

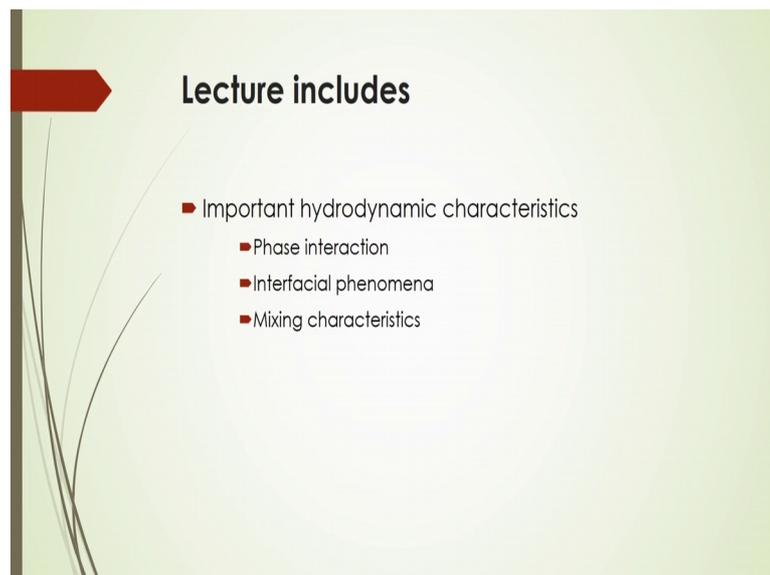
**Fluid Flow Operations**  
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**Module –12**  
**Multiphase flow phenomena and its application – Part 3**  
**Lecture – 32**  
**Hydrodynamics in multiphase flow-continued**

Welcome to massive open online course on Fluid Flow Operations. So, in this lecture we will continue the Hydrodynamic aspects of multiphase flow. We have already discussed something regarding that hydrodynamics of two phase flow and like a pro pattern and also some characteristics of what is that transition of that flow pattern and also the frictional resistance and how to actually analyze that fictional pressure drop. And, also hold up characteristics that is the volume friction of the phases in the two phase system.

Now, in this lecture will discuss something more about that what should be the phase interaction; that means, here if there is a two phase flow, if there is a flow of gas on liquid in a vessel, then there will be a formation of bubble liquid medium. Then what should the interaction of the bubble and bumble, how the bubble should be coalescence and bubbles should be braked up into a smaller refiner bubbles?

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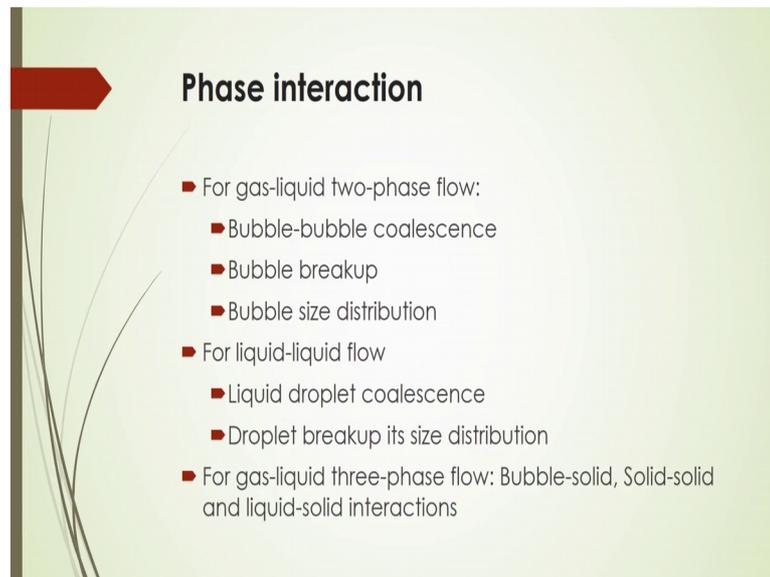
And you know that also after picking up the bubbles or coalescence will see the interfacial phenomena will change in that case after breaking the bubbles will get more interfacial area of the gas and liquid. And also what should be the mixing characteristics of the phases that you have to know because of this mixing characteristic the phases of course, will affect the yield of the processes.

And also interfacial phenomena it of course, one of the main important characteristic factor of multiphase flow systems because this interface through the interfacial area, there be a mass transfer when you are going to separate one phase from another phase or mixture of phases. Then you will see the mass transfer will be happened through the interface and in that case what should the interfacial area that should be accounted. And during that face interaction how this interfacial area will be actually changing that also to be known.

So, we will discuss very simple way what is that how to actually calculate that interfacial area having that interface what should be the size distribution of the particles like if it is bubbles, then how the bubbles will be forming and how interfacial area will be forming. And, also how to obtain that the size of that bubbles that ultimately that size of the bubbles will give you the interfacial area.

Now, what should be the distribution of the bubbles in a particular systems of gas and liquid, in that case gas will be dispersed in the dispersed phase of bubbles then all the bubble should not be uniform in size and also the number bubble number of bubbles will be changing. So, you have to know the distribution of the bubbles in the column, then you will be able to analyze and whose range of bubble this process particular process of the separation on chemical you have chemical engineering processes are happening. So, those things is to be a known.

