Mass Transfer Operations II
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Lecture No. 22
Electric field enhanced membrane separation processes

Welcome back to mass transfer operations II we were discussing on membrane separation technologies. Now, we will be discussing on electric field enhanced membrane separation processes.

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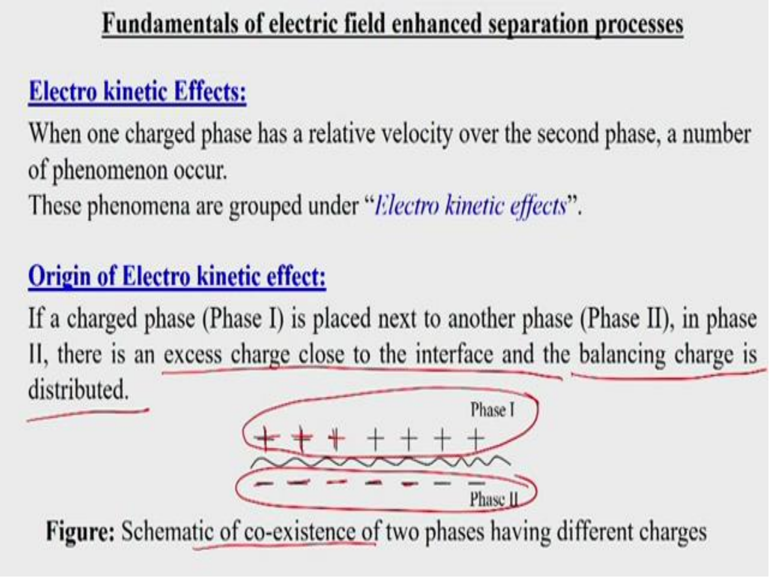

Fundamentals of electric field enhanced separation processes

Electro kinetic Effects:

When one charged phase has a relative velocity over the second phase, a number of phenomenon occur.
These phenomena are grouped under "*Electro kinetic effects*".

Origin of Electro kinetic effect:

If a charged phase (Phase I) is placed next to another phase (Phase II), in phase II, there is an excess charge close to the interface and the balancing charge is distributed.

Figure: Schematic of co-existence of two phases having different charges

So, before this entering into the details of this electric field enhanced membrane separation processes. We need to understand this electro kinetic effect and electro kinetic phenomena. So that is we can say these are fundamentals of the electric field enhanced processes. Though when one charged phase has relative velocity over the second phase, a number of phenomena occur. So these phenomena is actually are grouped under this electro kinetic phenomena.

So, so many phenomena will happen so, and we will be discussing one by one, so first we need to understand the origin of the electro kinetic effect like if phase suppose this phase 1 is placed next to another phase 2 like this suppose this phase 1 is the positively charged phase is like this, and this negatively charged phases are placed just one after another then there is an excess charge close to the interface and the balancing charge is distributed. So that is it is schematically shown the coexistence of 2 phases having different charges. So, whenever the 2 phases are allow to remain in this equilibrium condition then we can say the at the end of these phenomena this electro neutrality will prevail.

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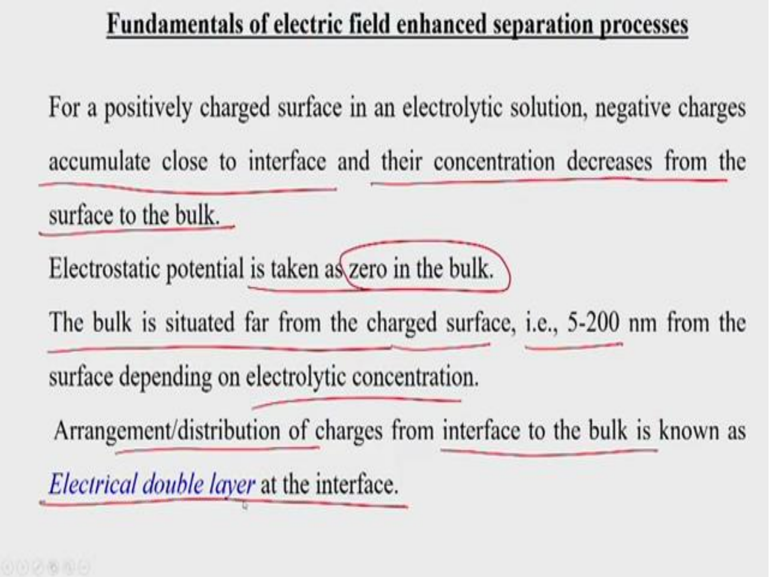

Fundamentals of electric field enhanced separation processes

For a positively charged surface in an electrolytic solution, negative charges accumulate close to interface and their concentration decreases from the surface to the bulk.

Electrostatic potential is taken as zero in the bulk.

The bulk is situated far from the charged surface, i.e., 5-200 nm from the surface depending on electrolytic concentration.

Arrangement/distribution of charges from interface to the bulk is known as *Electrical double layer* at the interface.

So, for a positively charged surface like in an electrolytic solution so negatively charged particles will be accumulate close to the interface due to their we can say electrostatic force of attraction and their concentration decreases from the surface to the bulk. So we will be showing this an schematically that how this co ions and counter ions will be changing their concentrations from the charge surface, say suppose positively charged to the bulk or negatively charged surface to the bulk.

So electrostatic potential is taken as 0 in the bulk so that actually we can say that is nothing but the electro neutrality condition. So the bulky situated for from the charge surface, so that sometimes, 5 to 200 nanometer from the surface depending on the electrolytic concentration. So this arrangement or distribution of the charges from this interface to the bulk is known as the electric double layer at the interface. So at any in the surface when suppose positively charged particles are there and just after that or we can say in the vicinity these negatively charged particles will be forming one layer so that is called we can say electrical double layer.

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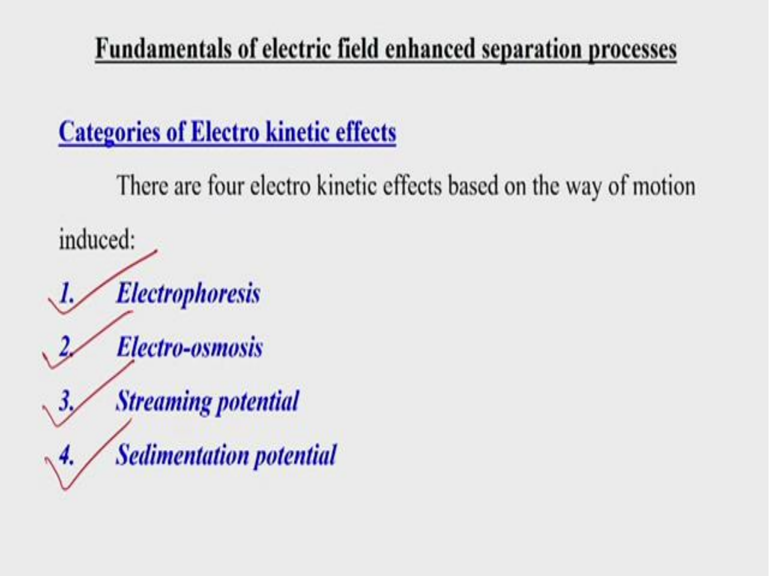
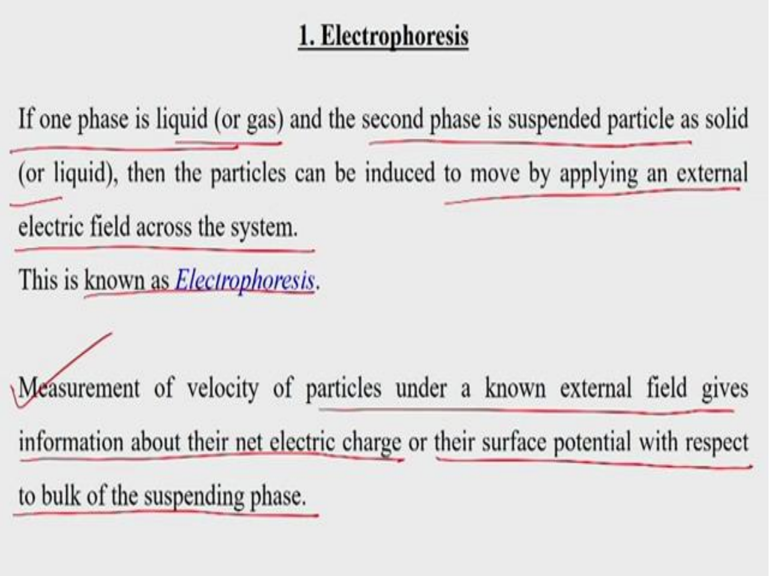


Fundamentals of electric field enhanced separation processes

Categories of Electro kinetic effects

There are four electro kinetic effects based on the way of motion induced:

1. *Electrophoresis*
2. *Electro-osmosis*
3. *Streaming potential*
4. *Sedimentation potential*



1. Electrophoresis

If one phase is liquid (or gas) and the second phase is suspended particle as solid (or liquid), then the particles can be induced to move by applying an external electric field across the system.

This is known as *Electrophoresis*.

Measurement of velocity of particles under a known external field gives information about their net electric charge or their surface potential with respect to bulk of the suspending phase.

So there are 4 different types of electro kinetic effects, one is this electrophoresis, second one is electro-osmosis, third one is streaming potential and fourth one is sedimentation potential. We will be discussing one after another the briefly we will be discussing all these 4 different electro kinetic phenomena. The first one is this electrophoresis, if one phase is liquid or gas and the second phase is suspended particles as solid or may be liquid then particles can be induced to move by applying an external electric field across the system. so this is known as the electrophoresis. Like measurement of velocity of the particles under a known external field gives the information about their net electric charge. Or their surface potential with respect to the bulk of the suspended phases.

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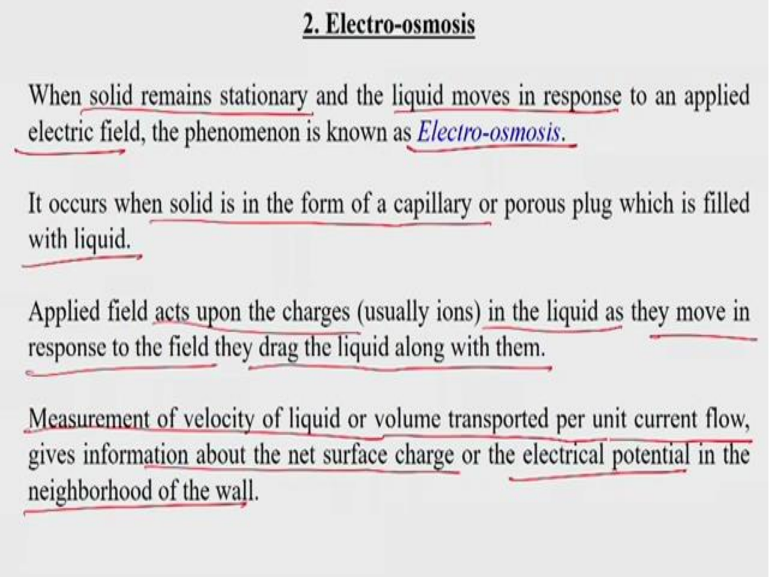

2. Electro-osmosis

When solid remains stationary and the liquid moves in response to an applied electric field, the phenomenon is known as *Electro-osmosis*.

It occurs when solid is in the form of a capillary or porous plug which is filled with liquid.

Applied field acts upon the charges (usually ions) in the liquid as they move in response to the field they drag the liquid along with them.

Measurement of velocity of liquid or volume transported per unit current flow, gives information about the net surface charge or the electrical potential in the neighborhood of the wall.

So this next one is this electro-osmosis like in this case the solid remains stationary and the liquid actually moves in response to an applied electric filed like this, so if the electric field is applied and liquid will move and solid remain in the porous material like this so solid remains in a stationary field. Then the phenomena is known as electro osmosis it occurs when solid is in the form of a capillary or porous plug which is filled liquid like this.

So applied field acts upon the charges like this in the liquid as they move in response to the field they drag the liquid along with them so in side this suppose, porous material suppose liquid is filled then suppose applied electric field is there. Then we say, this is the phenomena of electro osmosis. Like measurement of velocity of the liquid or volume transported per unit current flow gives the information about the net surface charge or the electrical potential in a neighborhood of the wall. So that is we can say the phenomena electro kinetic phenomena as electro osmosis.

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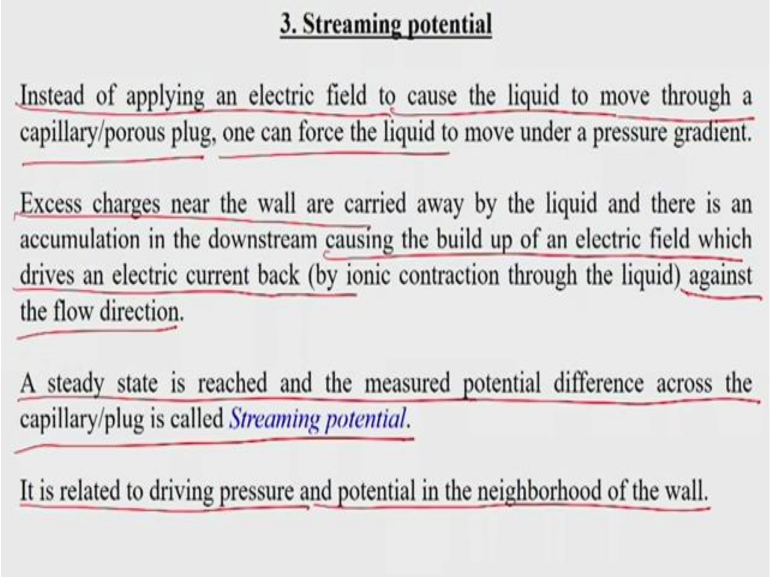

3. Streaming potential

Instead of applying an electric field to cause the liquid to move through a capillary/porous plug, one can force the liquid to move under a pressure gradient.

