocity that earlier we have you know given here in this slide in equation number I think yeah in this case equation number 16 we have given that relationship actual velocity how it is related with the superficial velocity. Now if we substitute that you know Up a value here for this Up a value that will be Ut into 1 minus epsilon f into epsilon f to the power n.

Here the Equation

$$U_{ps} = U_{p,a}(1 - \varepsilon_f) = U_t(1 - \varepsilon_f)\varepsilon_f^n$$

So, this you know after substitution we are getting this equation number 26. So, from this equation we can write Ups by Ut that will be equal to 1 minus epsilon f into epsilon f to the power n or we can write epsilon p into 1 minus epsilon p whole to the power n like this.

Here the Equation

$$\frac{U_{ps}}{U_t} = (1 - \varepsilon_f)\varepsilon_f^n$$

equation okay in a graph where x-axis will be is equal to 1 minus epsilon f or epsilon p and in y-axis it will be you know that ratio of this superficial particle velocity to the terminal velocity of the single particle then we can have this type of profile here okay this type of profile here. So, in this case you will get some maximum value maximum value of this Ups by Ut this ratio and also there will be some inflection point at which that a concentration layer will be changing okay from this upper layer to the lower layer here as per that particulate concentration or particle concentration will be changing during that settlement. So, what will be the maximum point at which that maximum you know Ups by Ut value we can get. So, that maximum will be there after you know getting derivation of this Ups by Ut with respect to epsilon f then we can get that maximum value will be at 1 minus epsilon f will be is equal to 1 by n plus 1

Here the Equation

$$1 - \varepsilon_f = \frac{1}{n+1}$$

and that inflection point will be happened 1 minus epsilon f is equal to 2 by n plus 1.

Here the Equation

$$1 - \varepsilon_f = \frac{2}{n+1}$$

So, here in this case we can get this you know maximum or inflection point just by taking first and second derivatives of the equation 27 which will give you the plot of dimensionless particle settling flux versus suspension volumetric concentration 1 minus epsilon f with this value of that 1 by n plus 1 for the maxima and 2 by n plus 1 for the you know inflection point okay.

So, this is basically the representation of that you know solid flux during the settling relative to that terminal velocity and how it will be you know changing with respect to that particle concentration that can be represented by this you know plot. Now, let us do an example based on this you know observation of that hindered settling velocity. Here let us consider that a 30 percent by volume suspension of spherical sand particles in a viscous soil has a hindered settling velocity of 4.

44 micrometer per second. If the research on Jackie hindered setting index is 4.5 then what is the terminal velocity of the sand grain there. So, this problem is given in gate 2000s there. So, let us solve this problem as per whatever we have learned here for this hindered settling velocity. Now, we know that according to research on Jackie relationship the general equation for hindered settling velocity can be expressed by this equation here U p a will be is equal to U t into epsilon f to the power n is equal to U t into 1 minus epsilon p to the power n. So, here as per problem here U p a that means hindered settling velocity is given to you that is 4.

44 micrometer per second. So, here U p a will be is equal to 4.44 and U t value that you have

to find out that your problem is what is the terminal velocity of the sand grain. So, terminal velocity to be found and then epsilon p value is given to you what is that epsilon p value 30 percent by volume it is given to you. So, here epsilon p is equal to 0.30. So, after substitution of this U p a and epsilon p and solving we can get here U t will be is equal to 22.

10 micrometer per second. So, based on this you know example we can understand how to calculate what is the settling velocity in a suspension. Then another concept that concentration of interface in sedimentation whenever particles will be falling downward in the vessel you will see that the sedimentation of that particles will happen you will see that there will be a formation of interface that means there will be some layer some demarcation point of concentration layer. So, you will see that with respect to time whenever settled down that particles will be settled down the concentration in one layer will not be same as the you know upper layer or it from its you know downward layer as per you know shown in the picture here you see if you consider any layer here. Okay, so above this layer the concentration of the particles here will not be same what will be the concentration of the you know solid here.

Okay, so there will be a difference. So, let us consider that concentration at this layer of this you know interface is C 1 and below this interface the concentration is C 2. What is that concentration? Basically for convenience we are considering C 1 and C 2 instead of epsilon p 1 or epsilon p 2. Okay, this is basically epsilon p particle concentration C. So, defining the symbol C to represent the particle volume fraction that is C that is epsilon p this is basically 1 minus epsilon f. So, whenever the settling down there you will see that there will be some interface that interface will be forming because of that concentration difference.