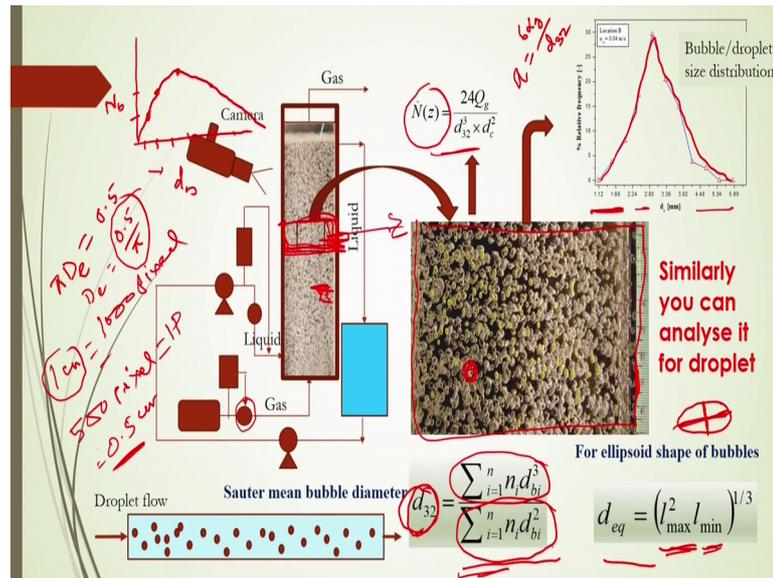
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Now, a phase interaction means your if you are considered gas liquid to phase flow, then other bubble-bubble coalescence, bubble breakup, bubble size distribution even, if you are considering liquid-liquid flow systems. Then what should be the liquid droplet coalescence, droplet breakup and its size distribution. For gas liquid three phase flow systems then bubbles solid, solid-solid among liquid-liquid and liquid-liquid interactions. So, in this case we are having the interfaces between the phases as a bubbles or droplets.

We will see on examples like a droplet if kerosene is flowing through the pipe in a continuous water medium then you see that there will formation of kerosene droplet; that means, which is dispersed phase. Generally, the droplet will be forming for that lighter phases kerosene is lighter than water and in that case kerosene droplet will be performing just by distribution of the energy of the liquid. And there will be the change of that droplet size based on that interfacial phenomena like is there any physical property change or not even also the inertia effect will be on the droplet surfaces. Then what should be the stable droplet bubbles that we discussed here.

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Now, have a look on this diagram what is this. Let us experiment of gas liquid operation, in this case here if you observe that there is a flow of gas in a column the gas is supplied from the bottom of this column as a disperse phase of bubbles. And then gas is actually distributed to the distributor and distributor will have certain whole size and through who is this gas will be passing. And then a gas will be dispersed in this continuous liquid medium here disperse phase of bubbles.

See here bottom part from the bottom part of this column gas is supplied by a compressor and from that compressor gas is coming through a rotameter; this rotameter will give you the measurement of gas flow rate. And after rotameter it will come through the what is that solenoid valve and whenever it will be a solenoid valve is being used for certain stop of this operation. And it will come to that a distributor and to the distributor gas is distributing.

And here also liquid is actually poured into the liquid by a pump from a storage tank and it may be continuous, it may be best wise, so if you are considering that here best wise what will happened, then only some amount of liquid pull will be in the column and in that liquid pull this gas will be dispersed phase and dispersed phase of bubbles.

And then gas is coming from the bottom of this column to the top and then what happened this gas is disengaging and it will be collapsing at the top of the surface of this gas liquid mixture and then it will be coming out the atmosphere. Now, this is an

example like this if you are actually applied this process for carbon dioxide gas absorption from the atmosphere, then you have to supply the mixture of carbon dioxide gas.

And air that is from the atmosphere and it will be passing through the distributor and as it is (Refer Time: 08:08) phase of bubbles that is carbon dioxide gas bubbles and their bubbles will be coming out and those carbon dioxide gas bubbles will be actually the flowing through this column and during that flow since carbon dioxide gas have some interfacial area between that bubble and liquid.

So, through that interfacial area the carbon dioxide gas will be absorbed or transferred from this here from carbon dioxide gas bubbles to the what is that liquid medium. Here liquid maybe, here sodium hydroxide solution you can use or some (Refer Time: 08:07) solution of you can use where the solubility of the carbon dioxide is more compared to the water that solution also you can used as a solvent. So, in that solvent you can absorb this carbon dioxide here.

So, this is one example now some hydrodynamic characteristics that you have to study for these. Now, we have already discussed what should be the hold up of this gas or carbon dioxide gas here that is one of the important hydrodynamic characteristics. Because, holdup will you give you the interfacial area that is because specific interfacial area is related to this hold up of these a gas.

And more the hold up of the gas you can expect more interfacial area more mass transfer and more absorption there. And other characteristics like what will be the friction of pressure drop that already we have discussed and how to estimate the frictional pressure drop that you can measure by manometer or some other sophisticated instrument can be used to measure these pressure drop.

And after getting that pressure drop and the particular that pressure drop you can apply this process here. Even sometimes if you increase the pressure drop or control the pressure drop you can expect more final size of bubble and in that case you can expect more interfacial area and more mass transfer for their. But sometimes higher pressure may decrease the mass transfer efficiency.

In that case you have to optimize at who is pressure drop you have to actually perform this experiment or this process. So, there will be an effective pressure also for controlling this bubble size there and also the flow rate of the gas is also one important variables by which you can get a that the size of the bubbles whether it will be more finer or courses that depend on the inertia force for kinetic energy distribution in this column whenever flow will be there.

So, in that case based on that may be at higher a flow rate you will see there will be a more probability of coalescence of the bubbles and more breakage of the bubbles, in that case which one is the more dominant factors whether it should be a coalescence or break up. But simultaneously that coalescence and breakup will happen in that case may be a more finer bubbles may obtained or courser bubbles also may obtained that variables are that controlling of that variables at a particular condition that you have to actually analyze and based on who is your performance of the process will be calculated.

Now, let us here consider that bubbles here whatever size is formed during this operation. So, what you will see that how to actually analyze this bubble size there because, this is one of the important factor by which you can get the mass transfer efficiency. Now that how to actually estimate this bubble size? Very interesting that what to do that you have to use one camera that can be placed that placed in front of the column and background of the column you can make it black, so that you can get more contrast bubbles there.

And if you are making this column as a rectangular column then it will be more actually accurate for measuring this size of the bubbles. So, one method is very common method that is photo graphic method, in this case on high speed camera you have to use and you have to capture the bubbles whenever it will be moving there.

So, during that movement of the bubbles if you capture the bubbles at a particular position like here shown at this position if you capture the bubbles and if you magnify that bubbles, after actually refinement, then you will see this type of bubbles you can expect there this window. So, these are the bubbles that you can get.