Excess charges near the wall are carried away by the liquid and there is an accumulation in the downstream causing the build up of an electric field which drives an electric current back (by ionic contraction through the liquid) against the flow direction.

A steady state is reached and the measured potential difference across the capillary/plug is called *Streaming potential*.

It is related to driving pressure and potential in the neighborhood of the wall.

The third one is the streaming potential so instead of we can say the applying an electric filed like this, in case of this electro osmosis, instead of that to cause the liquid to move thrho capillary or porous plug, one can force the liquid to move under the pressure gradient so, that time that steady state will be reached actually and the major potential difference across the capillary plug is called streaming potential. So what will happen this excess charges near the wall are carried away by the liquid and there is an accumulation in the downstream causing the buildup of an electric field which drives an electric current back against the flow direction. So it is related to driving pressure and potential in a neighborhood of the wall. So that is we can say it is a streaming potential.

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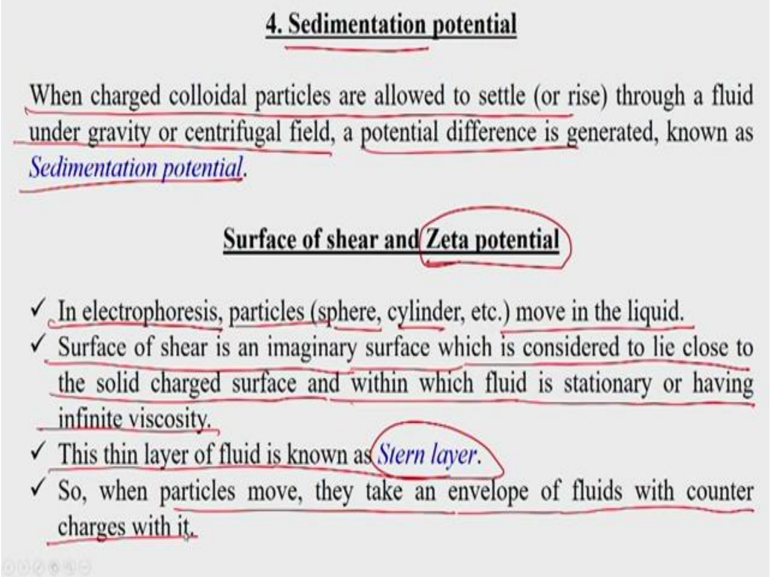

4. Sedimentation potential

When charged colloidal particles are allowed to settle (or rise) through a fluid under gravity or centrifugal field, a potential difference is generated, known as *Sedimentation potential*.

Surface of shear and Zeta potential

- ✓ In electrophoresis, particles (sphere, cylinder, etc.) move in the liquid.
- ✓ Surface of shear is an imaginary surface which is considered to lie close to the solid charged surface and within which fluid is stationary or having infinite viscosity.
- ✓ This thin layer of fluid is known as *Stern layer*.
- ✓ So, when particles move, they take an envelope of fluids with counter charges with it.

The fourth one is the sedimentation potential the name suggested that something is sedimenting or is allowed to settle like this when charged colloidal particles are allowed to settle like this, through fluid under the gravity or under the centrifugal field also a potential difference or gradient actually is generated that is known as the sedimentation potential. Now, we need to understand about the surface of the shear and zeta potential specially this zeta potential that is very important parameter for the electric field enhanced in separation processes.

And for that actually we need to know that in electrophoresis, particles or we can say they may be sphere cylindrical or any type of materials move in this in the liquid and surface of the shear is an imaginary surface we can say which is considered to be lie closed to the solid charged surface and within which fluid stationary or having infinite viscosity. This thin layer of fluid is called stern layer or in the stern layer the effect of electric field is pronounced. So when particles move they take an envelope of fluid with counter charges with it.

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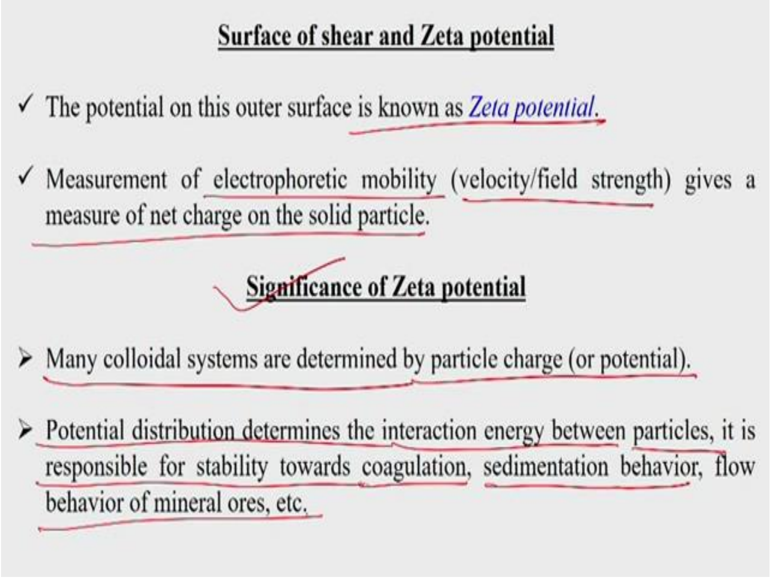

Surface of shear and Zeta potential

- ✓ The potential on this outer surface is known as *Zeta potential*.
- ✓ Measurement of electrophoretic mobility (velocity/field strength) gives a measure of net charge on the solid particle.

Significance of Zeta potential

- ➢ Many colloidal systems are determined by particle charge (or potential).
- ➢ Potential distribution determines the interaction energy between particles, it is responsible for stability towards coagulation, sedimentation behavior, flow behavior of mineral ores, etc.

So the potential on the outer surface is known as the zeta potential we will be discussing on so in detail about the zeta potential how it is measured for different charged particle systems. And measurement of electrophoretic mobility or we can say this is a velocity of field strength gives a measure of net charge on the solid particle.

Now the significance of the zeta potential value is very much pronounced and if we know the zeta potential value of any system from there we can predict that as how the solute particle will be behaving towards the charged surfaces or cathode or anode or what type of charged plates are used. So many colloidal systems are determined by particle charges or a potential, potential distribution determines the interaction energy between the particles it is responsible for stability toward this coagulation sedimentation behavior then flow behavior of the mineral ores so many things.

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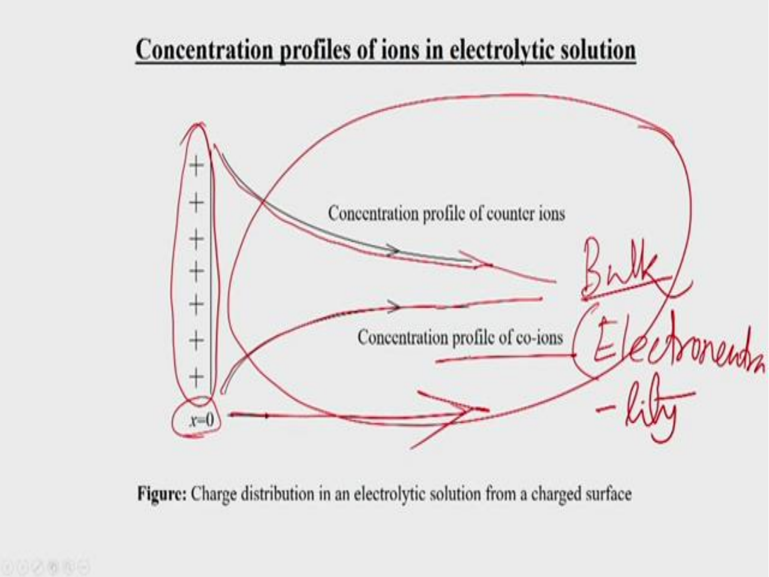

Concentration profiles of ions in electrolytic solution

Figure: Charge distribution in an electrolytic solution from a charged surface

Now, we will be discussing about the concentration profile of the ions in electrolytic solution. Let us take this positively charged surface is here, so we can say this is the bulk so this is bulk, and this is x is equal to 0 means where surface starts and it is in this direction the concentration of the concentration profile of the co-ions or we can say the plus ion so we can say is increasing, whereas this concentration of the we can say counter ion or negatively charged ion will be decreasing here, and it will be meeting somewhere, where electro neutrality will prevail. So electro neutrality, electro neutrality will be prevailed actually at the bulk.

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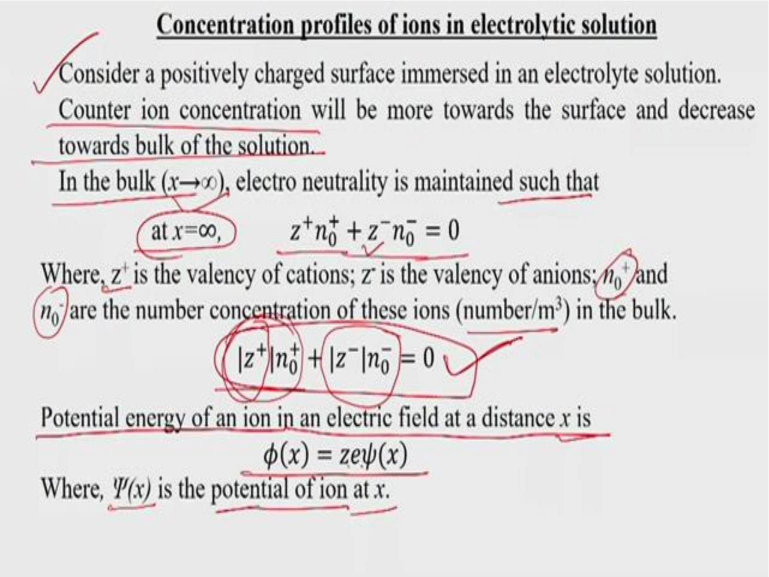

Concentration profiles of ions in electrolytic solution

Consider a positively charged surface immersed in an electrolyte solution. Counter ion concentration will be more towards the surface and decrease towards bulk of the solution.

In the bulk ($x \to \infty$), electro neutrality is maintained such that

at $x = \infty$, $$z^+ n_0^+ + z^- n_0^- = 0$$

Where, z^+ is the valency of cations; z^- is the valency of anions; n_0^+ and n_0^- are the number concentration of these ions (number/m^3) in the bulk.

$$|z^+| n_0^+ + |z^-| n_0^- = 0$$

Potential energy of an ion in an electric field at a distance x is

$$\phi(x) = ze\psi(x)$$

Where, $\Psi(x)$ is the potential of ion at x.

So the concentration profile of the ions in side this electrolytic solution will be like this now we say this a, consider positively charged surface immersed in an electrolytic solution like this and counter ion concentration or negatively charged concentration will, be more towards the surface and decreases towards the bulk of the solution that is why counter ion concentration actually decreases and at the bulk the concentration of both the co ion and counter ions will be counter balanced. So in the bulk where x tends to infinity or where electro neutrality actually is maintained such that at x is equal to infinity or where at the farthest part of this bulk the z plus into n0 minus plus z minus into n0 minus that will be 0, where z plus is the valency of the cations and z minus is the valency of the anions.

And n0 plus and n0 minus actually are the number of concentration of these ions like this we can say cations and anions that we can say number per meter cube or may be concentration in other form also in the bulk. So the according to this electro neutrality principle this how much amount of these cations are presents or the net amount of the positively charged particles and net amount of negatively charged particles will give this 0 potential or no cationic or anionic charges will be excess and it is also true that in the universe the electro neutrality prevails.

Where we have this positively charged particles then definitely in somewhere this negatively charged particles are present and net positively charge is equal to net negatively charge. So the potential energy of an ion in an electrolytic field at a distance of x is 5x is equal to z into e into psi x where we can say psi x is we can say is the potential of the ion at distance x is the electric charge.

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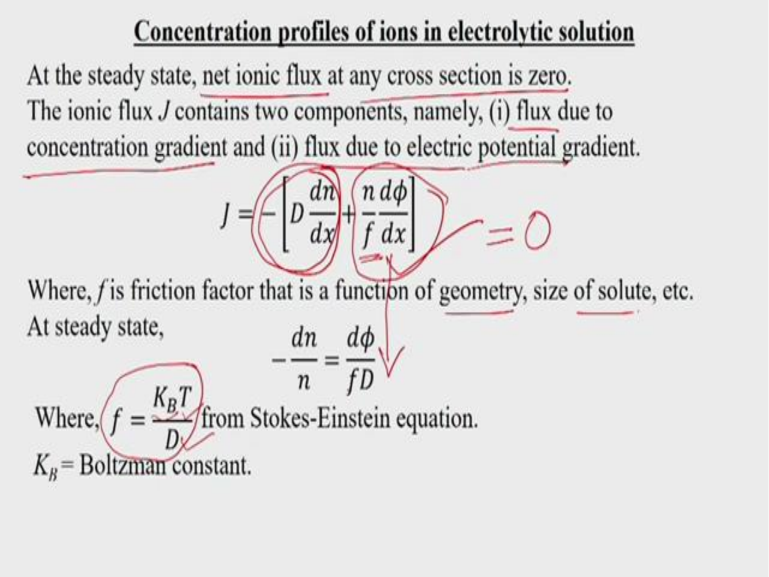

Concentration profiles of ions in electrolytic solution

At the steady state, net ionic flux at any cross section is zero.
The ionic flux J contains two components, namely, (i) flux due to concentration gradient and (ii) flux due to electric potential gradient.

$$J = -\left[D\frac{dn}{dx} + \frac{n}{f}\frac{d\phi}{dx}\right] = 0$$

Where, f is friction factor that is a function of geometry, size of solute, etc.
At steady state,

$$-\frac{dn}{n} = \frac{d\phi}{fD}$$

Where, $f = \frac{K_B T}{D}$ from Stokes-Einstein equation.
K_B = Boltzman constant.

So at this steady state we can say this net ionic flux at any cross section will be 0, and say ionic flux contains two parts like this one part will be, first part will be like this, due to this concentration gradient like as per the Darcy’s law and the second part will be this is first part as per first law of Fick’s first law and the second part will be this flux due to electric potential gradients that is this part is due to electric potential gradient.

