Lec 11: Terminal velocity of single particle

Hello everybody. Welcome to this massive open online course on solid-fluid operations. So in this lecture, we will try to learn about the terminal velocity of single particle. In the previous lecture, we were discussing about the drag force of single solid particles that is acted on its by fluid flow over the solid surface and also we were discussing about the lift force and also how the drag force will be acting at a different flow velocity and also at a different Reynolds number range. So in this lecture, we will try to learn that terminal velocity but it will be only of single particle. So in the next lecture, we will try to again learn about what will be the terminal velocity for multi-particle if they are suspending in a liquid or other medium.

So in that case, first of all let us learn about the terminal velocity of single particle where there will be no interaction of that single particle with the other particle. So here see that suppose a particle is falling from that it is a still position freely. So in that case, the particles will be falling under gravity in the fluid and also if fluid is moving upward then what will be the relative velocity of that particle that will be fall down there. So the relative motion of the particles in a fluid is generally under the action of the forces of buoyancy, drag and gravity.

So whenever a particle will be falling downward freely, there are different forces will be acting on that solid particle like what is that one will be called buoyancy force and another will be called drag force that we have already discussed in the previous lecture, how the drag force will be acting on a single particle and then gravity force. So these 3 forces will be acting simultaneously there. But whenever you will see that particles will gain that steady or speed, these 3 particles will be balanced to each other. So in that case, that as the particle accelerates, you will see that the drag force will be increased, whereas causing that acceleration to reduce. So interesting is that whenever the particles will be falling with a certain velocity change with respect to time that is with acceleration, then you will see that due to this acceleration, you will see that the drag force will be increases because there velocity continuously will be increasing with respect to time and due to that increase in velocity, their drag force also increase because the drag force depends on that velocity as well as the projected area of the particle.

So that is why you will see that whenever these particles will gain that acceleration or falls under an accelerative motion, then the drag force will be acting on that particle and it will increase. So when this drag force will be increasing continuously, there at a certain time, you will see that this particles acceleration will be decreasing. So the velocity at which the acceleration will be 0 that means there will be no acceleration at which velocity that velocity will be called as a single particle terminal velocity. In this case, so we are having these 3 forces to represent that single particle terminal velocity there and since these 3 forces will be acting in such way that there will be no acceleration. So at that acceleration point that means the particles will gain that steady speed.

So that steady speed will be called as a terminal velocity. So what are those forces that we have discussed that that will be gravity force, buoyancy force, drag force. Now if we do the balance of these 3 forces and at that steady velocity or that there will be no change of velocity, then that means we can say that acceleration force will be 0. Otherwise, the particles will be falling downward with a certain acceleration force. So that is why we can have that what will be the net force acting on that solid particle whenever it will be falling downward.

So that net force will be is equal to gravity force minus buoyancy force minus drag force. That means what will be the gravity force from that gravity force that will be acting downward as shown in the picture by arrow sign here, this gravity force that is Fz we can say that will be Mz whereas the drag force will be acting upward in the upward direction. So here the drag force then the direction of this drag force will be opposite to that gravity force. So that is why gravity force minus drag force will be coming. Whereas, buoyancy force will be acting also that opposite to that gravity force.

So that is why again from that gravity force we have to subtract that buoyancy force. So the net force will be basically that gravity force minus drag force minus buoyancy force and whatever net force will be there with that net force the particles will be falling downward.

Here the Equation

$$\text{Gravity Force} - \text{Buoyancy Force} - \text{Drag Force} = \text{Acceleration Force}$$
$$F_G - F_B - F_D = F_a$$

Now if during that falling down the acceleration is maintained then there will be acceleration force that can be represented by F A. So we are having this force balance like this Fg minus Fb minus Fd is equal to F A. Now see here interesting point that you will see that suppose if you are talking about a man is falling from the aeroplane with parachute.

