

Lec 10: Flow Past a Cylinder and Spherical Particle

Hello everybody. Welcome to this massive open online course on solid fluid operations. So from today on the next three successive lecture we will try to understand the module of about flow through immersed bodies. So here in this lecture we will try to learn something about the flow past cylinder and spherical particle. So here we will try to know about the drag when flow around body, the component of drag and drag in various cases of spheres and cylinders. Now we will see that when a flow around a body will be running what will happen that flow around the body if it is placed in a uniform flow.

So there we will develop a thin layer along the body surface with largely changing velocity that is the boundary layer due to the viscosity of the fluid. And also you will see that the flow separates behind the body discharging a wake with eddies. So here we can say that flow around a body. So that may be any type of body.

Here if we consider that a body the general form if it is placed in a position where over this body we will see that or surrounding this bodies you can say that there will be a stream flow of the fluid, uniform flow around the body. And this flow will develop a thin layer along the body surface. And there itself you will see that there will be change in velocity whenever it will be flowing over these surfaces. And this change of velocity will be due to the viscosity effect of the fluid. There will be some frictional resistance will be incurred by that fluid on the surface and because of which the velocity of that component of the fluid will be changing.

Now whenever it will be flowing over the surface there will be formation of boundary layer due to this viscosity of the fluid. And the flow whenever it will be flowing you will see that that will separate behind the body and whenever it will be separating behind the body there will be formation of wake with eddies. Now you will see that a flow around say cylinder and a flat plate. Here in the picture it is shown cylinder and flat plate. The flow from an upstream here or here point upstream point here suppose a is stopped at point b whenever it will be flowing from this A point and it will be whenever touching at the surface at this point b you will see that that upstream point A whenever that flow will be coming from which it will be stopped at point B on the body surface with its velocity where it will be decreasing to 0.

Now here this b is called a stagnant point there will be no flow of fluid element at this surface that is why it is called stagnation point. The flow then divides into the upper and lower flows at point B. Here in the cylindrical case and here in this that flat surface. For a cylinder you will see that the flow separates at point c producing a wake with eddies here you will see that from this point the flow will be producing eddies. Eddies means that mass of fluid element will be circulating in that particular region after separating it from this cylindrical surface.

So there will be formation of eddies. There may be several eddies will be formed according

to the flow degree of flow you can say or you can say the flow rate. Whereas in the case of flat plate you will see that there will be no formation of you know eddies. So here in two cases we are getting different that results of that formation of eddies. For a cylinder here the flow separates at point c that will produce a wake with eddies.

Now important point that what will be the pressure at that stagnation point if the pressure at the uniform flow region is P_∞ and the pressure at this stagnation point P_0 you have to find out what will be the pressure at this P_0 . Whereas uniform velocity is U . So how to calculate that P_0 ? As you have already heard that what is the Bernoulli's theorem. In fluid mechanics I think you have learned about that Bernoulli's theorem that it states that the total energy will be constant. If you are considering that point a and the point b in these two points you will see that the sum of pressure energy, kinetic energy and another energy it is called potential energy will be same in both the points.

So you will see that if we consider that concept here so what will be the total energy at this point A? What will be the first you can say that pressure energy? This is basically P_∞ you can say that at this point pressure is P_∞ at point a and pressure energy and what will be the kinetic energy here at this point a? It will be ρU^2 where U is the uniform velocity at this point a.

Here the Equation

$$p_0 = p_\infty + \frac{\rho u^2}{2}$$

This is the kinetic energy plus what will be the potential energy since it is horizontal and if you are considering a datum point from some certain base then what is the distance here? This distance will be is equal to what is that certain height let it be Z_a . So here the potential energy will be is equal to what? ρg into Z_a minus 0. So this is your potential energy. The same way if we consider at point b we are having that pressure as a stagnation pressure as P_0 kinetic energy here ρU_b^2 here what will be the velocity at this point that will be 0.