So when, where this f actually is called is a friction factor that is the function of geometry, size of the solute and this f is equal to as per the Stokes Einstein equation KB into T by D where KBB is the Boltzmann constant T is the temperature D is the diffusivity. So at steady state this j will be equal to at steady state this j will be equal to 0 and after this manipulation we will be getting this minus dn by n is equal to d phi by f into D.

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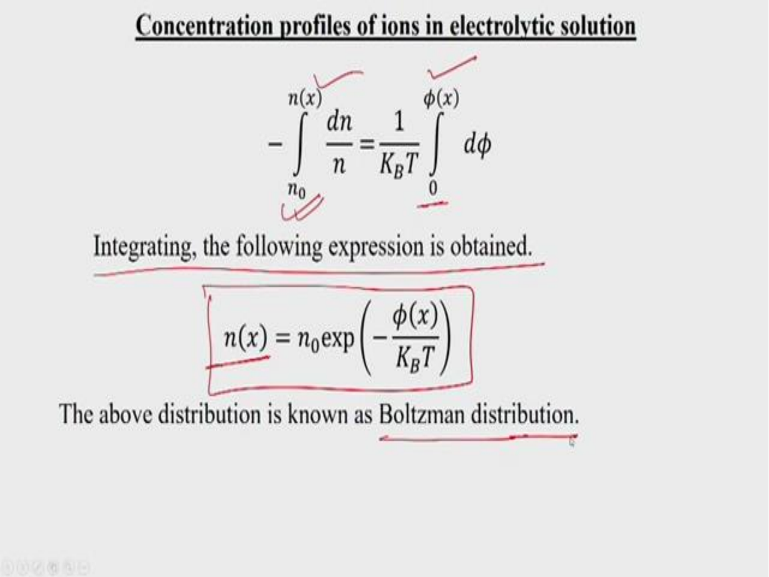

Concentration profiles of ions in electrolytic solution

$$-\int_{n_0}^{n(x)} \frac{dn}{n} = \frac{1}{K_B T}\int_{0}^{\phi(x)} d\phi$$

Integrating, the following expression is obtained.

$$\boxed{n(x) = n_0 \exp\left(-\frac{\phi(x)}{K_B T}\right)}$$

The above distribution is known as Boltzman distribution.

So we will because getting this one and if we integrate this equation like this minus dn by n is equal to d phi by KB into T from distance 0 to 5x and where this charge will be from n0 to nx so then if we integrate this expression then we will be getting nx is by n0 is equal to nx minus (ln) nx by n0 is equal to this one phi by KBB into T. just after that nx by n0 is equal to x potential minus 5x by KBB into T or nx is equal to n0 into x potential minus 5x by KBB into T. this equation is known as Boltzmann distribution.

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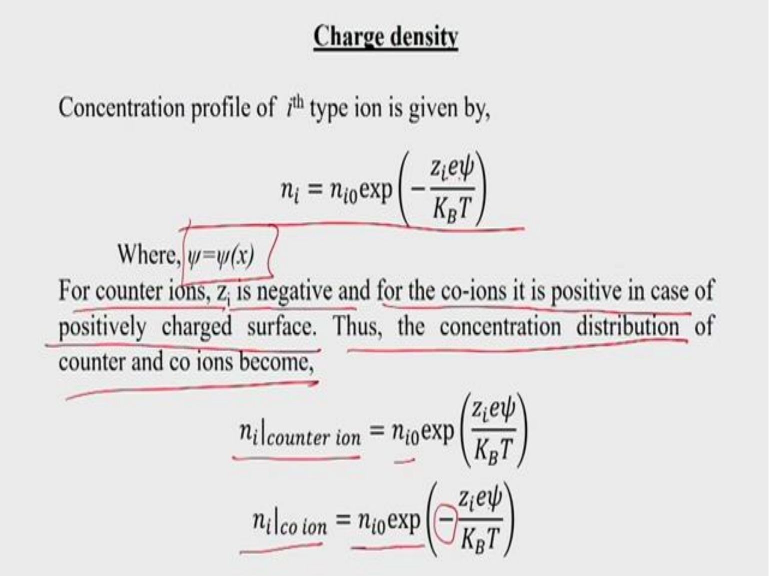

Charge density

Concentration profile of i^{th} type ion is given by,

$$n_i = n_{i0}\exp\left(-\frac{z_i e\psi}{K_B T}\right)$$

Where, $\boxed{\psi=\psi(x)}$

For counter ions, z_i is negative and for the co-ions it is positive in case of positively charged surface. Thus, the concentration distribution of counter and co ions become,

$$n_i|_{counter\ ion} = n_{i0}\exp\left(\frac{z_i e\psi}{K_B T}\right)$$

$$n_i|_{co\ ion} = n_{i0}\exp\left(-\frac{z_i e\psi}{K_B T}\right)$$

Now, we see concentration profile of the i th type ion actually is given by ni is equal to ni0 into exponential minus zi into e into 5 by KB into T. so where psi we can say is a function of x. so for counter ion this zi is negative and for co ion it is positive in case of the positively charged surface. So if it is reverse one or we can say if we say that the negatively charged surface is there we can say zi will be say we can say positive and we can say for the co ions if it is we can say negatively charged surface.

Thus the concentration distribution of the counter ions and co ions will become like this ni counter ion is equal to ni0 into exponential zi into e into psi by KBKB into T. And for co ion that will be this one negative sign actually is there ni0 into exponential into minus zi into e into 5 by KB into T. So this is this is the case where we can say the surface is positively charged if it is negatively charged then the notation will be reverse.

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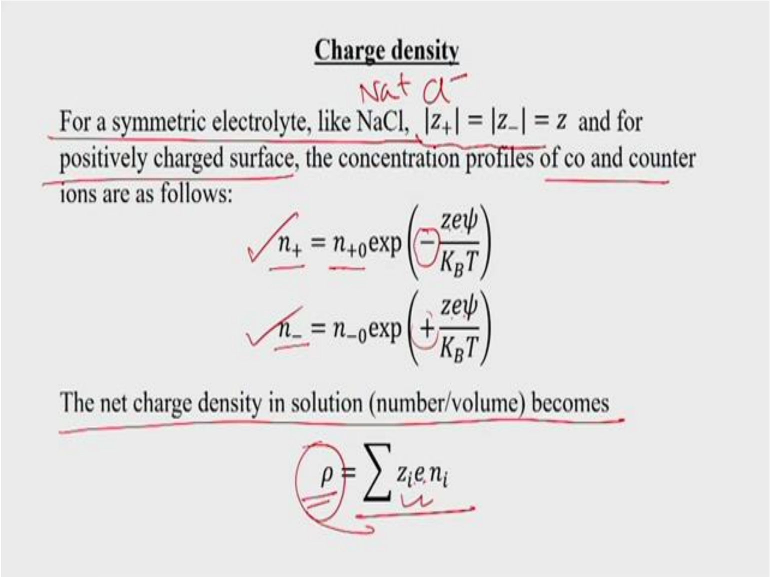

Charge density

$Na^+ \ Cl^-$

For a symmetric electrolyte, like NaCl, $|z_+| = |z_-| = z$ and for positively charged surface, the concentration profiles of co and counter ions are as follows:

$$n_+ = n_{+0}\exp\left(-\frac{ze\psi}{K_BT}\right)$$

$$n_- = n_{-0}\exp\left(+\frac{ze\psi}{K_BT}\right)$$

The net charge density in solution (number/volume) becomes

$$\rho = \sum z_i e\, n_i$$

And for symmetric electrolyte like NaCl like Na plus and Cl minus so z plus will be z minus is equal to z like this both are 1 and for positively charger surface the concentration profile of co ion counter ion will be like this, so only the difference will be this minus sign like this so n plus will be n plus 0 into exponential minus z into e into psi by KB into T. where n minus will be equal to n minus 0 exponential plus this one or we can say z into e into psi by KBKB into T. So the net charge density in the solution becomes this we can say the rho actually that is we can say this charged density that is summation of zi e into ni. So this ni will be like this ni will be like this and this, summation of this. So that is we can say rho is a net charge density.

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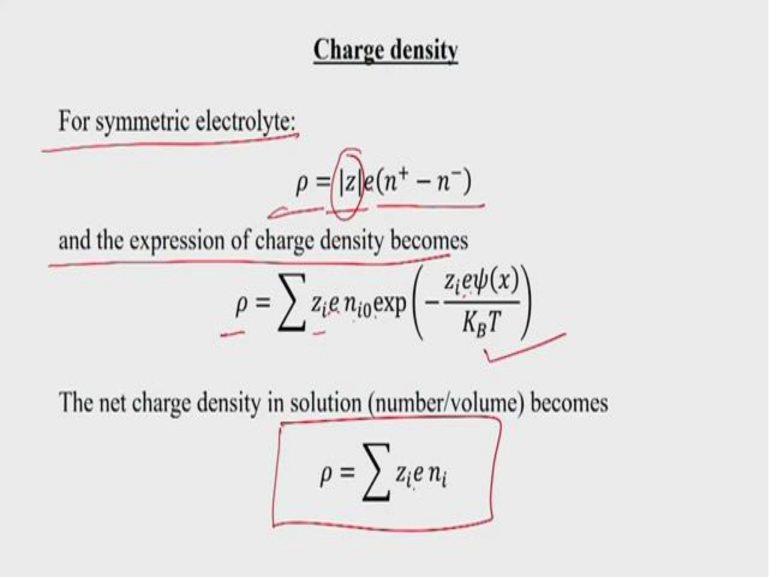

Charge density

For symmetric electrolyte:

$$\rho = |z|e(n^{+} - n^{-})$$

and the expression of charge density becomes

$$\rho = \sum z_i e\, n_{i0} \exp\left(-\frac{z_i e \psi(x)}{K_B T}\right)$$

The net charge density in solution (number/volume) becomes

$$\boxed{\rho = \sum z_i e\, n_i}$$

So that for symmetric electrolyte like Na plus Cl minus like this one where numbers of cations and anions are same that case we can say rho is equal to this modulus z into e into n plus minus n minus. So hence our expression of the charge density rho will be like this summation of zi into e into ni0 into exponential minus zi into e into psi by KB into T. so the net charge density in the solution that is we can say number per volume becomes like rho is equal to summation of zi into e into ni.

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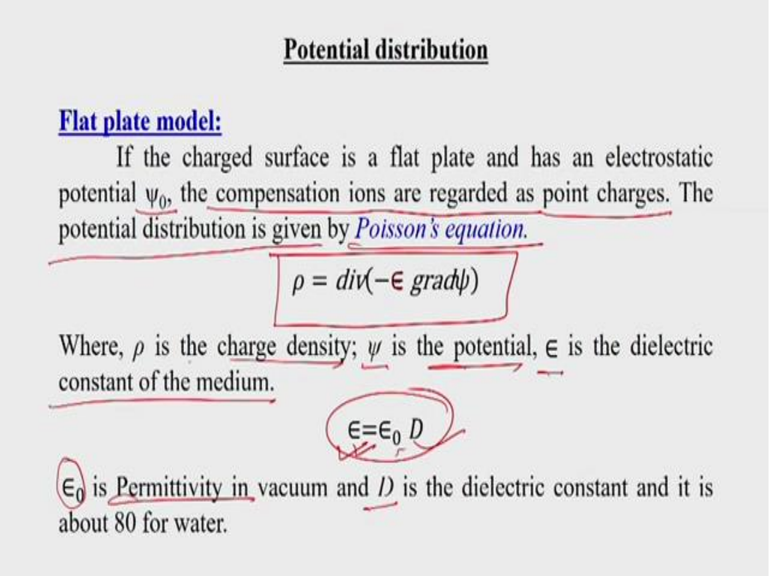

Potential distribution

Flat plate model:

If the charged surface is a flat plate and has an electrostatic potential ψ_0, the compensation ions are regarded as point charges. The potential distribution is given by *Poisson's equation.*

$$\boxed{\rho = div(-\epsilon\ grad\psi)}$$

Where, ρ is the charge density; ψ is the potential, ϵ is the dielectric constant of the medium.

$$\epsilon = \epsilon_0 D$$

ϵ_0 is Permittivity in vacuum and D is the dielectric constant and it is about 80 for water.

Now we say the we will be discussing about the potential distribution in the flat plate model like this if the charged surface is a flat plate and has an electrostatic potential psi 0 the composition ions are regarded as point charges so the potential distribution is given by these

Poisson's equation, this rho is equal to diversions of minus this eta into grad of psi where this rho we know the if we charge density. The psi is the we say electrochemical potential and eta is equal to is the dielectric constant of the medium. Where this eta is equal to eta 0 into D that is the diffusivity. D is the dielectric constant where eta0 is the permittivity in vacuum so, that is standard one compared to vacuum how much amount of permittivity is there in the electrolytic media so that is actually we can say this D is 80 for water.

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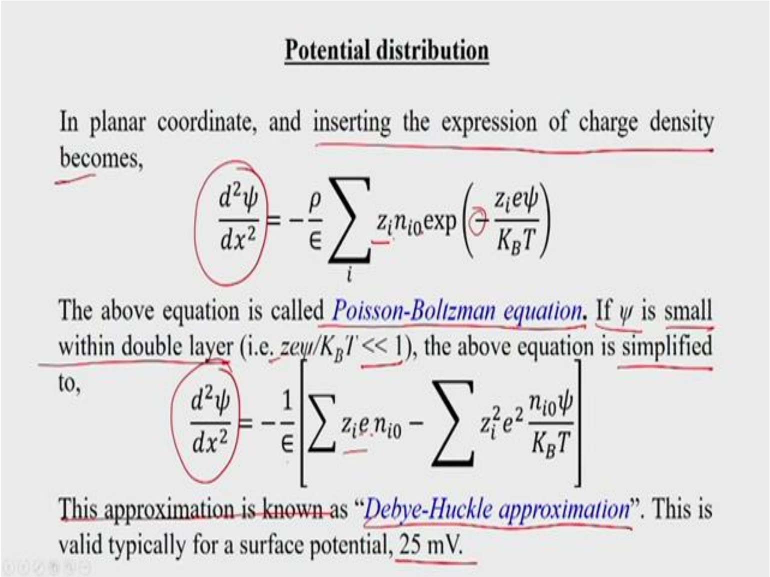

Potential distribution

In planar coordinate, and inserting the expression of charge density becomes,

$$\frac{d^2\psi}{dx^2} = -\frac{\rho}{\epsilon}\sum_i z_i n_{i0} \exp\left(-\frac{z_i e\psi}{K_B T}\right)$$

The above equation is called *Poisson-Boltzman equation*. If ψ is small within double layer (i.e. $ze\psi/K_BT << 1$), the above equation is simplified to,

$$\frac{d^2\psi}{dx^2} = -\frac{1}{\epsilon}\left[\sum z_i e\, n_{i0} - \sum z_i^2 e^2 \frac{n_{i0}\psi}{K_B T}\right]$$

This approximation is known as "*Debye-Huckle approximation*". This is valid typically for a surface potential, 25 mV.

