Computer Aided Applied Single Objective Optimization
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Lecture - 01
Introduction to Optimization

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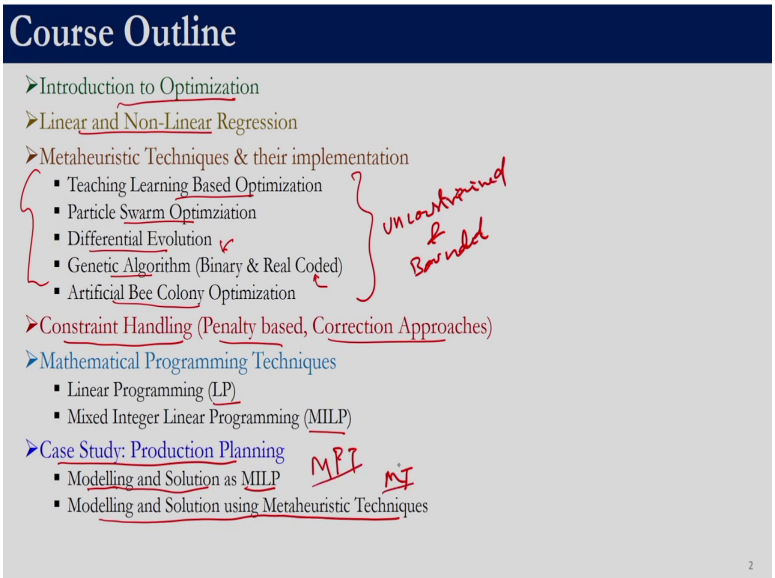

Welcome, this is going to be our first session. So, in this we will be doing Introduction to Optimization. We will follow this up with a linear and non-linear regression, right. So, in this we will be basically doing that given a set of data points and a model; how do we find out the model coefficients, right. Then we will be discussing this five metaheuristic techniques teaching; learning based optimization, particle swarm optimization, differential evolution, genetic algorithm, and artificial bee colony optimization, right.

So, in genetic algorithm, we will be discussing both binary and real coded g a. So, we will not only be discussing this meta heuristic techniques; we will also implement them in MATLAB, right. So, we will be using MATLAB; once you understand the techniques, you are free to implement it on whichever programming language you are comfortable with. So, the assignments and quiz will not involve any coding exercise, right.

So it is not necessary for you to know MATLAB or learn MATLAB to be able to do the assignments or give the exam; but we will be doing considerable portion of this course in MATLAB, right. And we will also introduce you to a couple of other softwares which we will look into, right. So, we will also be looking at constraint handling. So, initially when we discuss these problems, we will be, initially when we discuss this meta heuristic techniques we will be looking at only at unconstraint problems; unconstrained and bounded problems, right.

So, we will assume that there are no constraints and then we will discuss these techniques. Then we will let you, then we will discuss how do we handle constraints; in this meta heuristic techniques if the problem has constraints, how do we handle the constraints using this meta heuristic techniques. So, broadly we will be looking into penalty based and correction approaches, right.

So once you know penalty based constraint handling or correction approach for constraint handling, you can use that with all of this five techniques to solve any constrained optimization problem, right. We will also be looking at two mathematical programming techniques; linear programming and mixed integer linear programming, right. So, we are also bringing in this mathematical programming techniques into this, so that you can actually also understand the limitations of meta heuristic techniques, right.

So, given an optimization problem, there are multiple ways to solve it, right. So, we should be in a position to select the best possible set of techniques that can help us to solve the problem. Then we will be looking at a production planning case study, right. So, in this we will take one problem and we will model it as a MILP, right. We will model and solve it as an MILP, also we will model and solve it using meta heuristic techniques.

So, the reason for doing this is, you will be able to understand the strength and weakness of both mathematical programming techniques and meta heuristic techniques, right. So, we will take a problem which you can consider it to be a real life optimization problem; and then we show how do we solve it with both mathematical programming as well as meta heuristic techniques, so that you can understand the strength and weakness of both of them.

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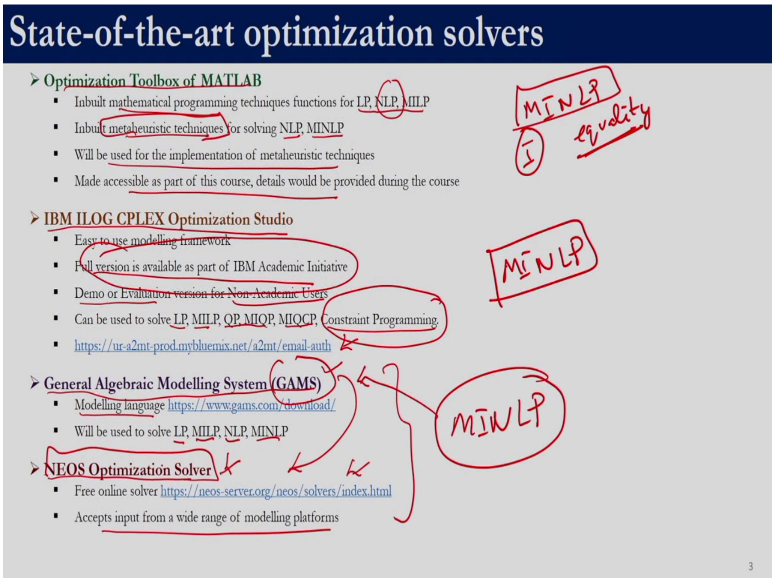

As part of this course we will be introducing you to three different state of the optimization solvers, right; one is optimization toolbox of MATLAB. We will be looking at IBM ILOG CPLEX optimization studio, we will be looking at general algebraic modeling system GAMS, right and we will also look at NEOS optimization solver, right.

So, both of this we will be using together, right. So, optimization toolbox of MATLAB, it has inbuilt mathematical programming techniques; for linear programming, non-linear programming and mixed integer linear programming. It has inbuilt meta heuristic techniques which can solve non-linear programming, for which mathematical programming techniques are also available and it can solve certain versions of mixed integer non-linear programming.

Also we will be using MATLAB, right for implementing the meta heuristic techniques that we will discuss. IBM, ILOG, CPLEX optimization; the full version is available as part of IBM academic initiative, right. So, non academic users can use either the demo or evaluation version, right. So, IBM, ILOG, CPLEX optimization studio can be used for solving linear programming, mixed integer linear programming, quadratic programming and mixed integer quadratic programming, mixed integer quadratic constrained program constrained programming as well as constrained programming, all right.

So you can go to this link and you will have to register right and then you will be able to freely download that software, ok. So, GAMS is primarily a modeling language, right. So, very new merely state what is the problem right and then there are a set of solvers which can be used to solve it, right. So, the solvers which come with the free, with the demo version of GAMS is limited right; like it cannot solve problems of very large dimension.

So, what we will do is, we will code the problem in GAMS right and then we have this NEOS solver; which can accept the GAMS files all right and then it can solve the problem for us. Once the problem is solved, we will be getting the results; we can get the results over email or it will even be displayed on your browser where in you submit the when you submit the problem, right.

So GAMS can be used for linear programming, mixed integer linear programming, non-linear programming and mixed integer non-linear programming, right. And this NEOS optimization solver, we will be using it only for GAMS; but if you go and have a look at this website, NEOS server dot org, you will see that it accepts input from a wide range of modeling platforms not only from GAMS, right.

So, the reason for taking three different solvers and not restricting to one solver is, each of the solver has their own limitation, right. So, for example, optimization toolbox of MATLAB cannot solve mixed integer non-linear programming when equality constraints are involved, right. So, when equality constraints are involved, it does not support integer variables; there is no function as of now that is in 2019 which can solve a proper MINLP problem.

The good thing about MATLAB is programming is much easier in MATLAB, right. So that is why we are going to use it to code the meta heuristic techniques in MATLAB. This IBM, ILOG, CPLEX optimization studio again cannot solve MINLP problems, right; a generic MINLP problem it cannot solve. But the good thing about this particular software is it is full version is available as part of IBM academic initiative plus this constraint programming is available which is not there in MATLAB, right.

Similarly the good thing about GAMS is no matter what is the problem size, we will be able to use it to model it and we can solve it using the freely available NEOS optimization solver. So, this can solve MINLP also right; the drawback of GAMS is that the demo version does not solve bigger larger problems. So, we overcome that limitation with this NEOS optimization solver, ok. So, each of the software has their own set of benefits and drawbacks, right.

So we are looking into the best possible set of tools, so that we can solve most of the optimization problems that are encountered in real life. In this session we will be looking into what are the components of an optimization problem and also how do we classify our optimization problems and also into how we classify optimization techniques.

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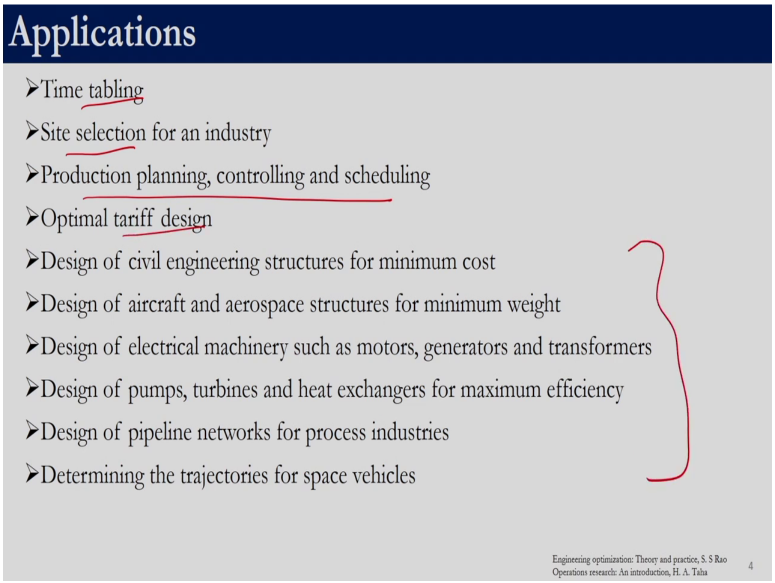

Some of the common applications of optimization are timetabling, right. So, for example, our timetabling wherein we need to schedule the classes right, we need to decide which course would be taught in which class at what time, right. So, that is a classical application of timetabling.

Another common application is site selection, right. So, if you want to establish an industry, you have lot of options right to establish the industry. So, you need to decide where you want to establish the industry. So, depending upon the industry, a large number of factors will play a role into how we decide the optimal location of an industry. Optimization is also widely used in production planning, controlling and scheduling; it is used in tariff design and these are some engineering applications of optimization.

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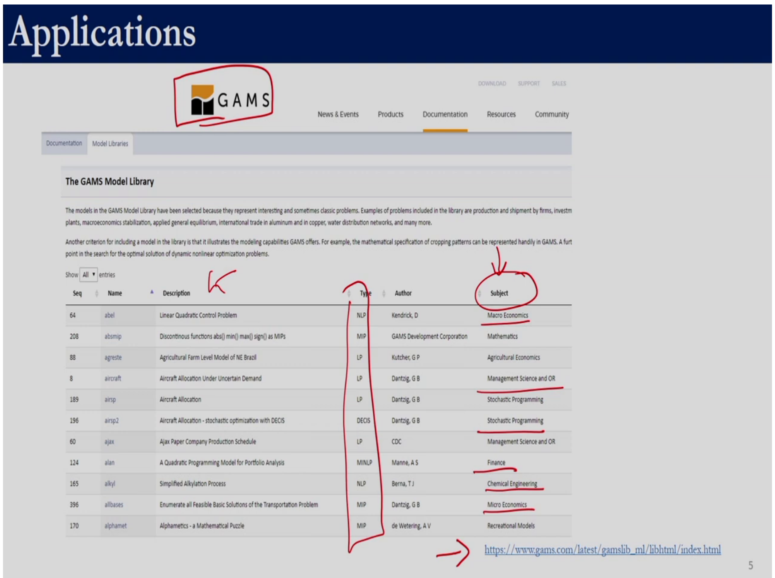

You can also look into various other applications, right. So, if you go to this website, this is a GAMS website, right. So, this is one of the software that we will be learning as part of this course, right. So, it has a collection of problems right; depending upon the subject, you can look into the problems depending upon the subject. So, right here if we see it has been used in macroeconomics, management science and OR, finance, stochastic programming, microeconomics chemical engineering, right.

So, this is just a very small subset of problems which we are showing over here, right. So, if you go into this; if you go to this website, you will be able to see a large collection of problems and you will be able to see that it is used in various areas, right. So all these files are freely available; so you can even sort them as per the type of problem right and this briefly describes what is the problem.

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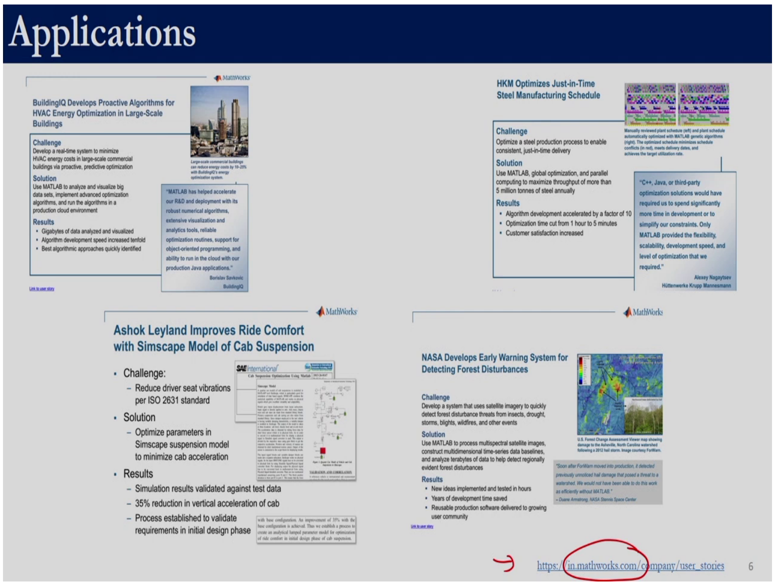

So, these are some commercial success story reported by mathworks. So, mathworks is the company which owns MATLAB, right. So, here we have shown four commercial applications. If you are interested you can go and have a look at the individual story and see how it has been applied in that particular industry.

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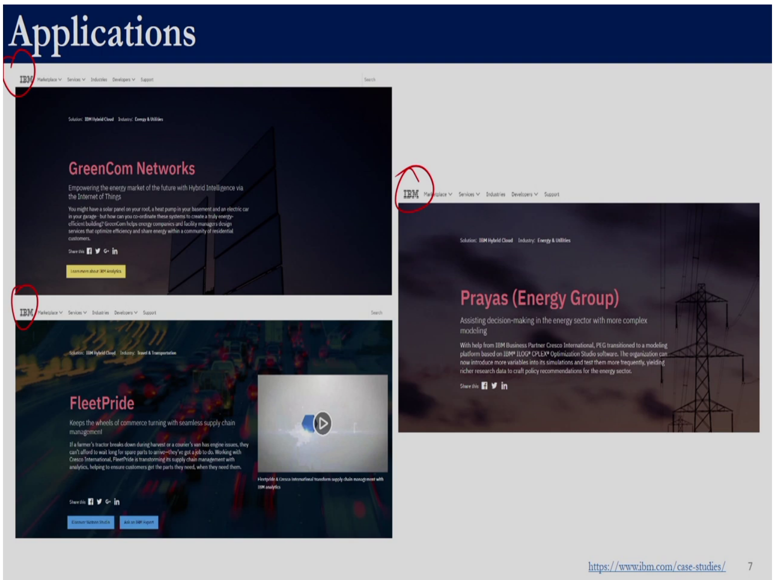

These are some of the applications from IBM; here we are only here we are restricting ourselves with the success stories of the three softwares which we are going to study, but optimization has been used in many commercial applications apart from this.

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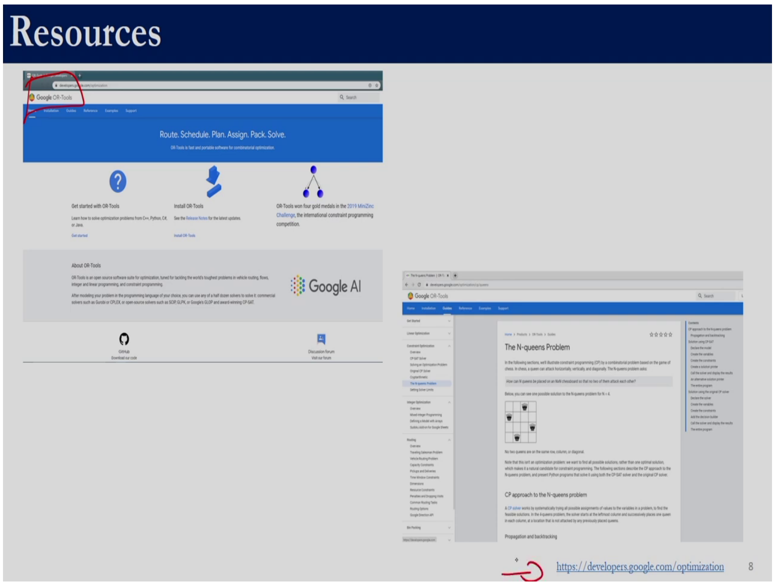

So you can also get some resources from this Google OR tools. So, Google has this web page, right; so wherein they have given a lot of OR tools which are freely available. In the last three to four slides we just showed you the commercial applications right, where the tools which we are going to discuss as part of this course have been used.

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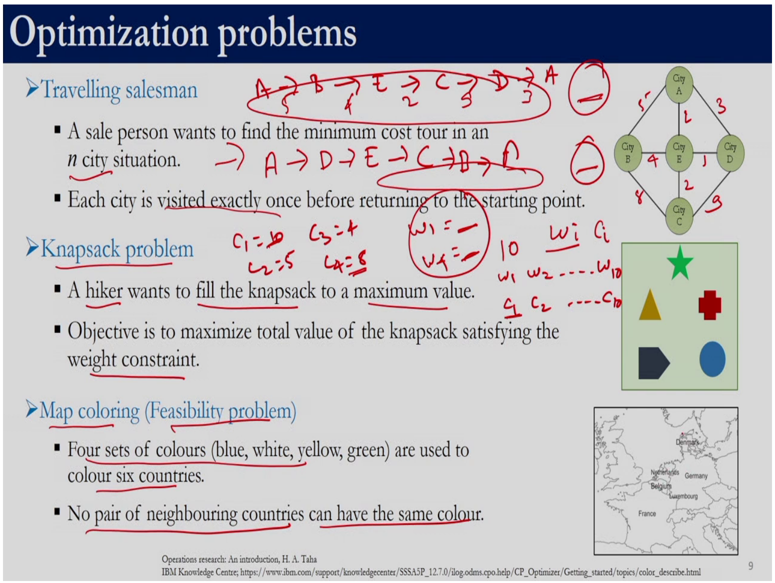

So, now let us look into some of the interesting optimization problems that are widely stated, right. So, this traveling salesman problem; a traveling person has to tour n cities. So, in this case; so for example, let us considered this case where in there are E cities. So, there are five cities city A, B, C, D, E right and there is a cost associated with each city. So, if we travel from city A to city B the cost is 5; if we travel from city A to city E the cost is 2; if we travel from city A to city D the cost is 3.

So if you are traveling from city E to city D the cost is let us say 1; this is let us say 4, this is let us say 8 and this is let us say 9 and this let us say it is 2, right. So, this is the cost of traveling in between two cities. So, the task is, the traveling salesman has to visit all the cities right and come back to the city that started with and each city is to be visited exactly once.

So, one of the solution can be from city A to city B, city B to city E, city E to city C and city C to city D and returned back to city A; we know the costs associated with each tour. So, A to B we know the cost is 5; B to E the cost is 4; E to C the cost is 2; C to D the cost is 9; D to A right D to A the cost is 3, right. So, the total cost is the summation of all this, right.

Similarly another tool can be, instead of this person going to city B let us say he decides to go to city D right, and then to city E, then to city C, then to city B and then he comes back to city A. So, again there is a cost associated with each of this, right. So, again we can calculate what is the total cost, right. So, there is a different cost over here, there is a different cost over here; so depending upon the route the person takes, the cost is going to be different.

So, the task here is to find out this path; that which path is to be chosen such that the total costs required to tour all the cities without repeating any of them and again come back to the original starting point that cost has to be minimum. So, this is a classical problem, it is known as traveling salesman problem, right. So, there are various ways to solve this traveling salesman problem, right.