So 0^2 plus P_∞ plus $\rho g Z_b$ divided by 2 plus what will be the potential energy here the same distance from this datum point. So here it will be again ρg into ZB minus 0. So ultimately we are getting here so it will be 0 and here we can say that this $\rho g ZB$ and $\rho g ZA$ both are same because here ZA will be equals to ZB since this is the horizontal flow. So in this case we are having simply after simplification P_0 will be is equal to P_∞ plus ρU^2 . So this is your stagnation pressure stagnation pressure.

So whenever that fluid will be getting obstruction at the surface and nullify at the surface and dividing into two parts and then forming eddies behind this solid surface you will see that there will be stagnation pressure at this B which can be calculated by this equation where it depends on what will be the pressure at this point A and what will be the velocity at this point A. So it depends on pressure and velocity at this uniform zone of velocity. So in

this way you can calculate what will be the pressure at stagnation point. Then we are coming to discuss about that here realization of drag and lift. Suppose any solid surface any shape it may be, maybe that wing shape maybe you know that here other type of shape here.

So at A body it will realize a force from the surrounding fluid whenever it is placed in a flow of fluid. So in this case you will see A body where some force will be applied and it will be realizing that force from that surrounding fluid by which you are applying whenever it is placed in the flow of fluid. A flat plate realizes a force only in the downstream direction when it is placed in the flow direction. However a wing realizes the force R here in this direction R you will see that as shown in the slide which is inclined to the flow as shown in the figure. So in this case in general the force R will be acting on a body that will have two components one will be in the direction of flow another will be perpendicular to the direction of the flow.

So in the direction of flow that will be represented by F_D and in the perpendicular to the flow or normal to that flow it will be represented by F_L . Since F_D is called the drag force. So drag force is basically what the force which is realized by the flow of fluid on any bodies where the force will give the component in the direction of flow that force will be regarded or referred as drag force. Whereas in the normal direction of the flow that force component will be referred to as lift force. So in any body whenever any force will be applied at an angle to a direction of that flow you will see that in the direction of flow what will be the component of that force that will be called as drag force whereas normal to that flow that force will be called as lift force.

So how that drag force and lift force will be defined here. Then if you consider suppose how to calculate the drag force let us see here suppose any object like this yellow marking as like this as shown in the picture. Suppose surface that is total area is A . Now consider that pressure of fluid acting on a given smallest area or minute area you can say dA on the body surface which is actually P that is which is called or represented or denoted by P . So on this surface the pressure will be acting which is denoted by P and the friction force per unit area will be defined as a shear stress τ .

So at this surface you will see that there will be a pressure force will be acting here P and the shear stress will be acting along the surface of the body that will be regarded as shear stress that will be denoted by τ . And the force you will see that $P dA$ due to the pressure P act normal to dA act normal to dA . So here you will see that what will be the dA will be there that on that surface normal to that surface P will be acting here. So P is P and P into dA will be regarded as force here. The force acting normal to that minute surface will be called as P into dA .

Whereas the force due to the friction stress that is τ that will be acting tangentially. So we are having here that see one is pressure force that will be acting on the surface of the

body and the force acting on that due to that pressure it will be P into dA since surface area is dA small surface area and along the small surface area or tangential to that smallest surface area is τ into dA . τ is basically shear stress that means what is the force applied tangentially per unit area that is called stress. So stress into smallest area that is dA it will be regarded as frictional or friction stress that is τ or shear stress and τ into dA it will be shear force. Then you will see that as per definition of that drag force if you are having that pressure force as $P dA$ in this direction and it will make this θ angle with the direction of flow.

Then in the direction of flow what is the component of this $P dA$ that will be P into dA into $\cos \theta$ that will be called as drag force because drag force is basically defined that force applied in the direction of flow. So here the pressure force is applied in this direction which is making θ angle the direction of flow. So in the direction of flow the component of this $P dA$ will be is equal to $P dA$ into $\cos \theta$.

Here the Equation(s)

$$\text{The drag } (F_D) \left\{ \begin{aligned} F_{D,p} &= \int_A p dA \cos \theta \\ F_{D,f} &= \int_A \tau dA \sin \theta \end{aligned} \right.$$

Whereas normal to this flow normal to this flow the component of $P dA$ the component of $P dA$ that will be your what is that another component of this $P dA$ what will be that. So that will be regarded as lift force so we are not coming here as a lift force here only that drag force.