Now, we see in planar coordinate the this one an inserting the expression of the charge density d square psi by dx square is equal to minus rho by epsilon and summation of zi into ni 0, into exponential minus zi e into psi KB into T. so this equation actually is called Poisson's Boltzmann constant we need to remember this one frequently will be using this one we have frequently also.

So the if this psi is small within this electric double layer that is z into e into psi KB by T will be much less than 1 or we say much much lesser than 1, the above equation will be simplified like this we can say d square psi by dx square is equal to minus 1 by epsilon into zi e into ni0 minus zi square into e square into ni0 psi by KB into T. So this approximation actually is known as Debye Huckle approximation so we will be doing this approximation this one for the calculation of Debye Huckle length also. This is valid typically for a surface potential at 125 millivolts.

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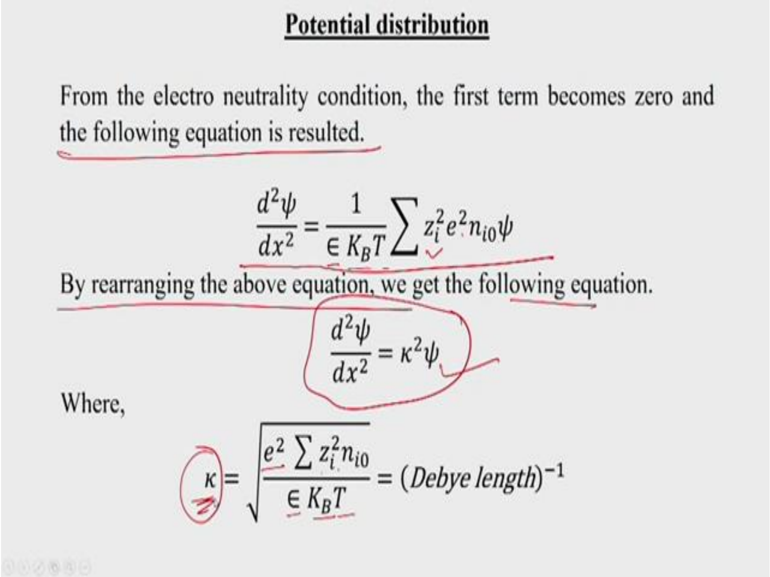

Potential distribution

From the electro neutrality condition, the first term becomes zero and the following equation is resulted.

$$\frac{d^2\psi}{dx^2} = \frac{1}{\in K_B T}\sum z_i^2 e^2 n_{i0}\psi$$

By rearranging the above equation, we get the following equation.

$$\frac{d^2\psi}{dx^2} = \kappa^2\psi$$

Where,

$$\kappa = \sqrt{\frac{e^2 \sum z_i^2 n_{i0}}{\in K_B T}} = (Debye\ length)^{-1}$$

So from the electro neutrality condition the first term becomes 0 of the previous equation and the following equation actually it is resulted like this d square psi by dx square will be equal to suppose this from the first equation this part will be going and this minus minus will be this plus one and then it will be like this one by epsilon KB into T and summation of zi square into ni0 into psi. By rearranging the above equation, we get the following equation like this d square psi is equal to just from there actually will be getting d square psi by dx square is equal to kappa square into psi.Where, kappa actually new term, we can say the reverse of the Debye length is equal to root over e square into summation of zi square into ni0 by epsilon into KB into T so this is , 1 by Debye length. This kappa is equal to 1 by Debye length.

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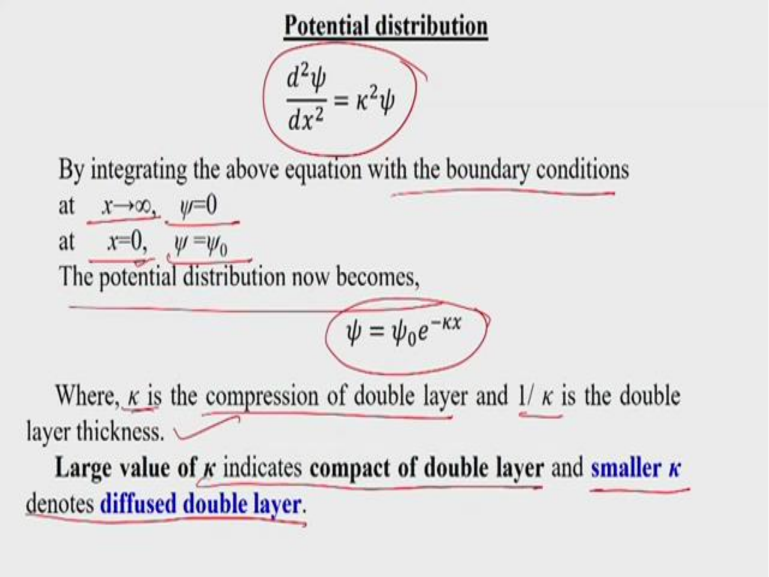

Potential distribution

$$\frac{d^2\psi}{dx^2} = \kappa^2\psi$$

By integrating the above equation with the boundary conditions

at $x \to \infty$, $\psi = 0$

at $x = 0$, $\psi = \psi_0$

The potential distribution now becomes,

$$\psi = \psi_0 e^{-\kappa x}$$

Where, κ is the compression of double layer and $1/\kappa$ is the double layer thickness.

Large value of κ indicates **compact of double layer** and **smaller** κ denotes **diffused double layer**.

Now we see that is from there if we put this kappa value in the previous equation will be getting d square psi by dx square is equal to kappa square into psi. Because a kappa is regarded as root over e square into summation zi square into ni0 by epsilon KBB into T. So by integrating the above equation using this boundary condition like this when x tends to infinity then psi will be 0, where x is equal to 0, means we can say at the interface then psi will be psi 0.

So the potential distribution will be like this from this above equation psi is equal to psi 0 into exponential minus kappa into x. the kappa called is compression of the double layer or by kappa is the double layer thickness. So large value of kappa actually indicates the compact double layer and smaller k value denotes the diffused double layer so from this kappa value we will be able to predict that whether this double layer is compacted or whether the double layer is diffused. So that we will be able to understand.

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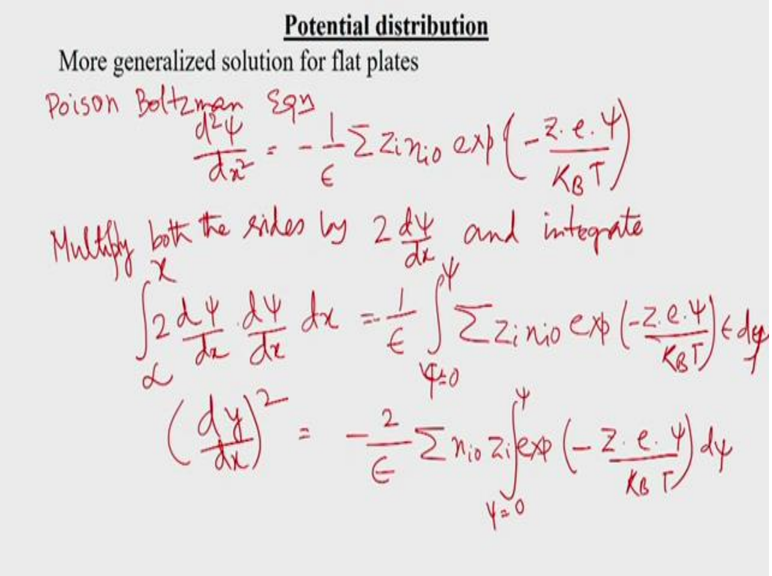

Potential distribution

More generalized solution for flat plates

Poison Boltzman Eqn

$$\frac{d^2\psi}{dx^2} = -\frac{1}{\epsilon}\sum z_i n_{io} \exp\left(-\frac{z \cdot e \cdot \psi}{K_B T}\right)$$

Multiply both the sides by $2\frac{d\psi}{dx}$ and integrate

$$\int_{\infty}^{x} 2\frac{d\psi}{dx}\frac{d\psi}{dx}\,dx = -\frac{1}{\epsilon}\int_{\psi=0}^{\psi}\sum z_i n_{io} \exp\left(-\frac{z \cdot e \cdot \psi}{K_B T}\right)\epsilon\, d\psi$$

$$\left(\frac{d\psi}{dx}\right)^2 = -\frac{2}{\epsilon}\sum n_{io} z_i \int_{\psi=0}^{\psi} \exp\left(-\frac{z \cdot e \cdot \psi}{K_B T}\right) d\psi$$

Now for generalized solution also we will be learning with this kappa value also we will be getting like this for, more generalized solution for the flat plate will be like this. This Poisson’s Boltzmann equation whatever we have derived now, this d square psi by dx square that is equal to 1 by epsilon into summation of zi ni0, and exponential minus z into e into psi by KB into T.

So now we need to multiply, both the sides by 2 d psi by dx and then integrate so, it will be like this integration 2 into d psi by dx into d psi by dx into dx is equal to minus 1 by epsilon this integration that is we can see this infinity to x that will be we can say psi is equal to 0 to psi into summation of zi ni0, exponential minus z into e into psi divided by KB into T into epsilon into dy. Or , dy by dx square is equal to minus 2 by epsilon into summation of ni0 into zi exponential that is take the integration exponential minus z into e into psi by KB into T that is 0 to psi is equal to 0 to psi into d psi. So that is actually d psi.

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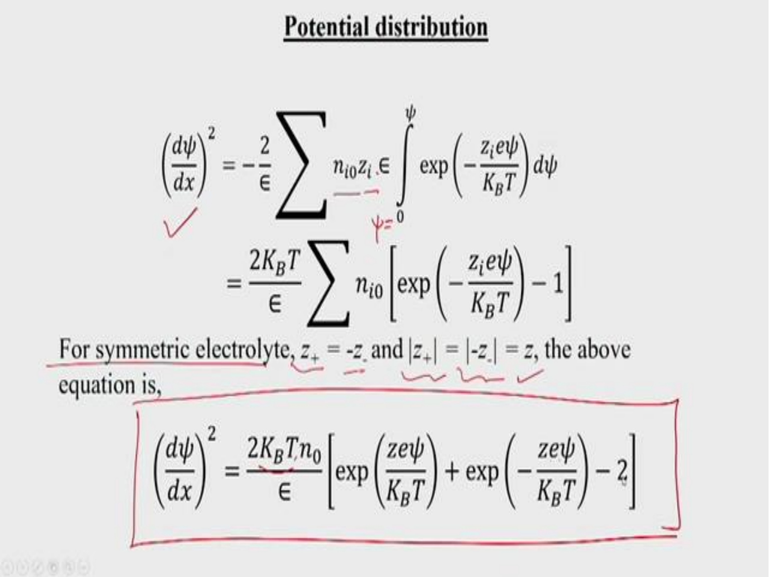

Potential distribution

$$\left(\frac{d\psi}{dx}\right)^2 = -\frac{2}{\epsilon}\sum n_{i0} z_i\, \epsilon \int_{\psi=0}^{\psi} \exp\left(-\frac{z_i e\psi}{K_B T}\right) d\psi$$

$$= \frac{2K_B T}{\epsilon}\sum n_{i0}\left[\exp\left(-\frac{z_i e\psi}{K_B T}\right) - 1\right]$$

For symmetric electrolyte, $z_+ = -z_-$ and $|z_+| = |-z_-| = z$, the above equation is,

$$\boxed{\left(\frac{d\psi}{dx}\right)^2 = \frac{2K_B T n_0}{\epsilon}\left[\exp\left(\frac{ze\psi}{K_B T}\right) + \exp\left(-\frac{ze\psi}{K_B T}\right) - 2\right]}$$

So from there actually we will be getting d psi by dx square is equal to minus 2 by epsilon into summation of ni0 into zi into epsilon into the psi is equal to 0 to psi and exponential minus zi e psi by KB into T into d psi. That after manipulation actually we will be getting this one say if we expand this one so we will be getting like this 2 KB into T by epsilon into summation of ni into exponential minus zi e into psi by KB into T minus 1.

For symmetric electrolyte, where zi is equal to minus z minus and then numbers will be same means say modulus of z plus and modulus minus modulus of z minus that will be z like this then the above equation will be converting into d psi by dx whole square is equal to 2 KB into T into n0 by epsilon into exponential. This z into e into psi by KB onto T plus exponential minus z into e into psi by KB into T minus 2.