So, another problem is now what is called as knapsack problem. So, in this knapsack problem we have a hiker right, who wants to fill the knapsack to a maximum value. So, let us say there are 10 items, each item is associated with the weight and it has a cost, right. So, I have 10 items; i denotes the index w 1, w 2 all the way up to w 10 and then we have c 1, c 2 all the way up to c 10. So, c indicates the cost, right.

So, now the person wants to fill as many items as possible in the bag, right and there is a weight constraint; that the total weight of the knapsack has to be less than a given value, right. And for whatever value whatever item he or she chooses to place in the knapsack, we can calculate the total value that he is carrying. Let us say the we have c 1 is equal to 10, c 2 equal to 5, c 3 is equal to 4, c 4 is equal to 8; let us say he decides to carry just c 1 and c 4.

So, the value that he is carrying is only 18; 10 plus 8, 18. And corresponding to the first item there is also going to be a weight and for the fourth item there is going to be a weight. So, the weight that is carrying going to carry is w 1 plus w 4, right. So, that total weight has to be less than what is prescribed, right. So, now the task is which of the items can be chosen, so that the weight constraint is respected as well as the person is able to carry the maximum value with himself or herself. So that is known as knapsack problem.

Both traveling salesman and knapsack problem are optimization problem. There is something called us feasibility problem. So, in feasibility problem, we do not have an objective that has to be minimized or maximized right; but we have a set of constraints that need to be satisfied, right. So, map coloring is one such classical problem. So, here there are six countries right; as shown over here there are six countries Netherland, Belgium, Luxembourg, Germany, Denmark, and France.

So, the task is to color each country such that no two countries which are neighbors are colored with the same color, right. So, we have four sets of color blue, white, yellow, green. So, each of the six city is to be colored by one of these four colors right and the constraint is that no pair of neighboring countries should have the same color, right.

So here if you see there is no objective which has to be minimized or maximized; but we have a set of constraints that have to be satisfied. So, the color of Denmark and Germany cannot be the same; because they are neighboring. The color of Germany Netherlands or Germany-Belgium, Germany-Luxembourg cannot be same as they have they are neighbors, right. Similarly, Netherlands cannot have the same color as Belgium, Germany or Denmark, right.

So, this is a feasibility problem. So, here we need to find a set of decision variables; as long as it satisfies the constraints which are given, it is sufficient, there is no objective which is to be maximized or minimized. So, feasibility problems are a subset of optimization problem. So, if we solve an optimization problem; the solution that which we get will be feasible, right. But the reverse is not true; the solution of a feasibility problem will be feasible for an optimization problem, but it need not be the best solution.

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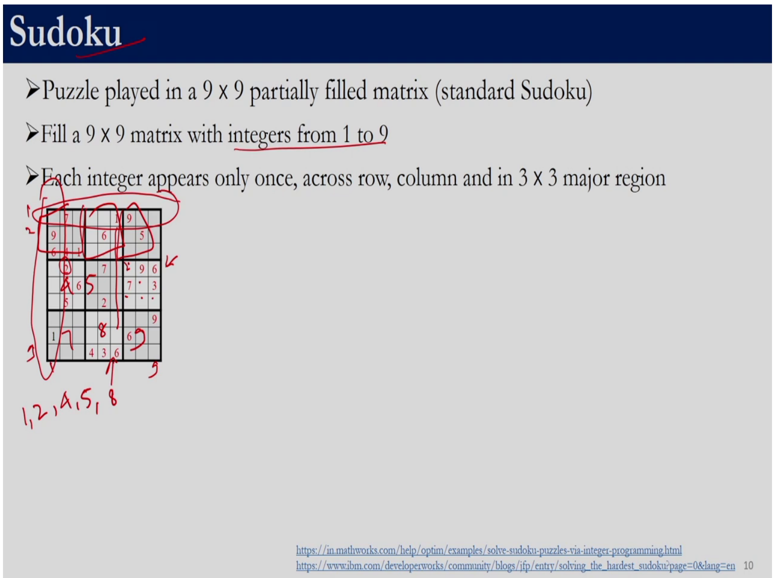

Another classical feasibility problem is Doduko problem; many of you would have solved the Sudoku problem. So, in the Sudoku problem, we have 81 squares; there are 9 rows and 9 columns, right; of 1 to 9 rows and 1 to 9 columns. So, there are a total of 81 cells, right. So, each cell is to be filled with integers from 1 to 9 right; remember 0 is not allowed, 1 right.

So, each row has to be filled with the numbers any number from 1 to 9, such that no two numbers in the same column or no two numbers in the same row are identical, right. So, for example, this is already filled with 9. So, I cannot use 9 to fill any other empty cell in row 1, right.

Similarly, this column if you see, this particular column it already has a 6 and 1. So the rest of the cells which we have over here, can take any value from 1 to 9 except 1 and 6, right. So, that is the constraint that, no two values in any row should be identical and no two values in any column should be identical, right. And there are these 9 squares. So, here if you see this is the 1st square, 2nd square, 3rd, 4th, 5th, 6th, 7th, 8th and 9th, right.

So, each of the square is going to have 9 cells and the values in the 9 in these 9 cells need to be unique. So, for example, this cell if you see, right. So, there are 9 cells and we have already used 3, 6, 9, 7, right. So, the remaining values are 1, 2, 4, 5, 8, right 1, 2, 3, 4, 5, 6, 7 is already there 8, 9.

So these five boxes 1, 2, 3, 4, 5 have to be filled with these values. Again the row and column constraints need to be respected, right. So, for example, 2 we cannot fill this box with 2, because 2 is already there in that row, right. So, this is Sudoku problem; there is no objective as such over here, there are only constraints. So, a Sudoko problem can have more than one solution, right.

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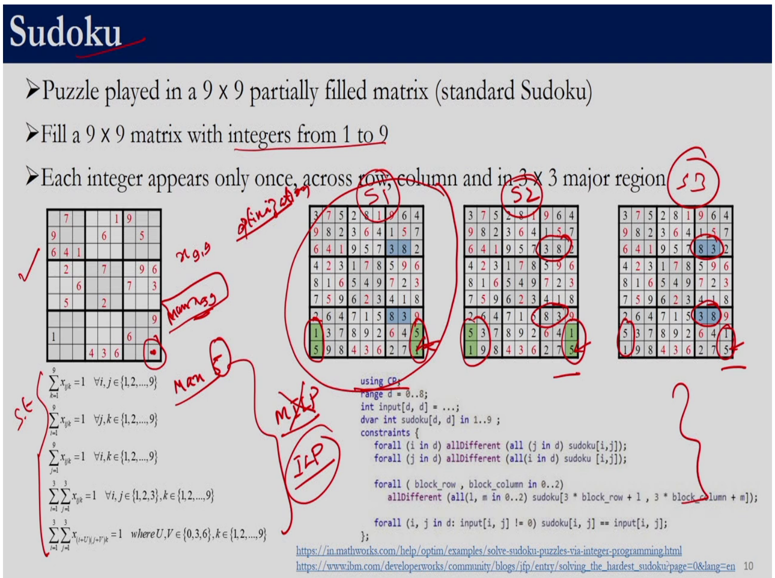

So, for example, for this problem there are three solutions, right. So, all these three solutions are unique right. So, consider these two solutions here it is 1, 5; here it is 5, 1 and again here it is 5, 1 and here it is 1, 5 right. So, these two solutions are unique solutions, right. So, both of these solutions are equally good for this feasibility problem, right.

Similarly, if we compare the solution 3, if we call this a solution 3 and if we call this a solution 2, right. So here if we see this is also 5, 1, this is also 5, 1, this is also 1, 5, this is also 1, 5 right; but the change happens here. So, here it is 3, 8, here it is 8, 3 and this is 8 3 over here and this is 3 8 over here, right. So, all these three solutions S 1, S 2, S 3 are feasible solutions for this Sudoku problem right; however, any feasibility problem can be converted into an optimization problem, right.

Right now I can say that I want to fill this box right, satisfying all the constraints which we discussed so far and we want this value to be maximum, right. So, let us say if this box is represented since there are 81 cells right; let us say I this box is represented by 9 comma 9 or x 99, this is the name of a variable right. So if I denote the value filled; if I denote the cell as x 99, right, so it can take either the value of 1 over here as shown over here, 5 as shown over here or 5 as shown over here, right.

Then this solution is not optimal right because we wanted to maximize this value right; where as these two solutions are equally good for this optimization problem. Remember first we discussed feasibility problem; feasibility problem we had to obey only the constraints, right. So, all the solutions all the three solutions are equally good; then we converted into an optimization problem. In optimization problem we said that, all the constraints need to be satisfied; on top of that we said that the last box right denoted by x 9, 9 should have the maximum possible value, right.

Once we say that, then S 1 is no longer the optimal solution; because it has a value 1, whereas solution 2 and solution 3 have a value of 5. So we would not prefer solution 1, if we had an objective maximize x 9, 9 ok; whereas solution 2 and solution 3 are equally good solution even for that optimization problem. So, that is the difference between a feasibility problem and an optimization problem. Now we just saw that this problem can be converted into an optimization problem, right.

So, here what we have are two approaches, right. So, this is one approach one and this is approach two; this is an mixed integer linear programming formulation, right. So, this is an optimization problem; here we can either add a objective function right, that one of the variable has to be maximum or minimum or we can just say maximize 5, right. So; obviously, this does not make sense because this is a scalar it is going to be 5; but these are constraints, right.

So, this problem can be mathematically formulated into a mixed integer linear programming problem. In fact, this is actually ILP, it is not even mixed integer linear programming, it is just integer linear programming; because all the variables will take only integer values, right. So, that is going to be one approach right; wherein this whatever we discussed can be mathematically translated into these equations, right.

So, that is the modeling part; any optimization problem is going to have a first a modeling part right. Once we have the model, it is only then we apply any optimization techniques to solve. So, this is one approach; wherein the Sudoku problem is converted into a integer linear programming and then solved. This is another approach right called as constraint programming. We will not be discussing constraint programming as part of this course right; but we thought since this is an introduction lecture, we will at least let you know that there are other techniques also, right.

So CP stands for constraint programming and the Sudoku problem is much easier to model for constraint programming, right. So, you do not need to worry about how did these equations come as of now, right. When we are discussing a mixed integer linear programming, we will let you know how these equations were arrived at, right. So, right now from the slide you just need to understand that, there are feasibility problems, there are optimization problem.

Any feasibility problem can be converted into an optimization problem, right. So, if we know how to solve an optimization problem, we can solve feasibility problems, right.

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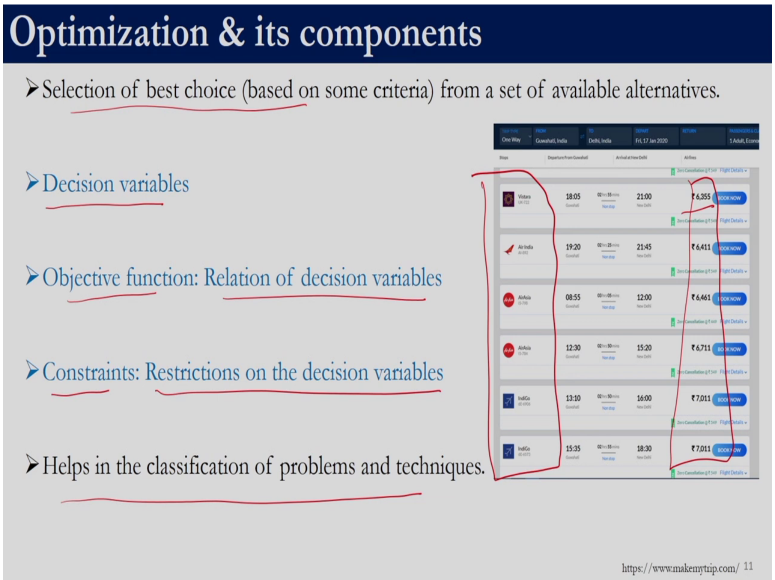

So, now let us formally look into the optimization and its components. So, optimization as you know, it is the selection of best choice right; when we say best choice, it has to be based on some criteria. If we are going to say select the best choice that invariably means that, we have a set of alternatives from which we have to select the best choice, right.

So that decision is called as decision variables. So, we need to decide something. So, in optimization we will be calling it as a decision variable, right. Objective function is a relation of decision variables, which is what we want to either minimize or maximize or in general we say optimize. And we have constraints which are restriction on the decision variables. So, broadly we have three components; one is decision variables, other is objective function and the third one is constraint.

So, constraints limit what we can do right; objective function is what we want to maximize or minimize and decision variables are those which we can actually change, right. So, the point is that, we need to find out the optimal value of the decision variables, right. So, one classical optimization problem that many of us might have solved is selecting the cheapest flight to travel from one city to another city, right.

So in that case the decision variable is to select a flight, right. So, these are the decision variables, right. So, the decision variable as such is which flight to select. So, in this case if we see the minimum cost is 6355, right. So, that is the optimal decision with respect to minimizing the cost. So, the nature of the decision variable objective function and constraints helps in the classification of problems and in many times and at times even it helps in the classification of techniques.

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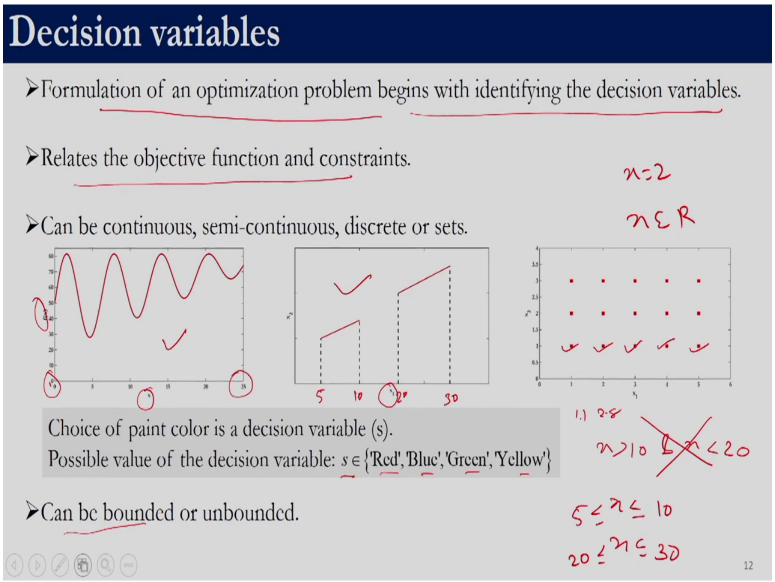

So, let us look into each of the component in a little bit more detail. The formulation of an optimization problem starts with identifying the decision variables. So, because we need to know what is that can be changed, so as to improve the objective function. So, if we are not allowed to alter anything, in that case there is nothing to optimize.

So for example, if we tell that x has to take a value of 2 right; then there is no choice that has to be made for x. But if it is said that x can take any value in the real domain, then we have a set of alternatives for from which we need to choose what is the best value for x, right. So, this decision variable relates the objective function and constraints. So, the decision variables can be continuous; so for example, here if x is the decision variable and if f of x is the objective function, right.

So, x can take any value between 0 and 25. So, in that case x is continuous; the lower bound of x is 0 and upper bound of x is 25. So, x is x can vary from 0 to 25. Our task is to find out the value of x for which the function is either minimum or maximum. At times the variable can be discontinuous; so for example, if let us say this is 5, this is 10, this is 20, and this is 30, right.

So, in this case this variable x 1 can take any value between 5 to 10 right and can take any value between 20 to 30; but it cannot take a value which is greater than 10 right and we cannot take a value which is less than 20 right greater than 10. And so, x greater than 10 and x less than 20 is not allowed right; whereas, x in between 5 and 10 is permissible and x between 20 and 30 is a permissible, right.

So, between 10 and 20 there is a discontinuity. So, for integer variables only the integer values are allowed. So, for example, one is allowed, two is allowed, threes is allowed for x 1, four is allowed, five is allowed; but 1.1 is not allowed or 2.8 is not allowed right, only the integers are allowed, right. So, in this case x 1 can vary between 1 and 5, the only the integer values, right.

So, the decision variable can be continuous, semi continuous or it can be integer or it can even be a set. So, for example, we need to decide which color is to be used to paint a country on the map, right. So, there we have options of colors, right; so red, blue, green, yellow. So, the decision variable can be set also. In most real life problem or the decision variables are bounded; but mathematically a decision variable can also be an unbounded variable.

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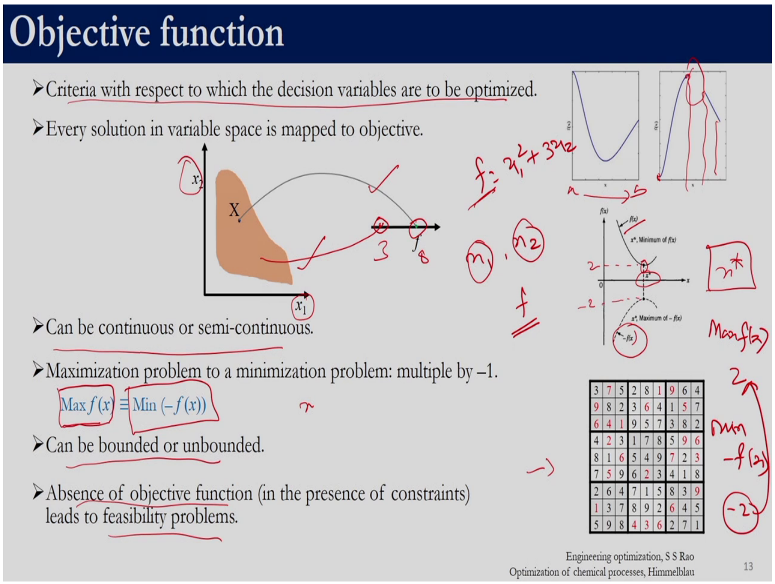

So, now that we have done a decision variable. Let us move onto the objective function right. So, the objective function is the criteria with respect to which the decision variables are to be optimized. So our task is to determine the values of the decision variable, so that a function known as objective function is either minimum or maximum, right.

So, every point in the decision variable space; so for example, if we have a two variable problem x 1 and x 2 right and let us say if this portion is the feasible region. Then every point in the feasible region has a particular scalar value called as objective function, right. So, for example, let us say f is equal to x 1 squared plus 3 x 2; if this is our objective function, then for any value of x 1 and x 2 there is a unique value of f, right.

So, a point in the search space, so x 1 and x 2 constitutes this search space; because we are looking for the optimal values for x 1 and x 2 in it is domain, right. So every point over here has a corresponding scalar value for it is objective function. So, now if we are given two solutions; let us say this solution is over here it is corresponds to over here and the solution corresponds to over here and let us say this is 3 and this let us say this is 8, right.

So, if the optimization problem is to maximize f, then this solution is better and if the optimization problem is to minimize f, then this solution is better right. So, comparing two solutions has nothing, but finding out the minimum of those two objective function values. So, similar to decision variables, the objective function can be continuous or semi continuous. So, here we have an example of a continuous function, right.

So, as x varies from let us say point a to point b, f of x is continuous. Over here we have shown a discontinuous function; that the objective function is defined from this particular x to this particular x, but there is no definition for the objective function in this region, right. So, over here again it is defined in this region; but there is a discontinuity in between. So, the objective function can be continuous or semi continuous.

So, for this course we will be most of the time we will be either talking about a minimization problem or a maximization problem. So, any minimization problem can be converted to a maximization problem or vice versa by multiplying the objective function with a negative sign, ok. So, for example, if the problem is to maximize f of x; the solution for this problem is the same as the solution for minimizing minus f of x, right. So, the value of the objective function might differ, but the value of the solution x would be the same.

So, for example, you can look at this figure. So, this is the curve that top curve is for f of x right; this curve is for f of x and the bottom curve is for minus f of x, right. So, if you see that is an mirror image, right. So, the minimum of this function f of x is located over here; the value at which the minimum occurs is x star, right. So, at x star if you see, it is also the same point at which the maximum of this minus f of x occurs, right. So, x star remains the same, right.

So, the objective function would be this, if you are minimizing f of x. So, let us say if you get 2 over here, over here you would get minus 2 right so; but x star would be the same. So, if we know that we are actually, let us say we had optimization problem it said a maximize f of x and we got a value of let us say 2 and if we minimize it, minimize if you do minus minimize; if you do minimize negative of f of x we will get a solution minus 2.