Now see what are the two components of this pressure force one is in the direction of flow $P dA$ into $\cos \theta$ another will be normal to that flow and also in the direction of flow we are having another component from the τdA that means shear force. So shear force again it will have another component in the direction of flow that may be that τdA into $\sin \theta$ because here the θ will be here and then it will be that 90 minus θ here so it will be coming as τ into dA $\sin \theta$. So in the direction of flow due to the pressure force there are two components one will be coming from one will be coming from that pressure force and then due to that shear force another component will be coming to the direction of the flow that will be you know τdA $\sin \theta$. So in the direction of flow we are getting two parts of force one is from pressure another is from shear stress or frictional stress you can say. So the drag force will have two components one is called pressure drag another will be called as frictional drag.

Pressure drag will be represented by $F_D P$ and frictional drag will be represented by $F_D F$. So if you add these two components the total drag will be is equal to F_D and this $F_D P$ will be then what that means due to that pressure force the drag force for the whole surface area then you will get after integration for that whole surface area of this $P dA \cos \theta$ and then frictional drag it will be again for the whole surface area just by integrating $\tau dA \sin \theta$. So you are having the total drag force one will be from the pressure component another will

be from the frictional resistance. Now the drag F_D on a body is the sum of the pressure drag F_{D_P} and the friction drag F_{D_F} whose proportions vary with the shape of the body. Now you will see that the pressure drag and frictional drag F_{D_P} and F_{D_F} that will be contributed by different shapes in different way.

Now for the shape like this here flat plate you will see the pressure drag will be almost zero whereas frictional drag will be 100 percent. With this type of figure you will see that here the pressure drag will be some extent that is around 10 percent whereas this frictional drag will be around 90 percent. So most of the part will give you the contribution to the frictional drag whereas this type of cylindrical you will see that 90 percent will be that pressure drag whereas 10 percent will be the friction drag. But for this type of vertical plate you will see that around 100 percent will be the pressure drag whereas the frictional drag will be negligible almost zero. Now you will see that out of these you can express the drag force in general form.

So drag force F_D can be expressed in the general form as F_D will be equal to that will be $C_D A \rho U^2$.

Here the Equation

$$F_D = C_D A \frac{\rho U^2}{2}$$

In this case drag force will be proportional to the projected area of the body as well as the kinetic energy of the flow. So that is why we can say that F_D will be proportional to cross sectional area of body and F_D is proportional to kinetic energy of the flow. So proportionality constant will be regarded as drag coefficient like this. So this C_D will be is equal to drag coefficient.

Here A is called the projected area of the body on the plane vertical to the direction of the uniform flow. So how you will get suppose this is your body so this part this is your projected area. So projected area basically you will see that this is your projected area. So for the spherical particle projected area itself is this cross sectional area that is simple circle circular area. And for any rectangular shape you will see that the projected area is equal to rectangle like this.

So A is the projected area of the body on the plane vertical to the direction of the uniform flow. Here uniform flow like this here in this direction flow. So projected area will be normal to this flow. And C_D is the non-dimensional number which is called drag coefficient. Now here in this table for different body shape you will see that different dimension ratios and according to their what will be the projected area it is shown and their respective drag coefficient are given.

But this drag coefficient can be calculated from the correlation itself also that developed by different investigators based on that shape of the particles. Now let us do an example for

this. Suppose a fluid having density equals to 5 kg per meter cube which is flowing with speed 2 meter per second along the length of the cylinder. The diameter of the cylinder is 10 centimeter. If the drag coefficient for this case is 0.91.

91 what will be the drag force by liquid exerted on the cylinder. So what we have to do here? For this you can solve this problem. So as a solution you can say how can we solve it. So drag force what is the drag force formula that we know? F_d that will be is equal to what? $C_d A \rho u^2$ by 2. We need to know that drag coefficient we need projected area we need that velocity of the fluid and density of the fluid.