You will see initially whenever he will fall down from the aeroplane you will see that he will fall down with an acceleration and this velocity of the person you will see that will be changing with respect to time. So there will be of the profile of that velocity change with respect to time that can be represented by this profile like this. So up to a certain time you will see that you will gain that increase in acceleration okay that means velocity will increase with respect to time and during that period there will be some drag force acting on his body and due to that drag force he will realize that acceleration coming to reduce okay and at a certain time you will see that he will gain the steady speed that means at a constant velocity he will be falling downward. So you will see that at a constant velocity he will be falling downward okay and that velocity will be called as a terminal velocity and then immediately he just open his parachute and when he open that parachute you will see that the parachute will gain again the drag force there along with his body then there will be again increasing the drag force and due to the drag force the acceleration will again reduced that will be represented by the deceleration. So whenever he will open his parachute and falling down with that parachute the drag force will be acting on this parachute as well as his body and the speed of him you know you will see that it will be decreasing that means he will be gaining the deceleration.

Now at a certain time you will see that this deceleration will come to 0 that means he will gain again steady speed and this steady speed again it will be called as a terminal velocity that means he will fall down with that terminal velocity again. So what happened initially he will gain acceleration and after a certain time he will get the constant velocity that constant velocity will be called as terminal velocity after that whenever he will open the parachute he will again gain the drag force and due to that drag force his acceleration will be reducing that is called deceleration and after a certain time he again will gain the constant speed that constant speed will be called as terminal velocity 2. So with that terminal velocity again he will be falling downward to the ground. So this is the case you will see that here also in animation it is shown that so initially that velocity will be increasing with respect to time and after that it will gain a constant velocity and then after opening parachute he will gain the reduction of velocity with respect to time and after a certain time again he will gain the constant velocity. So in this case you have to remember that whenever he will get the constant velocity that means the velocity does not change with respect to time at that velocity it will be regarded as terminal velocity.

So this is the concept of terminal velocity. So here he actually will gain two terminal velocity initially up to a certain time at this higher speed he will falling down before opening his parachute that will be higher terminal velocity whenever he will open that parachute and after certain time whenever he will gain the constant speed that will be your terminal velocity 2 but it will be less than that earlier terminal velocity because here he is getting more drag force on his body as well as by parachute. So that is why here deceleration will be there. So this is the concept of terminal velocity. Now let us derive that what will be the terminal velocity there.

So here how we can then derive the terminal velocity. Let us consider this part apple is falling down from the apple tree. So here whenever apple is falling downward this apple will will be under the forces of gravity drag force and buoyancy force and at the terminal velocity condition there will be no acceleration force that means the particles will be falling down at a steady speed. So in that case we can write this force balance here as earlier that gravity force minus buoyancy force minus drag force will be is equal to acceleration force.

Here the Equation (s)

$$\text{Gravity Force } (F_G) - \text{Buoyancy Force } (F_B) - \text{Drag Force } (F_D) = \text{Acceleration Force}$$

$$\frac{\pi d_p^3}{6}\rho_p g - \frac{\pi d_p^3}{6}\rho_f g - C_D \frac{\pi d_p^2}{4}\frac{1}{2}\rho_f U_t^2 = 0$$

$$U_t = \sqrt{\frac{4gd_p}{3C_D}\left(\frac{\rho_p - \rho_f}{\rho_f}\right)}$$

$$C_D = \frac{4}{3}\frac{gd_p}{U_t^2}\left[\frac{(\rho_p - \rho_f)}{\rho_f}\right]$$

What is the gravity force? How will calculate that gravity force? So if the particle diameter if we consider that it is a spherical particle and its diameter is dp.

So in this case we can say that what will be the volume of this particle very simple it will be 1 by 6 pi dp cube this is the volume and if you multiply this volume with its density if the density of the particle then you will get the mass of this particle and mass into gravitational acceleration it will be called as gravity force. So this gravity force will be is equal to pi dp cube by 6 rho p into z and then coming to the buoyancy force. Buoyancy force is basically what the force that is exerted by the same volume of liquid which will be displaced by that volume of the particle that is called buoyancy force. So this buoyancy force again what will be the mass of that fluid which is replaced by that particle. Now for that first what will be the volume of that particle and then if you multiply the density of that fluid then you will get that mass of the fluid which is displaced by the particle then into z.