So, before actually taking this that is snapshot of the bubbles you have to make a or you have to attach a scale on the what is that one of the column because, that scale will be actually actual in size what should be that you have to calibrate that bubble size with

respect to this scale and you can calculate the actual size of the bubbles. Now what to do? So, after capturing this bubble by this camera you have to actually refine that snapshot by some suitable software.

That is maybe images software or maybe what is that a photo shop software you have to refine that software and contrast and even some color contrast to be matched in such a way that you can get the finer contrast bubble and also the interfaces of the bubbles will be more actually visible. And in that case after getting that actually refinement of this picture of these bubbles you have to analyze the size of the bubbles.

Now, what to do? We will see in this snapshot of this bubble you will not get only single bubbles there will be more than I think 500 bubbles even more also a n number of bubbles you can expect there. So, in this case how to actually measure that bubble size? So, what you have to do? You have to individually identify bubble in this picture and what you have to do, then marking out what is that the surface of the bubbles here.

Let it be here one consideration, let it one bubbles here this is your one bubbles; so, in this case you mark it out this bubbles and you just measure the perimeter of this bubbles and how? So, you have to use one software image analyzing software like image plus even you can say image pro plus software, even some other commercial software there are several different types of softwares are available in the market.

And so, you can use that software and by the software you can directly actually measured this perimeter of this bubble. Now, this software may give this perimeter in terms of pixel or some other in terms of parameter. In that case what you have to do? you have to first calibrate this size of this bubbles in this place. So, you have to mark it out this to this length this length is suppose 1 centimeter. So, this centimeter will give you certain pixel as per the software.

Now, this length will give you the pixel of suppose 1 centimeter is giving you equivalent to suppose 1000 pixel. Now, in this case then 1000 pixel is equivalent to 1 centimeter, this 1 centimeter is the actual length. Now, if you are calculating the perimeter of this bubble, then what should be the pixel of these bubbles? That is pixel of the perimeter of this bubble. So, pixel here in this case may be it will be 500 pixel.

So, this 500 pixel is equal to 1 perimeter; that means, here then what should be that in actual length, then 1000 pixel is 1 centimeter, then 500 pixel will be equals to 0.5 centimeter, so it will be 0.5 centimeter, so this is your actual length. Since it is a perimeter of this bubble, then what should be the diameter of the bubble? And in this case you may not get the spherical bubble. It may be different types of different shapes bubbles, it may be droplet, it may be spheroid, it may be some other ellipsoidal bubble.

So, in that case you have to consider that equivalent diameter of the bubble what is that equivalent diameter of the bubble? In this case you have to actually equalize the equivalent volume of the bubbles which is coming here in this case, let it be considered here if you are considering the perimeter of that bubble here this perimeter. So, if you consider that one bubbles whose volume will be the same volume of this bubbles, then what would be the perimeter of that equivalent bubble?

So, for the calculation of that perimeter it will be  $\pi D_e$ ;  $D_e$  the equivalent diameter of the bubble. So, this  $\pi D_e$  will be is equal to what in this case this 0.5 centimeter, so  $D_e$  will be is equal to 0.5 by  $\pi$ . So, this is will be your bubble diameter as 0.5 by  $\pi$ . So, this will be your actual equivalent diameter of the bubble. So, in this way you can calculate what should be the size of the bubbles.

Now in this case you have to have 1000 bubble or more than 1000 bubbles to calculate because, all the bubbles will not be same in size. So, in that case you have to calculate what would be the mean of that bubble. Generally, for any physical operations, so chemical engineering operations this type of discrete interfaces in that case Sauter bubble diameter that is called volume to surface area ratio mean diameter is considered.

So, volume to surface area ratio mean diameter is defined as this as  $d_{32}$  is denoted by  $d_{32}$  is defined as summation of  $n_i d_{bi}^3$  divided by  $n_i d_{bi}^2$ . Here  $d_{bi}$  means  $i$ th bubble diameter that will be equivalent for that individual bubble equivalent diameter and based on this equivalent bubble diameter if you consider suppose a 200 bubbles, then individually 2 for 200 bubbles you have to measure the perimeter and then equivalent diameter and then getting the cube of that equivalent diameter and square of that equivalent bubble diameter.

Then, after substitution of these you can get this  $d_{32}$  that is Sauter mean bubble diameter or it is called volume to surface area mean diameter. So, this will be your

actually mean diameter. So, out of may be 1000 or 100 more be more than 100 or maybe that is as per you suitable but it should not be less than 100 bubble then it will be somewhere yes, but if you are considering more than 100 bubble, then we around will be less.

So, after getting more than 100 bubbles and the calculating the mean diameter of the bubbles by this formula you can analyze what should be the number of bubbles here in this particular snapshot of this window. And in that case you can calculate that number plucks of the bubbles, number plucks means here per unit time how many bubbles will be crossing that cross section at which you are taking this bubble. So, in this case that number plucks can be denoted by  $N \cdot z$ , the  $z$  is the particular height of that a column.

So, at this column this is your  $z$ , so at this height if you are taking this bubbles and analyzing and getting the Sauter mean bubble diameter. Once you know the Sauter mean bubble diameter, then you will be able to call how many number of bubbles actually formed in that particular or moving in that particular cross section. So, it will be  $24$  into  $Q_g$  by  $d_{32}$  this is Sauter mean bubble diameter cube into  $d_c$  square. What is that actually? This is nothing, but that number of bubbles.

First of all you have to calculate the what should be the volume of that gas here that volume it will come what will be the volumetric flow rate of the gas that you are supplying there. Now since this window will be a mixture of gas and liquid, so in that case what should be the amount of particular gas there. If you know the gas holdup already we have discussed how to calculate the gas holdup, if you know the gas holdup there then what will be the amount of gas that is passing through this particular cross section.