So here C_d is given 0.91 and projected area A this is called I think this is the length of the cylinder. So here this projected area will be is equal to πd^2 by 4. Projected area where diameter of the cylinder it is given d . So it will be coming as what around 10 centimeter after calculation. So here then you need to know what is the density.

Density I think it is given to you. Density is 5 kg per meter cube and also u , u is the velocity it is given 2 meter per second. So after substitution of those values so F_d will be is equal to 0.91 into A is 10 centimeter that means 0.10 you have to convert it to meter and into what is that ρ , ρ is here I think 5 kg per meter cube into what is the value of u , u is given 2 so 2 square divided by 2.

So after calculation it will be coming as 0.07 Newton like this. So here we are talking about that drag force. Now similarly the development of lift whenever fluid will be flowing over the body there will be a lift force acting on that body. So the lift force can be regarded as L . So total lift force will be again due to that pressure force and lift force due to that frictional force.

Now that if we consider again normal to the flow. In the normal to the flow what will be the component of pressure force that will be $P da \sin \theta$ and also in the normal direction of the flow what will be the component of frictional resistance that will be $\tau da \cos \theta$, θ here given so here it will be θ again here it will be what is that 90 minus θ . So accordingly you will get what will be the component of $P da$ that means your pressure force component in the normal to the flow and also the component of that τda in the normal to the flow of fluid. So here L_p can be called as lift force in the direction of normal to the flow due to that pressure that will be for the whole surface that can be obtained by integrating over the surface of the value of $P da \sin \theta$. Similarly the lift force due to the frictional resistance in the direction to the normal of that fluid flow so that will be for whole surface again by integrating of this $\tau da \cos \theta$ we can get. So here also the lift force L on a body is the sum of the pressure lift force and frictional lift force that means L_p plus L_f whose proportions vary with the shape of the body.

Here the Equation(s)

The lift (L) $L_p = \int_A p dA \sin \theta$ $L_f = \int_A \tau dA \cos \theta$ So drag for a cylinder in case of viscous fluid for low

Reynolds number is less than 1. Here Reynolds number is basically defined as $\rho u d_p / \mu$ where ρ is the density of the fluid, u is the velocity of the fluid and d_p is the particle diameter not that conduit diameter here. Here only that over which that flow is flowing that is particle diameter divided by μ viscosity of the fluid. So here Re if it is less than 1 Reynolds number this is basically what is that Reynolds number what is the significance of Reynolds number this is the ratio of inertia force to the viscous force. So in case of viscous flow behind the cylinder for very low Reynolds number that means less than 1 the streamlines come together symmetrically as at the front of the cylinder as indicated in the figure.

So here see that from that front side this tangentially that velocity component will be there and the streamlines come together here and the front side like this. But whenever Reynolds number will be higher suppose 2 to 30 then if Reynolds number is increased to this range the boundary layer separate symmetrically at position A as shown in the picture and 2 eddies will be formed which will be rotating in the opposite directions in the opposite directions like this. And behind the eddies the main streams come together and flowing like this. So here if Reynolds number will increase there will be a formation of eddies that means circulation cell behind this surface. So this happens if Reynolds number is greater than 1 that means 2 to 30.

Here in this case 2 eddies will be formed within this range of Reynolds number. Now with an increase in the Reynolds number beyond 30 what will happen 30 to 90 if it is this value the eddies elongate, eddies will elongated and periodic oscillation of the wake will be resulted and these eddies will be called as twin vortices like this. These 2 vortices here eddies will be elongated like this and then here the circulation of the fluid will be happening here and these 2 vortices will form and whenever this type of vortices or eddies will be formed it will be called twin vortices. And it is happened only if you are having that Reynolds number within this range. Now if you Reynolds number beyond 90 you will see that the Reynolds number will be very high so in that case eddies are continuously shed alternatively you will see that here it will be shedding like this alternatively it will be shedding from the 2 sides of the cylinder.