So it will be called as buoyancy force. And then drag force, drag force already we have discussed in the previous class that what is drag force? So drag force is basically that CD into projectional area into kinetic energy. So that is your drag coefficient into projectional area into kinetic energy per unit volume. So this is your drag force as per earlier definition of the drag force. And here u is the velocity of the particle since we are considering that the particles falls with the terminal velocity so we can consider here that ut as a terminal velocity.

Then that will be is equal to 0, why? At this terminal velocity there will be no acceleration force. So from this force balance we can simplify and after simplification we can get ut will be is equal to root over 4 g dp by 3 cd into rho p minus rho f by rho f. Here dp is the particle diameter, c is the drag coefficient and rho p is the particle density and here rho f is the fluid density. So under this terminal velocity we can say that the terminal velocity will be is equal to this that you have to remember. So this is very important you have to remember throughout your life you can say that because this is the very important equation that you have to remember.

Now it depends on what the drag coefficient. You will see that also the particle diameter. It depends on particle diameter drag coefficient and you can say that fluid and particle properties. So under this terminal velocity we can also calculate what will be the cd value from this equation. Simple from this equation just simplifying this equation and finding the

C_D value from this equation.

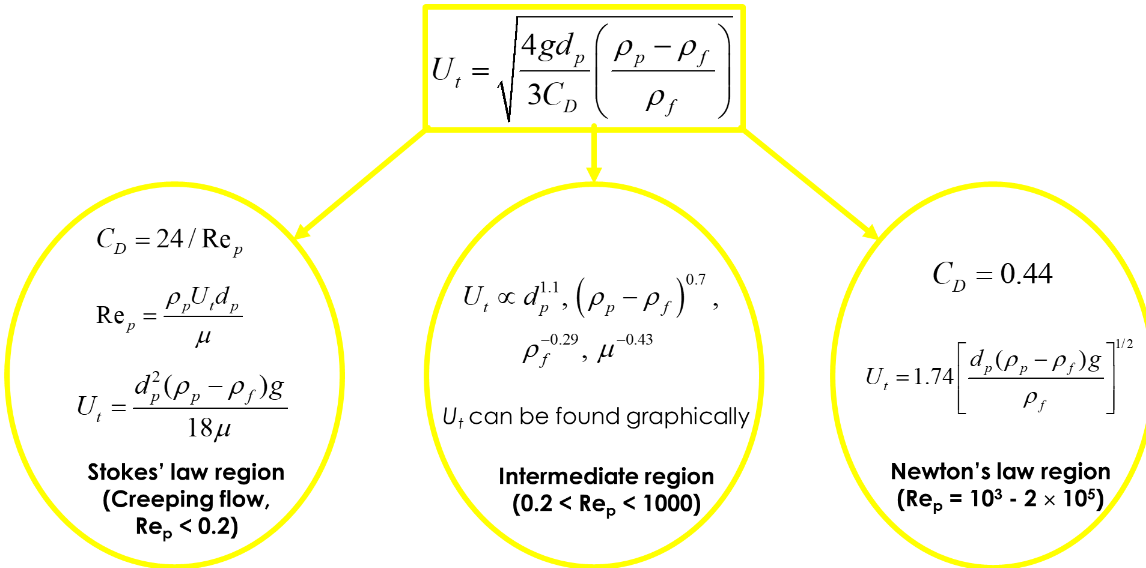
So interesting that we are having that terminal velocity is the proportional to the square root of the particle diameter. Terminal velocity is inversely proportional to the drag coefficient, the square root of drag coefficient we can say and also this terminal velocity is proportional to the square root of density difference of particle and fluid and also you can say that it is inversely proportional to the square root of the fluid density. So here let us consider the different flow conditions like Reynolds number and that other condition like that. You will see that if we consider that Stokes flow condition that we have already discussed about that Stokes flow condition where the flow will be laminar and Reynolds number at this flow condition it will be less than 0.