So, that will be simple by  $Q_g$  into that is hold up of that, so that is the fraction of that total mixture of gas and liquid here that friction depends on the gas holdup. So, once you know that  $Q_g$  volume or volumetric flow rate of the gas there and if you divide it by the volume of the individual bubble then you will be able to calculate what should be the number of bubbles in that particular cross section. So, in this way you can calculate what would the number of bubbles.

Now, you will see there may be 100 bubbles more than 100 bubble even 1000 bubbles you do not know, but if you are considering suppose only 200 bubbles, let us consider only 200 bubble. So, individual you have to calculate what should be the size of that 200 bubbles. So, you have to segregate all this bubble size into a different classes, like if you are getting the bubble size range from 0.1 millimeter to 5 millimeter.

So, in that there is a wide range 0.1 to 5 millimeter range of this bubble size. So, you just classify this bubble size range into a 10 classes, like here 0.1 to 0.5 one class, 0.5 to 0.7 that is one class, 0.7 to 0.9 another class, 0.9 to 1.2 another class, 1.2 to 1.5 another class, 1.5 to 1.8 another class, 1.8 to 2.1 another class, 2.1 to 2.4 another class, 2.4 to 2.7 another class, 2.7 to 3.0 another class, 3.0 to 3.3 another class, 3.3 to 3.6 another class, 3.6 to 3.9 another class, 3.9 to 4.2 another class, 4.2 to 4.5 another class, 4.5 to 4.8 another class, 4.8 to 5.1 another class, 5.1 to 5.4 another class, 5.4 to 5.7 another class, 5.7 to 6.0 another class. So, you can consider just divide 0.1 2 point 0.1 to 5 millimeter then; that means, here 0.3, 0.1 to 0.31, 0.3 to 0.6 another class 0.3 to 0.9 another class like this you will get 10 classes.

So, out of that 10 classes what you have to do that you have to count how many number of bubbles are coming within that particular class. So, in that particular classes suppose 0.3 to 0.6 millimeter diameter range we are getting only 5 bubbles. Similarly 0.6 to 0.9 we are getting 10 bubbles, 0.9 to 1.2 millimeter we are getting what is that only 25 bubbles.

Similarly 1.2 to 1.5 in that class we are getting here 20 bubbles and then 1.5 to 1.8 millimeter range we are getting maybe only 6 bubbles. So, in this case you see there will be a number of bubbles that will be it depends on that classes that used in a certain classes how many bubble that depends on the size of the bubbles.

So, if you are getting the; that means, you see some particular very very small region of bubble size you see there may be more number of bubbles are many more number of bubbles in the higher range of that is bubble size is that depends on that size of the bubbles.

Now, if we represent this number of bubbles graphically what you can say that here, let us do a graph here in the x axis you just represent the size of the bubbles as  $d$  and accordingly here within a certain range of this  $d$  you are getting this number of bubbles in the y axis you have number of bubbles  $n$  number of bubbles.

And, here again another range bubbles you are getting these number of bubbles and another range you are getting these number of bubbles and other range for getting these number because another in these number of bubbles. So, in this case if you add this

number of bubbles here in axis we are getting this type of distribution. Here one example of this distribution from the experimental results it is given here in this case like this.

So, this is one distribution how say you are see this is a classification classified are different in different classes how number of bubbles are there, but here relative of numbers here. So, number of bubbles it will be represented as a frequency that is number of bubbles within a certain range of bubble sizes how many numbers of bubbles is coming that are analyzing that will be your frequency of the bubbles.

And you can represent the size distribution based on this frequency that is number of bubbles or you can represent the density of the bubbles, what is that density? That means here, per unit cross sectional area or you can say per unit volume of that cross sections that you are considered, how many number of bubbles are coming there? So, if you divide that number of bubbles divided by that is volume of that particular cross sections, then you can have the density of the bubbles.

So, you can represent the in terms of density of the bubbles as a function of a bubble diameter. Also you can represent the size distribution based on what is that relative frequency. What is that relative frequency? You will see if you are considering that 200 a number of bubbles, then for individual classes how many numbers of bubbles are you are getting.

If you divide that number of bubbles by the total number of bubbles, then you will get the relative frequency and accordingly as per the bubble class based on the size you can get the what is that relative frequency accordingly. Then you can represent it in a graph, then it will be called as size distribution of that bubble. And this bubbles formation of course, depends on the what type of distributor you are using. If you are using a more you know that finer (Refer Time: 30:11) based that is the distributor you can expect more finer bubbles there.

But sometimes this size may depend on physical properties of the system also because; if you are using more viscous liquid in that case you may expect sometimes finer bubbles there. Even, if you are using a sum surfactant or if you reduce the surface tension of the solution you will see you may get the more finer bubbles their.

Size will be reduced if you add some surfactant there. And during that addition of the surfactant you may expect, the foam also because foam is something different from the bubbles that foam will have certain shape of that what is that hexagonal some structure, but in the bubbles not have some structure in that case it will be having that what is that round shape maybe a different shape, but it not have the particular structure. So, foam that is air bubble so, in that case it will have some particular regular a structure.

So, foam also can be measured in that way because that is also important for application of prop rotation process and in that case for a mineral beneficiation the prop rotation is important. So, in that case this bubble size of foam size analyses also important; another important point, that if you are using like a distributor in such a way that nozzle type distributor in that case you may expect bigger bubbles.

But sometimes due to the shear effect from the nozzles there will be a small bubbles will form, but after a certain time it may coalesce to each other based on the physical properties of the system and forming a big bigger bubbles. Sometimes from the jet also you will see their formation of elongated bubbles, so you not get that a spherical bubbles there. So, in that case you have to carefully analyze the weather you should consider that Sauter mean bubble diameter not.

Generally for that it is not suggest to consider that Sauter mean bubble diameter. So, only that length and what is that cross section of the bubbles you have to consider like bullet shaped bubble sometimes it is forming in (Refer Time: 32:50) flow system. So, with that case what is the length of the bullet, what are the cross section of the bullet that will be analyzed not like that mean diameter will be considered.