So this will happen and there are n number of vortices will form. Now if your Reynolds number above 100 but less than 10^5 the separation occurs near 80 degree from the front stagnation point here. Here this is the front stagnation point and the separation as shown in here in the picture the separation will be happening from this region the fluid streams will be separated here from this region and this angle will be 80 degree. So the separation of the layer fluid layer will occurs near 80 degree from the front stagnation point and this arrangement of vortices is called that Karman vortex street. So this way from this there will be a separation and this wake will be forming like this.

There will be a formation of wake like this. So this type of wake formation and making a you will see that there is a street like that wake will be forming in this way and like a street and that wake will be elongated and this arrangement of these vortices is called the Karman vortex street. So laminar boundary point before separation you will see that it will be less than 80 degree from this stagnation point and separation will not happen but whenever this angle will be beyond that 80 degree the separation will occur and it will happen only the Reynolds number within this range. And we will see that there will be a certain Reynolds number whose value is 3.8×10^5 this Reynolds number will be called as critical Reynolds number and at this critical Reynolds number boundary layer becomes turbulent. So before that the boundary layer will be acting as simply that streamline or you can say that there will be a laminar flow.

But beyond this critical value of 3.8×10^5 you will see that the boundary layer becomes turbulent and once this turbulent boundary layer will reach that it will becomes turbulent the separation position will be moving further downstream the separation point will not be here at this region the separation point will be coming further beyond here beyond this point so it will be coming from this point. So it will make 130 degree here from this stagnation point. So for a rear half of the cylinder just like the case of a divergent pipe the flow gradually decelerates with the velocity gradient which will be reaching zero and this point is called the separation point and downstream flow get reversal and developing eddies here. You will see that from this point they will be getting turbulent and they will be moving reverse direction whenever they are getting that turbulent movement in this way. And the fluid particles seen and around the boundary layer will be mixing with each other by the mixing action of the turbulent flow.

So this is the case that at the turbulent point what will be the separation point what is the degree of that separation point will be making or angle that will be making from that stagnation point. So that will be 130. Now let us do an example of using the drag force again. It is said that a cylindrical chimney 1 meter in diameter and 25 meter tall is exposed to a uniform 50 kilometer per hour wind at standard atmospheric conditions.

End effect and gust may be neglected. In this case estimate the bending moment at the base of the chimney due to wind forces. The drag coefficient is to be assumed as 0.35. So here basically a chimney whose diameter is given 1 meter and height is given 25 meter. So in this case they told that estimate the bending moment at the base of the chimney when the fluid or wind fluid as here wind will be flowing at a velocity 50 kilometer per hour.

So you will see what then we can have here. First of all we have to calculate what will be the drag force. Then we can calculate we will be able to calculate what will be the bending moment because to calculate the moment you have to have that force. Here this force will be as a drag force. So how to calculate the drag force? So here drag force is basically what? Drag force is equal to F_D that means what is the formula for the drag force that is $C_D \times A \times \rho U^2$ by 2.

Here CD value is given 0.35 and A is the projectional area. What is that projected area here? This will be cylindrical of course. So see here whether it will be that πD^2 square by 4 or not. So this projected area since this is a chimney so projected area is basically that rectangle. So projected area will be is equal to what here 1×25 . One is the diameter and height is the 25 meter and into rho, rho is what density of the wind let it be 1.

2 since wind here and U square is 50 kilometer per hour it is given. So it will be 50 kilometer means here 10 to the power 3 you have to multiply it by thousands and then you have to convert it to second then 3600 this will be your meter per second square divided by 2. So ultimately it will be coming as what is that FD will be is equal to what. And then bending moment can be calculated as M. So M will be is equal to FD into what will be the average length of this chimney that will be considered here that means L by 2. That means here after substitution of this FD value and L by 2 value you can have maybe 13.

0 or else you please calculate it. So in this way you can solve this problem. And if you consider the drag coefficient for cylinders and other column shape bodies that you can get it from this graph here the profile as per that different shape is given for that CD value with respect to Reynolds number. So when Reynolds number will be is equal to 10 to the power 3 to 2 into 10 to the power 5 the CD value will be within a range of 1 to 1.2 or a roughly constant value.