2. So at this Reynolds number that means at this Stokes flow region sometimes it is called creeping flow that is very laminar flow you will see that this cd value that means drag coefficient will be is equal to 24 by Rep that means Reynolds number of the particle. How Reynolds number of the particle is defined this is rho p ut dp by mu. So in this case if you substitute the value of cd here in this terminal velocity equation so you will get this terminal velocity will be is equal to what this. So at a Stokes law region or creeping flow condition or where Rep less than 0.2 ut will be is equal to dp square that into rho p minus rho f into g by 18 mu.

Here interesting that this terminal velocity at this Stokes flow condition will be proportional to the square of the particle diameter and also it is inversely proportional to the viscosity of the fluid and directly proportional to that deviation of the fluid density compared to the particle density. And then coming to that point of intermediate region where the Reynolds number will be within the range of 0.2 to 1000. So there the terminal velocity will be proportional to dp to the power 1.1 and rho p minus rho f to the power 0.

7 and it will be proportional to the rho f to the power minus 0.29 and viscosity to the power minus 0.43. Here also we can say that this terminal velocity will be directly proportional to the 1.1 power of particle diameter whereas it will be proportional to that difference of that fluid density compared to that particle density to the power 0.

7. Whereas it will be inversely proportional to the fluid density and inversely proportional to that viscosity. Now in this case that ut you cannot get it from directly from this equation here because Cd value is not known to here. So in this case that ut to be found from the graph where that Cd value is given within a certain range of that Reynolds number. So from which you can calculate what will be the terminal velocity. So once you get that Cd value within this range then you can easily calculate what will be the terminal velocity from this equation.
Here the Equation(s)



Then coming to that higher Reynolds number range where the particle Reynolds number will be within the range of 1000 to 2 into 10 to the power 5 it is called Newton's law region. So at this region you will see that the drag coefficient will be coming almost constant in its value is 0.44. So once you substitute this value of 0.44 here instead of Cd then you can get it the simplified form of ut will be is equal to 1.

74 into dp into rho p minus rho f by rho f into g whole to the power half. So this is your terminal velocity. So at different flow regime we can get the terminal velocity based on the drag coefficient value at that different flow regimes. So we can easily calculate. Now coming to that point where if you do not know the regime at which the particle diameter or terminal velocity which is to be calculated.

So this is the very important point that you have to calculate the terminal velocity where you do not know the regime. So how to do that? There are two actually methods to calculate here. One is actually to calculate ut for a given particle size. This is the case one based on which you have to calculate what will be the terminal velocity if the particle diameter. So in this case what you have to do? You have to first consult a graph where log CD with respect to log Rep data given to you that is experimental data that is given to you that is available in the textbook that you have to consult.

After that what you have to do? You have to calculate that CD into Rep square as per your problem as per your known values. So what is that CD and Rep square? After substitution of CD and Rep square we are getting here like this. That means CD Rep square will be equal to 4 by 3 dp cube rho f into rho p minus rho f by mu square into Z. Now question is that from where we are getting this that CD value. CD value that we have calculated earlier here this is CD.

So here CD into Rep square after simplification you are getting this 4 by 3 dp cube rho f into rho p minus rho f into Z by mu square. Here it will be constant. Why it will be constant? Because here dp is fixed it is known to you rho f density of the fluid also fixed here particle density that means rho p is fixed g is constant mu is also constant because the same fluid here it is not changed. So here this value will be coming as constant value.

So let it be M1. Now taking logarithm on both sides we can get log CD that will be is equal to log M1 minus you will see that 2 into log Rep.