So, based on that bullet volume and surface area what should be the interfacial area for that mass transfer that is generally being considered, but for other type of bubbles that you have to consider in this way what should be the Sauter mean diameter there. Generally, for discrete bubble it is considered and generally bubble diameter up to 5 millimeter this type of the Sauter mean bubble diameter is considered as a mean diameter and further analysis.

And what about then liquid-liquid operation this is supposed this to gas liquid operation what droplet formation in liquid-liquid operation. Suppose this kerosene is supplied in a

pool of what is that water, so from the bottom then we will see the kerosene will be a flowing in that water medium as dispersed phase of droplet.

So, in the same way by camera you can I take the snapshot of the droplet and then analyze what should be the size of the droplet and also in the same way you can calculate Sauter mean bubble diameter and then what should be the interfacial area. Once you know that Sauter mean bubble diameter, then interfacial area; that means, the specific interfacial area it is generally interfacial area of surface area divided by whole volume of what is that gas liquid mixture or liquid-liquid mixture there.

So, in that case for gas liquid operation that specific interfacial area is denoted by  $a$  and it will be  $a = 6 \alpha g$  by Sauter mean bubble diameter  $d_{32}$ . So, this is your formula to calculate the interfacial area. Once you know that the interfacial area, then you will be able to calculate what should be the volumetric mass transfer coefficient because there will be a mass transfer from the gaseous phase to the liquid phase to the interface.

And overall concentration differences will be the what is that driving force and based on phase there will be a that is how much moles or mass will be transferred due to this concentration gradient that to be proportional to this account solution differences. So, based on which you can get the proportionality constant at the volumetric mass transfer coefficient. This volumetric mass transfer coefficient actually is a product of the individual mass transfer coefficient and what is that specific interfacial area.

So, in that case this is specific interfacial area to be actually estimated by this method or other method like a chemical method you can calculate. So generally photographic method by photographic you can have this specific interfacial area as  $a = 6 f_s \alpha g$  by  $d_{32}$  for that you have to calculate the Sauter mean bubble diameter to calculate the Sauter mean bubble diameter you have to use camera taking a snapshot of that a bubble and then analyze by what is that software images software or some other suitable software you can use.

And after analyzing that individual bubble size then you have to calculate the Sauter mean bubble diameter, so in this way you can get interfacial. Now, one important point you have to remember here since the bubbles all the bubble size are not in spherical in shape we are just considering the equivalent volume of the sphere and then we are

calculating equivalent bubble diameter. But in other way also you can calculate the equivalent diameter of individual bubbles.

Suppose, if you are getting the ellipsoidal shape of bubbles, then what you have to do? You just mark it out what should be the here suppose ellipsoidal bubble, here just measure this measure axis and measure the minor axis. If you know that major axis and if you know the major minor axis of that, then you will be able to calculate equivalent diameter as  $l_{max}^2$  into  $l_{mean}$  to the power  $1/3$ , here this will be measure axis is  $l_{max}$  and minor axis will be  $n l_{mean}$ .

So, if you know the  $l_{max}$  and  $l_{mean}$  then you will be able to calculate what will be the equivalent diameter. So, in this way we are getting the bubble size and its distribution. Now, once you know this a distribution you can feed this distribution and you can analyze by different model of a size distribution. Generally lognormal distribution is one important model by which you can analyze this type of bubble size distribution or droplet size distribution.

So, I am not going to details on that lognormal distribution here up to this it is fine you have to know this is the way to find out the bubble size. And what is that let us consider what should be the rise velocity or terminal rise velocity of that individual bubbles. Whenever bubbles are actually allowed to float in the continuous liquid medium or in a liquid medium gas liquid medium what will happen? You see the bubbles will try to go up due to their Buoyancy effect. Now, due to the Buoyancy effect it will be moving up with a certain velocity, this velocity is called terminal velocity of the individual bubble or particle.

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### Bubble/droplet rise velocity

**The terminal rise velocity of a bubble as**

$$u_b = \frac{\Delta L}{\Delta t}$$

**As per Lahrer (1976)**

$$u_b = \sqrt{\frac{3\sigma}{\rho d_{b,eq}} + \frac{d_{b,eq} g \Delta \rho}{2\rho}}$$

**The size of bubbles produced at the orifice**

$$d_{b,eq} = 2.9 \left( \frac{\sigma d_o}{g \rho_l} \right)^{1/3}$$

**Equation for bubble diameter:**

$$d_{b,eq} = \left( l_{\max}^2 l_{\min} \right)^{1/3}$$

Lahrer, H. G. A rational terminal velocity equation for bubbles and drops of intermediate and high Reynolds number. J. Chem. Eng. Jpn. 1976, 9, 237.

Now, how to actually estimate this terminal rise velocity of a bubble or droplet? Now, if you consider this bubbles here as shown in your here in the video you will see a from the bottom part of the column bubbles are bubbles suppose individual bubble which is allowed from a series or nozzles that individual bubbles is coming and producing it is going up due to the Beyonce effect.

Now if you consider this bubbles movement and recording it to highest speed camera and then you can measure what will be the length or distance that is travelled by this individual bubbles there. Now, suppose this bubbles is here see this bubble is moving up due to the Beyonce; due to Beyonce effect and it follows a certain pot and if we measure this length; if we measure this length with respect to time.

Then within a certain period of time like delta t if you are considering does this to this length is travelled and then we can say that what would be the rise velocity of these bubbles. So, this rise velocity is will be is equal to delta L by delta t. So, here if suppose this bubbles is go straight then only simple this to this length to will be there, so that will be your rise velocity.

So, delta L that is distance that is travelled by the bubble per unit time delta t, so this will be your terminal rise velocity. Now this rise velocity maybe some times faster may be slower it depends on the size of the bubbles, if suppose bubble size is larger then you will see that it will move fast, it will be moving at high speed and high Beyonce effect will be

there and because of which it will be going higher. So, in that case the residence time of the bubbles will be; that means, how long it will take in the column in the liquid media that is called residence time of the bubble.