So, this can be brought back to this actual value by just multiplying a with a negative sign. There would not be any change at the point at which the maxima or the minima occurs, right. So, x star remains the same and that is the task right; to find out the value of x at which a function is either maximum or minimum. So, the objective function just like the decision variables can be bounded or unbounded, right.

And there are some special problem called as feasibility problems, in which there is no objective function right; but there are a set of constraints that need to be satisfied. So, for example, Sudoku problem is a example for feasibility problem. We did briefly discuss how this feasibility problem can be converted into an optimization problem, right. So, in feasibility problem we only have constraints; in an optimization problem we may or may not have restrictions on the decision variable.

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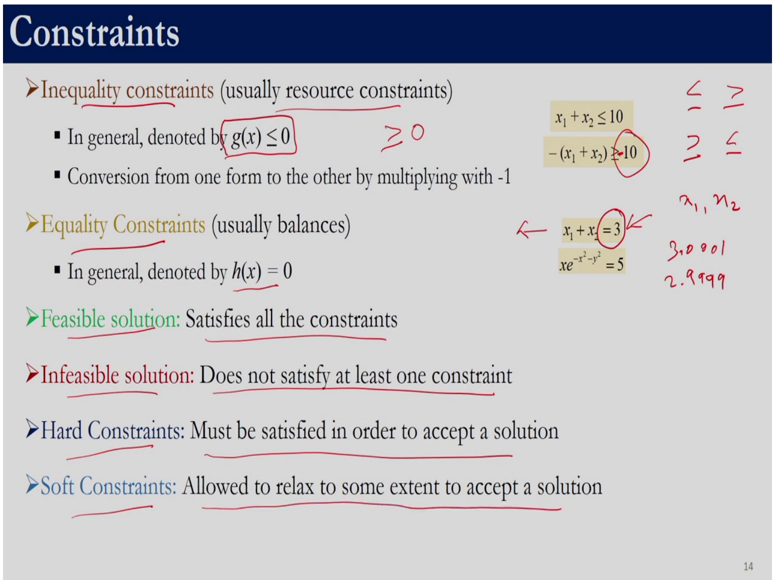

Now that we have discussed decision variables and objective function, let us move on to constraints, right. So, broadly constraints can be of two types; one is inequality constraints and the other one is equality constraints. So, inequality constraints usually arise when we have resource constraints that something has to be less than or equal to something or something requires to be greater than or equal to something, right.

So, in general they are denoted by g of x less than 0; but it can be converted into greater than 0, greater than equal to 0 by just multiplying with the negative sign, right. So, if x 1 plus x 2 is less than equal to 10; if this is the requirement, then it is the same as prescribing minus x 1 plus x 2 is greater than or equal to minus 10 not plus 10, but a minus 10 over here, right.

So, just like objective function the constraints can also be converted from less than equal to form to greater than equal to form or vice versa greater than equal to form to less than equal to form by multiplying with the negative sign on both the sides of the equation. So, equality constraints are usually considered a very difficult to satisfy; in general they are denoted by h of x is equal to 0, right.

So, here we have two examples that x 1 plus x 2 is equal to 3 and over here x e to the power minus x square minus y square should be equal to phi. So, the task over here is to come up for; if this is our constraint the task here is to come up with the values of x 1 and x 2 such that if we add both of this number it should be exactly equal to 3, right. So, 3.00000 1 is not allowed and 2.99999 is also not allowed, right. So, that is why these constraints are very difficult to satisfy.

So, usually some tolerance is given, right. So, this equality constraint usually tolerance is given to that. So, when we actually start solving a problem we will look into that. So, based on this constraints; a solution can be classified either as feasible solution or an infeasible solution. So, if a solution satisfies all the constraints in the problem, it is said to be a feasible solution and if a solution does not satisfy even one constraint, it is said to be an in feasible solution.

So, we may have a 100 constraints, a solution may satisfy 99 constraints and maybe failing on a 1 constraint; failing in the sense it is not able to satisfy the requirement, even in that case the solution is to be classified as infeasible solution, right. And again constraints can be classified as hard constraints and soft constraints, right. So, hard constraints are those that have to be satisfied, so that a solution has to be accepted. Soft constraints are those which can be allowed to relax to some extent to accept the solution, right.

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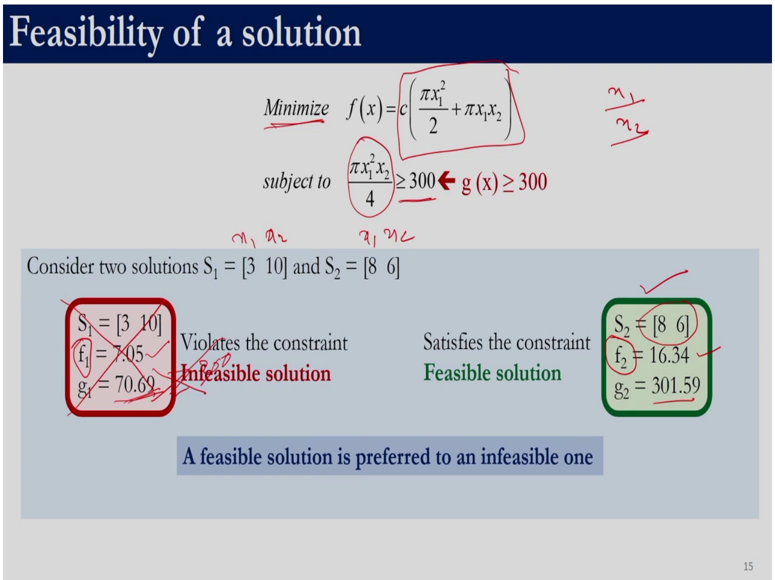

So, just let us look at couple of examples to understand the feasibility of solution. So, here we have a objective function, right. So, the decision variables are x 1 and x 2, right. So, now we are supposed to find out the values of x 1 and x 2 such that when those values are plugged into this part that should result in a value which is greater than or equal to 300, right. So, if it does not happen then the solution is said to be an infeasible solution.

So, for example, consider the solution; x 1 is equal to 3 and x 2 is equal to 10 and another solution x 1 is equal to 8 and x 2 is equal to 6. So, if we calculate the objective function for both of this, right. So, if I plug 3 comma 10 in this expression right, we will get a 7.05 and if we plug this 8 comma 6, we will get a value of 16.34. And if we calculate this right hand side of this constraint that pi x 1 square x 2 by 4; if we calculate that part, it turns out to be 70.69 for solution 1 and in solution 2 it turns out to be 301.59, right.

So, if we look at the objective function f 1 and f 2, 7.05 is better than 16.34; because our objective is to minimize. However, this solution does not satisfy the constraint, that 70.69 is not greater than equal to 300, right. So, this solution is an infeasible solution, right, it violates the constraint, it is an infeasible solution; whereas, this solution the solution is to it is a feasible solution, since it satisfies the constraint, right.

So, solution 2 would be preferred to solution 1, right; despite the fact that f 1 is better than f 2, because the problem is minimizing. So, a feasible solution is to be preferred to an infeasible solution, right; no matter how good is the objective function value for an infeasible solution, it does not matter, because it does not satisfy the constraints which is a requirement.

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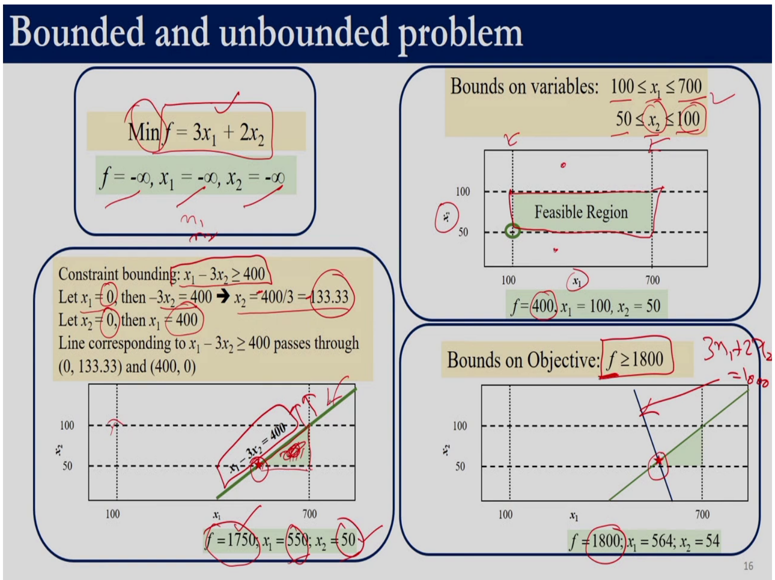

So, now let us look at what is a bounded and an unbounded problem, right. So, let us say we have a function f which is 3 x 1 plus 2 x 2 right and we are supposed to minimize that, right.

So, and if there are no restrictions of on x 1 and x 2, then x 1 and x 2 can take a value of minus infinity and minus infinity right and the problem the objective function value itself will be minus infinity.

So, this problem is known as an unbounded problem. So, the minimum value of f s minus infinity, right. So, now, if we put constraints on the decision variables, so the decision variables x 1 and x 2; here they were unrestricted, right. So, x 1 and x 2 could have taken any values between minus infinity to plus infinity. Now we have restricted that x 1 has to be between 100 and 700 and x 2 has to be between 50 and 100, right.

So, we have this line this, this is x 1, this is x 2. So, x 1 has to be greater than or equal to 100, so we have this line right and then x 1 has to be less than equal to 700, so we have this line right. So, x 1 the feasible region of x 1 is from here to here. Now, we plot the feasible region of x 2. So, x 2 has to be greater than or equal to 50. So, any value over here is infeasible with respect to x 2; with respect to x 1 it is feasible, but with respect to x 2 it is infeasible.

Similarly, x 2 has to be less than equal to 100, right. So, any value over here is infeasible with respect to x 2, right. So, the feasible region now is bounded. So, it is in this region that we are supposed to locate the best solution right. So, in this case it happens that this objective function 3 x 1 plus 2 x 2 will have a minimum value of 400 at this point; any other point if you calculate inside this entire feasible region, you would see that it has objective function value which is actually greater than 400.

So, since we are looking at a minimization problem; the least value that we can have for this problem with these constraints is 400, right. So, now let us add one more constraint; that the values of x 1 and x 2 should satisfy this constraint that x 1 minus 3 x 2 should be greater than or equal to 400. So, to graphically plot what we will plot is x 1 minus 3 x 2 is equal to 400; remember our constraint is greater than or equal to 400, but we are plotting this line x 1 minus 3 x 2 equal to 400, this line can be plotted. This is a region right, greater than equal to 400 is a region.

So, let us. So, in order to plot this line; what we can do is we can take x 1 to be 0 right, if you take x 1 to be 0, right. So, then minus 3 x 2 is equal to 400, which implies x 2 is equal to 133.33. Similarly, so that gives one point 0 comma 133.33 that is one point. The second point is we will keep x 2 as 0 and calculate the value of x 1. So, it will come turn out to be 0 comma 400. So, now we have two points, right. So, we can draw this line over here which is shown, right.

So, that line passes through 0 comma 133.33 and 0 and 400 comma 0, right. So, that line is plotted and so this feasible region which we had over here, this rectangle has now been reduced to this particular region right, this shaded part which is shown here, right. So, that is because in this region x 1 minus 3 x 2 is greater than 400; whereas, in the region about this right, it will not satisfy this constraint, x 1 minus 3 x 2 will be less than equal to 400. So, you can try out some values.

So, for example, if you substitute this particular value 100 comma 100 in this equation, right. So, a 100 minus 3 times 100 right that will be minus 200, which is not greater than 400. So, this point is to it is not feasible. So, similarly all the points above this line are infeasible right and over here we are restricted by this line x 1 is 700 and over here we are restricted by this line x 2 equal to 50. So, this triangle which we see here that is the feasible region. So, as you can see, as we add a constraint the feasible region decreases, right.

So, here there are two types of constraints; one is redundant constraint and one is non redundant constraint, right. So, a redundant constraint does not help you to reduce the feasible region and non redundant constraint helps you to reduce the feasible region. So, the optimal solution for this problem is x 1 is equal to 550, x 2 is equal to 50. So, if we plug these two values in this objective function 3 x 1 plus 2 x 2; we will get a objective function value of 1750, right.

So, when we do linear programming, this problem is a linear programming problem; when we actually do linear programming, you will see that how did we arrive at this solution. Right now what you need to understand is, what is a feasible region and what is an infeasible region. So, at times we can even have bounds on the objective function though it is very rare, but it is possible, right.

So, in that case if you see this is the line 3 x 1 plus 2 x 2. So, just like we plotted x 1 minus 3 x 2 is equal to 400; we can also plot a line 3 x 1 plus 2 x 2 is equal to 1800, right. So, if we plot this line, it will turn out to be this line, right. So, now the feasibility region is even reduced. So, here if you see the star which is over here has actually moved over, right. So, the optimal solution is 1800; here we are limited by the objective function.

1750 is possible that solution is possible, but since we have this additional constraint that the fitness function has to be greater than or equal to 1800, right the solution actually is 1800, right. So, this solution becomes infeasible, 1750 becomes infeasible; because there is a constraint that the fitness the objective function has to be greater than or equal to 1800.

So, in this case we had bounds on the decision variable. So, over here there is a small typo, right. So, minus 3 x 2 is equal to 400. So, if we are to calculate x 2 it has to be minus 400 by 3 and this has to be minus 133 by 3, right. So, the line is still drawn correctly, but just that we had a typo over there.

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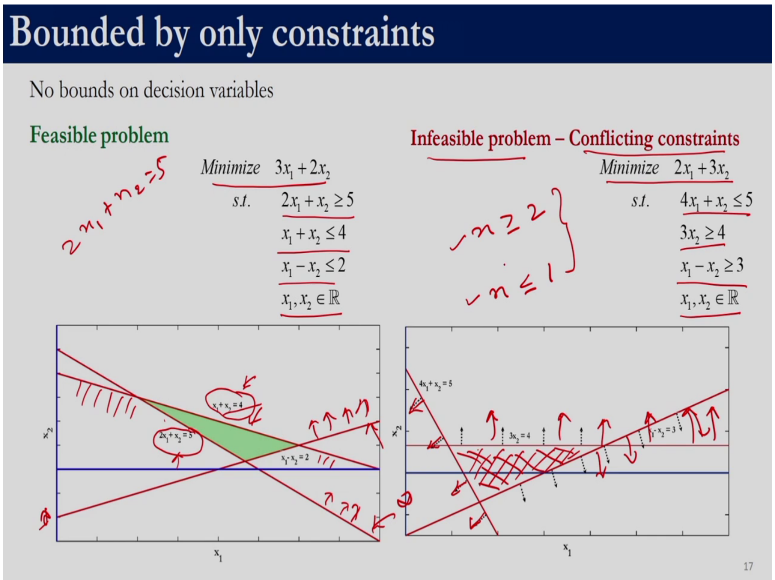

So, in the previous example we had seen the decision variables they were explicitly bounded; however, it is not necessary that the decision variables need to be explicitly bounded for the problem to be a bounded problem. So, the objective function is minimized 3 x 1 plus 2 x 2, the constraint is 2 x 1 plus x 2 greater than 5. So, we do the same thing that we took we take it as t x 1 plus x 2 is equal to 5.

And then take two values of x 1 calculate the values of x 2 and we will be able to plot this line, right. So, this line indicates 2 x 1 plus x 2 equal to 5. And the similar to the calculation that we discussed in the previous slide; this is this would be the area of feasibility. Similarly, we can also plot these two lines x 1 plus x 2 is equal to 4 and we take two values of x 1 determine what is the value of x 2 and plot it, right. So, this line is going to be a x 1 plus x 2 is equal to 4 and this line is going to be x 1 minus x 2 equal to 2.

So, any region below this x 1 plus x 2 curve x 1 plus x 2 equal to 4 line is feasible region. So, this is a feasible region, right. So, this is also a feasible region. Similarly for the other line x 1 minus x 2 is less than equal to 2. So, this is the feasible region, right. So, if we plot all the three all the feasible regions of the constraint; we will be left with only the shaded regions right. Because let us say any value which is feasible for this x 1 plus x 2 equal to 4 need not be feasible for 2 x 1 plus x 2 greater than equal to 5.

That is why some of the area which is feasible for let us say this constraint is not feasible for this constraint, right. So, the common area which is feasible for all the three constraints is our feasible region. So, the optima has to be located in this feasible region. So, here if you see x 1 and x 2 can vary from minus infinity to infinity right; there is no explicit bound on x 1 and x 2, still the problem is a bounded problem.

So, for us to declare a problem is feasible or not; we need to actually look into the nature of the constraints, right. Merely by looking into the bounds of the variables, it is not possible to say whether the problem is bounded or unbounded for an arbitrary optimization problem. So, this is a case wherein we show a infeasible problem that the constraints are conflicting.

So, over here also the, over here the objective function is minimized 2 x 1 plus 3 x 2 and the constraints are 4 x 1 plus x 2 is less than 5, 3 x 2 is greater than equal to 4 and x 1 minus x 2 is greater than or equal to 3 and again x 1 and x 2 can vary from minus infinity to plus infinity, right. So, if we plot these three lines, again plotting the lines is as we discussed previously we take we convert this into a equality constraint generate two points and since it is a linear equation we draw straight lines, right.

In this case the feasible region for each constraint is indicated by this dotted arrows, right. So, for the constraint x 1 minus x 2 greater than 3, this is the feasible region, below the constraint; for this curve 3 x 2 for this line 3 x 2 greater than or equal to 4 right this is that line 3 x 2 equal to 4, right. So, basically what we are saying is x 2 is equal to 1.33. So, x 2 has to be greater than 1.33, so it is this region. In this region x the value of x 2 is greater than 1.33.

Similarly we plot the line x 1 minus x 2 equal to 3, right. So, we this region is the infeasible, this region is the feasible region, right. So, now, if we see; there is no common area for this three constraints, in this case we call the problem as infeasible problem. So, another easy way to understand is let us say there is a constraint which says x has to be greater than or equal to 2 and another constraint which says x has to be less than or equal to 1, right.

So, any value of x which is less than or equal to 1 is feasible for this constraint; for this constraint any value of x which is actually greater than or equal to 2 is feasible, right. So, if I have to satisfy both this constraints; there is no single value of x which will satisfy both of these equations, right. So, in this case these two equations put together make the problem infeasible.

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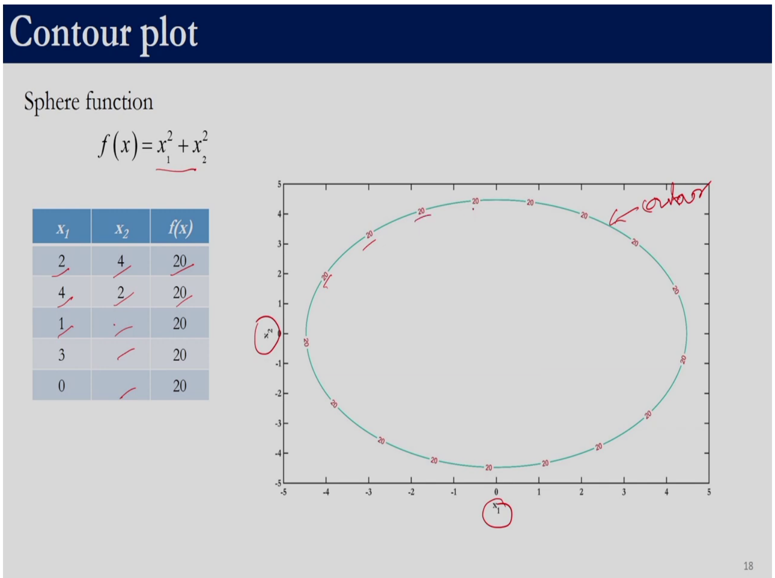

Another term which will be commonly using is contours. So, contours are lines which have identical objective function value. So, for example, here if we see if I if we take x 1 is equal to 2 and x 2 is equal to 4; it will give objective function value of 20. If we take x 1 is equal to 4 x 2 is equal to 2; we will get an objective function value of 20. If you take x 1 is equal to 1 and fix f to be 20 right; we can actually find out what is the value of x 2.

So, similarly we can generate many points and then we can plot. So, this is x 1, this is x 2, right. So, all these values can be plotted 2 comma 4, 4 comma 2, 1 comma whatever we get over here, 3 comma something, 0 comma something. So, we would get a curve like this. So, this is called as contour. So, all the points on this curve have an function value of 20, right.