Here the Equation(s)

$$C_D \operatorname{Re}_p^2 = \frac{4}{3}\frac{d_p^3\rho_f\left(\rho_p-\rho_f\right)g}{\mu^2} = M_1 (= \text{Const.})$$
$$\Rightarrow \log(C_D) = \log(M_1) - 2\log(\operatorname{Re}_p)$$

So this equation you will get from this CD into Rep square value. So from this you can say that this is a straight line with negative slope of 2. So this equation to be drawn in that graph which you have consulted that is from the text or some other sources like where log CD versus log Rep data are given. So in that graph that you have to draw this line of this log CD is equal to log M1 minus 2 log Rep.

So here you will get this equation like this. This will be your equation. So this equation wherever it will be intersect with that log CD versus log Rep data that is on the chart from that intersection point you will be able to find out what will be the value of log Rep value. If that log Rep value here then you will be able to find out what will be the Rep value just by taking antilog and once you get that Rep value you will be able to calculate what will be the terminal velocity. What will be the terminal velocity? So this is here actually this graph is here negative slope not here this will be here. So here it will be your intersection point and what will be the Rep value.

So once that Rep value so Ut will be is equal to what in that case Rep mu by rho p dp. In this case dp is known to you so you have to find out Ut value here just correction is there this is the equation with negative slope of minus 2. Now next one is that to calculate the size dp here in this case it will be size dp for a given Ut value. So in this case what you have to do you have to first calculate CD by Rep value. So CD by Rep again after substitution of value of CD and Rep you will get this.

Here the Equation(s)

$$\frac{C_D}{\text{Re}_p} = \frac{4}{3}\frac{g\mu\left(\rho_p - \rho_f\right)}{U_t^3 \rho_f^2} = M_2 (= \text{Const.})$$
$$\Rightarrow \log(C_D) = \log(M_2) + \log(\text{Re}_p)$$

$$\text{Re}_p = \frac{\rho_p U_t d_p}{\mu}$$

This is also a constant. Now from this you can get the logarithm on both sides and you will get this equation here and this equation also is a straight line with a slope 1 positive 1. So in this case you will see that this line will be coming with a positive slope of plus 1 and wherever this straight line would be intersecting with that given log CD versus log Rep data and from that intersection point corresponding value of dp can be calculated from the log Rep value. So here from this log Rep value again the dp can be calculated like this here Rep this is known from this value and then into mu by rho p into Ut because here Ut is given to you dp to be found. So in this way you can easily calculate the terminal velocity and particle diameter at any flow regimes. Now let us do an example based on this concept of calculating the terminal velocity at a particular range of flow.

Now in this case a problem is given that a sphere of density 2500 kg per meter cube that falls freely under gravity in a fluid of density 700 kg per meter cube and viscosity 0.5 into 10 to the power minus 3 Pascal second. Given that the terminal velocity of the sphere is 0.15 meter per second. Calculate its diameter what would be the edge length of a cube of the same material falling in the same fluid at the same terminal velocity.

So let us consider first what would be the diameter of that particle. Here terminal velocity is given even density of the particle is given and the particles on which fluid to be falling downward it is given also this is 700 kg per meter cube and also terminal velocity is given to you. So in this case you have to find out what will be the particle diameter. So in this case we know that terminal velocity Ut and need to find the particle diameter dp. Since we do not know which regime is appropriate here we must first calculate the dimensionless group like Cd by Rep as discussed earlier.

So Cd by Rep will be is equal to what as per substitution of Cd and Rep we are having this value. So as per this that if you change the Rep value respective Cd value you will get. For Rep is equal to 100 you will get the Cd value 0.712 for Rep 1000 then you will get Cd is equal to 7.

12 and for Rep 10,000 this Cd value will be is equal to 71.2. Now plot this log Cd versus log Rep and then you will get there will be a certain intersection and of course this will be a straight line of positive slope 1 in the log-log coordinates of that standard drag curve. So after having this here let us see this one. So this is the standard Cd versus Rep curve and here you will see that this is log-log graph. So that is why it is not that log Cd and log Rep

here log-log graph itself. So here this is the intersection point that we have obtained this value here this is the straight line as per that different value of Cd and Rep.