But when Reynolds number is the critical one that is 3.8 into 10 to the power 5 or so the CD value suddenly will be decreasing to 0.3. This is due to the fact that the location of the separation point suddenly changes as it recess to this Reynolds number. Then coming to that drag of a sphere. Now if we consider that there will be a slow flow if Reynolds number is less than 0.

2 around a sphere instead of cylindrical shape or other shape we are considering a spherical shape. And if that slow flow around a sphere is known to you that means where Reynolds number is less than 0.2 this slow flow is called Stokes flow where Reynolds number is less than 0.2. From the Navier-Stokes equation and the continuity equation generally this drag force can be derived at this low flow that will be defined as already you have learned I think earlier that drag force will be is equal to $3 \pi \mu U D$.

Here if we consider the general equation here as $CD \times A \rho U^2$ then in this case that means for the Stokes flow the CD value will be is equal to what $24 \text{ by } Re$. That means if you substitute the value of CD as $24 \text{ by } Re$ here then you will get this FD value is equal to $3 \pi \mu U D$. So for this Stokes flow the CD value will be is equal to $24 \text{ by } Re$ value. So this is known as that Stokes equation that FD will be is equal to $3 \pi \mu U D$ or FD is equal to $CD A \rho U^2$ where CD will be is equal to $24 \text{ by } Re$.

Here the Equation(s)

$$F_D = 3\pi\mu Ud = C_D A \frac{\rho U^2}{2} C_D = \frac{24}{Re}$$

Let us do an example for this problem again that in Stokes regime or Stokes flow condition the drag force on a sphere of a diameter 10 millimeter that will be moving with a velocity 5 meter per second in a fluid having viscosity 0.

0.004 Pascal second. What is the value of drag force? So how will you solve this problem? So in this case again that you can easily solve this problem just by that formula of that F_D will be is equal to what $3\pi\mu D U$. Here μ is given to you that means 0.004 Pascal second and D is given to you diameter of the sphere it is given to you that means what is the diameter is given 0.01 meter and U is given to you U is given here 5 meter per second. Now putting these values in this equation of this F_D here we can have this F_D will be is equal to 1.

1.88 into 10 to the power minus 3 Newton. So this way you can easily calculate. Other way you can calculate also first you have to find out what will be the C_D value that is 24 by Re as per the since it is Stokes regimes so you have 24 by Re , Re means $\rho U D$ by μ then what will be the value it will be coming as per Reynolds number will be less than 0.2 so within that region C_D value will get after getting C_D value you can calculate it from this equation again that F_D is equal to $C_D A$ into ρU^2 by 2. So in that case C_D value you can substitute here and A value projected area again πD^2 square by 4 and then ρ then U^2 square by 2 after substitution of these values you can get the same value of F_D will be is equal to 1.88 into 10 to the power minus 3 Newton. Another problem this problem is given in GATE 2015 GATE examination we are told that a spherical solid particle of 1 millimeter diameter is falling with a downward velocity of 1.

1.7 millimeter per second through a liquid whose viscosity is 0.04 Pascal second at a low Reynolds number that means Stokes regime the liquid is flowing upward at a velocity of 1 millimeter per second. All velocities are with respect to a stationary reference frame neglecting the effects what will be the drag force per unit cross sectional area of the projected area of the particle in Pascal. So here the particle is flowing downward at a velocity that means 1.7 meter per second whereas this flowing through a fluid whereas this fluid velocity will be upward at 1 millimeter per second 1 millimeter per second and particle is falling 1.

1.7 here it is 1.7 millimeter per second. So in this case what will be the drag force per unit projected area that you have to calculate. So we know that F_D is equal to here $C_D A \rho u^2$ square by 2 that means you have to find out F_D by A that will be is equal to $C_D \rho u^2$ square by 2 this you have to find out. First of all you have to calculate what will be the C_D value since it is a Stokes regime 24 by Re that means you can say 24 by Re means what $\rho u d$ by μ . that what is the velocity here. So velocity see here effective velocity you have to consider here since solid particle is flowing downward whereas liquid is flowing upward.