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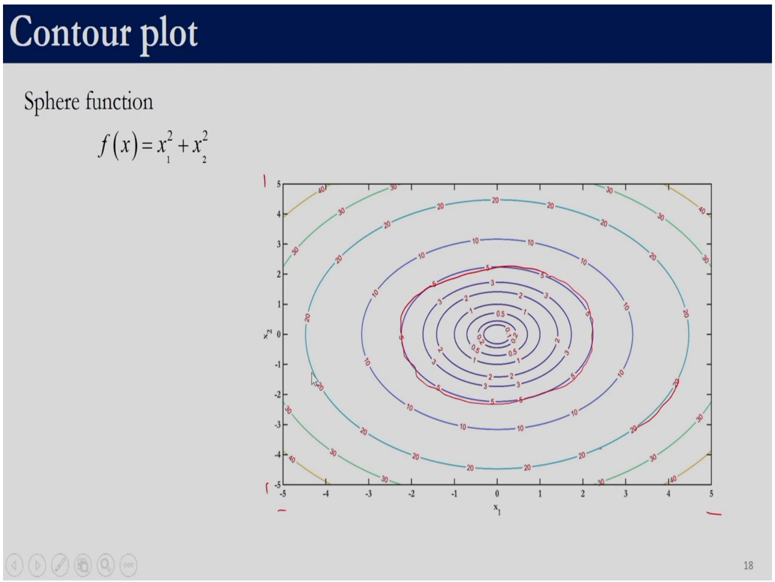

So, similarly we can plot contours for 10, we can plot contours for 5. So, similarly we can plot counters for other values, right. So, this is known as contour plot. So, this shows as how the objective function behaves in the search space. So, the our search space is x 1 is equal to minus 5 to 5 and x 2 is equal to minus 5 to 5. So, in this region how the objective function varies, right.

So, all the points in this curve we will have an objective function value of 20; all the points in on this contour, we will have a function value of 5. So, very often we will be referring to this contour plot.

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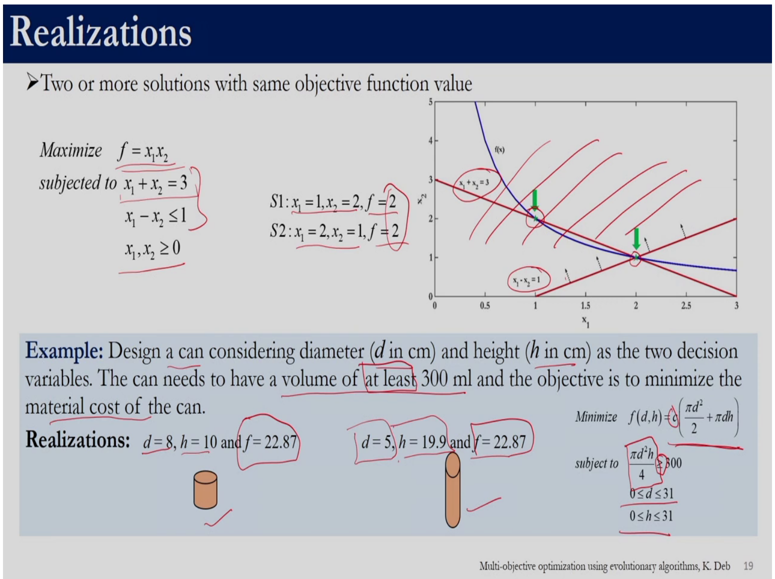

And then we have something called as realizations, right. So, realizations are those solutions which have the same objective function value, but different decision variable value. So, for example, if you remember the slide in which the flight costs were given; you may have two different flights which have which may have the same cost, right.

So, from the perspective of the objective function, which is cost in our case the value is same; but the decision variables can be different, right. So, those are called as realizations, right. So, for example, if we have this problem maximize f is equal to x 1 x 2; subject to these two constraints x 1 plus x 2 should be equal to 3 and x 1 minus x 2 should be less than equal to 1 and bounded by the constraint that x 1 and x 2 should be greater than or equal to 0.

So, if we plot that. So, this is the line x 1 plus x 2 equal to 3, right; and this is the line x 1 minus x 2 equal to 1, right. So, since this is a less than equal to 1. So, it this is the feasible region as indicated by these dotted arrows, right. And the feasible region is also this line; this x 1 plus x 2 equal to 3, because the points that are lying on this straight line x 1 plus x 2 equal to 3 only those points satisfy this constraint, right.

So, the intersection point of these two constraint is over here and remember this entire region is feasible, right. This entire region is feasible because it has x 1 minus x 2 is less than equal to 1. So, we have one more point over here in addition to this, right. So, those two points are x 1 is equal to 1, x 2 is equal to 2 and x 1 is equal to 2 and x 2 equal to 1. So, at both these places the objective function value is 2, right. So, these are called us realizations right; same objective function value, but different decision variables, right.

In one case x 1 is in one case the solution is 1 comma 2 and in the other case the solution is 2 comma 1. So, this is an example again for a realization, right. So, here we need to design a can right; what we can vary is the diameter and the height right and the diameter and height has to be between 0 and 31. So, the constraint is that the can should have a volume of at least 300 ml, right.

So, the volume is given by pi d square h by 4. So, this has to be greater than equal to 300; because we have at least, right. So, if it is more than 300 it is fine, but we want at least 300 ml. So, that is why we have this greater than equal to constraint and the objective is to minimize the material cost of the can, all right. So, that if you see a see it denotes the cost and this will correspond to the material cost of the can.

So, in this case if we take the diameter as 8 centimeter and if we take the height as 10 centimeter; the objective function value would be 22.87. And if you take the diameter is 5 centimeter and the height as 19.9 centimeter; the objective function value which is the cost would still be 22.87. So, the can either look like this or it can look like this, right.

So, here the height is smaller then the this can; but in both cases the cost is 22.87, right. So, depending upon other constraints; one can choose either this solution or this solution right without compromising on the objective function value. So, that is why it is good to know the realizations; not only the optimal solution, but also many instances in which the same optimal costs can be obtained.

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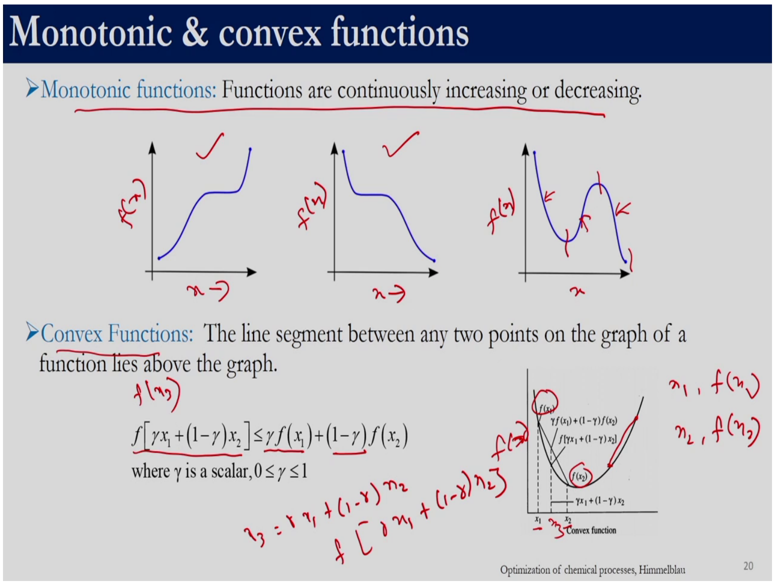

Now, let us look at a monotonic function and convex functions, right. So, we will be using this term very often. So, monotonic functions are functions which are either continuously increasing or decreasing with respect to x. So, for example, in this case if we increase as we increase x, the value of let us say this is f of x keeps increasing, right.

So, it has a monotonic nature to it; here in the second plot if we in the second figure if we see, if we keep increasing x, f of x continuously decreases, right. So, this is also monotonic, this is also monotonic; it does not matter whether it is increasing or decreasing as long as it is consistently increasing or decreasing. Whereas, this third plot is an example of a non-monotonic function, right. So, here if we see as we increase x, f of x is first decreasing till here and as we keep increasing x, f of x is increasing and again as we keep increasing x, f of x is decreasing, right.

So, here if you see from here to here, from in this region the function is monotonically decreasing; in this region the function is monotonically increasing and again in this region the function is monotonically decreasing. When we have functions behaving like this, those are known as non monotonic functions. We will also be using this term convex functions very often, right.

So, convex functions most of you would be knowing convex function, right. So, convex function if you take any two points in the line, right. So, for example, if I take this point and if I take this point and if I were to draw a straight line connecting these two points, right. So, the line would be above the curve, right. So, if that happens that is known as a convex function. So, that is the geometrical interpretation of convex function.

Mathematically let us say if this is if I do it two points x 1 and x 2, right. So, they are their corresponding fitness function value is the f of x 1 and f of x 2 right. So, we have two points x 1, it is corresponding y value or the function values f of x 1; we have x 2 it is corresponding function values f of x 2. And if we take any point right; so let us say we take this point x 3, right. So, x 3 is a linear combination of x 1 and x 2, right. So, we can express x 3 is equal to gamma x 1 plus 1 minus gamma x 2, right.

So, and again we can calculate f of x 3. So, f of x 3 is nothing, but f of gamma x 1 plus 1 minus gamma x 2, right. So, that is what is written over here. So, this is nothing, but f of x 3, right. So, that would be less than if that is less than gamma times f of x 1 plus 1 minus gamma times f of x 2, then the function is a convex function. So, as you can see a convex function will have only one point at which the gradient vanishes, right. So, we will have only one optimal solution, right.

So, the local optima itself will be the global optima. So, whenever we have a convex function, we it is sufficient if we find the stationary point and for at that stationery point the function is bound to be minimum, right. So, it is not necessary to calculate the higher derivatives.

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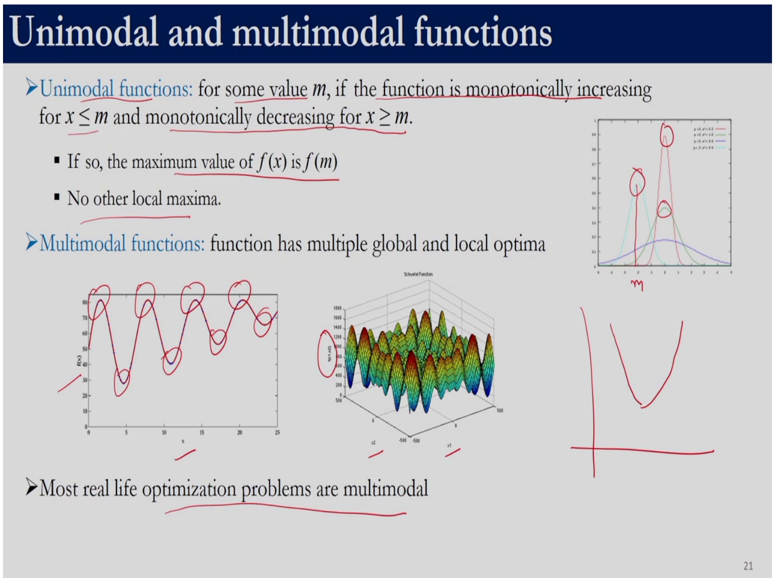

And then we have unimodal functions and multimodal functions, right. So, unimodal functions are those for some value of m, if the function is monotonically increasing for x less than m and monotonically decreasing for x greater than or equal to m, right. So, these are examples of monotonically, these are examples of unimodal function. So, if you see we basically have one peak, right. So, till this point the function is. So, this is our m.

So, till that point the function is actually increasing after that it is decreasing, right. So, we basically have this a one peak. So, those are called as unimodal function, right. Similarly what we have shown is over here for maximization right. So, for minimization it can be, it continuously decreases till a particular value and then increases beyond that value. So, this is also a unimodal function, right. So, for the unimodal function the maximum value of f of x is f of m, right. So, here if we see the value of the function is maximum at m, right.

So, that is a nice property about a unimodal function, right and it has only one peak, right. So, there are no other local maxima. These are examples of multimodal functions, right. So, for example, as x increases; here it is increasing, then it is decreasing and still if x is increasing this is starting to increase, right. So, here if we see we have multiple peaks over here and multiple valleys over here, right. So, this plot chosen, but one dimensional; so here the plot is between x and f of x; here it is between x 1 x 2 and f. So, it is a 3 D plot.

So, now, if we see traps are there, right. So, there are lot of valleys, there are lot of peaks; if our problem is to maximize is to find out the peak that is that has the best value, despite there being so many other peaks. And if our problem is to minimize the objective function, then our task is to find out the values of x 1 and x 2 at which the objective function has a lowest value. So, that is unimodal and multimodal functions. So, most real life optimization problems are multimodal in nature.

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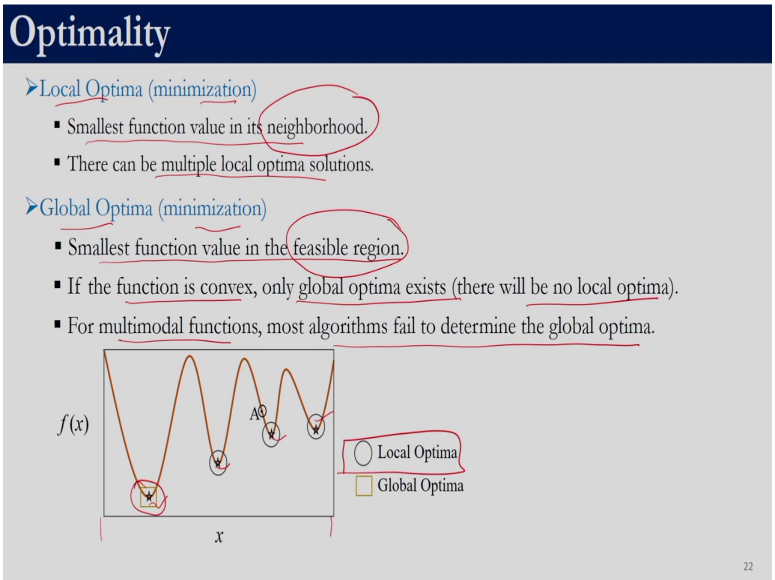

So, optimal solutions can be said to be either a local or global, right. So, what we are discussing here which is with respect to a minimization problem; but the same thing can be extended to maximization problem. So, local optima means the smallest function value in it is neighborhood, right. So, for example, if we take this point; in it is neighborhood it has the least value right, same thing this point it has the least value in it is neighborhood, right.

So, all those four points are local optima, right. So, far a problem there can be multiple local optima; for global optima it is the smallest function value in the entire feasible region. So, the differences over here, so local optima is the smallest function value in it is neighborhood; whereas, global optimize the smallest function value in the entire feasible region, right.

So, here if we see out of these four points 1 2 3 and 4 right, all the four points are local optima, right; because they are the they have the least value in their neighborhood. Whereas this particular point is actually lowest among the entire feasible region. So, if our feasible region is from here to here for f of x, for a single variable optimization problem; then we can see that the best solution is over here.

These are also local optima, but this is the global optima, right. So, if the function is convex right; only one global optima exists and there will be no local optima, right. So, that is why we looked into a convex function, just to understand what is a global optima. So, if our function is convex in nature, then if we are able to find an optima, we do not need to worry about whether it is global optima or local optima; because there is only one optimal solution, right. If the function is not convex or the function is multimodal; then most algorithms fail to determine the global optimal solution that is why non-linear programming is still a open area of research.

To consolidate whatever we have seen so far; so we had seen what are the components of an optimization problem? We have decision variable, objective function and constraints, right. So, decision variable can be either continuous or it can be semi continuous; it can be integer or a set. Objective function can be continuous or discontinuous. We saw how we can convert a maximization problem into a minimization problem or vice versa.

In constraints we saw basically there are two types of constraints; one is in inequality constraints and other that one is equality constraints. So, inequality constraints are of the form g of x is less than equal to 0. If we happen to have a constraint which is actually having let us say 3 x squared plus 5 y is greater than 10, greater than or equal to 10; then we can multiply that equation by a minus sign on both sides and the inequality sign would get reversed, right. So, that is what we saw as components of optimization problem.

Then we looked into certain features about a optimization problem; we saw what is boundary region, what is a feasible solution, what is an infeasible solution, what are realizations. Then we looked into what are a unimodal functions and multimodal function; we saw what is a convex function. And then we also tried to understand what is a local optima and the global optima.

The global optimize what we are interested; because we are interested in the value of the objective function, which is the least in the entire feasible region. Now, let us look into how do we classify optimization problems, right. This is a typical optimization problem, right.

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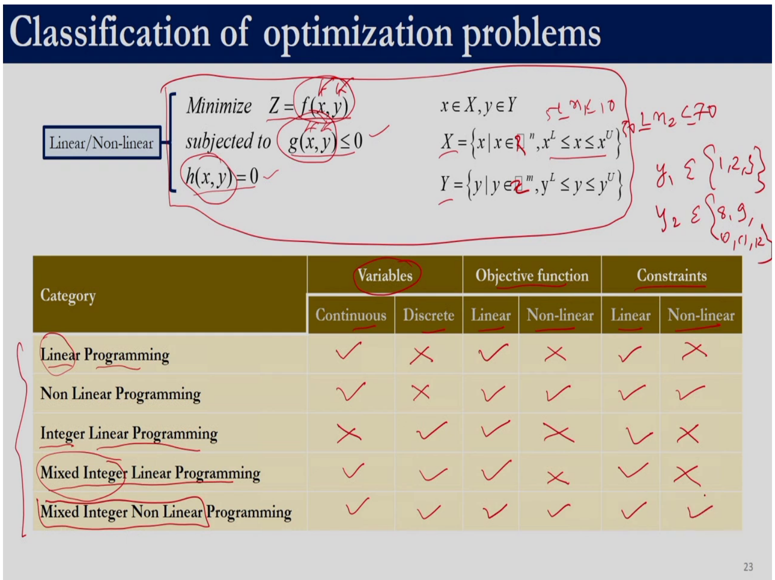

So, let us say we have an objective function which involves x which stand for the continuous variable and y which stands for a binary variables or integer variables. So, even if we have sets that can be converted into a integer variable, right. So, basically if we have a objective function which involves both the continuous variable and binary variable. Similar the we have constraints g which involve the continuous variable and binary variable and inequality constraints. So, this is inequality constrains, this is equality constraint.

And then we have a bounds for each of the variables. So, x belongs to the real domain or we can have a bound constrains that a particular value of x can be between let us say 5 and 10.

Let us say if you have two variables x 1 is between x 1 has to be between 5 and 10 and x 2 has to be between let us say 50 and 70, right. So, those are the bound constraint. Similarly for the integer variables or binary variables we can have that y can y 1 can take values 1, 2, 3; whereas, y 2 another variable can take values 8, 9, let us say 10, 11, 12 and so on, right.

So, this is a generic representation of an optimization problem. We have continuous variable, we have discrete variable, we have inequality constraints, we have equality constraints, right. So, here if we see these three are actually functions; f of x comma y, g of x comma y and h of x comma y are actually functions, right. So, function we can classify as linear or non-linear.

And decision variable we will just restrict ourselves to whether it is continuous variable or integer variable, right. So, we are as of now we are ignoring for classifying this problems, we are ignoring the semi continuous variable. So, variables can be continuous or discrete right or integer right; discreet as stands for integer also. The objective function can be linear or non-linear; the constraints can be linear or non-linear. So, depending upon this we have five classes of problem, right.

Broadly, five classes of problem; one is called as linear programming. So, linear programming as the name indicates right, the functions are going to be linear, right. So, the objective function is going to be linear, the constraints are going to be linear, right. So, nonlinearity is not allowed either in the objective function or in the constraints and since it does not mention anything about the nature of the decision variables.

So, when we say linear programming, we are not refer we are not explicitly specifying what is the nature of the decision variable. So, by default it means only continuous variable, discrete variables are not allowed. So, a linear programming is a problem in which the objective function and the constraints are linear and the variables are continuous; no non-linearity is allowed in the constraints or the objective function and the variables cannot be discrete. So, that is linear programming.

So, similarly non-linear programming the variables cannot be discreet; we are not explicitly specifying anything about the nature of the decision variable in the name, right, so the decision variable have to be only continuous. The objective function can be either linear or non-linear, and the constraints can be either linear or non-linear, right. So, it is possible that some of the constraints are linear, some of the constraints are non-linear; but if we have even one single constraint which is a non-linear, it falls under the category of non-linear programming.

For integer linear programming, the name itself specifies integer right so; that means, only discreet variables are allowed continuous variables are not allowed. And since it is linear programming, objective function and constraint should be only linear right; nonlinearity is not allowed in integer linear programming. Then we have mixed integer linear programming.