So this is the intersection point and from this intersection point corresponding value of this Rep corresponding value of Rep. Now question is why this line is considered? Here since it is a spherical particle as per problem it is given. So for spherical particle we know that sphericity is equal to 1. So for this sphericity this line is is appropriate. So here from this line intersection point we are getting the corresponding value of Rep.

Once we know that Rep value once we know that Rep value we will get the dp value. So Rep is here 130 and from this 130 value so other things Ut is given to you, dp is given to you, mu is given to you, also rho p is given to you. So from this dp will be is equal to what 130 into mu divided by rho p by Ut. So that will be is equal to 619 micrometer. Similarly for a cube having the same terminal velocity under the same condition the same Cd versus Rep relationship to be applied.

Only the standard drag curve is that for a cube psi will be is equal to 0.806. So here if suppose this is the cubical particle so here that you have to consider this drag curve. So in the next here what would be the edge length of a cube of same material falling in the same fluid at the same terminal velocity. So here in this case we are having that for this cuboidal substances so there the intersection point is here and for this cube the respective value here is 310.

So from this 310 so dp value will be is equal to 148 since others parameters are known to you. So at this condition we are having that what is the edge length of that cube. This is basically that dp value to be found so it will be 148. So in this way you can solve that whenever the terminal velocity is known to you how to find out that particle diameter this is the method. Whereas if the particle diameter and find out the terminal velocity then you have to consider this equation and from this equation you have to find out what will be the intersection point.

From this intersection point you have to find out what will be the respective Rep value and then Rep from that Rep value what will be the terminal velocity. I think you understood this problem. Next another problem let us have a particle of equivalent volume diameter 0.5 millimeter density is given 2000 kg per meter cube and sphericity is 0.

6 that will fall freely under gravity in a fluid of density 1.6 kg per meter cube and viscosity is given of that fluid 2 into 10 to the power minus 5 Pascal second. You have to calculate what will be the terminal velocity that means reached by the particle. In this case we know the particle size and also we know that density of the fluid density of the particle even viscosity of the fluid. So it is very easy to calculate what will be the terminal velocity whenever the particle will gain it. So here also without knowing that regime in which regime actually it is there so you have to find out that terminal velocity.

So we have to follow this equation here CD into Rep square that you have to calculate first and it will becoming constant this value will come. So then consider that Rep value then respective CD value will be calculated from this equation then what CD value respective Rep is equal to 100 it will becoming 1.307 what will be the CD value 0.013 for the respective Rep value of 1000. So for different Rep value different CD value you are getting now plot it here.

So plot it and here the particle sphericity is given 0.6 so we have to follow this line and after plotting those data Rep versus CD in the plot we are having this line and this line will intersect this drag coefficient profile here and what will be the respective Rep value this is like this 40. So the plotted line intersects the standard drag curve here for a sphericity of psi at the 0.6 the corresponding Rep value is 40 and then this corresponding value of 40 of this Rep value you will be able to find out what will be the terminal velocity. So this terminal velocity will come Ut will be is equal to what 40 into mu divided by rho p into dp it is given to you that means 40 into mu, mu value is I think what is the value is given 2 into 10 to the power minus 5 divided by rho p, rho p value that means particle density is given I think 2000 and into what is that dp particle diameter it is given 0.

5 millimeter that means 0.5 into 10 to the power minus 3. So it will be coming as almost 1.0 this will be meter per second so this is your terminal velocity. So in this way for any flow regime we can calculate the terminal velocity if the particle diameter. Now let us solve some other problems which is very important that is given in gate examination you will see that here problem it has given in gate 2020 as rigid spherical particle that undergoes free settling in a liquid of density 750 kg per meter cube and viscosity 9.

81 into 10 to the power minus 3 Pascal second. Now density of the particle is 3000 kg per meter cube and the particle diameter is given 2 into 10 to the power minus 4 meter acceleration due to gravity is 9.81 meter per second square assuming Stokes law to be valid what will be the terminal velocity of the particle. Very simple here the already flow regime is given since it is Stokes flow condition so here CD value will be is equal to 24 by Rep. So once that CD by Rep then you will be able to calculate. Now at that the Stokes law condition also we have given that after substitution of CD by Rep in that terminal velocity equation for that CD and after simplification this equation is coming.