So in this case the effective velocity will be is equal to what? The effective velocity u

effective that will be is equal to what? Downward 1.7 minus of fluid velocity that will be upward direction you have to consider in downward direction itself and it will be minus of 1 here that will be is equal to 2.7 this will be millimeter per second. So therefore we can say that F_D by A that will be is equal to what? Here after substitution of C_D value here F_D by A will be is equal to C_D means here $24 \rho u d_p$ by μ into ρu^2 by 2 then it will be coming as $12 \mu u^2$ by d_p .

D_p means particle diameter. So here u will be is equal to effective velocity after substitution you can have then $2L$ into μ value is given I think 0.00 what is that value it is given 0.04 and here u is given what is that 2.7 this is millimeter per second you have to consider into meter per second then you have to divide it 10 to the power then minus into multiply into 10 to the power minus 3 then square divided by d_p what is that d_p value this is given in I think 1 millimeter d_p is given 1 millimeter so you have to convert it to meter then 1 into 10 to the power minus 3.

So finally its value will come 1.296 Pascal so this is your answer. So in this way you have to calculate what will be the drag force if any solid body is falling through a liquid where liquid will have the upward velocity of certain value. In this case you have to consider the effective velocity of that particle. Then another problem what will be the drag coefficient of a bacterium moving in a water at 1 millimeter per second assume the size of the bacterium to be 1 micron and kinematic viscosity of the water to be 10^{-6} meter square per second. This problem is given in GATE examination 2002.

So here again that you have to find out what will be the value of drag coefficient. So drag coefficient let us see here first what is the values are given according to the problem. Here μ by ρ this is called kinematic viscosity. So it is given 10^{-6} meter square per second and U value is given 1 millimeter per second that means 10^{-3} meter per second and diameter of that bacterium it is given 1 micron that means 10^{-6} meter. Then what will be the Re value Reynolds number of this particle that will be $\rho U D$ by μ that means $U D$ by μ by ρ we can write like this. So here U is given I think 10^{-3} what is the U value 10^{-3} into D , D value is given 10^{-6} and by μ by U is also 10^{-6} .

So it is coming ultimately what is that 10^{-3} that means 0.001 which is less than 0.2 that we can say that it is a Stokes flow condition. So at the Stokes flow condition C_D will be is equal to 24 by Re .

So what is that Re value that 24 by Re value we have calculated here 0.001. So its value is coming 24000. So this is your final value of C_D . Again if you drag coefficient of the sphere will have that range other than that Stokes flow what will happen? For the flow if Reynolds number will be in the range of 0.225 around a sphere then you know there will be a modification of Stokes equation by Oisín by partly taking account the effect of inertia terms and he found that C_D will be is equal to 24 by Re into $1 + \frac{3}{16 Re}$ and the drag force

accordingly it will be like this or F_D will be is equal to $24 \text{ Re} + 1 + \frac{3}{16} \text{ Re}$ into a into ρU^2 by 2.

Here the Equation(s)

$$C_D = \frac{24}{\text{Re}} \left[1 + \frac{3}{16} \text{Re} \right]$$

$$F_D = C_D A \frac{\rho U^2}{2}$$

$F_D = \frac{24}{\text{Re}} \left[1 + \frac{3}{16} \text{Re} \right] A \frac{\rho U^2}{2}$ So here C_D value will be within this range of 0.225. Again if your range will be

Cilar and Norman they have given that drag coefficient and they have suggested one correlation which is actually giving the well predicted value of the drag coefficient from their experimental data and they have suggested this correlation to predict that drag coefficient within this range of Reynolds number.