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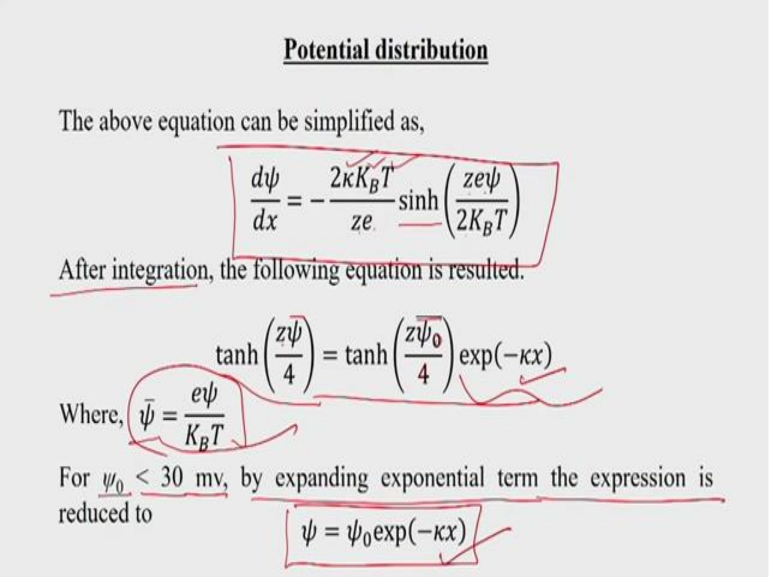

Potential distribution

The above equation can be simplified as,

$$\boxed{\frac{d\psi}{dx} = -\frac{2\kappa K_B T}{ze}\sinh\left(\frac{ze\psi}{2K_B T}\right)}$$

After integration, the following equation is resulted.

$$\tanh\left(\frac{z\bar{\psi}}{4}\right) = \tanh\left(\frac{z\bar{\psi}_0}{4}\right)\exp(-\kappa x)$$

Where, $\bar{\psi} = \frac{e\psi}{K_B T}$

For $\psi_0 < 30$ mv, by expanding exponential term the expression is reduced to

$$\boxed{\psi = \psi_0 \exp(-\kappa x)}$$

So that we will be getting this one and so the above equation can be simplified like this for the just if we remove d square then d psi by dx will be equal to minus 2 kappa into KB into T by z into e into sin hyperbolic z into e into psi by 2 into KB into T. After this integration the following equation will be resulted like this tan hyperbolic z into psi prime by 4 is equal to tan hyperbolic z into psi prime bar this one 0 by 4, into exponential minus kappa into x. where the psi prime is equal to e into psi by KB into T.

So when this psi 0 will be less than 30 millivolts this by expanding the exponential term actually this one, like this in the expression will be reduced psi is equal to psi 0 into exponential kappa into x. That we obtained from the this one normal electro neutrality condition. This psi by psi 0 is equal to exponential minus kappa into x, that kappa is equal to 1 by Debye length.

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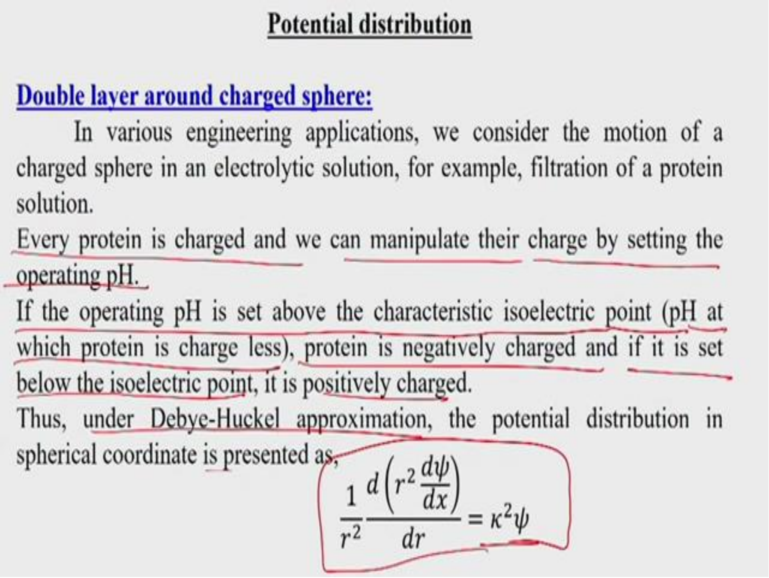

Potential distribution

Double layer around charged sphere:

In various engineering applications, we consider the motion of a charged sphere in an electrolytic solution, for example, filtration of a protein solution.

Every protein is charged and we can manipulate their charge by setting the operating pH.

If the operating pH is set above the characteristic isoelectric point (pH at which protein is charge less), protein is negatively charged and if it is set below the isoelectric point, it is positively charged.

Thus, under Debye-Huckel approximation, the potential distribution in spherical coordinate is presented as,

$$\boxed{\frac{1}{r^2}\frac{d\left(r^2\frac{d\psi}{dx}\right)}{dr}=\kappa^2\psi}$$

Now we will be discussing about the potential distribution like this in the double layer around the charged sphere suppose, till now we have discussed about the flat plate now we are discussing about double layer around the charged sphere like this one sphere is there in various engineering applications we consider the motion of charged spheres, so say may be in the vertical direction may be in the horizontal direction or any direction suppose charged particles are traveling inside the electrolytic solution.

For example, filtration of the protein solution like these proteins will be chemically positively charged or negatively charged above or below this isoelectric point like this, so every protein charged we can manipulate their charge by setting the operating pH like this what pH actually we will be maintaining. Like this if the operating pH is set above the isoelectric point so we can say the pH at which this protein is charged or , isoelectric point means the protein is neutral.

So protein is negatively charged and if it set below this isoelectric point so it is positively charged. So thus, this under this Debye Huckle approximation the potential distribution in spherical coordinate will be like this, 1 by r square into d into r square into d psi dx by dr is equal to kappa square into psi. Just in the spherical coordinate we will be getting this expression for this we can say electric double layer.

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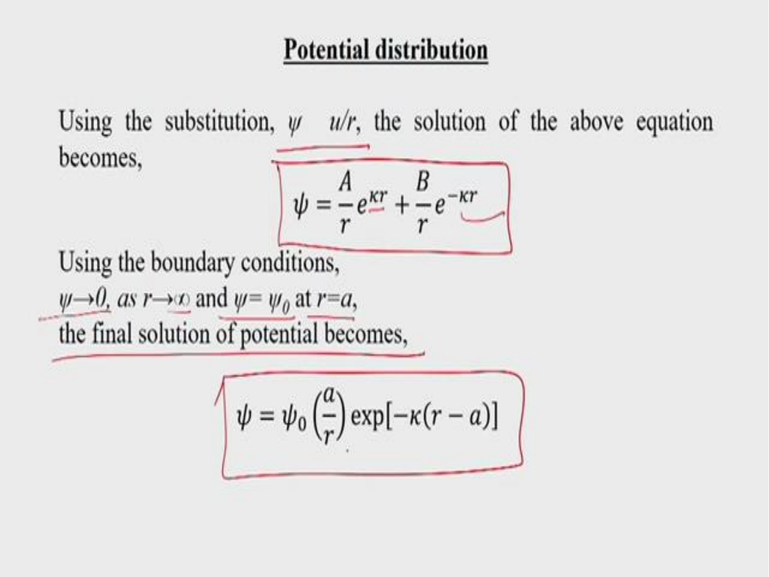

Potential distribution

Using the substitution, ψ u/r, the solution of the above equation becomes,

$$\boxed{\psi = \frac{A}{r}e^{\kappa r} + \frac{B}{r}e^{-\kappa r}}$$

Using the boundary conditions,

$\psi \to 0$, as $r \to \infty$ and $\psi = \psi_0$ at $r = a$,

the final solution of potential becomes,

$$\boxed{\psi = \psi_0\left(\frac{a}{r}\right)\exp[-\kappa(r-a)]}$$

Using the substitution like this psi is equal to u by r the solution of the above equation will be like this psi is equal to A by r into exponential kappa into r plus B by r into exponential minus kappa into r. So using the boundary condition like psi will be 0, that r will be infinity like this so, at the bulk and psi will be psi 0, at the r is equal to A means A in particular radius. So the final solution of the potential will be like this psi is equal to psi 0 into A by r into exponential minus kappa into r minus A. It is to some estimation different from the flat plate where psi is equal to psi 0 and so we obtained this one in terms of (()) (30:36) into kappa r.

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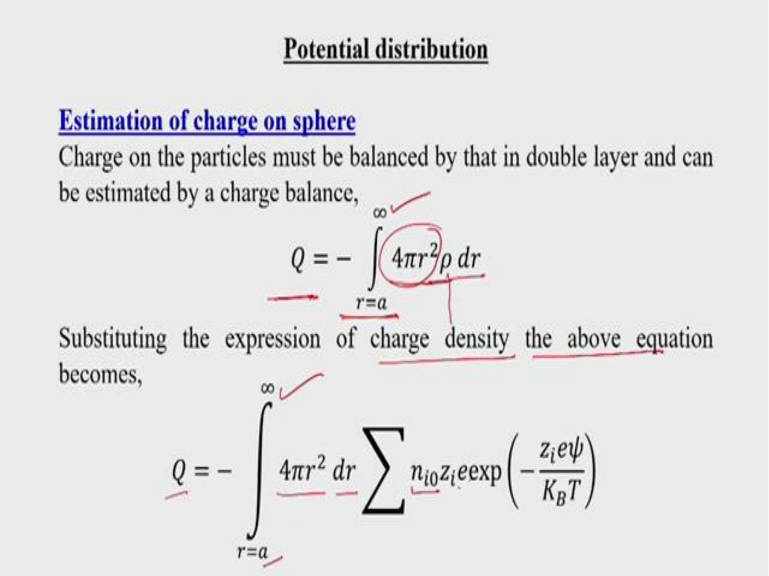

Potential distribution

Estimation of charge on sphere

Charge on the particles must be balanced by that in double layer and can be estimated by a charge balance,

$$Q = -\int_{r=a}^{\infty} 4\pi r^2 \rho \, dr$$

Substituting the expression of charge density the above equation becomes,

$$Q = -\int_{r=a}^{\infty} 4\pi r^2 \, dr \sum n_{i0} z_i e \exp\left(-\frac{z_i e \psi}{K_B T}\right)$$

So now, we will be doing the estimation of the charge on the sphere like this so, like , charge on the particle must be balanced by the double layer and can be estimated by a charge balance like this Q will, be equal to minus 4 pi r square that is this total area into charge density this one into dr where r we can say will be from the radius means from the top surface to the infinity.

Substituting these expressions in the charge density where charge density rho the above equation will be like Q is equal to minus 4 pi r square into dr into summation of ni0 into zi into e into exponential minus zi into e into psi by KB into T. where this r will be ranging from a to infinity. Means a top surface of the sphere to the infinity or means till the bulk where electro neutrality is prevailed.

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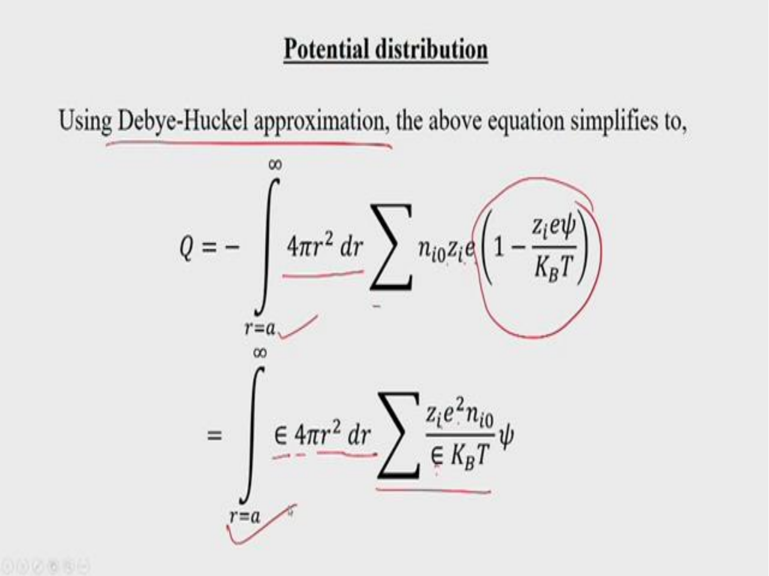

Potential distribution

Using Debye-Huckel approximation, the above equation simplifies to,

$$Q = -\int_{r=a}^{\infty} 4\pi r^2\, dr \sum n_{i0} z_i e \left(1 - \frac{z_i e \psi}{K_B T}\right)$$

$$= \int_{r=a}^{\infty} \epsilon\, 4\pi r^2\, dr \sum \frac{z_i e^2 n_{i0}}{\epsilon\, K_B T} \psi$$

So from there will be getting Q is equal to 4 pi r square into dr into summation of ni0 into zi into e that just using this Debye Huckle approximation we will be converting that part the exponential term as 1 minus zi into e into psi by KB into T. so where that is it will be ranging from a to infinity. Or top surface to the infinity. So that will be like this epsilon into 4 pi r square into dr and summation of zi into e square into ni0, by epsilon into KB into T into psi. There from a to infinity.

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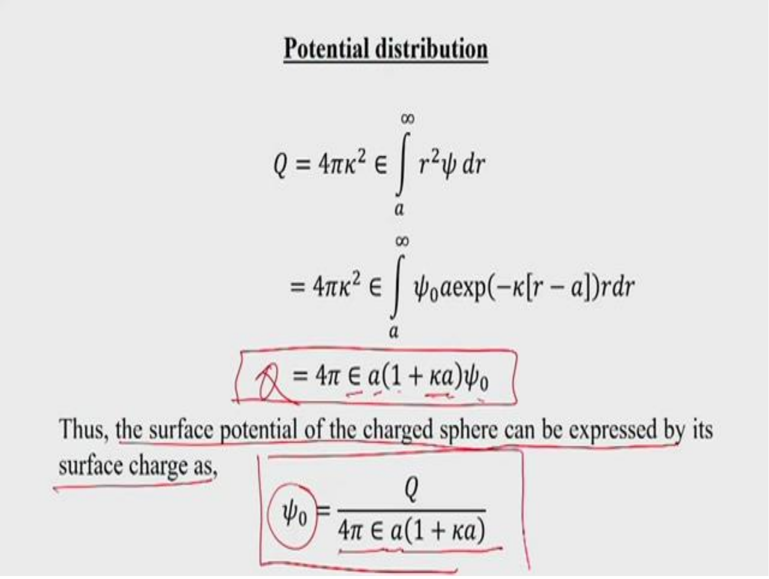

Potential distribution

$$Q = 4\pi\kappa^2 \in \int_a^\infty r^2\psi\, dr$$

$$= 4\pi\kappa^2 \in \int_a^\infty \psi_0 a\exp(-\kappa[r-a])rdr$$

$$\boxed{Q = 4\pi \in a(1+\kappa a)\psi_0}$$

Thus, the surface potential of the charged sphere can be expressed by its surface charge as,

$$\boxed{\psi_0 = \frac{Q}{4\pi \in a(1+\kappa a)}}$$

So that will be like this after integration we will be getting this Q is equal to 4 pi epsilon into a into 1 plus kappa into a into psi 0 that a is the radius thus, the surface potential of the charge spheres can be expressed by its surface charge like psi0 is equal Q by 4 pi into epsilon into a into 1 plus kappa into a. So that surface potential we can measure so if we know the charge.