Earlier that we have given now everything is given to you dP value that is 2 into 10 to the power minus 4 square into 3000 minus here rho f is given 750 and also the g value is given and 18 into mu, mu value is given also 9.81 into what is that 10 to the power minus 3. So ultimately after calculation the terminal velocity is coming at 0.005 meter per second. Now this problem here what is the terminal velocity in meter per second calculate from Stokes law for a particle of diameter here 0.

1 into 10 to the power minus 3 meter density 2800 kg per meter cube settling in water of density 1000 kg per meter cube and viscosity of 10 to the power minus 3 kg per meter

second. So in this case assume g will be equal to 10 meter per second square. So in this case again at Stokes law condition the same equation that can be used so after substitution of all parameters here you will get this terminal velocity as 0.01. Again another example it is given for the calculation of terminal velocity and comparison of terminal velocity of two particles two types of particles there.

Here two identically sized spherical particles A and B having densities rho A and rho B respectively are settling in a fluid of density rho assuming free settling under turbulent flow condition what would be the ratio of terminal settling velocity of the particle A to that of particle B. This problem is given in gate 2000 set. In this case terminal velocity of a particle is basically the definition that u t will be equal to root over 4 gd by 3 cd into rho p minus rho f by rho f. Here the particle diameter for those two particles here particle A and particle B are same so we can write dp A will be equal to dp B. Now after substitution of this dp A and dp B value and for constant fluid density the same fluid but particle density will be different.

So in that case after substitution of those values and then taking a ratio of this then we are getting this value ut A by ut B so in this way. And then finally we are getting after simplification root over rho A minus rho by rho B minus rho so you can get it. Another problem based on this terminal velocity is given in gate 2009. The problem is that the terminal velocity of a 6 mm diameter glass sphere whose density is 2500 kg per meter cube in a viscous Newtonian liquid whose density is 1500 kg per meter cube is 100 micrometer per second. If the particle Reynolds number is small and the value of acceleration due to gravity is 9.

81 meter per second square then find the viscosity of the liquid. So, using the Stokes law condition to be used so here as per Stokes flow regime that ut will be equal to dp square into rho P minus rho F into g by 18 mu. So from this you have to calculate what will be the viscosity of the fluid so others are given to you. So mu will be equal to dp square into rho P minus rho F into g by 18 ut. So after substitution of all values here and calculation that will give you that 196.

2 Pascal second. And this is also one important problem that is also given in gate 2016. Consider a rigid solid sphere that will be falling with a constant velocity in a fluid. The flowing data are known at the conditions of interest. So the density of the fluid is 0.1 Pascal second, acceleration due to gravity is 10 meter per second square and the density of the particle is given, the density of the fluid also is given.

What is the diameter of the largest sphere that settles in the Stokes law regime. So you will see that as per problem again the statements Reynolds number of largest particle is found to be from this equation. It is given because Reynolds number is equal to what is that rho ut dp by mu, rho is given to you, dp is not known to you, ut also not known to you, mu is not known to you. But its value will be what 0.

1 maximum for that your Reynolds condition it will be you know less than 0.1. So ut will be is equal to what like this. Now under Stokes law regimes we can say that ut will be is equal to dp square into rho p minus rho f into g by 18 mu. So that will be is equal to 0.

1 mu by 1180 into dp. So from this after solving the dp will be coming as 2.05 millimeter. Then coming to the point where the terminal velocity of single particle of centrifugal sedimentation to be found. Now centrifuges are extremely used for separating fine solids from suspension in a liquid. Here the centrifuges are used for separating fine particles and droplets. It is necessary to consider only the Stokes law regime in calculating the drag between the particles and liquid.