So, as the name indicates mixed integer right so; that means, continuous variables are also there discrete variables are also there linear programming, right. So, non-linear objective function is not allowed, non-linear constraint is not allowed; the objective function has to be linear and the constraint have to be linear. In mixed integer non-linear programming right, the variables some of the variables can be continuous, some of the variables can be integer; the objective function can be either be linear or non-linear and the constraints can either be linear or non-linear, right.

So, these are the five categories of problem. Obviously, you can also have integer non-linear programming; wherein the objective function or the constraints are a non-linear in nature.

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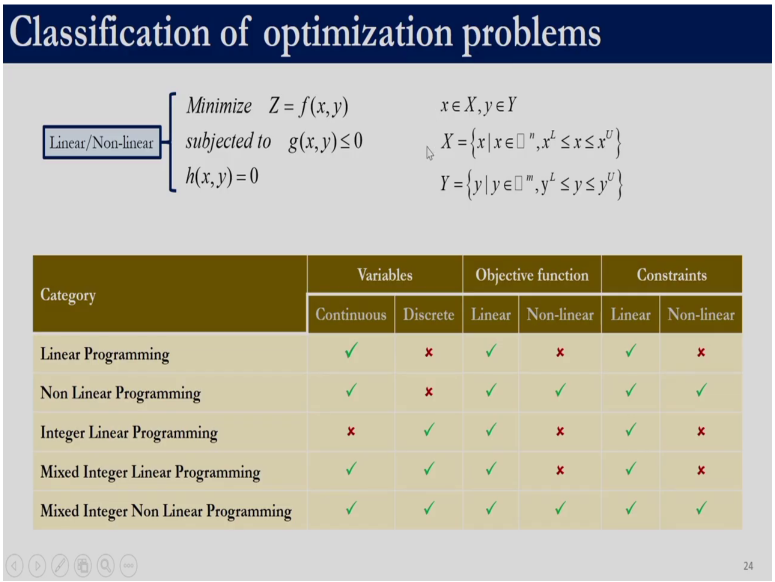

So, this gives the consolidated picture of whatever we have discussed. Now that we have classified problems; there are various ways to classify optimization techniques, right.

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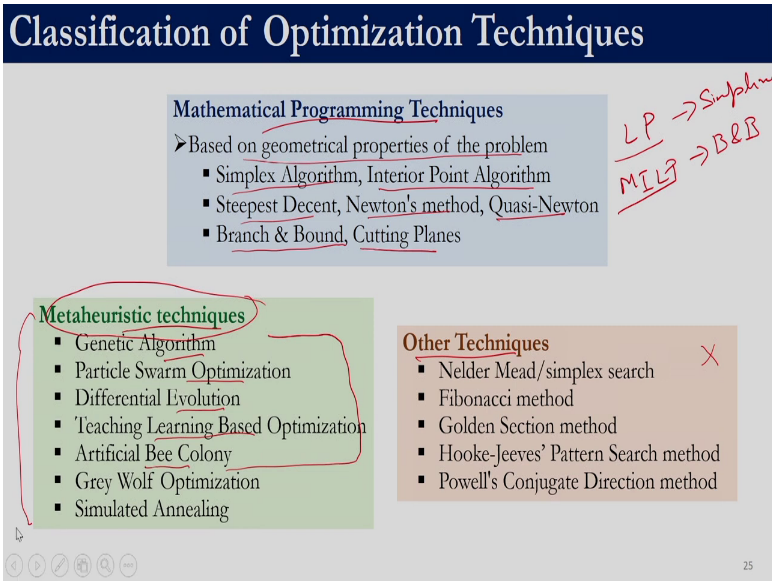

For this course we will classify broadly into three categories. One is mathematical programming techniques, metaheuristic techniques and what we call it as other techniques, right. So, this is something that we are not going to see as part of this course. So, in this course we will be primarily focusing on metaheuristic techniques, we will also touch up on a mathematical programming techniques, right.

So, mathematical programming techniques as based on geometrical properties of the problem. So, if it is the algorithm themselves are designed taking into account the nature of the problem. So, for example, for linear programming problem, we have something called as simplex algorithm. So, that explicitly exploits the fact that, the objective function and the constraints are linear in nature, right.

Similarly non-linear programming make use of the nature of the function to design the algorithm. So, some of the non-linear programming techniques are steepest decent, Newton's method, Quasi Newton method; for integer programming problems we have branch and bound and cutting planes, right.

Interior point algorithm can be used for linear programming as well as non-linear programming, right. So, here for this course we will only be looking at linear programming from the perspective of mathematical programming techniques. We will be looking into linear programming only with simplex algorithm. And for a mixed integer linear programming, we will be looking into a branch and bound.

There are various other mathematical programming techniques for non-linear programming and mixed integer non-linear programming, which you would not be covering in this course, right. From the perspective of metaheuristic techniques, so these are nature inspired techniques, right. So, we will be looking into genetic algorithm, particles form optimization, differential evolution, teaching learning based optimization, artificial bee colony, optimization. We will be looking into this five techniques.

There are; obviously, lot of other techniques which come up every year right, but we will be restricting it to these techniques. So these techniques do not necessarily exploit the structure of the problem. So, the way we solve a linear programming problem using metaheuristic technique is the same way we will be solving a non-linear programming problem or a mixed integer linear programming problem or a mixed integer non-linear programming problem.

So, these techniques consider the problem as a black box optimization problem; so we will look into it in detail as an when we start looking into metaheuristic techniques.

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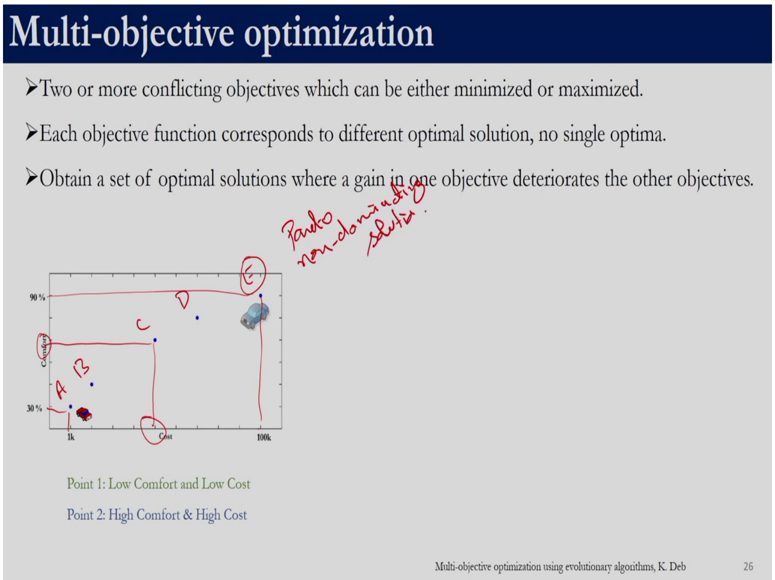

So, just for the sake of our knowledge, let us also look into multi objective optimization, right. So, in single whatever we discussed so far was with respect to single objective optimization. In multi objective optimization we have more than one objective, when we say more than one objective; that means, the objectives are conflicting in nature. Conflicting in the sense like, if we try to improve in one objective, we end up deteriorating the other objective.

And both the objectives are equally important, right. So, that is when we have multi objective optimization, right. So, classical example from Professor Kalyanmoy debs book, right. So, here you know someone wants to buy a car, five options A, B, C, D, E, right. So, all the cost of the cars are different and the comfort that it provides is also different.

So, the cost of car A is somewhere around 1 k in some arbitrary monitory units, and the comfort level that it provides is 30 percent right; whereas, car E is has a cost of a 100 k and the comfort level it provides is 90 percent, right. So, if someone were to say that their objective is to maximize let us say the comfort; then obviously, car E is the best choice, because the solutions A, B, C, D, E have a lower value of the comfort, right.

So, that is easy that is single objective optimization. Same way if someone were to say that minimizing cost is their objective, then it is easy to choose car A, because it has the lowest cost. But if someone were to say that maximizing comfort and minimizing costs is their objective, then all these five solutions A, B, C, D, E are equally good, right. So, for example, if we take a if we compare car C and car E, so car C has a lower cost right, but a lower comfort.

So, C cannot be eliminated when compared to car E that is because the cost is lower for C and we want to minimize the costs; whereas car E cannot be eliminated, because the comfort is better than that for car C, right. So, all these five points are equally good and we have what is called as pareto solutions, right or the set of non dominating solutions, right. So, there is no solution which dominates these solutions. So, non-dominating solutions.

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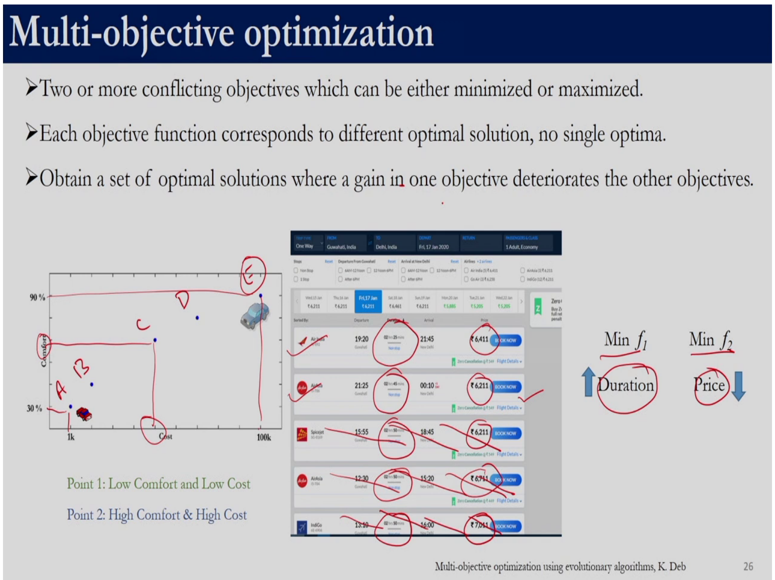

So, similarly if we go back to that previous example which we had that we want to select a flight, right. And the objective is to have the duration of the flight should be minimum as well as the cost should be minimum that both we want minimize f 1 and minimize f 2; f 2 is price and f 1 is let us say duration. So, for example, these two if we consider 6211, 6211; this takes 2 hours 15 minutes, and this flight takes 2 hour 45 minutes.

So, this solution can be eliminated, because we are not compromising on the cost right; whereas, we have a better duration, I mean lower duration with this particular flight. And if you compare 6411 and 6211; obviously, this is better in cost; but this solution has a lower duration, right, so between these first two flights it is difficult to eliminate any particular solution. Again this solution can be eliminated this right, because we have a solution which says 2 hours 45 minutes and the cost is 6211; here the cost is also higher, the duration is also higher.

So, this solution can be eliminated. So, here if we see again the solution can be eliminated, because we have a higher cost as well as the duration is longer than these two. So, in this case we have only two solutions for two objectives; whereas, in this case in this car problem even for two objectives, we had five solutions, right.

So, it is not a in multi objective optimization very often students have this misconception that if there are two objectives, then there are two solutions that is not the case, right. With two objectives you can have even zero solutions, you can have in finite solutions in the Pareto front or you can have a discrete Pareto front defined by a finite number of points.

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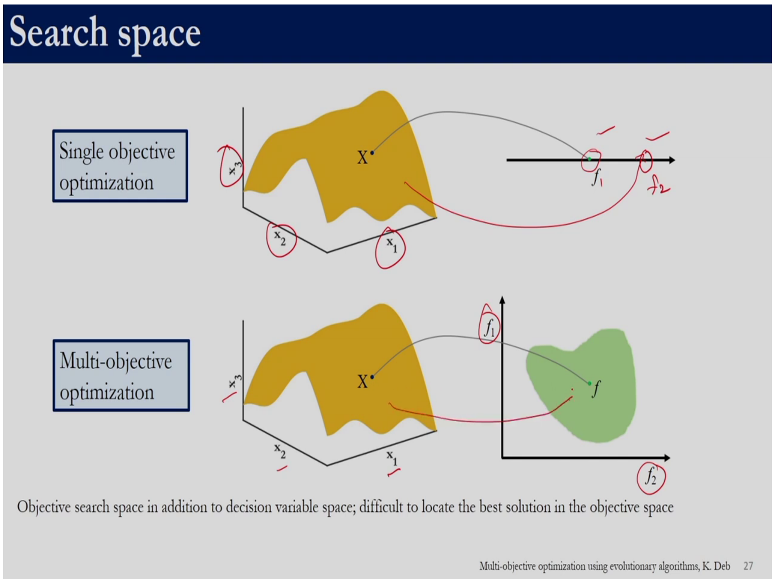

So, the search space in multi objective optimization and single objective optimization. So, in single objective optimization, let us say we have a 3 variable problem x 1 and x 2, x 3 and the shaded region let us say it is the feasible region. Then any point in this feasible region actually corresponds to a scalar value in the objective function; in the objective function space right. So, this is just a linear line, right. So, it is much easier to say which solution is better.

So, for example, let us say one solution is over here and another solution corresponds to this particular value; if this is f 1 and this is f 2 and if the objective is to maximize, then f 2 is better than f 1. So it is easy to make a make that call, because we are comparing only two scalar values, all right. In multi objective optimization every point in this search space x 1, x 2, x 3 are the decision variables. So, every point over here actually corresponds to a point in a; if we have two objectives, then it corresponds to a point in a two dimensional space, right.

So, this point is over here and this point is over here, right. Here the search processes little bit more complicated, because we intend to get the set of non dominating solutions or the Pareto solutions or the trade of solutions.

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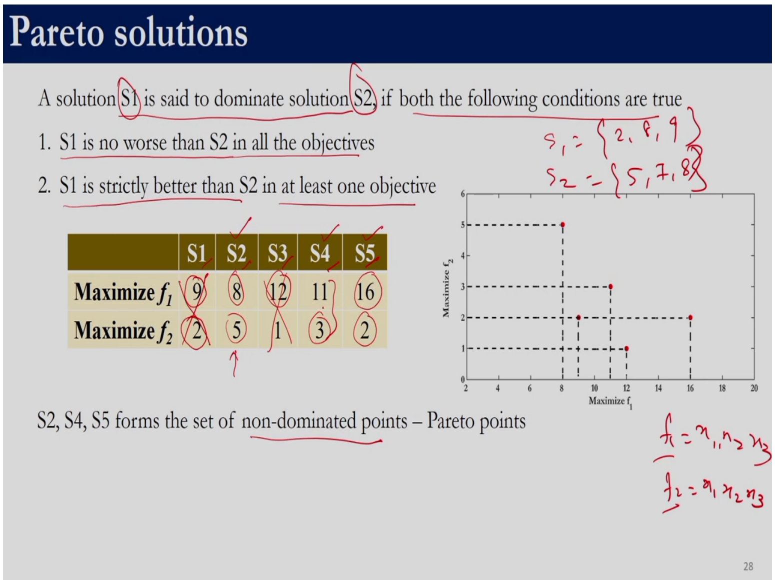

So, formally we can define a Pareto solution as that solution S 1 is said to dominate solution S 2. So, if you have two solutions S 1, S 2; S 1 is set to dominate S 2, if both the following conditions are true. So the first condition is that; S 1 is not worse than S 2 in it any of the objectives, right. So, it is not definitely bad then S 2 in any of the objectives; and that S 1 has to be strictly better than S 2 in at least one of the objectives, right. So, only then we can say S 1 is dominating the solution S 2.

So, if we are given these points, again there is a common misconception that for two objective functions to be conflicting; one has to be maximum, the other has to be minimum or it has to be min max, right. So, that is not correct; you can have two objective functions which both need to be maximized and yet they can be conflicting, right. So, this example shows you that thing. So, for example, these are the solutions; these are the solution S 1, S 2, S 3, S 4, S 5.

What is shown over here is their objective function values, right; these are not the decision variable values. For some let us say if it is a three variable problem; S 1 has let us say 2, 8, 9; S 2 has let us say 5, 7, 8. And then we have two objective function f 1 is equal to let us say some relation between x 1, x 2, x 3 and f 2 is a some relation between x 1, x 2, x 3, right. So, if we plug these decision variable values into these objective function values, we will get f 1 and f 2 for each of the solutions.

So, that is what is shown over here; solution S 1 has an objective function value of 9 and an objective function value of 2, similarly the other solutions how the respective objective function values. If we compare all the solutions; so let us say let us compare S 1 and S 2. So, we want to maximize. So, between S 1 and S 2 it is difficult to pick a solution, because S 1 is better than S 2, right in f 1; whereas S 2 is better than f 1, right.

So, if we do with 3, the same thing that S 3 is better in f 1, S 1 is better than f 2; because it has a value of 2. Over here if we see S 4, completely dominates S 1; because here we have 11 and 3 and we want to maximize both of this. So, this S 1 is definitely not a part of the Pareto solutions, because there is this solution which will definitely out rank f 1. So, we would never choose S 1 if we have S 4, all right. So, now that we have ruled out S 1, let us look at S 2, right.

So, between S 2 and S 3, we cannot make a call; because S 3 is better than f 1, better in f 1 and S 2 is better in f. Let us compare with S 4, so S 2 does not lose out to S 4 also; because we have a 5 over here and the 3 over here, right. Between S 2 and S 5, again S 2 does not lose out, right. So, S 2 is definitely going to be the part of a Pareto front. So, similarly you can do the other calculation, right.

So, S 3 will not lose out to S 4, because of this 12 being better right; but S 3 will lose out to S 5, because we have a 16 which is better than 12 and we have a 2 which is better than 1. So, S 3 loses out to this. Now we can compare S 4 and S 5, right. So, 11 and 16; 3 and 2 right, so obviously, S 4 does not lose out to S 5 right and other comparisons were made. So, these three solutions form a part of non dominated points, right.

So for example, here if you see this solution S 2; it had a poor objective function than S 1 and S 3, right; because it is 8 and this is 9 and 12. But still S 2 goes into the Pareto point; S 1 and S 3 lose out, because this S 2 has better value of f 2 which is why it survives, right. So, suppose for example, if I had to select S 1, then instead of selecting S 1 I would as well select S 4; because it is good in both the objectives, right.

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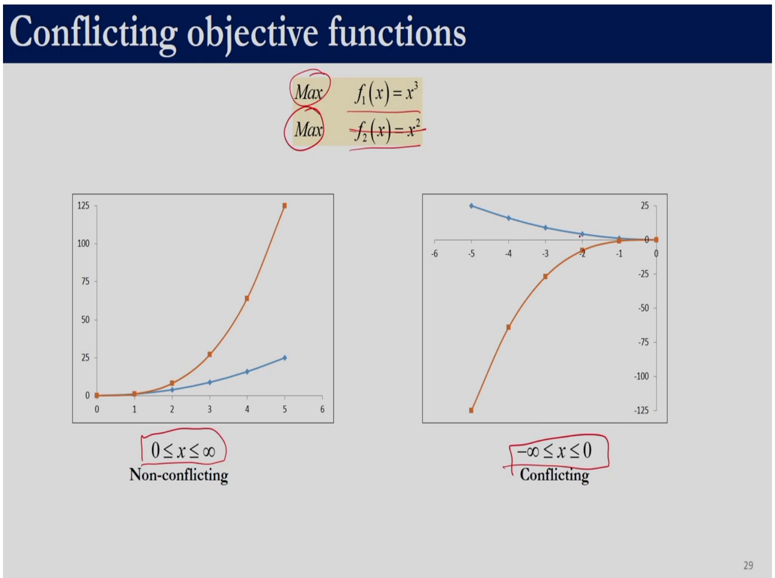

So, this is an example which helps you to establish that the nature of the function is to be studied before declaring them to be conflicting or not. So, for example, let us have this function that f 1 is x cube; f 2 is x squared and we want to maximize both this function, right.

So if the feasible region is let us say the domain of x is between 0 and infinity right; then these two variables then these two functions are not conflicting, right.

I can ignore one of the objective function and just solve with respect to the other objective function; whatever is the optimal solution I get that, would be the same for this one also. But if we change the domain of the variables from instead of from 0 to infinity; if we do it from minus infinity to 0 right, then they become conflicting, right.

So, one has to study the nature of the objective function, the constraint, and the domain of the decision variable to determine whether two objective functions are conflicting or not right. So, this is just to give you the difference between single objective and multi objective optimization. In this course we will be restricting ourselves to single objective optimization, right.