The residence time of the bubbles will be here in this case lower because of higher buoyancy effect due to the largest size bubbles, but if you produce the smaller bubbles what will happen that there buoyancy effect will be low and because of its terminal velocity will be very lower or low. So, in this case you can expect that the bubbles will reside a longer time in the column. So, in that case retention time or residence time of the bubbles will be more than earlier.

So, here retention time or you can say that residence time of the bubbles that depends on the size of the bubbles. So, that is why sometimes some processes it is one of the important factors why you have to consider the residence time. Suppose, if the same example carbon dioxide gas absorption in the liquid, now you have to allow the bubbles carbon dioxide gas bubbles for longer time in the column. So, that your mass transfer will be more, if size is higher, if the bubble size is higher what will happen immediately it will go up and to be collapsed at the surface.

So, in that case you may not get the 100 percent transfer of the carbon dioxide gas to the liquid medium. So, in that case you need more residence time. Even some specific reactions also gas liquid reactions, sometimes it needs more residence time for slow reaction. So, in that case you have to produce more finer bubbles, so that the residence time will be more. So, this is an important factor, so all this residence time in bubble rise velocity all depends on the medium properties or you can say that physical properties of the system like surface tension, like density like viscosity all those things.

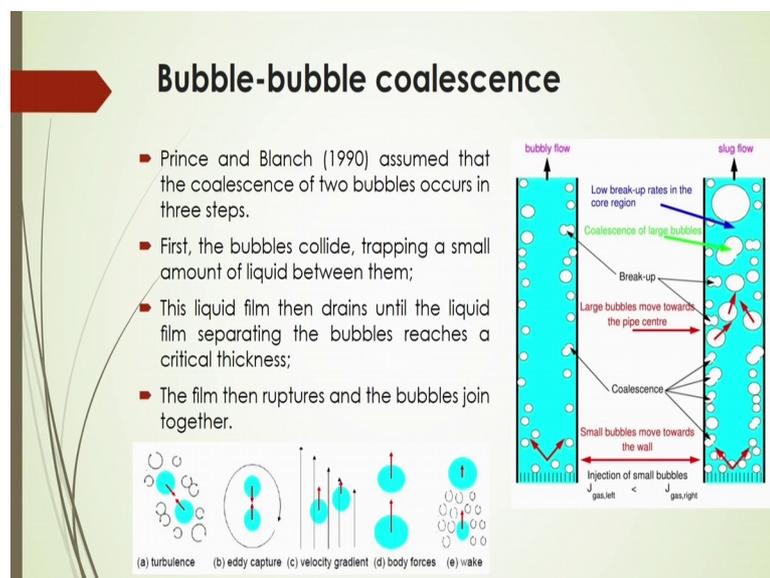
So, here one important correlation by which you can calculate what should be the rise velocity of the bubbles? Once you know the surface tension of the system, once you know the size of the bubble, once you know the density difference of the gas and liquid and then you will be able to calculate what should be the bubble rise velocity. Once you know the bubble rise velocity, then what should be the residence time of the bubbles in the column provided that you have the length of the column or length of the liquid medium in the column. So, in this way you can calculate what should be the bubble rise velocity and residence time of the bubble.

Similarly for droplet also it will be the same way to calculate this residence time and the terminal velocity of the droplet and another one important correlations I have to say here for equivalent bubble diameter, if suppose bubble is produced and it will be distributed through a distributor and the distributor whole size is  $d_0$  that is orifice diameter it is called to the orifice the gas is coming out from the distributor.

And then in this case the orifice diameter if it is the  $d_0$  once you know that orifice diameter and the physical properties of the system then we will you be able to calculate what should be the equivalent bubble diameter that is coming out from the distributor. So, this is the formula that you can directly use to calculate the equivalent diameter.

So, this equivalent diameter if you substitute the substitute is equivalent diameter here in this equation and surface tension and density of the systems, then you will be able to calculate easily what should be the bubble rise velocity and also the residence time distribution.

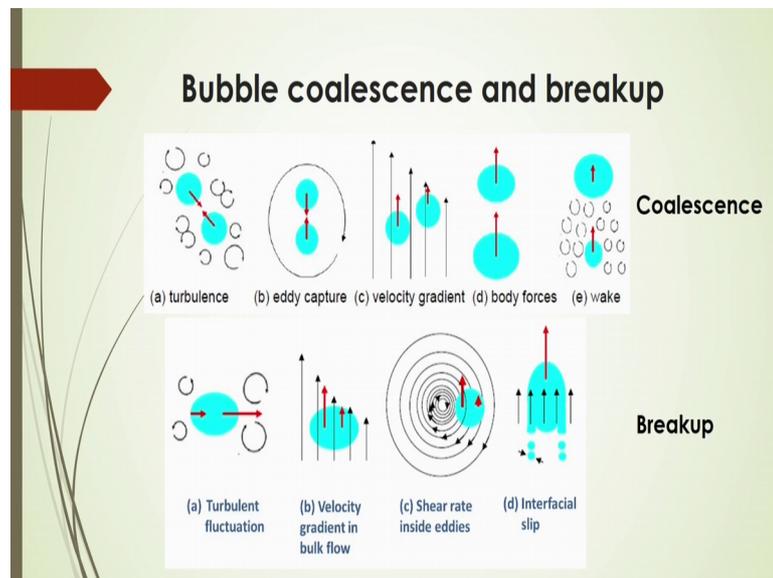
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And bubble-bubble coalescence one important aspect is this case because you have to know how actually this coalescence of the bubbles happened. Here you see there are several methods are actually explain by different investigators that when two bubbles are coming to each other and how they are actually colliding to each other and their joining to each other and making a big bubbles. Suppose two small bubbles are coming to each other and they collide first and they were trapped a small amount of liquid between them.

After that what will happen, the when they will attracting between them the small liquid film will be there and that liquid film drains until the liquid films separating the bubbles and reaches a critical thickness. And the film then ruptures and bubbles join together and then bubble coalescence will happen. The coalescence of the bubbles; that means, the two bubbles will come or two or more than two bubbles also may come to each other at a particular conditions and getting joining to each other and making a what is that big bubbles, so this is called coalescence.