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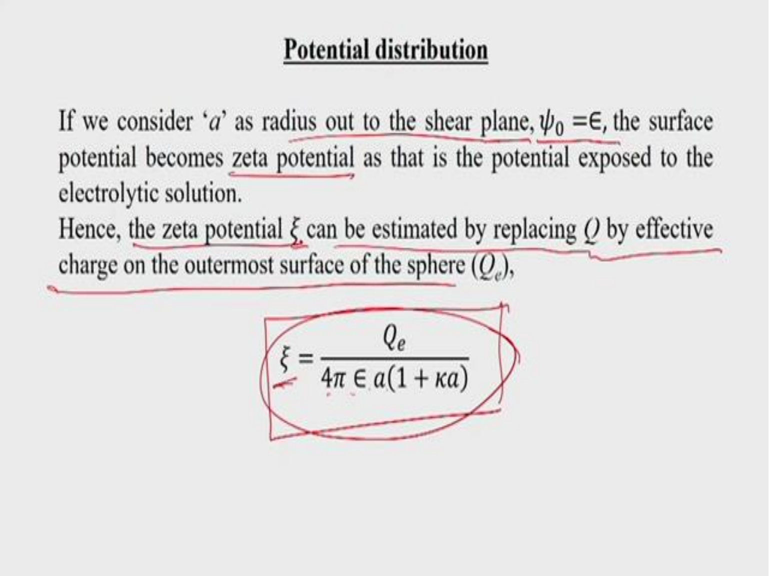

Potential distribution

If we consider '*a*' as radius out to the shear plane, $\psi_0 = \in$, the surface potential becomes zeta potential as that is the potential exposed to the electrolytic solution.

Hence, the zeta potential ξ can be estimated by replacing Q by effective charge on the outermost surface of the sphere (Q_e),

$$\boxed{\xi = \frac{Q_e}{4\pi \in a(1+\kappa a)}}$$

So if we consider a as the radius out of the this one to the square plane then psi 0 will be epsilon the surface potential becomes this zeta potential. As that is the potential expressed to the electrolytic solution. So hence, the zeta potential this zeta can be estimated by replacing this Q by the effective charge on the out most surface like this there we can say this zeta is equal to Qe by 4 pi into epsilon into a into 1 plus kappa into a. So we will be using this formula for solving this zeta potential problems frequently.

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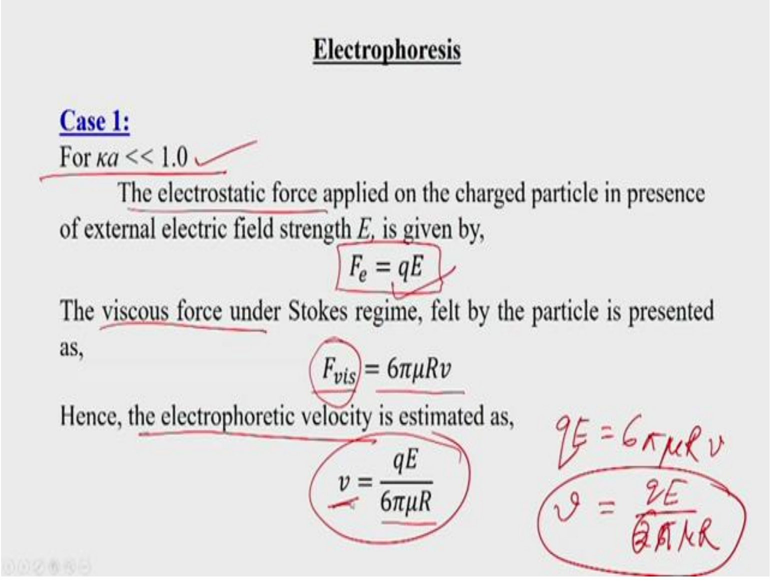

Electrophoresis

Case 1:

For $\kappa a << 1.0$

The electrostatic force applied on the charged particle in presence of external electric field strength E, is given by,

$$F_e = qE$$

The viscous force under Stokes regime, felt by the particle is presented as,

$$F_{vis} = 6\pi\mu Rv$$

Hence, the electrophoretic velocity is estimated as,

$$v = \frac{qE}{6\pi\mu R}$$

$$qE = 6\pi\mu R v$$

$$\vartheta = \frac{qE}{6\pi\mu R}$$

Now you see there are two cases where kappa a will be much much lower than 1 or where this kappa into r will be much much greater than 1. So this the first case is when kappa into a is much much lower than 1, then electrostatic force applied on the charged particles in presence of the external electric field strength E is given by this Fe is equal to Q into E.

The viscous force under the Stokes regime, felt by the particle is presented by F viscous is equal to 6 pi mu into R into v. So if we replace this viscous force with this Q into e that term we can say this electrophoretic velocity will be obtained like this Fe is equal to Fviscous is equal to Q into e is equal to 6 pi mu into R into v, where v will be like this, q into E by 6 pi into mu into R. So from here Q into e is equal to 6 pi mu into R into v, where v is equal to Q into e by 6 pi mu into R. So that is actually we can say this electrophoretic velocity we will be obtaining.

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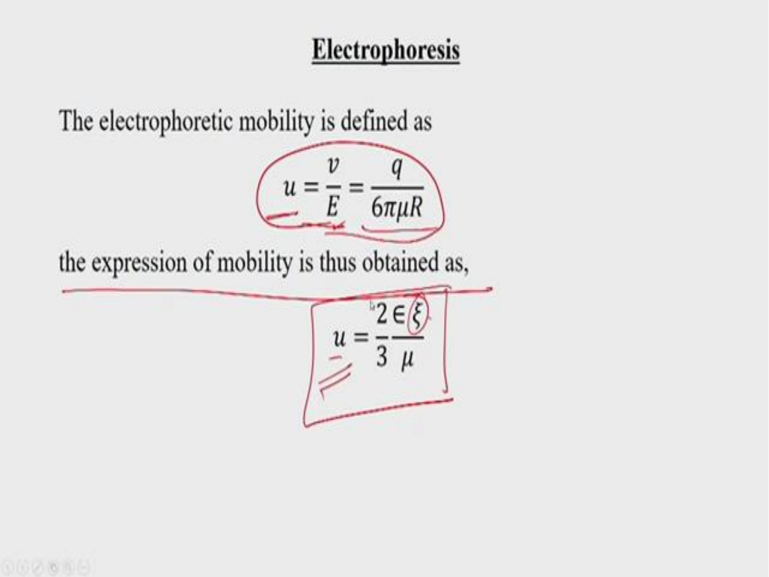

Electrophoresis

The electrophoretic mobility is defined as

$$\boxed{u = \frac{v}{E} = \frac{q}{6\pi\mu R}}$$

the expression of mobility is thus obtained as,

$$\boxed{u = \frac{2}{3}\frac{\epsilon \xi}{\mu}}$$

So, this electrophoretic mobility you will be like this v by E that is we can say this will be simply whatever be we have this one, will be dividing by this with E then it will be like just E will be eliminated from this numerator so q by 6 pi mu into R. the expression of mobility is thus obtained as this in terms of zeta so u is equal to 2 by 3 into epsilon into zeta by 3 into mu. So, this electrophoretic mobility is now, expressed in terms of zeta or zeta potential.

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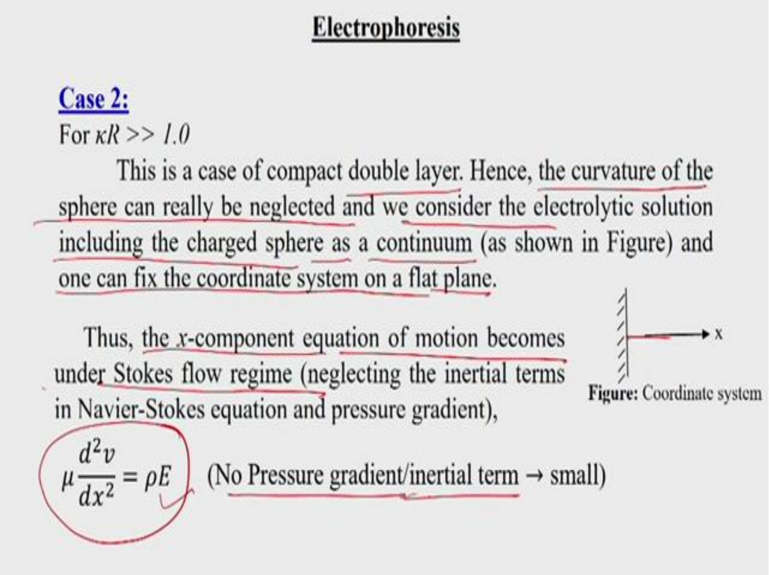

Electrophoresis

Case 2:

For $\kappa R >> 1.0$

This is a case of compact double layer. Hence, the curvature of the sphere can really be neglected and we consider the electrolytic solution including the charged sphere as a continuum (as shown in Figure) and one can fix the coordinate system on a flat plane.

Thus, the *x*-component equation of motion becomes under Stokes flow regime (neglecting the inertial terms in Navier-Stokes equation and pressure gradient),

Figure: Coordinate system

$$\boxed{\mu\frac{d^2v}{dx^2} = \rho E}$$ (No Pressure gradient/inertial term → small)

The case 2 we told that this if kappa into R is much much greater than 1. So this is the case of this compact double layer like this so hence, the curvature of the sphere can be really be neglected and we consider the electrolytic solution, including the charged sphere as a continuum so, like this from here, and one can fix the coordinate system on a flat plane like this so , it is converting into sphere is converting into the flat plane like this thus, x component equation of the motion becomes under the Stokes flow regime that is why we will be mu into d square v by dx square is equal to rho into E. So no pressure gradient or no inertial term, or very small also we can say.

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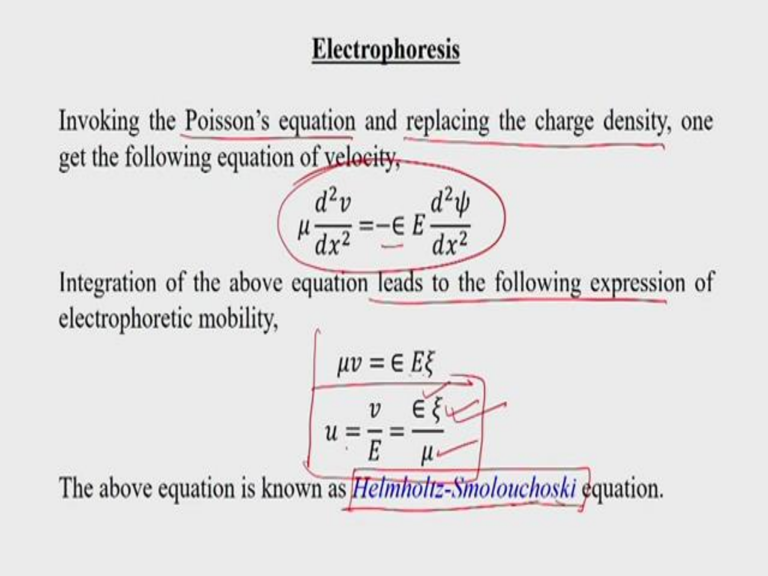

Electrophoresis

Invoking the Poisson's equation and replacing the charge density, one get the following equation of velocity,

$$\mu \frac{d^2 v}{dx^2} = -\in E \frac{d^2 \psi}{dx^2}$$

Integration of the above equation leads to the following expression of electrophoretic mobility,

$$\mu v = \in E \xi$$

$$u = \frac{v}{E} = \frac{\in \xi}{\mu}$$

The above equation is known as *Helmholtz-Smolouchoski* equation.

So, this will now invoking the Poisson's equation whatever we have derived and replacing the charge density we can be get this one mu into dv square by dx square is equal to minus epsilon into E into d square psi by dx square. Now we need to integrate this above equation like this following expression we will be getting that mu into v is equal to epsilon into E into zeta and u is equal to this v by E that will be like this, epsilon and zeta by mu. So this equation actually is called Helmholtz Smolouchoski equation so, we will be using this one to calculate the zeta potential or velocity also of the charged particles.

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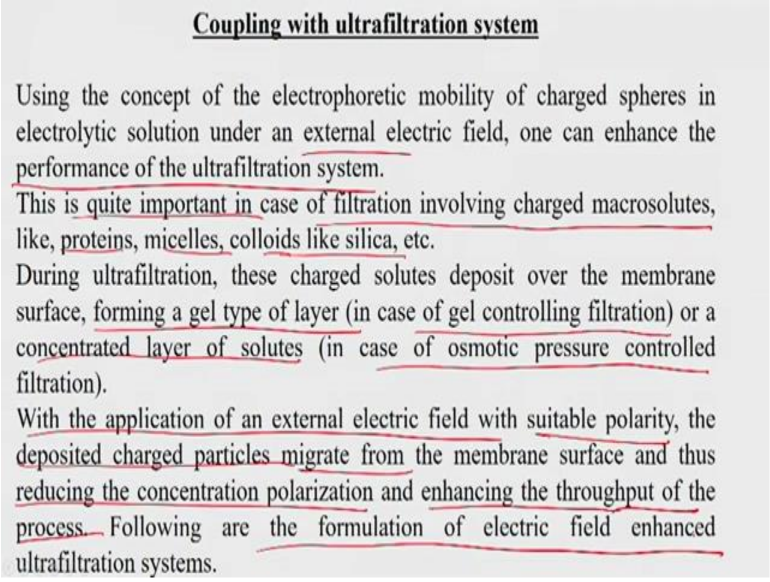

Coupling with ultrafiltration system

Using the concept of the electrophoretic mobility of charged spheres in electrolytic solution under an external electric field, one can enhance the performance of the ultrafiltration system.