Here the Equation

$$C_D = \frac{24}{\text{Re}} \left[1 + \frac{3}{20} \text{Re} \right]$$

So you have to use this correlation for the drag coefficient if your Reynolds number is coming within this range and other than this range that means above 1000 that means within the range of 10 to the power 3 to 2 into 10 to the power 5 the resistance is proportional to the square of the velocity and then drag coefficient will be almost constant it will be regarded as 0.44. As Reynolds number reaches 3 into 10 to the power 5 and so the boundary layer changes from the laminar flow to the turbulent flow and their separation will happen according to that this critical Reynolds number will give that respective value of C_D and it will be decreasing to 0.

1 or less. And on reaching that higher Reynolds number the C_D value again gradually approaches to 0.2 as per this diagram. Another example here it is given as per that correlation how to calculate that drag force. Now here what will be the drag force on a sphere of diameter 5 millimeter moving with a velocity 6 meter per second in a fluid of density 2 kg per meter cube if the value of Reynolds number is equal to 1. So in this case the Reynolds number range is 0.

2 to 2 to 5 that means here Re is equal to 1 it is given. So if this Reynolds number range is coming in this range then we can use this correlation here the equation number 9 to calculate the C_D value. So C_D will be is equal to what $24 \text{ Re} + 1 + \frac{3}{16} \text{Re}$ into Re this will be C_D so accordingly that F_D will be coming so if you substitute the value of Re as 1 so here C_D value will be equal to $1 + \frac{3}{16} + 1$ it will be coming so it will be coming as $24 + 19 + 16$. So 57 by here I think to this value. So accordingly that F_D

value will be is equal to C_D into A into ρu^2 by 2 after substitution of those values you will get that F_D value is equal to 0.0201 Newton.

So this way so drag coefficient all about this for spherical particle and then cylindrical particle shape whereas if you are having that non spherical particle then you can easily calculate the drag coefficient or you can have the drag coefficient with respect to Reynolds number from this chart. Where this these are the profile of this drag coefficient for different Reynolds number and also based on that sphericity value of different shape of body for ψ it is basically that sphericity if it is 1 that means spherical shape body so you can have this drag coefficient from this data from this graph you can easily calculate or read what will be the drag coefficient. Similarly for other shape of particles that you can have this drag coefficient. Here again you will see that if you are having that problem of that other than spherical particle then you can easily calculate from that C_D value from the chart. Now one example is given here like this that an aeroplane passenger weight of 100 kg comes down to the around from an aeroplane with the help of parachute against the resistance of air the shape of the parachute is hemispherical of 2 meter diameter find the velocity of the parachute with which it comes down the drag coefficient C_D is equal to 0.

5 the density of the air is given viscosity of the air is given here. So you have to calculate what will be the velocity of the parachute. So basically here first of all you have to find out the what will be the projected area projected area A that will be is equal to π by 4 into 200 square what is that 200 square the diameter it is given hemispherical shape of that it is given 2 meter so here 200 centimeter it is given so it is coming as 3.14 into 10 to the power 4 centimeter square and then you will see that you have to calculate the F_D value that is $C_D A$ into ρu^2 by 2. Now you will see that the u of the parachute will be then calculated after substitution of this F_D value. Now what will be that F_D value this F_D value of course will be equals to the weight of this parachute weight of this parachute.

So this F_D must be equal to the weight of the man with that parachute falls down. So from this you can have after substitution of this F_D value and C_D value A value and ρ by 2 then solving this u that will be coming as around 31600 centimeter per second. You try once I think you will be having this value after substitution of all those values there C_D value is given here 0.5 projected area that already here and then ρ density of the fluid also given and then accordingly what will be the u value.

So it will be 31600 centimeter per second or you can say that 31.6 meter per second. So in this way you can calculate for the drag force drag coefficient for different type of body and accordingly you can have that values of lift force also if other parameters are given to you. So I think this lecture we have discussed what is the drag force how it is defined conceptually how that lift force is defined drag force is defined how to calculate what will be the general terms of the drag force what are the different shape of the particles and their respective projected area and from that projected area what will be the that drag force how to calculate even what will be the drag coefficient within a different range of Reynolds number. So I think you understood this and you go through once again if you have any doubt

you can contact with this email.

So in the next lecture we will try to discuss about the terminal velocity. So thank you have a nice day. Thank you.