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And breakup means what due to the inertia effect that is turbulence of the floor you will see two bubbles will come to each other and colliding to each other sometimes gaining some initial effect and it will break up into a two bubbles. So, it is called breakup efficiency, so there is a coalescence and the breakup. So, in a particular process when you are doing that process based on this what is that interfacial production by gas liquid operation or liquid-liquid operation some (Refer Time: 47:16) coalescence and bubble breakup happen in the system.

So, that what would be the rate of coalescence what would be the rate of breakup that you have to know of our further analysis of the process and modeling of the system. So, here we are not going to detail of that rate of that bubbles and breakup and this is not the scope of this courses. So, this will be discussed in for the other courses and the MOOCs I think multiple pro systems it is I think even in details there, so you can follow.

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### Maximum stable size of bubble/drop

- The size of stable bubbles can be determined by the forces acting on the bubble.
- In low viscosity liquids the bubbles are deformed by forces arising from liquid velocity fluctuations acting over distances of the order of the bubble diameter,  $d_b$ .

$$\overline{u'^2} = \lambda \left( \frac{\dot{E}_v d_b}{\rho_l} \right)^{2/3}$$

- The deforming force resisting the deformation of the bubble is due to surface tension acting at the gas-liquid interface.

$$d_{b,max} = \left( \frac{We_c^3 \sigma^3}{\lambda^3 \rho_l} \right)^{1/5} \dot{E}_v^{-2/5}$$

$We_c$  = critical Weber no. = 1.18  
 $\lambda = 2$   
 $\dot{E}_v$  is the average energy dissipation rate per unit volume  
 $\sigma$  = Surface tension  
 $\rho$  = Density of liquid

Now, another important aspects that if you are producing this bubbles what should be the maximum stable size of the bubbles confirm that depends on the what is that viscosity of the liquids and what is that forces that acting on the surface of the bubbles and there will be a suppose flow rate of the liquid or gas in a system, then what are the liquid velocity fluctuation setting over the distance of order of that bubble diameter that is main important factor to actually breakup of the bubbles and also coalescence of the bubbles. So, if you know that velocity fluctuation that is generally represented by this average of this is a square of these velocity fluctuations, it is generally depends on the energy dissipation per unit volume in the systems and also size of the bubbles.

And the deforming force for the resisting of this deformation due to this velocity fluctuation imposing on the surface of the liquid and that will be balanced by that what is that surface tension acting that the gas liquid interfaces. So, surface tension is very important there, that surface tension you will actually resist to the deformation of the bubbles. Now, at a certain condition you will see these two forces will be in such a way that at a particular stable condition of the bubbles surface tension forces will balance the other forces, so that the bubbles will get the stable conditions.

So, in that case it will be calculated based on this equation, this will be at the critical condition that critical condition will be represented by the critical Weber number and this critical Weber number this is nothing, but  $We_c$  that will be is equal to what is that rho l

$u$  square  $d_b$  bubble diameter at that critical bubble diameter this is sorry this is  $d_b \rho u$  square  $d_b$  by  $\sigma$ .

So, this is your critical Weber number, so this critical Weber number it will be generally 1.18 for this stable bubble size and in this case what should be the energy dissipation this energy dissipation rate per unit volume will be  $E_v$ ; that means, how much energy is supplied there you have to calculate the kinetic energy and then you have to divide it by total volume of the gas liquid mixture, then you will get the energy dissipation rate per unit volume.

And then  $\sigma$  is the surface tension  $\rho$  is the density of the liquid. So, once you know this surface tension density and average energy dissipation rate and critical number is 1.18, then you can get the maximum stable bubble size there.

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**Interfacial area**

**Specific interfacial area**

$$a = \frac{\text{Surface area of bubble}}{\text{Volume of dispersed bubble bed}} = \frac{S_b}{V_b/\epsilon_g} = \frac{\pi d_{b,32}^2 \epsilon_g}{(1/6)\pi d_{b,32}^3} = \frac{6\epsilon_g}{d_{b,32}}$$

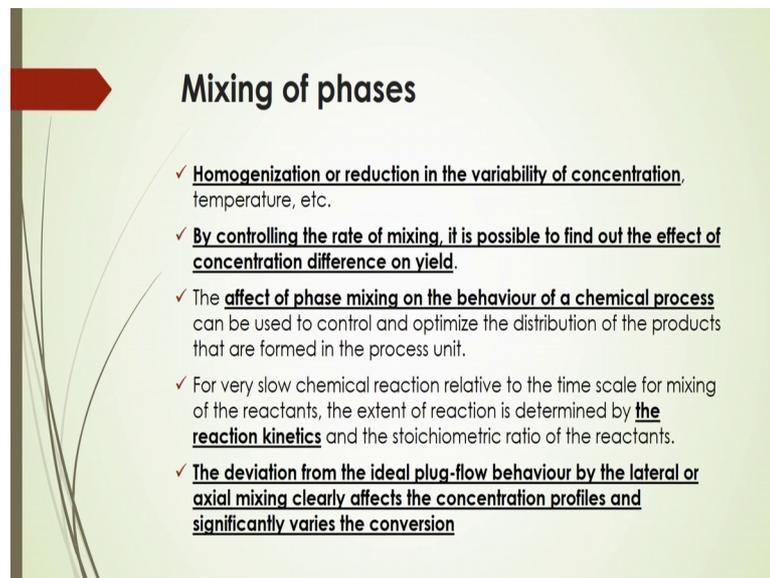
$$n = \frac{\epsilon_g}{(\pi/6)d_e^3} = \frac{\epsilon_g}{\pi \left(\frac{d_e}{6}\right)^3 d_{32}^3} = \frac{\epsilon_g}{\pi \left(\frac{d_e}{d_{32}}\right)^3 \left(\frac{6\epsilon_g}{a}\right)^3} = \frac{a^3}{36\pi \left(\frac{d_e}{d_{32}}\right)^3 \epsilon_g^2} = \psi \left(\frac{a^3}{\epsilon_g^2}\right)$$

$\psi = \frac{1}{36\pi} \left(\frac{d_{b,32}}{d_{b,e}}\right)^3$  depends on the shape of the bubbles.  
 For spherical bubble,  $\psi = 1/(36\pi)$

Now, interfacial once you know this bubble size then what should a specific interfacial area, the specific interfacial area is defined as surface area of the bubble divided by volume of this first bubble update in the column. So, that will be a  $S_b$  is the surface area of the bubble and this will be  $V_b$  by absolute this is your volume of this first double bed and finally, you can get this  $6 \epsilon_g$  by  $d_b d_{32}$  we have already told earlier that I have given that this specific interfacial area would be this.