This is quite important in case of filtration involving charged macrosolutes, like, proteins, micelles, colloids like silica, etc.

During ultrafiltration, these charged solutes deposit over the membrane surface, forming a gel type of layer (in case of gel controlling filtration) or a concentrated layer of solutes (in case of osmotic pressure controlled filtration).

With the application of an external electric field with suitable polarity, the deposited charged particles migrate from the membrane surface and thus reducing the concentration polarization and enhancing the throughput of the process. Following are the formulation of electric field enhanced ultrafiltration systems.

Now, we will be discussing the coupling of this electric field with ultrafiltration or with the this osmotic pressure control, filter separations or so many membrane separation processes and we will establish that whenever the electrophoretic effects are added inside this membrane separation processes in most of the cases the efficiency of the membrane process enhances. But, the condition is that the particles which are this under the separation process must be charged. So using the concept of electrophoretic mobility of the charged spheres in the electrolytic solution, under an external electric field one can enhance the performance of the ultrafiltration system like that is.

That is why we need this electric field will be added but, the charge particles should be moved through this ultrafiltration process or micro filtration or Nano filtration process so this is quite important in case of this filtration involving charge macromolecules like proteins, micelles, colloids like silica, etc. So during this ultrafiltration the charged solutes deposit over the membrane surface forming a gel layer like this in case of the gel controlling filtration or a concentrated layer of the solutes in case of the osmotic pressure control solution.

So with the application of the external electric field with suitable polarity the deposited charge particles migrate from the membrane surface to the bulk actually and reducing the concentration polarization and enhancing the throughput of the process like this that whatever the concentration polarization is there or deposition of the solute layer over the membrane surface was there it will be minimized because due to this charge interaction suppose solutes with charge will be moving from the membrane surface to the bulk. Then some formulations are there for electric field enhance ultrafiltration systems like this we can say suppose for gel layer controlling ultra-filtration process.

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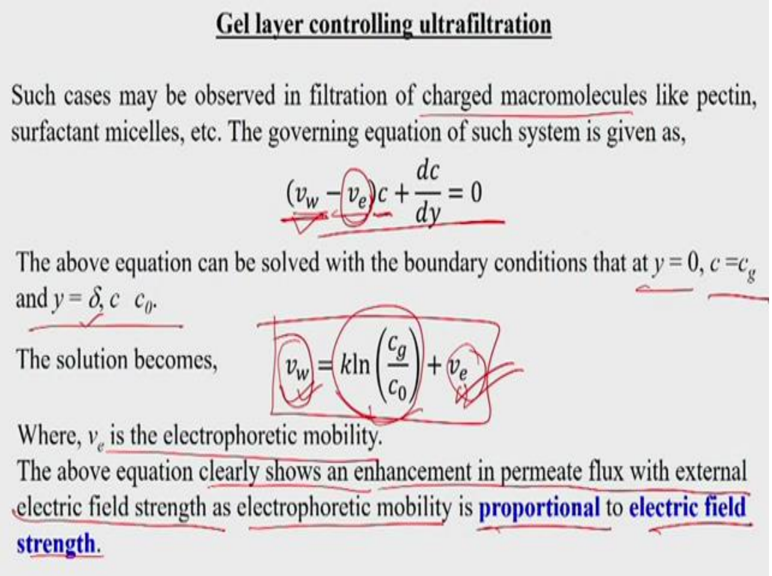

Gel layer controlling ultrafiltration

Such cases may be observed in filtration of charged macromolecules like pectin, surfactant micelles, etc. The governing equation of such system is given as,

$$(v_w - v_e)c + \frac{dc}{dy} = 0$$

The above equation can be solved with the boundary conditions that at $y = 0$, $c = c_g$ and $y = \delta$, c c_0.

The solution becomes, $\boxed{v_w = k\ln\left(\frac{c_g}{c_0}\right) + v_e}$

Where, v_e is the electrophoretic mobility.
The above equation clearly shows an enhancement in permeate flux with external electric field strength as electrophoretic mobility is **proportional** to **electric field strength.**

Suppose this is the vw is equal to we can say this is due to this only flux due to this pressure gradient but you see this due to this electric potential so, or we can say the charged macromolecules are there like we can say this pectin, for surfactant micelles, the governing equation will be like this vw minus v into c concentration plus dc by dy is equal to 0, now we need to kintegrate this one suppose this y is equal to 0, means at the membrane surface then c will be c g after certain period of time and y is equal to delta, where c is equal to c0, or we say initial concentration or delta is the gel layer concentration or we can say above this the solute concentration will remain in the as the bulk concentration.

Now we see this one this equation will be converting into vw is equal to k into ln cg by c0, plus ve. So this ve actually is the electrophoretic mobility that is the vw is now the summation of this two parts the whatever there that k into ln cg by c naught that we have already derived this one for this we can say gel layer control ultra-filtration process. That now ve is added quantity like electrophoretic mobility.

So the above equation actually clearly shows an enhancement in the permeate flux with this external electric field strength as the electrophoretic mobility proportional to the electric field strength so we can say this is one whatever the electric field strength is given so it will increase the electrophoretic mobility and as the as whole throughput is going to increase because that is we can say this added quantity.

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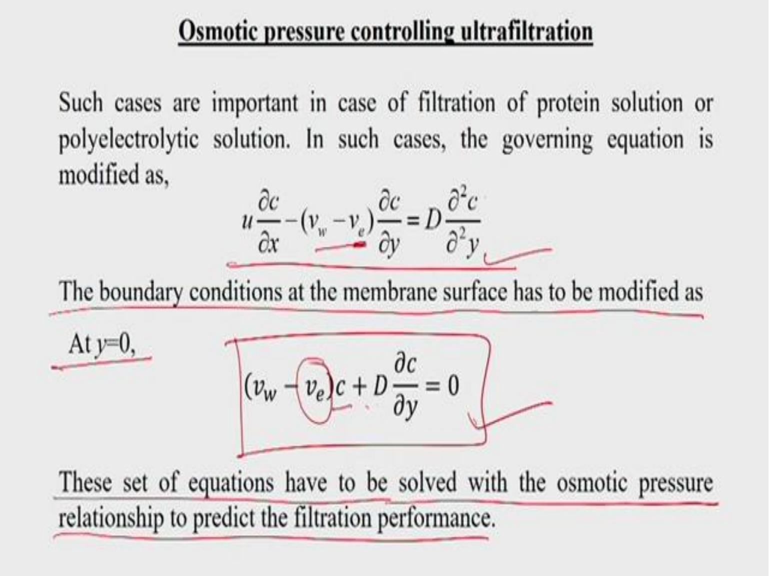

Osmotic pressure controlling ultrafiltration

Such cases are important in case of filtration of protein solution or polyelectrolytic solution. In such cases, the governing equation is modified as,

$$u\frac{\partial c}{\partial x}-(v_w-v_e)\frac{\partial c}{\partial y}=D\frac{\partial^2 c}{\partial^2 y}$$

The boundary conditions at the membrane surface has to be modified as

At y=0,

$$\boxed{(v_w - v_e)c + D\frac{\partial c}{\partial y} = 0}$$

These set of equations have to be solved with the osmotic pressure relationship to predict the filtration performance.

So such cases are important in case of the filtration of protein solutions or poly electrolytic solutions in such cases the governing equation is modified as u into del c del x minus vw minus ve into del c del y is equal to D into this del square c by del y square. So the boundary conditions at the membrane surface has to be modified as at y is equal to 0, this equation will be like this vw minus ve into c plus D into del c by del y is equal to 0, so this set of equations have to be solved actually with the osmotic pressure relationship to predict the filtration performance.

But, here also we can say as this vw actually ve actually is this will, be the added quantity in terms of throughput so we can say in case of this osmotic controlled osmotic pressure controlled ultra-filtration process also whenever the external electric field will be added their throughput will be going on increasing.

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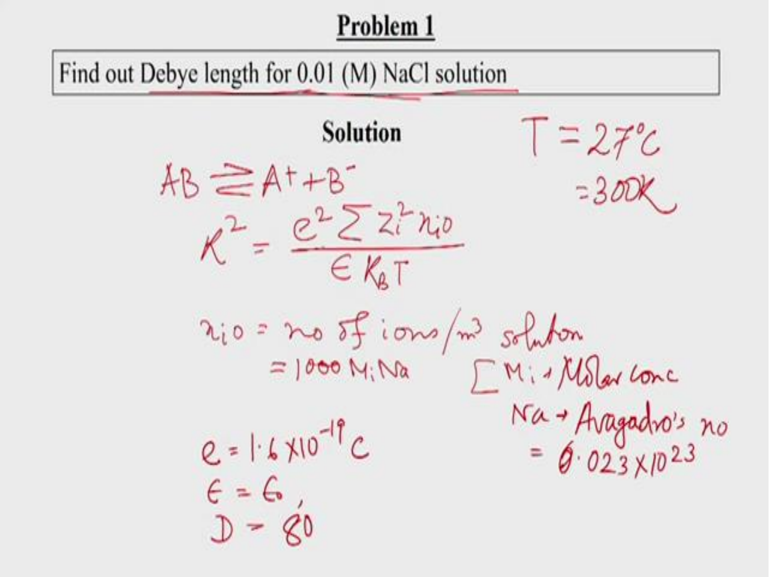

Problem 1

Find out Debye length for 0.01 (M) NaCl solution

Solution

$AB \rightleftharpoons A^+ + B^-$

$$\kappa^2 = \frac{e^2 \sum z_i^2 n_{i0}}{\epsilon K_B T}$$

$T = 27°C$
$= 300K$

n_{i0} = no of ions/m^3 solution
$= 1000 M_i N_a$

$M_i \rightarrow$ Molar conc
$N_a \rightarrow$ Avagadro's no
$= 6.023 \times 10^{23}$

$e = 1.6 \times 10^{-19} C$
$\epsilon = \epsilon_0$,
$D = 80$

Now, we will be solving one problem that the problem is given as find the Debye length actually of this 0.01 molar NaCl solution. This sis very simple problem so we will be trying to solve its one problem like in this case, let a see NaCl is will be like this say AB will be A plus plus B minus. This is the simple type thing so this kappa square that is inverse of Debye length that is we can say this we know that e square into summation of zi square into ni0, by epsilon into KB into T.

So here actually we can say temperature we will be taking as 27 degrees Celsius or 300 kelvin so we need to assume this one and ni0 will be equal to number of ions say per meter cube solution that is we can say is equal to 1000 into M i into Na. So that is Mi is the molar concentration and Na is the Avogadro’s number so that is nothing but 6.023 into 10 to the power 23. And e will be like this is equal to 1.6 these are all known 10 to power minus 19 coulomb and epsilon is equal to epsilon 0, and D is equal to dielectric constant that is for what are it is 80.

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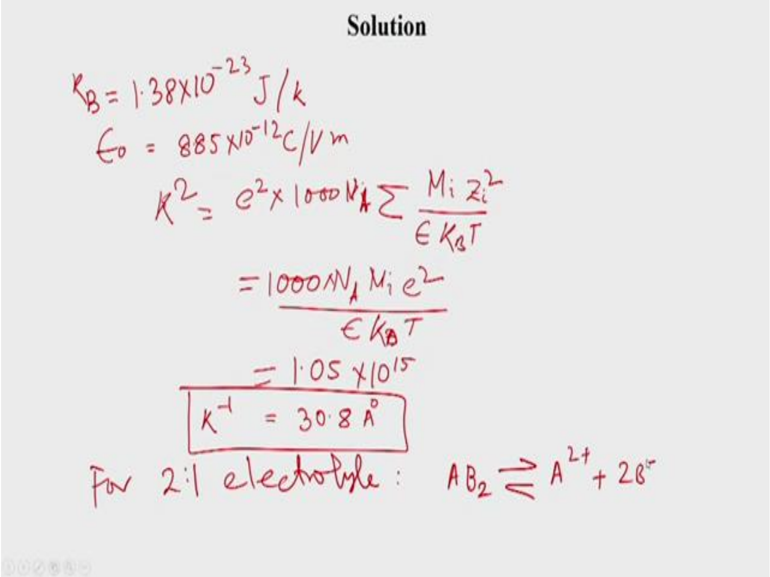

Solution

$K_B = 1.38 \times 10^{-23}\ J/k$

$\epsilon_0 = 885 \times 10^{-12}\ C/V\,m$

$$K^2 = e^2 \times 1000 N_A \sum \frac{M_i z_i^2}{\epsilon K_B T}$$

$$= \frac{1000 N_A M_i e^2}{\epsilon K_B T}$$

$$= 1.05 \times 10^{15}$$

$$\boxed{K^{-1} = 30.8\ \mathring{A}}$$

For 2:1 electrolyte : $AB_2 \rightleftharpoons A^{2+} + 2B^-$

Then, KB that is Boltzmann constant that is also known 1.38 into 10 to the power minus 23 Joule per kelvin. epsilon 0 that is 8.85 into 10 to the power minus 12 Coulomb per volt into meter. So now, we will be doing this kappa square is equal will be this one this e square into 1000 into Ni into summation of Mi zi square divided by epsilon into KB into T. so that will be coming out as 1000 into this, so this is actually Avogadro number actually this Avogadro number into Mi into e square by epsilon into KB into into T. That is coming out as 1.05 into 10 to the power 15. Now this K to the power minus 1 will be like this, 30.8 Angstrom. So that is inverse of Debye length that is 30.8 Angstrom. So for suppose 2 is to 1 electrolyte like AB2 will be A2 plus plus 2B minus so in that case.