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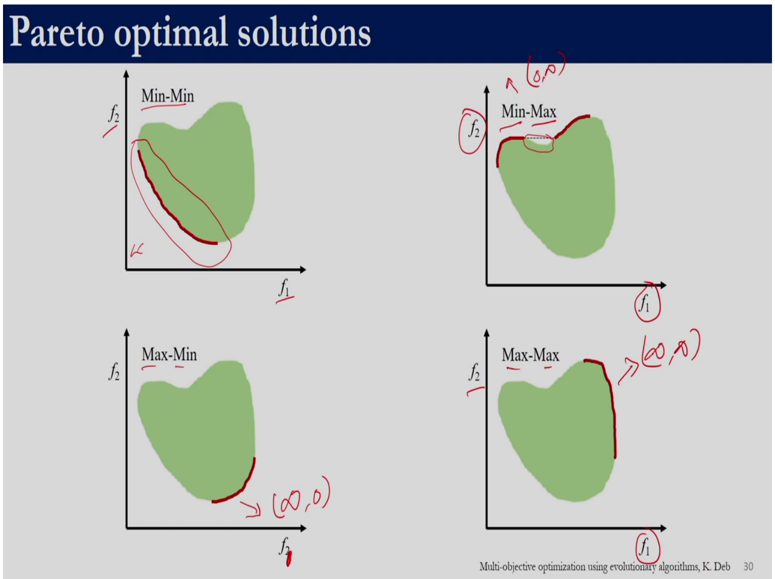

So, let us say we have these four cases, right. We have these two objective functions; f 1 and f 2, if both are to be minimized, right. So, you have Pareto front would look something like this. So, this highlighted part is what is the Pareto front, right. So, all the solutions in the direction of 0 0 would from the Pareto front, right.

If you are objective function f 1 is to be minimized and f 2 is to be maximized, right; so then we are looking at solutions which are pointing in the direction of 0 comma infinity. So, here as you can see the Pareto front here it was continuous, here it can be discreet, right. So, these points are actually inferior points you can work out; just like you can randomly select one point over here and compare, you will see that these highlighted points actually dominate the other points, right.

So, if you have a max min, then we are looking at; so this has to be f 1 right, then we are looking at infinity comma 0, right points. So, this direction is the Pareto front. So, if the both the objectives are to be maximized right; then we are looking at the points which are towards infinity comma infinity, right.

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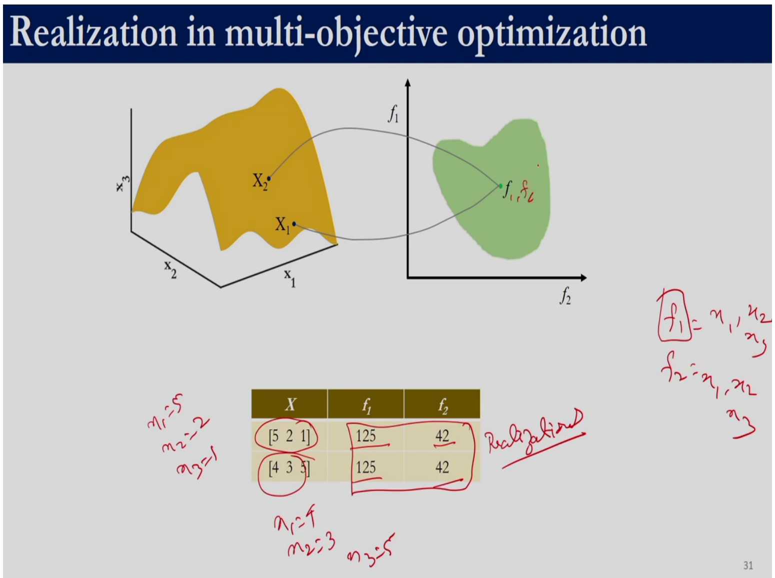

Just like we had realizations in single objective optimization, we can have realization in multiple objective optimization, right. So, let us consider two solutions x 1 and x 2 and there are some let us say there is some a function f 1 and f 2 which is a relation of x 1, x 2, x 3 and this is also a relation of x 1, x 2, x 3, right. So, here if we see if these two solutions are different right; here it means the x 1 is 5, x 2 is 2, x 3 is 1, right. So over here it indicates x 1 is 4, x 2 is 3 and x 3 is 5, right.

So, in this case what happened, in this case it can still happen depending upon what the objective function which we have, both f 1 and f 2 values are same, right. So, this is called this realization; these are not trade off solutions, the objective function is exactly identical both in x 1 and x 2, right. So, these are not trade off solutions, but these are realizations in multi objective optimization.

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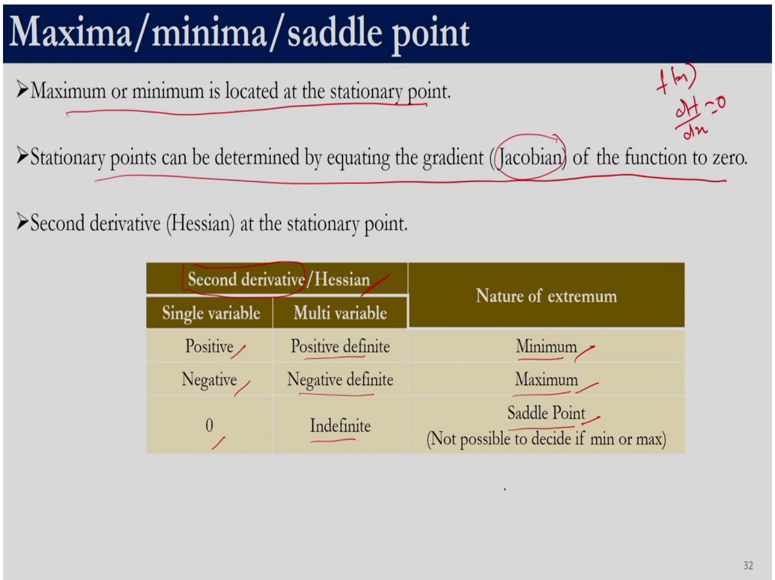

So, just like we had realizations in single objective optimization, we can have realization in multi objective optimization, right. So, here are two points correspond to the same value in the objective function space. So, this is f 1 comma f 2 right. Now let us come back to a single objective optimization; first we will see how to find out the optima for a single variable problem, right.

There is only one decision variable whose value is to be found right; again we are not considering the case wherein the constraints are there. So, we have an unconstrained optimization problem and single variable problem, right. So, in that case the maxima or minima is located at the stationary point. So, the stationary point you would have come across in your higher secondary education, right.

So, those can be determined by equating the gradient of the function to 0. So, if it is a single variable problem; if you have a function f of x right and if you want to find out the stationary point of this, then equate the derivative to 0. So, that will give the stationary point, right. Stationary point only tells that either a minima or maxima occurs at that point; you will have to check the condition of the secondary derivative, right.

So the second derivative has to be evaluated at this stationary point. So, if the second derivative is positive, then it is a minimum; if the second derivative is negative, then it is a maximum right; if the second derivative is 0, then it is a saddle point. So, in for a single variable we need to look at a higher derivatives to decide. So, in terms of multi variable problem, we will have what this Jacobian. So, the Jacobian has to be equated to 0.

So, we will solve a multi variable problem. So, we would better understand. And the second derivative is nothing, but Hessian matrix; again we will show you that. So, if the Hessian matrix is positive definite, then it indicates that the stationary point is minimum; if the Hessian is negative definite, then it indicates that the stationary point is maximum; and if it is indefinite, then again it is a saddle point. So it is not possible to decide whether at that point the function has a minimum value or a maximum value.

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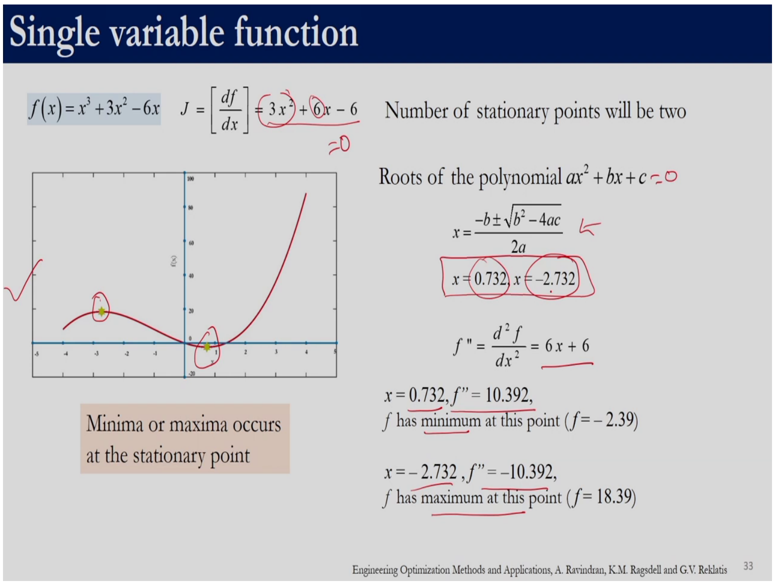

So, let us consider this single variable problem f of x is equal x cubed plus 3 x squared minus 6 x, right. So, if you take the gradient of this, that works out to be 3 x squared plus 6 x minus 6, right. So, we will have two stationary points. So, now, this has to be equated to 0. So, this is a quadratic equation. So, if we get, if we solve it we will get two stationary points, right.

So, the solution of the equation x squared plus b x plus c equal to 0 is x is equal to minus b plus or minus root of b squared minus 4 a c by 2 a. So, if we applied this, then we will get these two values, right. So, x is equal to 0.73; so a 0.732 is a stationary point. X is equal to minus 2.732 is a stationary point, right.

So, these are stationary points. So, with this information it is not possible to say whether the function is minima or maxima, right. So, we need to find out the second derivative, right. So, second derivative for this is 6 x plus 6, right. Now the second derivative has to be evaluated at each of the stationary 0.732 and minus 2.732, right. So, if we do if we evaluate the second derivative for 0.732 will turn out to be 10.392, right. Since it is positive, it is a minimum right; whereas the second point minus 2.732 since it is negative, it turns out it will be a maxima.

So, we have a minima at 0.732 and we have a maxima at minus 2.732, right. So, this plot shows the variation of x with respect to f of x, right. So, if we vary x from minus 5 to 5, this is how the function would look like. And here if we see this value is 0.732 where you can see it is actually a minima and here it will be the function will have a maxima which is minus 2.732.

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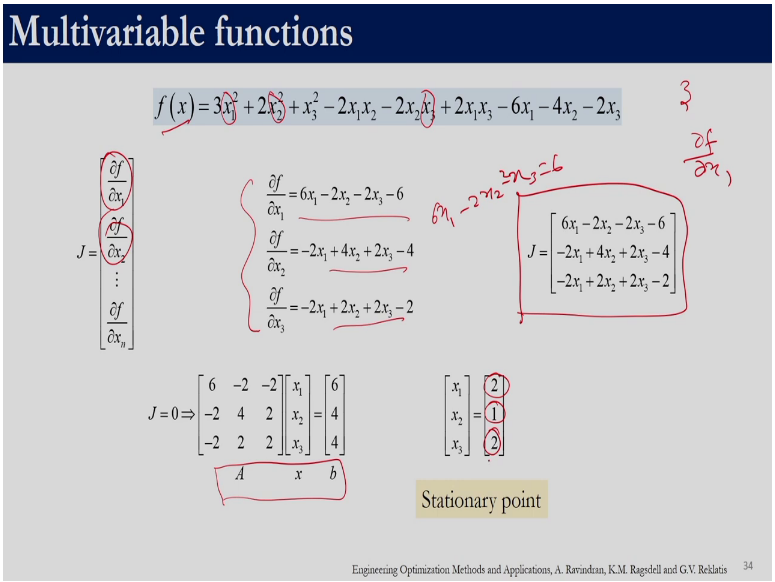

So, let us look into a multivariable function, right. So, here f of x right is involves x 1 as well as x 2 as well as x 3. So, we have three decision variables. So, the Jacobian is nothing, but the partial derivative of f with respect to x 1; partial derivative of f with respect to x 2 and the partial derivative of f with respect to x 3. Since we have three variables; if you have n variables all the partial derivatives are to be found, right. So, here we these are the three equations, right.

So if you differentiate it, you would get these three equations. So, this is our Jacobian now, right. So, this has to be equated to 0. So, this if you see it is simultaneous linear equation, right. So, this can be written as a 6 x 1 minus 2 x 2 minus 2 x 3 is equal to 6, right. So, this equation can be written as 6 x 1 minus 2 x 2 minus 2 x 3 is equal to 6 and the rest of the two equation also can be written.

And if we see, it will be a in the form of linear equation which can be said a x equal to b, right. So, if we solve this, we will get the values we will get the stationary point. So, in this case it happens to be 2, 1 and 2, right. So, this is a stationary point. Again with this we cannot say whether it is a minima or maxima, right.

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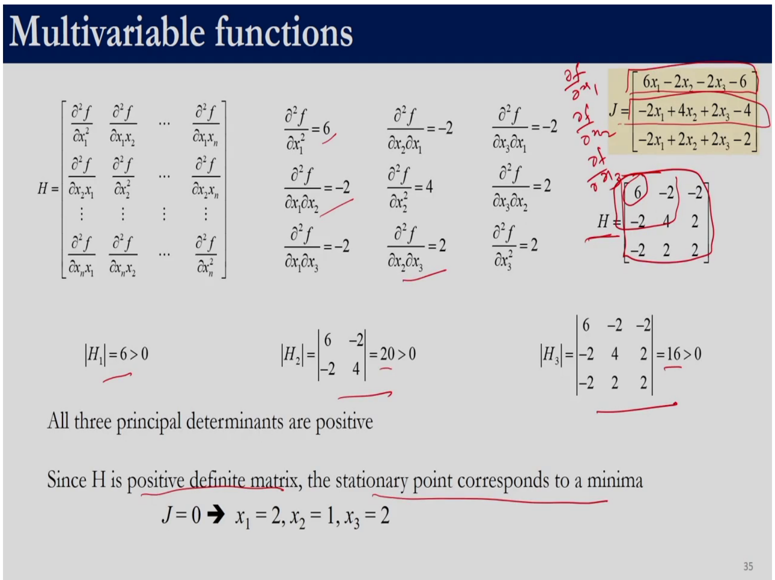

So, we need to determine the second derivative or the Hessian in this case. So, the Hessian is given by dou x squared f by dou x 1 square dou square f by dou x 1 dou x 2; all the way up to dou square f by dou x 1 dou x 1 x. Similarly we have dou x squared f dou x 2 x 1 dou x square f dou x 2 square and so on, right. So, in this case for the three variable problem, we will have to determine these partial derivatives. So, this is.

So, here we have dou f by dou x 1, dou f by dou x 2 and dou f by dou x 3, right. So, if we do dou x squared f by dou x 1 square. So, this equation if we further differentiate with respect to x 1, we will get only 6, right.

So dou x dou square f by dou x 1 dou x 2. So, this equation if we differentiate with respect to x 2, we will get minus 2 right. Similarly this one, so dou x 2 by dou x 3. So, dou x 2 is this equation, right. So, we will get only plus 2, right. So, if we plug that, we will get this is our

Hessian matrix, right. So, now we will have to see whether it is positive definite or negative definite.

So, positive definite; if the principal determinants, so this determinant, this determinant and the first 3 cross 3 determinant; if all of this happened to be greater than 0, then it is a positive definite matrix. So, in this case it happens that this is 6, right, so 6 into 4 minus 2 into minus 2 into minus 2, right. So, that would work out to be 20. In this case the all these three principle determinants happen to be greater than 0, right.

So, that is why we can display at this point to be, we can declare Hessian to be positive definite right and a stationary point which we found is actually corresponding to a minima.

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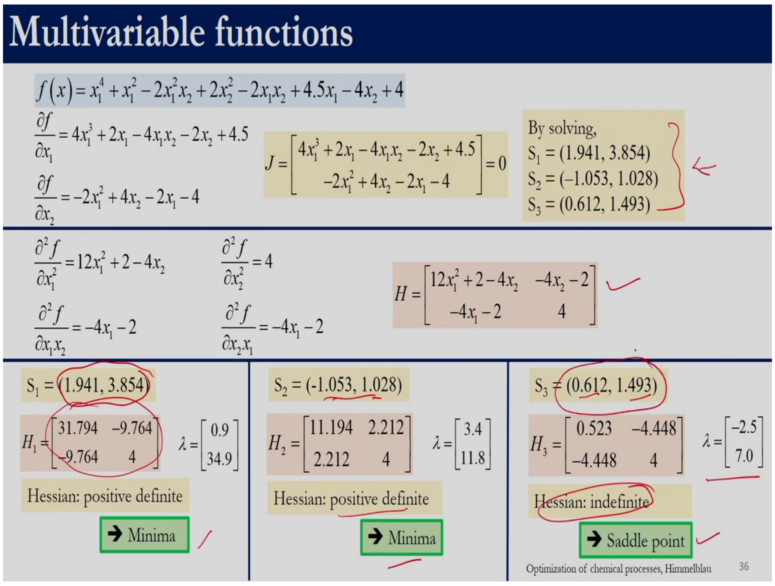

So, again this example we will give it to you, you can solve it, right. So, in this case we have a two variable problem x 1 and x 2, right. So, if we do dou f by dou x 1, dou f by dou x 2 that is our Jacobean if we equated to 0, right. So, these two equations can be satisfied by these three points, these three are our stationary points, right.

So for these three stationary points we need to find out; whether the Hessian is positive definite or negative definite, right. So, Hessian again is dou square f by dou x 1 square, dou square f by dou x 2 square, dou square f by dou x 1 dou x 2, dou square f by dou x 2 dou x 1. So, we will need this one and you can again compose the Hessian as shown over here, right. So, if you compose the Hessian you will get something similar to, we will get this matrix right and then if you find out the eigenvalues of this matrix.

In the previous slide, in the previous problem we only we saw the principle we saw that the principal determinants have to be positive right, for a Hessian matrix to be positive definite. Or we can use the eigenvalue properties; that if the eigenvalues of a matrix is positive, right. If all the eigenvalues of a matrix is positive, then the matrix is positive definite.

If all the eigenvalues are negative, then the matrix is negative definite; if some of the eigenvalues are positive and some of the values are negative eigenvalues are negative, then the matrix is said to be indefinite, right. So, in this case if you work out if you find out the eigenvalues or even if you calculate the determinant of this matrix; you will see that, you will get a positive which corresponds which tells that this is a minima, this particular stationary point is a minima.

So for the second point again if we plug in this x 1 x 2 values into this Hessian matrix right; if you calculate the eigenvalues right, you will see that all the both the eigenvalues are positive. So, again the session is a positive definite. So, this point corresponds to a minima right; whereas, the third point, here if we calculate again plug this x 1 and x 2 into the hessian matrix and if we calculate the eigenvalues it, one of the eigenvalues negative and the other is positive.

So, this is indefinite. So, here it is a saddle point. So, with this information whatever we have we cannot say whether this is a minima or a maxima. So, we will be using this concept while we are looking into regression right. So, the first derivative has to be equated to 0; we will get stationary point, at that stationary point we need to evaluate the second derivative, right. If the second derivative is greater than 0 for a single variable problem, it corresponds to a minima; if the second derivative is less than 0, it corresponds to a maxima for a single variable problem.

For a multi variable problem, we need to equate the Jacobian to 0, we will get the stationary point and we need to evaluate the Hessian matrix, right. So, for the Hessian matrix; if it is positive definite, then the corresponding stationary pointers minima; if it is negative definite, the corresponding a stationary point is maxima.

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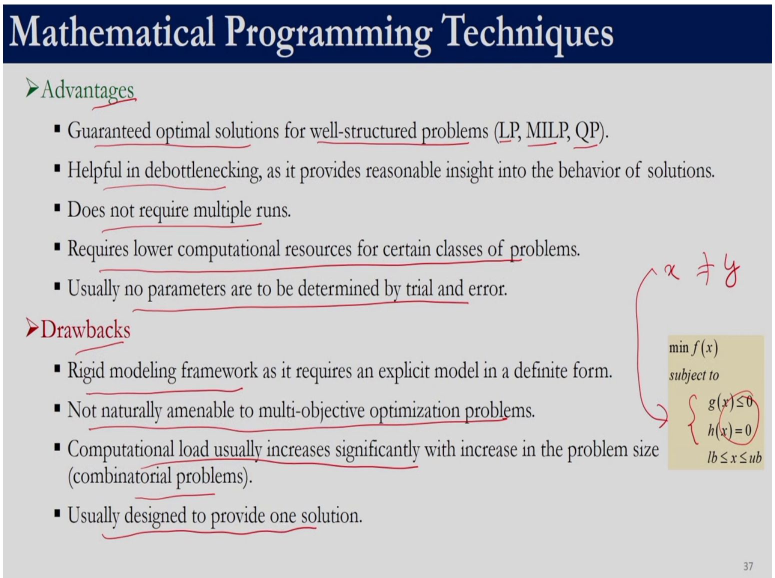

So, now let us look into the advantages and disadvantages of mathematical programming techniques. So, the advantages of mathematical programming technique is that, it has a guaranteed optimal solution for well-structured problems. So, if the problem is either linear programming, mixed integer linear programming or quadratic programming; there are algorithms which guarantee that the solution that we will obtain at the end of the procedure will be globally optimal, all right.