So, through this interfacial area you can expect the mass transfer there and what would be the number of bubbles that you can calculate based on this once you know this interfacial area and gas holdup, then you can get this what is the number bubbles and this year one terms is  $shy$ ;  $shy$  is defined as  $1 \text{ by } 35 \text{ by } d \text{ b } 32 \text{ by } d \text{ b } e$ , equivalent that depends on the shape of the bubbles and for spherical bubble  $shy$  should be  $1 \text{ by } 36 \text{ pi}$ . So, in this way you can calculate what would be the interfacial area and the number of bubbles.

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### Mixing of phases

- ✓ Homogenization or reduction in the variability of concentration, temperature, etc.
- ✓ By controlling the rate of mixing, it is possible to find out the effect of concentration difference on yield.
- ✓ The affect of phase mixing on the behaviour of a chemical process can be used to control and optimize the distribution of the products that are formed in the process unit.
- ✓ For very slow chemical reaction relative to the time scale for mixing of the reactants, the extent of reaction is determined by the reaction kinetics and the stoichiometric ratio of the reactants.
- ✓ The deviation from the ideal plug-flow behaviour by the lateral or axial mixing clearly affects the concentration profiles and significantly varies the conversion

Another important aspects of the multiphase flow system is mixing of the phases because mixing will give you the system whether it is homogeneous or heterogeneous system. Sometimes homogeneous of system means here for the concentration of the gaseous particles will be distributed throughout the column uniformly, so that will be your homogeneousness. And heterogeneous it may not be distributed in all sections of the column equally, so that is heterogeneous. So, it depends on this homogeneousness or heterogeneousness depends on the degree of mixing how this gas and liquids are mixing in the column.

So, you have to know what should the degree of mixing in the particular operations and based on that mixing what should be the performance of the particular process then you can calculate their. Generally, you will see that if you are getting a more mixing sometime some physical processes will give you the better performance, if you are

getting sometimes the heterogeneous mixing some specific process you may get the more advantage or more performance of that system.

Now, if you are having the physical absorption in that case homogeneousness; that means, more mixing sometimes it give you what is that fever of that process. And sometimes heterogeneousness also sometimes it will be some heat transfer operations it may give the more advantages of more preferable for the particular processes. But some processes like absorption processes like a gas and solid mix in that case back mixing of if it is happened, so higher mixing may not give the fruit full results for that may be there a reduction of that proficiency or efficiency of the particular process will be there.

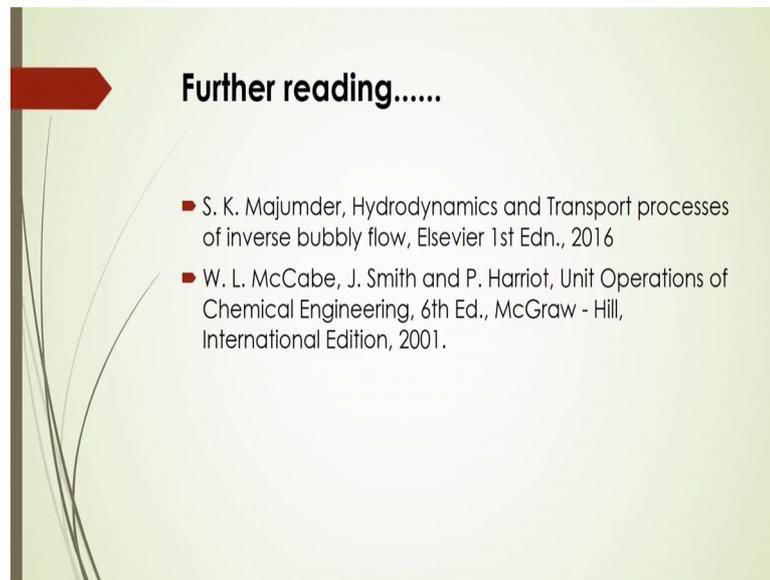
So, you have to then optimizing who is the condition of this mixing or what degree of mixing should be suitable for that particular processes, so in that way you have to know the mixing processes. And the dick to calculate the degree of mixing generally you see a two parameters are being estimated; one is residence time of the that is interfaces are like bubbles, droplets of particles through who is the mass transfer will happen.

And what is that other important parameter like it is called dispersion number; dispersion number is basically one parameter who is being used to calculate the degree of mixing. If this dispersion number is more higher than more degree of dispersion will be there, if dispersion number is lower it does means that there will be a less mixing there. So, how to estimate that mixing characteristics?

Generally, tracer techniques are being used to calculate that mixing characteristics and in that case one tracer particles are being insert into the column and then finally, the sample is being analyzed with respect to time. And then you get a certain profile of that and after analyzing that profile you can estimate the dispersion number their.

So, I am not going to deep on that particular estimation of this mixing processes here in this course and it will take I think another two three lecture to discuss this mixing and this is also not the scope of this process here this lecture also and so you can get it from other courses that mixing characteristics of the multiphase flow systems.

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So, here I want to stop I would suggest to go further reading if you are interested to analyze the process based on the mixing and model the process, then you have to go for the reading for that what is that the books that you have to follow for further understanding of this mixing process even multiphase flow systems and bubble size analyses all those things so.

Thank you for this.