Okay, below and above this layer. So, consider the suspension of concentration C 1 containing particles settling at a velocity U p A 1 and the suspension of concentration C 2 which is you know settling down at a velocity U p A 2 and during that you know sedimentation this you know demarcation layer or you can say the interface between this concentrated layer that will be also you know moving downward. So, that interface movement at which velocity it will be called as interface velocity. So, it will be represented by U interface. So, that interface is falling at a velocity U interface.

The mass balance over the interface now can be done. What will be that mass balance? So, you will see that mass balance over the interface gives U p A 1 minus U interface into C 1 that will be equal to U p A 2 minus U interface into C 2.

Here the Equation

$$(U_{pa,1} - U)C_1 = (U_{pa,2} - U)C_2$$

You will see that what will be the total solid here at this at this zone here if you multiply by A that means cross sectional area then you will see that A into U p A minus U interface that will give you the relative particle volume flow rate at which it will be going downward then into concentration that means concentration C 1 that means volume fraction of that particles that will give you the total amount of particles at this layer which is going downward similarly for this layer or zone we can say that the U p A 2 minus U interface into C 2 the same amount of solid will be coming downward. So, this will be the balance of that particles you know what this interface the mass above and below the interface will remain same. So, this is the mass balance you can say. Now, the interface is falling at a velocity U interface and the mass balance will be giving by equation number 28.

Here the Equation

$$(U_{pa,1} - U)C_1 = (U_{pa,2} - U)C_2$$

we can you know give the simplified form or other way that what will be the U interface that U interface that will interface velocity will be equal to U p A 1 to C 1 minus U p 2 into C 2 by C 1 minus C 2.

Here the Equation

$$U = \frac{U_{pa,1}C_1 - U_{pa,2}C_2}{C_1 - C_2}$$

So, in this case since U p A C 1 U p A C is particle volume metric flux that can be represented by U p S.

Then we can write U p S will be is equal to U p A 1 into C 1 or U p A 1 into epsilon p A 1

Here the Equation

$$U_{ps,1} = U_{pa,1}C_1 = U_{pa,1}\varepsilon_{p,1} = U_{pa,1}(1 - \varepsilon_{f,1})$$

or we can say like that you know U p A 1 into 1 minus epsilon f 1. Similarly, U p S 2 that will be U p A 2 into C 2 that will be U p A 2 into epsilon p 2 is equal to U p A 2 into 1 minus epsilon f 2.

Here the Equation

$$U_{ps,2} = U_{pa,2}C_2 = U_{pa,2}\varepsilon_{p,2} = U_{pa,2}(1 - \varepsilon_{f,2})$$

So, mass above and below the interface will remain same that is why you can write U interface by this equation by mass balance and here from this equation number 29 just we are substituting this U p A 1 into C 1 as a U p S 1 by equation number 30. Again U p A 2 into C 2 that can be replaced by U p S 2 by equation number 30 divided by C 1 minus C 2.

Here the Equation

$$U = \frac{U_{pa,1}C_1 - U_{pa,2}C_2}{C_1 - C_2}$$

this concentration will tends to 0 then will give you the derivative forms of U interface will be equal to d U p S by d C as 32 equation number 32.

Here the Equation

$$U = \frac{dU_{ps}}{dC}$$

So, this interface how it will be changing and also that what will be the solid flux with respect to concentration change how it can be represented. This graph will give you the representation of this U p S how it will be changing with respect to concentration. This yellow line will give you the profile. Now you will see that interface that is basically the slope that slope will give you the velocity of the layer of concentration or interface you can say.

So, in this case slope you will see that if we consider any point here this point from this graph it will give you the C 2 and then U p S 2 then what will be the joining line here and this line will give you that the slope will be giving you that U interface 1 and 2 between 1 and 2 layer and velocity of interface between clear liquid that is C 1 will be equals to 0 and the suspension of concentration C 2. Similarly, this slope will be giving you U interface 2 3 here this concentration C 2 and C 3 here that means your velocity of interface between suspension concentration C 2 and the settle bed of concentration C 3. So, in this case we are having slope as well as that interface velocity from that slope. The slope of a chord joining two points at concentration C 1 and C 2 is the velocity of interface between suspension of these two concentrations.