So, again here in optimization you need to remember, we have two things; one is that we want the best solution that exists in the search space, and the second thing is we want to reach that solution as quickly as possible, right. Given us a search space; obviously, you can evaluate the objective function at each and every point and see at which point it is minima or maxima. But then in a search space that can be infinite points, right. So, that is why we do not do an exhaustive search, we rely on optimization techniques, right.

So, far these two category of problem, these three category of problems there are algorithms right, which guarantee the optimality of solution right. And mathematical programming techniques are usual are helpful in debottlenecking, right. So, as in like it can provide reasonable insight into the solution, as to what is stopping us from having a better solution that can be inferred from for some problems using mathematical programming techniques, right.

It does not require multiple runs which is the case in metaheuristics techniques, right, the same technique does for the same problem will have to be run multiple times; because it involves stochasticity, right. So, that is not the case in mathematical programming techniques. Again it requires lower computational resources for certain classes of problems and there are no parameters that are to be set by trial and error, right. So, which is the case in metaheuristic techniques, right.

So, those are the advantages of mathematical programming techniques. The drawbacks include that it has a rigid modeling framework, right. So, all the constraints have to be expressed in this form. For example, in Sudoku problem the constraint that we have is that, let us say if we denote a sell by x 1 1 and it is neighboring sellers x 1 2; then the constraint that we have is actually x 1 1 is not should not be equal to x 1 2, right.

But that constraint is not of this form that is like this, right. So, x naught equal to y if we have a problem wherein we have this constraint that x should not be equal to y; that has to be transformed into these such constraints less than equal to or equal to, only then mathematical programming techniques can be used, right. So, that is a drawback of mathematical programming techniques like.

So, they are not naturally amenable to multi objective optimization problems. So, if you want to solve a multi objective optimization problem using mathematical programming technique; then the same problem has to be solved multiple times, right. And if it is a combinatorial problem, as in like if we have a lot of integer variables; as the size of the problem increases right the computational time increases exponentially, right.

So, it can become computationally intensive, particularly for combinatorial problem. Usually these techniques are designed to provide one solution, right. Some of these drawbacks have been overcome in the softwares like in the software; but in general these are the drawbacks of mathematical programming techniques.

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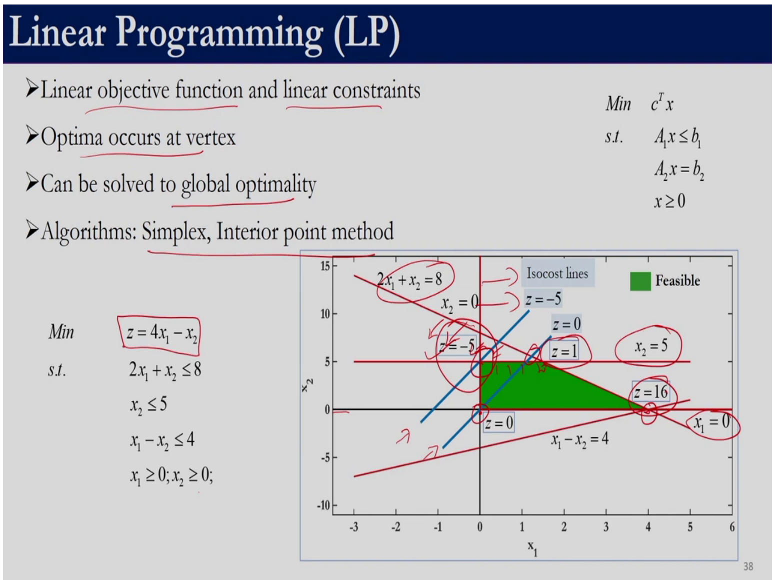

Let us now briefly look into the four categories of problem, right. So, linear programming in linear programming problem; as we discussed the objective function is linear and the variables are continuous. So, for linear programming problems it is guaranteed that the optima always occurs at the vertex, right. And linear programming problems using the simplex and interior point method can be solved to global optimality, right. So, that is an advantage of linear programming problem.

So, this is a linear programming problem that we have right. So, minimize z is equal to 4 x 1 minus x 2. So, our decision variables are x 1 and x 2; the constraint is that x 1 and x 2; 2 x 1 plus x 2 should be less than equal to 8, right. So, again we draw this line 2 x 1 plus x 2 is equal to 8. So, we will get this line; then we can another constraint that we have is x 2 less than equal to 5. So, that leads to this particular line x 2 less than x 2 equal to 5.

So, when we say x 2 less than 5, any value below this is the feasible region, right. So, 2 x 1 plus x 2 less than 8 is any value less than I mean in this region is actually a feasible solution with respect to this constraint right; but with respect to this constraint some of the feasible region is lost. Then we have this constraint x 1 minus x 2 is less than equal to 4. So, again we will draw the line x 1 minus x 2 equal to 4, so we get this line, right.

And then we have these variables that x 1 should and we have the bound constraint that, x 1 should be greater than equal to 0, x 2 should be greater than equal to 0, right. So, this lines we obtain, right. So, this x 1 is equal to 0, so the region above it and x 2 greater than 0, right, so this region, right. So, if we look at all the constraints.

So, this shaded region is actually the feasible region, right. So, for linear programming, it can be it has been theoretically established that the optima always occurs at the vertex. So, the optima is either here, here, here or here of this four points. So, there is no point searching at the interior point, only these four points have the optimal; one of these four points have the optimal solution.

This is our objective function line 4 x minus x 2. So, if we equate 4 x minus 4 x 1 minus x 2 as let us say minus 5 some value minus 5, then this is that line. So, throughout this line the objective function value is minus 5; throughout this particular line the objective function value is 0, right. So, these are known as Isocost lines, right. So, throughout this line z 1 z is minus 5, right.

So, that is an Isocost line; throughout this particular line z is equal to 0. In this case the optima occurs at this particular point, right. So, the other four points if you want you can evaluate; you will see that; the for the other four points the objective function value will work out to be 0 for this one, 16 for this one, again one for this point for this vertex and minus 5, right. So since our objective function is to minimize this vertex has the best possible value for the objective function.

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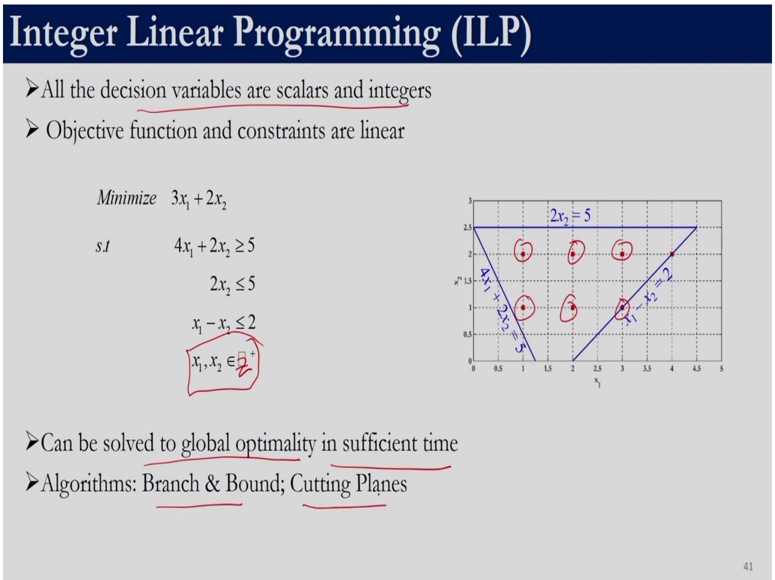

Coming to integer linear programming; so in integer linear the programming the decision variables are scalars and integers, like we can have multiple decision variable, but all of them are scalars right, but all of them are scalars. So, the objective function and constraints are supposed to be linear over here. So, here we have a linear programming problem, right. So, we want to minimize this function 3 x 1 plus 2 x 2 that is our objective function righ and the constraints are 4 x 1 plus 2 x 2 should be greater than equal to 5; 2 x 2 is less than equal to 5; x 1 minus x 2 should be less than equal to 2.

Again we convert all these constraints into equality and we can plot all these lines, right. So this is the feasible region right; but because of this constraint that, x 1 comma x 2 belongs to integer variables not real variable. So, not all values of x 1 and x 2 are acceptable, only the integer values of x 1 and x 2 are acceptable. So, that is why we have a. So, because of that the feasible points are only these six right; because other points do not have integer values of x 1 or x 2.

So, only these values are permissible values, right. So, it might seem that the search spaces smaller in case of integer variables; but integer programming problems are difficult to solve, because we do not have the property of continuity of the decision variables. So, most integer linear programming problems can be solve to global optimality given sufficient time, right.

But there are problems who sizes if the number of decision variables is too large, then it is difficult to solve in reasonable time, right. But theoretically given sufficient time these integer linear programming problems can be solved to global optimality. So, the common algorithms which are used for integer linear programming are branch and bound and cutting planes.

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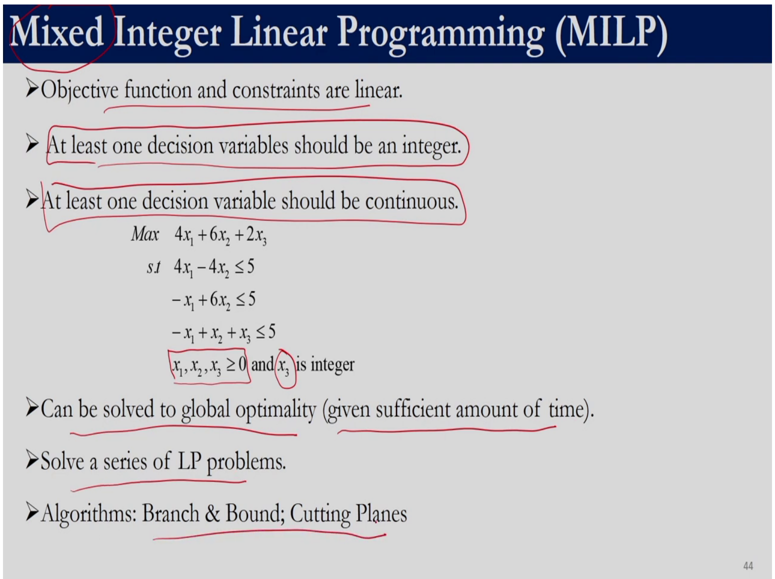

So, this is an example for mixed integer linear programming problems, right. In the previous slide we saw that all the variables were integer; over here the objective function and constraints are linear, at least one decision variable should be integer, right and at least one decision variable should be continuous that is why we have this mixed, right. So, these are the constraints, this is an example, right. So, in this case all the three variables are greater than or equal to 0; but x 3 can take only integer value.

So, x 1 and x 2 can take real values, but x 3 is an integer value. So, this is an example of mixed integer linear programming problem. Again mixed integer linear programming problem can be solved to global optimality given sufficient amount of time. So, mixed integer linear programming problems are solved as a series of LP problems and the algorithms that we commonly use are branch and bound and cutting planes.

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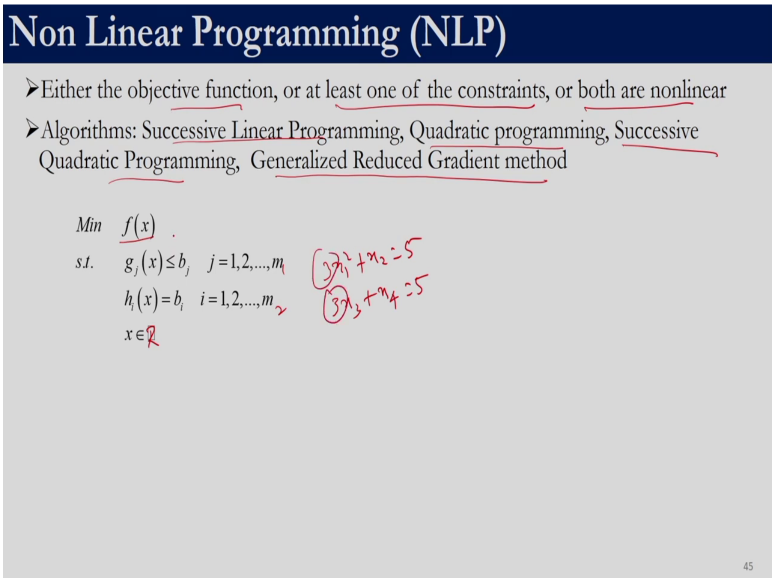

So, non-linear programming problems; in non-linear programming problems either the objective function or at least one of the constraints are both are non-linear, right. So, that the algorithms which are commonly used for solving non-linear programmings are; successive linear programming, quadratic programming, successive quadratic programming or generalized reduced gradient method, right. There are many other algorithms, we have this listed a few of them over here, right.

So, here we have an objective function minimize the f of x. We have m 1 constraints, m 1 inequality constraints. So, j is equal to 1, j is equal to 2, j is equal to 3, it can go all the way up to m 1, right. And we have m 2 equality constraints, right. So, we can have constraints which is 3 x 1 squared plus x 2 is equal to 5 a 3 x 3 plus x 4 is equal to 5. So, this is a non-linear constraint, this is a linear constraint, right. So, again inequalities are allowed, equalities are allowed in non-linear programming problem.

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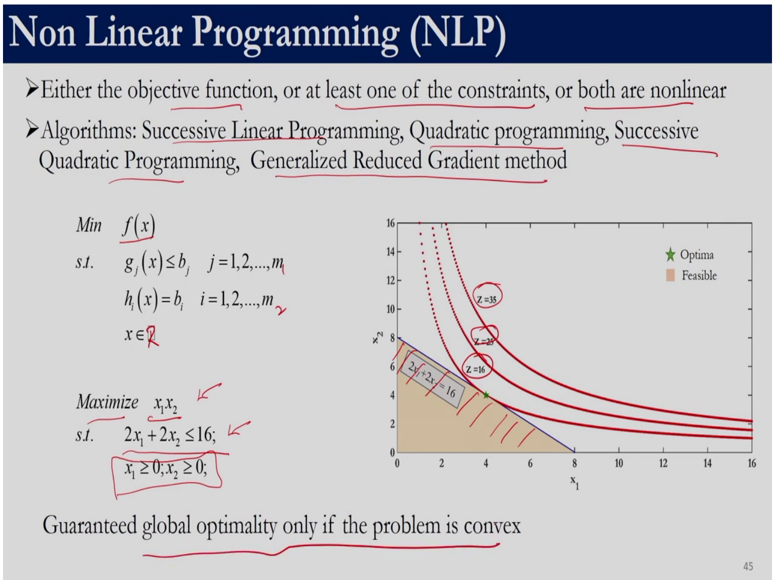

This is an example of non-linear programming problem. In fact, this is a quadratic programming problem, right. So, we have, we want to maximize x 1 x 2 subject to the constraint that 2 x 1 plus 2 x 2 is less than equal to 16, right. So, this is a linear constraint, the objective function is quadratic right and x 1 and x 2 should be greater than or equal to 0.

So, we can again draw this line. So any line any region below this is the feasible region. So, the shaded part is the region. So, these three lines show the value of the objective function right so; but for this line since the problem is to maximize even though z is equal to 35 is a better value than z is equal to 16, it is not in the feasible region, right. Whereas, this z is equal to 16 is in the feasible region, it touches this point, right. So, that is the optimal solution in this case, right.

So, again here we are not looking into how we are solving, how we are going to solve a non-linear programming right; here we are just introducing you to what is a non-linear programming problem. So in a non-linear programming problem global optimality is guaranteed only if the problem is convex or it has some special properties, right.

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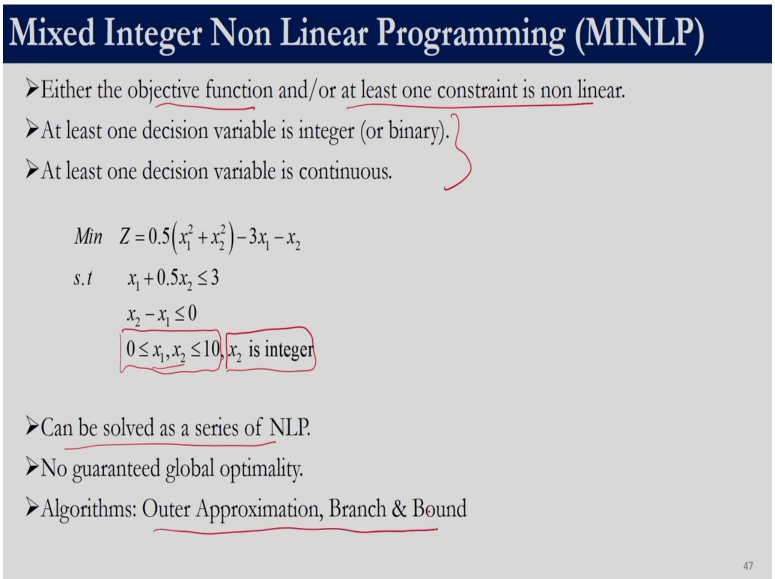

So, in a mixed integer non-linear programming either the objective function or at least one of the constraint is non-linear. Similar to mixed integer linear programming there is a either at least one a continuous variable and one at least one discrete variable. So, this is an example of an MINLP, right. So, it involves two variables x 1 and x 2 that, both x 1 and x 2 lie in the region 0 to 10; but x 2 is also an integer, right, so continuous values for x 2 is not allowed.

So, mixed integer linear programming problems can are usually solved as a series of NLP, right. For an arbitrary mixed integer linear programming, there are no algorithms which can you guarantee that the solution which has been found is globally optimal solution. So that is why MIN, some of the algorithms which are used to solve mixed integer linear programming or outer approximation and branch and bound.

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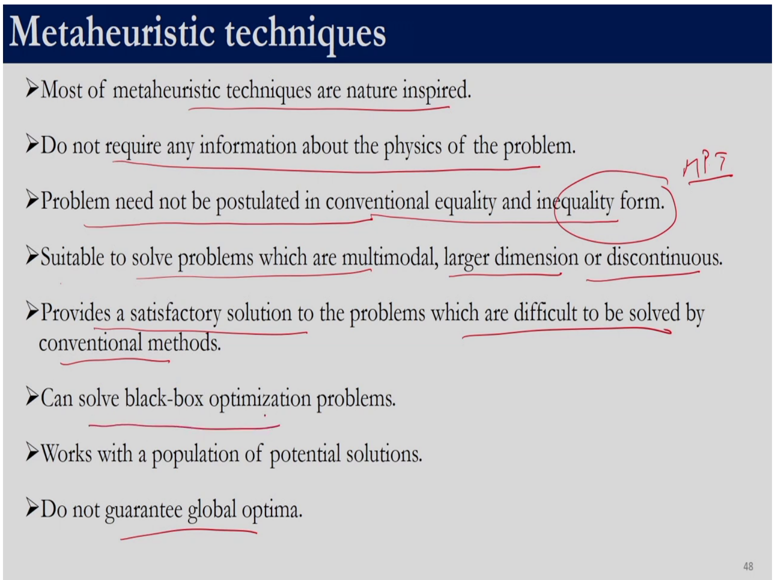

So, the other class of techniques is metaheuristic techniques, right. So, most of these techniques are nature inspired; they do not require any information about the physics of the problem. So, it is more like a black box communication, right. So, in this case the problem need not be potion the conventionally equality or inequality form, which was a drawback in the mathematical programming techniques right; here we do not have that requirement, right.

So, it is suitable to solve problems which are multimodal, they have a large number of decision variable and continuous, large number of decision variable or constraints and some of the functions involved in the problem are discontinuous. So, it does not give guarantee on the optimality of the solution; but usually it gives a satisfactory solution especially for problems which are difficult to be solved by a conventional method.