And in this case you will see that terminal velocity will be different from that free falling there. Here the centrifugal force will be acting. So based on which we can have this terminal velocity of single particle of centrifugal sedimentation. A particle here moves outwards as per figure towards the wall of a bowl of a centrifuge. The accelerating force progressively increases and therefore the particle never reaches an equilibrium velocity as is the case in the gravitational field.

Now neglecting the inertia of the particle then at Stokes regime we can write that UTC under this centrifugal action then UTC will be equal to dr by dt. So from which you can get this equation that dp square into rho p minus rho f into omega square r by 18 mu where it will be is equal to ut into omega square r by g.

Here the Equation

$$U_{t,c} = \frac{dr}{dt} = \frac{d_p^2(\rho_p - \rho_f)\omega^2 r}{18\mu} = U_t \frac{\omega^2 r}{g}$$

$$U_t = \frac{d_p^2(\rho_p - \rho_f)g}{18\mu}$$

Here you will see that ut is defined as per that gravitational field the dp square into rho p minus rho f into g by 18 mu as per the Stokes law condition. So here ut is the terminal velocity in centrifugal field, ut is the terminal velocity in absence of centrifugal field, r is the radius of the centrifuge, r is the radial distance from the central, dp the particle diameter, rho p particle density, rho f the fluid density, mu is the viscosity of the fluid, this is the gravitational acceleration and omega is the rotational speed. So based on which you can calculate what will be the UTC that means terminal velocity of the single particle at this centrifugal field.

Now what will be the time required to settle through a liquid layer of thickness h at the wall of the bowl is given by integration of that equation earlier equation here what will be the time required to settle. So here tr then it will be is equal to after integration then it will be coming like this.

Here the Equation

$$t_R = \int_0^{t_R} dt = \frac{18\mu}{d_p^2(\rho_p - \rho_f)\omega^2}\int_{r_0}^{R} \frac{1}{r} dr = \frac{18\mu}{d_p^2(\rho_p - \rho_f)\omega^2} \ln\left(\frac{R}{r_0}\right)$$

So where liquid layer of the thickness will be h that will be r minus r0 as per figure given in the slide. So tr is the minimum retention time required for all particles of size greater than dp to be deposited on the walls of the centrifuge bowl. So this is the things that you have to calculate what will be the minimum retention time that will be required for all the particles of size diameter less than dp to be deposited on the wall of the bowl.

So in this way you have to calculate. So here one example is given and in a bowl centrifugal classifier operating at 60 rpm with a fluid of viscosity like 0.001 kg per meter second the liquid surface is at a distance of 0.20 meter from the axis of rotation.

What would be the time required for a particle of specific gravity 2.5 and a particle diameter 0.0001 meter to traverse a distance of 0.05? meter from the liquid surface in the centrifuge. So, here this is the equation to calculate that terminal velocity at that centrifugal field. So all the parameters here given all the data are given as per problem and after substitution you will get this 6.

57 into 10 to the power minus 3 that will be meter per second. This is the thing so it will be your velocity at which that particles will be settled and what time it will be required for the particle of specific gravity of that it will be whenever it will be moving from distance of 0.05 meter from that liquid surface in the centrifuge. So that time to be calculated so that can be calculated just that just by what is the distance it is given 0.

05 meter. So 0.05 meter divided by this velocity then you will get simply what will be the time required. So it is around 7.6 second. So I think you understood that here what is the terminal velocity, what is the basic concept of terminal velocity and how that terminal velocity can be calculated by knowing that flow regime and without knowing also flow regime from the standard chart of that you know drag coefficient that you can easily calculate. Also how to calculate that terminal velocity of that single particle whenever it will be your under centrifugal field. So I think it will be enough for that terminal velocity here to understand and in the next class we will be discussing here again that what will be the terminal velocity or settling velocity of the particle will be multiple particles will be there.

There will be certain interaction between particle and particle and fluid and because of which that terminal velocity will be different from that single particle terminal velocity. So that terminal velocity will be discussing in the next class. Thank you.