So, this is the main concept from this the solid flux versus concentration from which you can get this. Now, let us do an example for this. Suppose a suspension in water of uniformly sized sphere whose diameter is 50 micrometer density of 1530 kg per meter cube has a solid concentration of 20 percent by volume. The suspension settles to a bed of solids concentration of 50 percent by volume. Now, in this case you have to calculate what will be the rate at which the water suspension interface settles and the rate at which the sediment or suspension interface rises assume water properties as density is 1000 kg per meter cube and viscosity 0.

001 Pascal second. So, this is your problem that you have to find out the interface velocity. So, how can we solve this problem? So, let us consider that case A in that case solid concentration of initial suspension is given to you this is 20 percent and as per equation 29 that we have described so that will allow you to calculate the velocity of interface between the suspension of different concentrations. Now, the velocity of the interface between initial suspension B and the clear liquid A interface we can have here this U interface AB will be equal to UPAA CA minus UPB into CB by CA minus CB.

Here the Equation

$$U_{AB} = \frac{U_{pa,A}C_A - U_{pa,B}C_B}{C_A - C_B}$$

So, here equation number 29 as per you know just see your slide that this is equation number 29. So, from this equation number 29 we are having this equation. Now, since here this as per problem CA will be equal to 0 the equation reduces to U interface AB will be equal to UPAB.

Now, UPAB is the hindered settling velocity of the particles relative to the vessel wall in basel settling and given by equation number 23 earlier that we have given. So, UPAA will be equal to UT into epsilon f to the power n. So, assuming Stokes law applies then n will be equal to what 4.65 and single particle terminal velocity can be calculated by this equation.

So, we know now the single particle terminal velocity and epsilon f. So, from which we can get this UPB value so which will be is equal to 2.56 into 10 to the power minus 4 meter per second. Now, in this case you have to check whether that we have considered here what is the n value that is as per Stokes law applies then this n value Jackie research on hindered settling index that will be equal to 4.65 whether it is coming this Stokes flow regime or not.

So, for that you have to calculate what will be the Reynolds number. So, as per that Reynolds number in terms of terminal velocity of the particles then we are getting 0.055. So, which is coming less than the limiting value for Stokes law that is 0.

3 less than 0.2 we can say and so the assumption is valid. So, hence we can say that interfacial or interface velocity between the initial suspension and the clear liquid will be is equal to 0.256 millimeter per second. And the fact that the velocity is positive here indicates that the interface is moving downward there. Now, case B you will see that here again we apply the equation 29 to calculate the velocity of the interface between suspension and different concentrations.

Now, the velocity of the interface between initial suspension B and the sediment here to be considered. So, in that case UPAB CB minus UPS CS divided by CB minus CS, S for here settlement that is sediment here you can say. So, in this case CB value is known to you CS value also known to you it is given to you and since the velocity of the sediment here UPS will be is equal to 0 here. So, U interface BS will be only this part.

So, that means here UPAB into 0.20 here this part will be is equal to 0 divided by 0.20 minus 0.50. So, it is coming here negative of 0.67 UPAB and from this part A we know that UPAB will be is equal to 0.256 millimeter per second.

So, U interface BS here after substitution of UPAB here it will give you that minus 0.171 millimeter per second. So, in this case negative value will be coming this negative sign signifies that the interface is moving upward. So, the interface between initial suspension and sediment is moving upwards at a velocity of 0.171 millimeter per second. I think you understood this you know problem how to calculate the interface movement at a certain velocity based on this equation. I think you understood the you know concept of this you know hindered settling velocity in presence of particles and how is it related to the terminal velocity whenever particles would be falling freely at its terminal velocity and also how that you know hindered settling velocity will be related to the free settling velocity and also particle concentration.

And how that interface of these two layers of that you know sediment in a particular vessel or column it will be changing or going downward and how that you know flux of that solid particle sedimentation will be changing with respect to you know concentration of the particle of the slurry that will affect that you know sedimentation efficiency or particle separation you can say. So, thank you for your attention. In the next lecture we will try to discuss another module. This is basically the flow through granular bed. So, in that case we will try to learn something basic law and terminology of that flow through granular bed in the next lecture. Thank you. Have a nice day. you