Over here we buy conventional method, we need the mathematical programming techniques and it can also be used to solve black box optimization problems, right. So, in black box optimization problem you do not have a mathematical formulation; but given a solution you have some kind of mechanism wherein the quality of the solution can be determined, right. So, those are black box optimization problems.

So, for black box optimization problem, it is not possible to use mathematical programming techniques, most of the times we rely on metaheuristic techniques. So, this is a Goldberg's view. So, Goldberg was one of the pioneer who worked in metaheuristic techniques, right.

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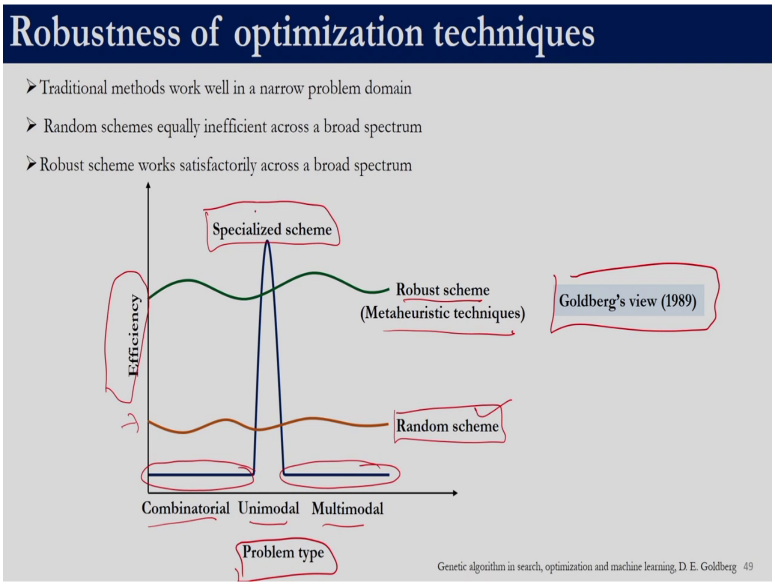

So, these are the various types of problem a unimodal problem, multi model. So, we have seen, what is unitmodal problem, multi model problem; combinatorial problems are those which involve large number of integer variables, right. So, this is problem type on the x axis; over here we have efficiency of the technique, right.

So if we employ some random scheme right, it is going to perform poorly, right. So, this is not a very high efficiency, it is going to perform very poorly on all the problems, right. Whereas, if we used specialized schemes; so for example, linear programming simplex method that is a specialized scheme. So, it is going to work extremely well for unimodal problems or that particular class of problems right; but over, but otherwise it is going to have a very poor performance, in fact performance a poor than any random scheme.

So, by random scheme we mean you give an optimization problem; let us say we randomly decide that these are the values of x 1, x 2 and x 3 the decision variables and it happens to satisfy the constraints, right. So, that is a random scheme right; whereas this metaheuristic techniques are known as robust scheme, right, so it does not have a very high efficiency. Similar to a specialized scheme, let us say for unimodal problems; but across a wide range of problems it can be used, right.

So, that is the advantage of metaheuristic techniques. It does not give guarantee even if for a special category of problem; but overall it gives a satisfactory performance for most of the optimization problems, right. So, our objective in this course is to have a knowledge of metaheuristic techniques as well as some of the specialized scheme, right. So, if we have a problem which actually falls into let us say linear programming or mixed integer linear programming; we would not choose metaheuristic techniques, if we are aware of the specialized schemes, right.

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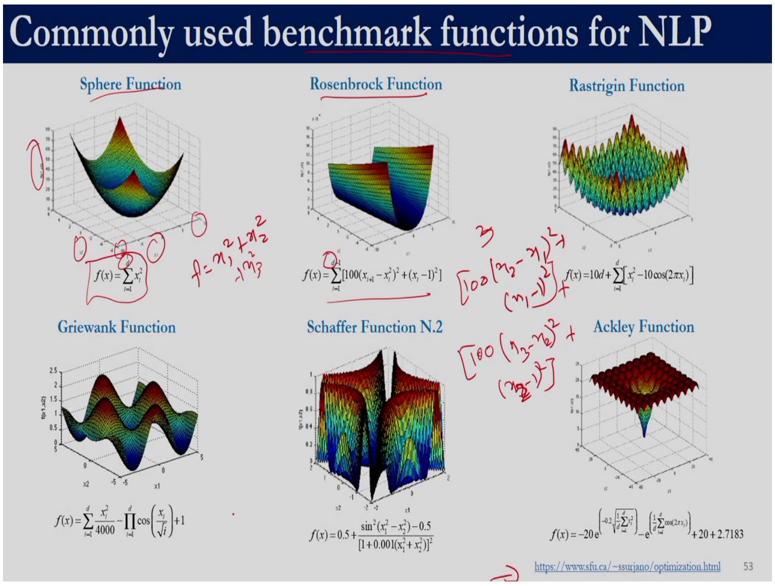

When I metaheuristic technique is designed, it is usually tested on what is called as benchmark functions, right. So for this benchmark functions the optimize already known. So, given any procedure if we want to save whether the scheme is working or not, right. So, what we will do is, we will test it on this benchmark functions, right. So, since the optimize already known for the benchmark functions; if the scheme is able to find the optimal solutions, then the scheme or the technique is said to be a reasonably good technique, right.

So, some of the benchmark functions that we will be referring in this course is listed over here. So, the spear function; so sphere function, the objective function is f is equal to x 1 squared plus x 2 square let us say, for a two variable problem x 1 and x 2. So, this is how the objective function will behave in the search space between minus 10 and 10, right for x 1 and x 2. So, this is the objective function, right.

So, these are called a scalable function. So, I can also have x 3 square, x 1 squared plus x 2 plus x 3 square; this figure is then no longer valid, but in comp, but the objective function for sphere function is written like this. So, these are called a scalable function, we can make this objective function for two variable problem, three variable problem, hundred variable problem, thousand variable problem, right.

So, that is what this d. So, this d determines the number of decision variables. So, that has to be fixed by the user, right. So, by fixing the d, we can test a metaheuristic techniques whether they are able to determine the optimal solution or not, right. So, this is another function Rosenberg function, right. So, the objective function here is as given over here, right.

So here if we have a let us say two variable problem, so the objective function is 100 x 2 minus x 1 the whole square plus x 1 minus 1 the whole square, right. So, this summation does not come into picture if we have two variable; but if we had three variable, then it will be this right plus 100 x 3 minus x 2 the whole square plus x 3 minus 1 the whole square x 2 minus 1 the whole square and so on.

So, again by changing this d, we can actually have a objective function of multiple variables, right. So, this is. So, here if you go into this link, you will be able to see all these functions right and MATLAB codes for all these functions are available. So, if you want to test a particular technique as we will do in the later half a later part of the course; we can use this functions to test the efficiency of a particular techniques. So these are some of the other commonly used benchmark functions, right.

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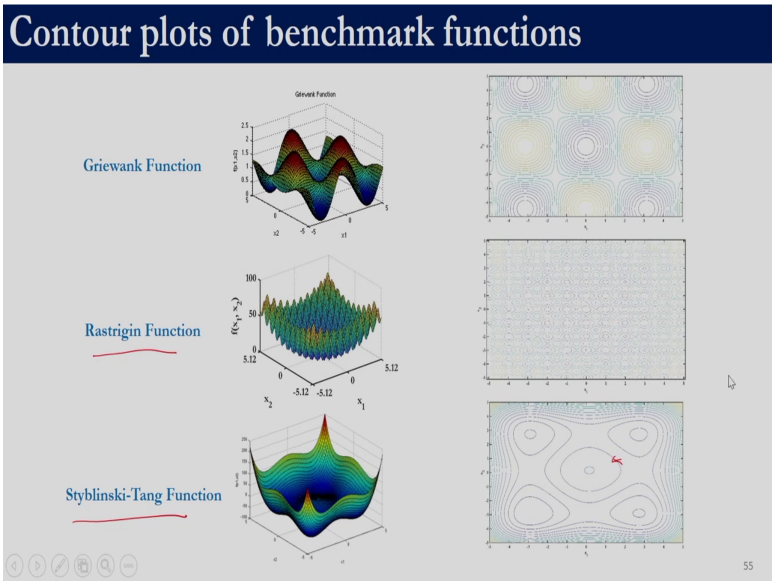

So, this shows the contour plots, right. So, Rastrigin function you can have a look at, it is objective function given in the previous slide. So, x 1 and x 2 this is how the search space looks like; remember the contour plots are lines of objective function which have the same objective function value, right. So, similarly over here for this function if you see, all the points in this particular on this particular curve have a similar objective function, right.

So, the our task is to find out the best value of x 1 and x 2 in this region. So, as you can see sometimes this region can be really complex, right. So, a random scheme would not be able to find out the optimal solution in this complex regions, right. So the metaheuristic techniques that we will discuss as part of this course would be able to find out the globally optimal solution even with this complex functions.

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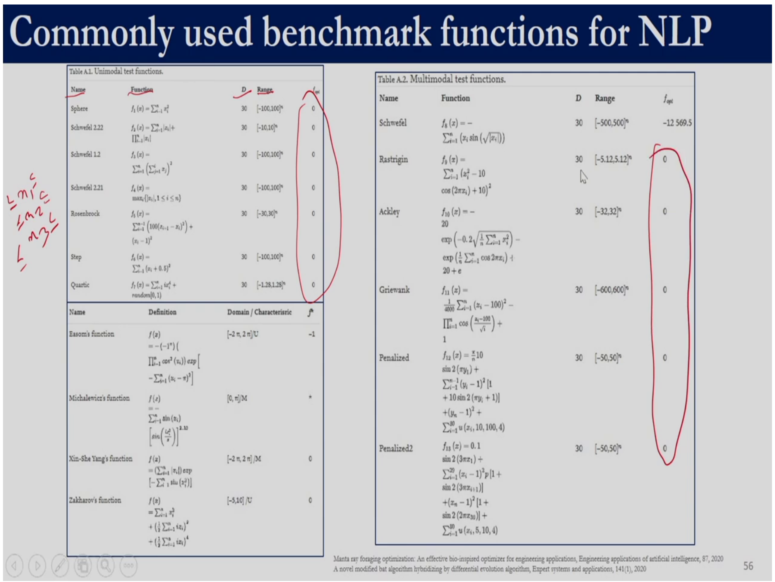

These are some of the functions. So, over here if we see this is the name of the function; the actual function has given d denotes the how many dimension, this is the range of the problem. So, if you have let us say three variable x 1, x 2, x 3; what is their domain? So, the domain here is minus a 100 to 100 for all the three variables. So, again in the later half of the course we will test the optimization techniques that we study on these problems.

So, for most of this problem if you see the optimize actually 0; the objective function value is 0, except in a few cases. So, these are the benchmark functions, right.

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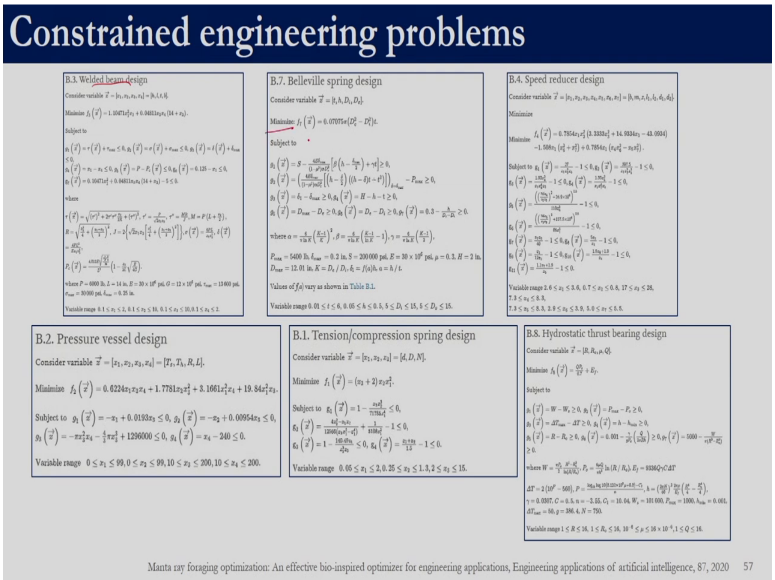

So, very often the techniques are also tested on engineering problems, right. So, right now we just want you to be aware that these problems exist, right. As and when required in the course we will be using them, right. So, these are some of the constraint problems, right so far welded beam, these are the equations. So, these are already available. So, the task for the technique is to find out the optimal value such that whatever that function is either minimum or maximum.

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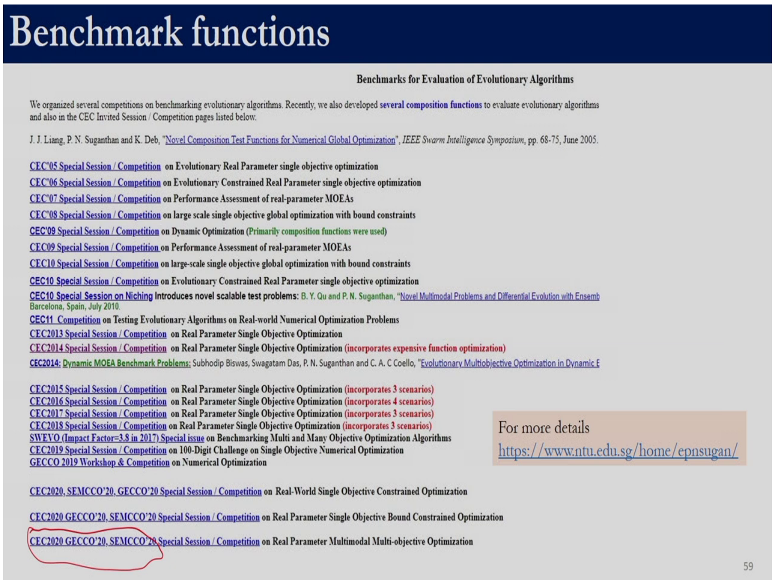

For metaheuristic techniques there are standard conference, there are prestigious conferences. So, for example, CEC, GECCO all these are names of special, all these are names of reputed conferences. So, every year when these conferences are conducted, they have competition. So, they give out problems, not the benchmark functions which you saw previously; there are some drawbacks with the benchmark functions which you have seen previously. We will discuss it later, right.

But these every year as part of these conferences; this benchmark functions are given to the user. So, a user is supposed to design techniques and demonstrate its performance on these problems; these are available for most of the programming languages like C, MATLAB the functions are already available, you can you do not need to code the objective function as such, you just need to evaluate whatever technique you have proposed on these functions.

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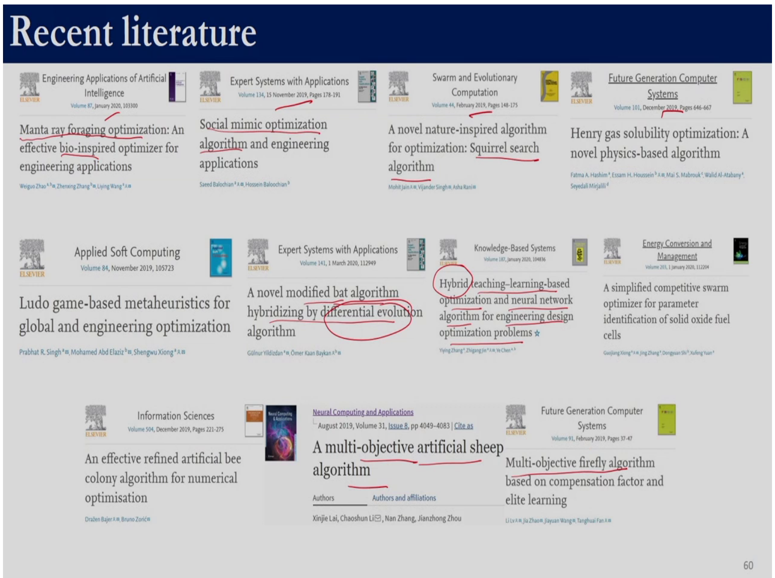

So, this is some of the recent literature right. So, metaheuristic techniques are hot area of research, right. So, all these are papers which are published in 2019-20, right. So, all this is 2020. So, this is again a new algorithm, new metaheuristic algorithms; so bio inspired right, where the name of the algorithm is manta ray foraging optimization, this is social mimic optimization algorithm, this is squirrel search algorithm, this is based on squirrel search algorithm, right.

And then there are algorithms which are modified often, right. So, for example, bat algorithm already exists. So, this paper proposes a hybridizing bat algorithm with differential evolutions. So, we will be looking into differential evolution as part of this course, right. So, we will be looking into teaching learning based optimization, right. So, over here in this work they have a hybridized with the neural network for engineering design of optimization problem.

These are a couple of examples for newly proposed multi objective optimization. So, for example, here if you see it is a multi-objective artificial sheep algorithm. So, this algorithm has been probably designed based on the behavior of sheep's, right. Here again we have multi objective firefly algorithm. So, you might know about fireflies, right. So, depending upon their behavior, this multi objective algorithm has been designed.

So the point that I am trying to reinforce is; once you learn this 4 or 5 once we learned few of the metaheuristic techniques, you will be in a position to actually understand many of this papers right and critically review this work and also possibly propose your own algorithm.

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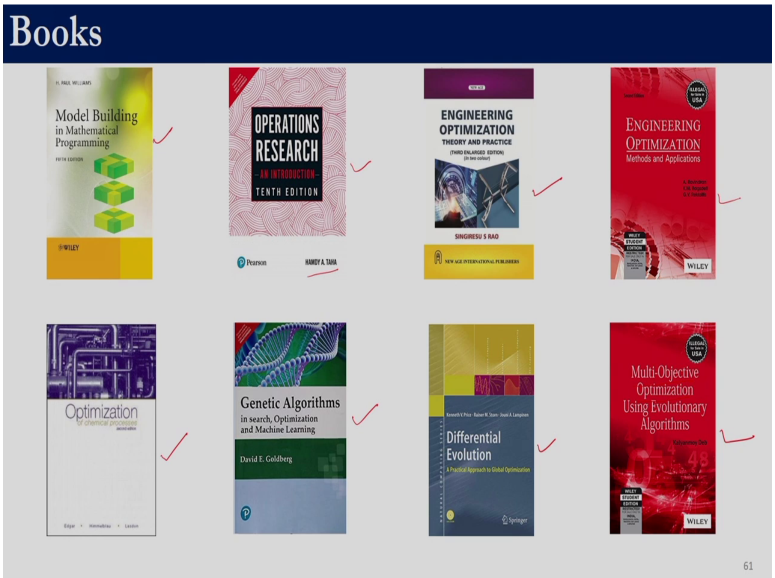

So, these are some of the classical optimization books, right. So, this book particularly helps in developing models, right. So, we will be touching upon developing models in this course; but greater details are available in this book, Model Building in for Mathematical Programming. This is a classical book by TAHA Operation Research this is again a standard textbook for a Engineering Optimization by S S RAo; this one is by Reklaitis and Coworkers, right.

So, this is Goldberg's book on a genetic algorithm; differential evolution by Rainser Storm, right. And this is another classical book by Kalyanmoy Deb, right. And we also have this optimization of chemical processes. So, since this course has also been listed under chemical engineering; those are few who are chemical engineers can actually look into this book wherein you have examples from chemical engineering.

Again whatever we are going to discuss as part of this course does not necessarily borrow any concepts from any particular engineering field right; whatever we are discussing is going to be very generic, you can apply it to your own area. With that we will be concluding this session; in this session we have seen components of optimization problem, classification of optimization problems as well as classification of optimization techniques.

Then we looked into multi objective optimization right; in this course we are not going to see how to solve a multi objective optimization problem, but at least broadly you know the difference between single objective optimization and a multi objective optimization. We followed this up with how to solve a single and multi-variable optimization problem in the absence of constraints, right. So, after that we looked into metaheuristic techniques, right.

So, we just give you a brief introduction to metaheuristic techniques and the major portion of this course is going to be on a metaheuristic techniques, right. So, the next session is going to be on a linear regression, right. So, the linear regression we have divided into three sessions, right. So, first we will be looking at linear regression, simple linear regression; then we will be extending that to solve polynomial regression problem and multi linear regression problem.

And we will also look into general linear least squares, right. So, the final session would be on how to use the inbuilt function of MATLAB to solve linear and non-linear regression problems. So, for those of you who are not interested in learning MATLAB, you can skip that particular session, right. So, the cuisine assignment will not have anything from that particular implementation session, right. So, with that we conclude the session.

Thank you.