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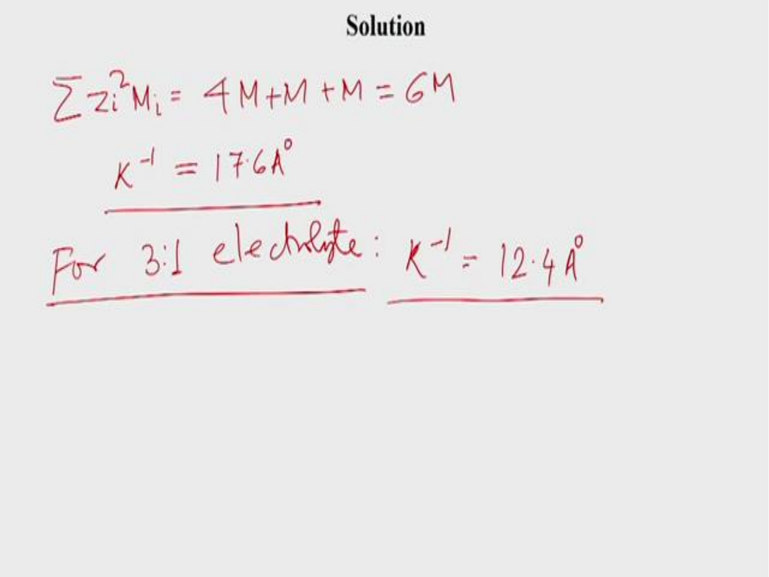

Solution

$\sum z_i^2 M_i = 4M + M + M = 6M$

$\kappa^{-1} = 17.6\,\mathring{A}$

For 3:1 electrolyte: $\kappa^{-1} = 12.4\,\mathring{A}$

Will be like this a summation of zi square into Mi that will be is equal to 4M plus M plus M so that will be coming out as 6 M so, anion dissociated with so kappa minus will be like this, 17.6 Angstrom and for 3 is to 1 electrolyte we need to this one do the same this solution and that time I will be getting kappa prime will be equal to 12.4 Angstrom so whenever whether moving from 1 is to 1 electrolyte so there actually you see this kappa inverse was 30.8.

So whenever we have this 2 is to 1 electrolyte then kappa is equal inverse of Debye length is 17.6 Angstrom so it is compressing so for 3 is to 1 electrolyte again it is compressing so it is becoming 12.4 Angstrom so with increase in the electrolyte concentration from 1 is to 1 to, 2 is to 1 to, 3 is to 1 or reverse one also say 1 is to 1, 1 is to 2, 1 is to 3, so electric double layer will going to be compressed.

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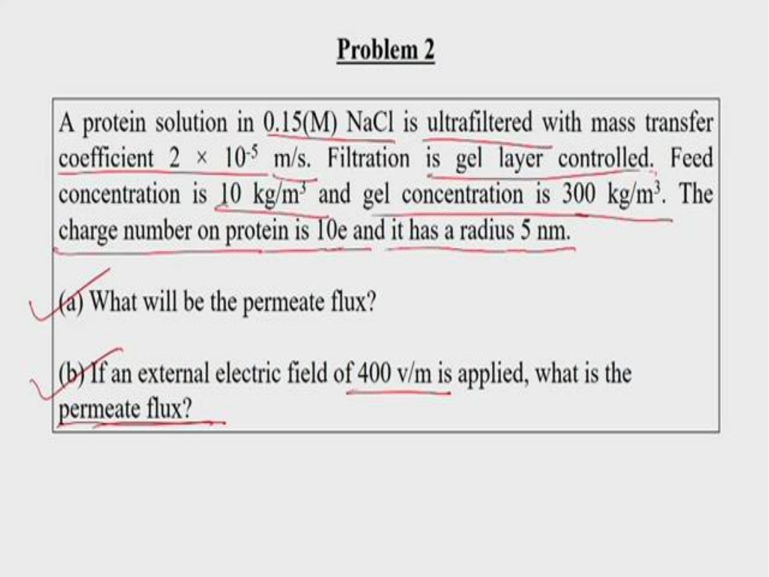

Problem 2

A protein solution in 0.15(M) NaCl is ultrafiltered with mass transfer coefficient 2×10^{-5} m/s. Filtration is gel layer controlled. Feed concentration is 10 kg/m^3 and gel concentration is 300 kg/m^3. The charge number on protein is 10e and it has a radius 5 nm.

(a) What will be the permeate flux?

(b) If an external electric field of 400 v/m is applied, what is the permeate flux?

Now we will be solving another problem, a protein solution in 0.15 molar NaCl is ultra-filtered with mass transfer coefficient of 2 into 10 to the power minus 5 meter per second. So filtration is gel layer controlled. Feed concentration is 10 kg per meter cube and gel concentration is 300 kg per meter cube the charge number on protein is 10e and it has a radius 5 nanometer we need to find at the permeate flux because this permeate flux in this problem the permeate flux values should be greater than this one permeate flux without the charging of the particle without the for the electrical neutral particles like this.

If an external electric field of 400 volts per meter is applied what is the permeate flux there. Actually, we say whenever the external electric field will be added or where the external electric field is not added so what will be the difference in the permeate flux values. So we will be calculating this one.

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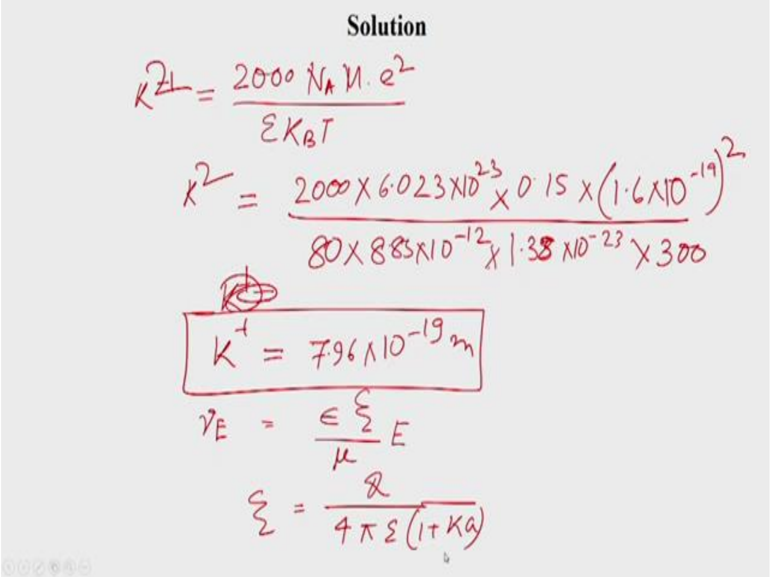

Solution

$$\kappa^2 = \frac{2000\, N_A M \cdot e^2}{\varepsilon K_B T}$$

$$\kappa^2 = \frac{2000 \times 6.023 \times 10^{23} \times 0.15 \times (1.6 \times 10^{-19})^2}{80 \times 8.85 \times 10^{-12} \times 1.38 \times 10^{-23} \times 300}$$

$$\boxed{\kappa^{-1} = 7.96 \times 10^{-19}\, m}$$

$$v_E = \frac{\epsilon \zeta}{\mu} E$$

$$\zeta = \frac{Q}{4\pi\varepsilon(1+\kappa a)}$$

First we need to calculate the kappa inverse 1 or inverse of Debye length so that is 2000 into Na into M into e square by epsilon into KB into T. so here M is equal to 0.15 molar so kappa prime actually will be like this sorry kappa square actually this one kappa square that will be 2000 into 6.023 into 10 to the power 23 into M will be 0.15 molar into charge of this one particle 1.6 into 10 to the power minus 19 coulomb that is whole square divided by that is for this and what it is 80, and KB is equal to 8.85 into 10 to the power minus 12 into 1.38 into 10 to the power minus 23 into 300.

So it is coming out as so from here, kappa inverse actually, kappa inverse is coming out as 7.96 into 10 to the power minus 19 meter so, that is reverse of the Debye length and say electrophoretic velocity is suppose VE will be like this this epsilon into zeta potential divided by mu into E so, that we can say so from there we have this zeta potential is equal to Q divided by 4 into pi into epsilon into 1 plus kappa into a so we can say.

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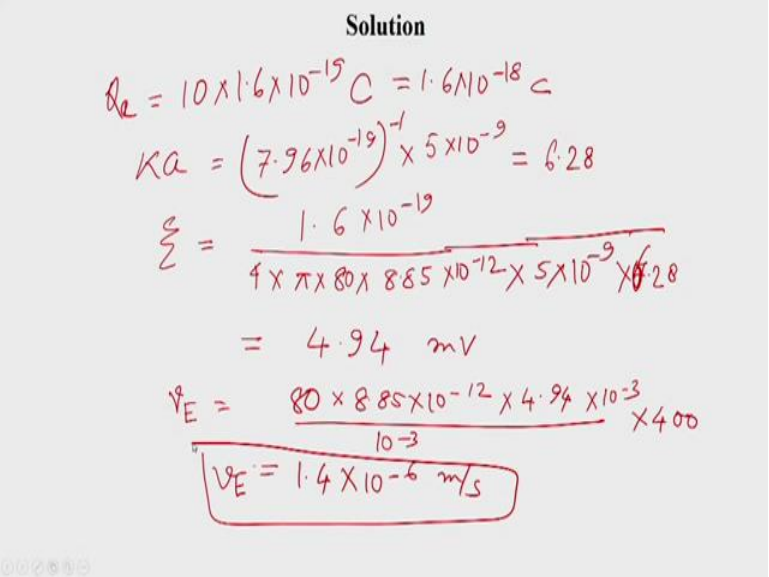

Solution

$$Q_e = 10 \times 1.6 \times 10^{-19}\,C = 1.6 \times 10^{-18}\,C$$

$$\kappa a = \left(7.96 \times 10^{-19}\right)^{-1} \times 5 \times 10^{-9} = 6.28$$

$$\zeta = \frac{1.6 \times 10^{-19}}{4 \times \pi \times 80 \times 8.85 \times 10^{-12} \times 5 \times 10^{-9} \times 6.28}$$

$$= 4.94\ mV$$

$$v_E = \frac{80 \times 8.85 \times 10^{-12} \times 4.94 \times 10^{-3}}{10^{-3}} \times 400$$

$$\boxed{v_E = 1.4 \times 10^{-6}\ m/s}$$

This Qe will be equal to 10 into 1.6 into 10 to the power minus 19 c that is coulomb that is equal to 1.6 into 10 to the power minus 18 coulomb. So the kappa into a will be equal to 7.96 into 10 to the power minus 19 to the power minus 1 into 5 into 10 to the power minus 9 that is coming out as 6.28 so this zeta potential value will be 1.6 into 10 to the power minus 19 divided by 4 into pi into 80 into 8.85 into 10 to the power minus 12 into 5 into 10 to the power minus 9 into 7.28, what is it 6.28 so, it is coming out as 4.94 millivolts.

So this electrophoretic mobility v will be is equal to 80 into 8.85 into 10 to the power minus 12 into 4.94 into 10 to the power minus 3 divided by 10 to the power minus 3. So into 400 so it is coming out as 1.4 into 10 to the power minus 6 meter per second. So that is the electrophoretic mobility we are getting.

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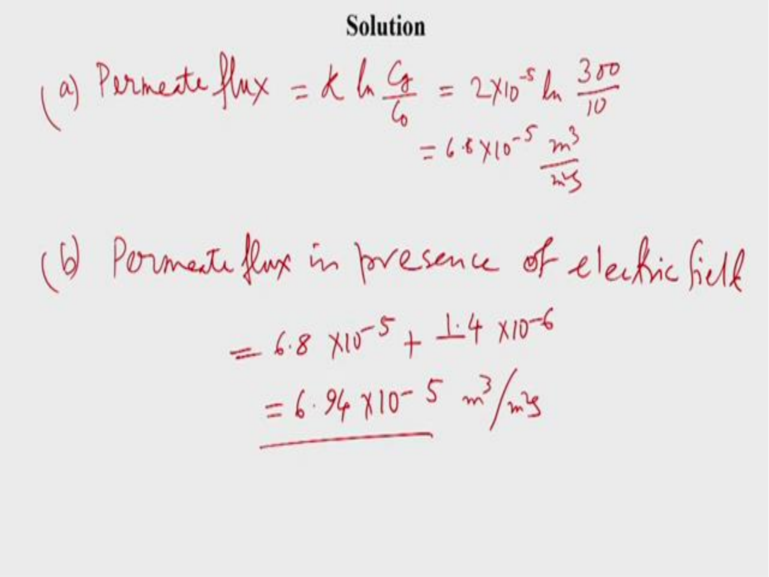

Solution

(a) Permeate flux $= k \ln \frac{C_g}{C_0} = 2\times10^{-5} \ln \frac{300}{10}$

$= 6.8\times10^{-5}\ \frac{m^3}{m^2 s}$

(b) Permeate flux in presence of electric field

$= 6.8\times10^{-5} + 1.4\times10^{-6}$

$= \underline{6.94\times10^{-5}}\ m^3/m^2 s$

So now we will be calculating the permeate flux value putting this in the final expression like this permeate flux for the first case so for the first case that will be like this k into ln cg by c0, that will be equal to say 2 into 10 to the power minus 5 into ln cg is equal to 300 by 10 that is coming out as 6.8 into 10 to the power minus 5 meter cube per meter square second. So that is the point so, in presence of electric field like this permeate flux in presence of, in presence of so electric field that will be like this so we can say 6.8 that will be equal to that is into the 10 to the power minus 5 that is there plus whatever the flux actually is due to electrophoretic mobility that is 1.4 into 10 to the power minus 6.

So it is coming out as 6.94 into 10 to the power minus 5 meter cube per meter square second. So that is whenever we have this electric field then the permeate flux value will be much higher than without electric field.

So in the next we will be discussing about the micellar enhanced ultra-filtration and that is the last chapter of the membrane separation technology.