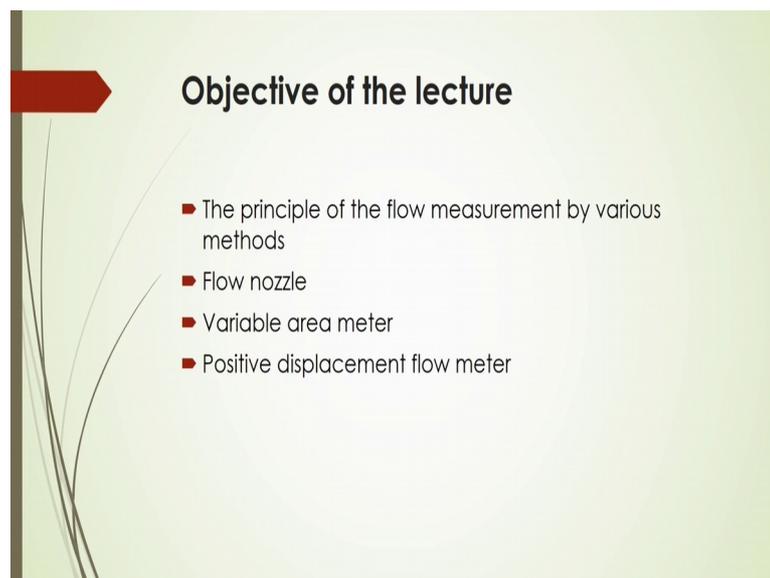


**Fluid Flow Operations**  
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**Module – 11**  
**Lecture – 28**  
**Measurement of Flow-Part 2**

Welcome to this massive open online course on Fluid Flow Operations. So, in this lecture, we will continue the module 11 as a Part 2 and I will discuss the phenomena of Flow Measurement. And we have discussed the flow measurement systems of venturi meter, pitot tube, orifice meter and also where by which how we can measure the flow rate and velocity based on the energy balance equation; mainly Bernoulli's equations are required to assess all those measuring devices.

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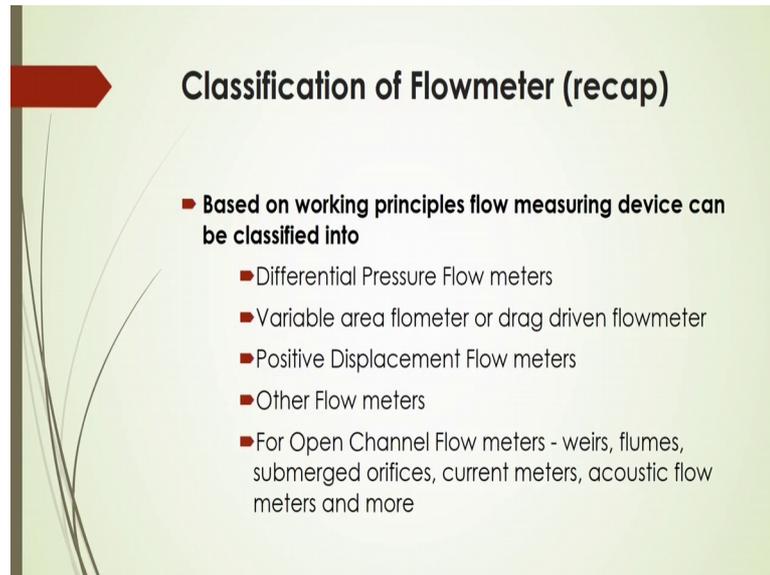
The slide features a light green background with a dark green vertical bar on the left side. A red arrow points to the right from the top of this bar. The title 'Objective of the lecture' is positioned to the right of the arrow. Below the title, there is a bulleted list of four items, each preceded by a red square bullet point.

**Objective of the lecture**

- The principle of the flow measurement by various methods
- Flow nozzle
- Variable area meter
- Positive displacement flow meter

Now, here in this lecture, we will also discuss the principles of flow measurements by other various methods like a flow nozzle, variable area meter, positive displacement flow meter.

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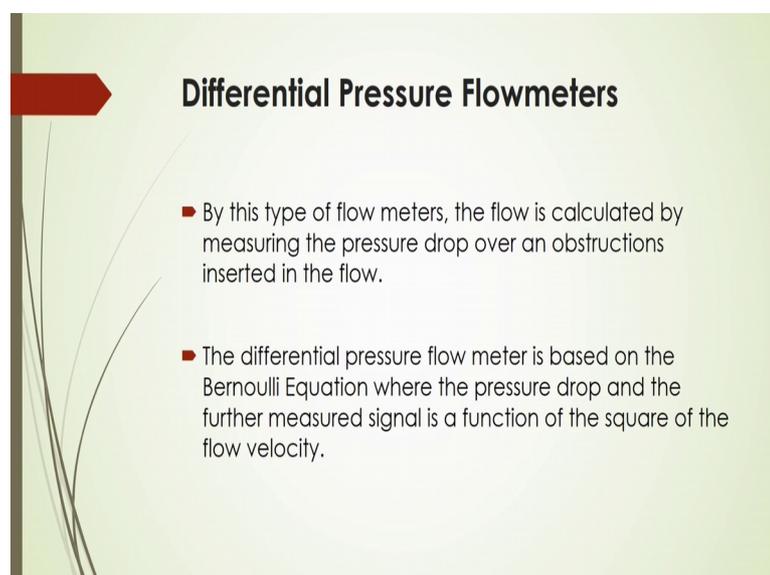


### Classification of Flowmeter (recap)

- Based on working principles flow measuring device can be classified into
  - Differential Pressure Flow meters
  - Variable area flowmeter or drag driven flowmeter
  - Positive Displacement Flow meters
  - Other Flow meters
  - For Open Channel Flow meters - weirs, flumes, submerged orifices, current meters, acoustic flow meters and more

So, as we have already classified the different types of flow devices based on the working principle of the measuring device so, we can classify it has differential flow meters, differential pressure flow meters, variable area flow meter or dark drag driven up flow meter. We have already talked about that different types of that flow meter and also what are the different type of positive displacement flow meters. Other different types of flow meters also we will discuss in the next lecture also. And for open flow channels, they are how to actually measure the flow rates that is by weirs, flumes, submerged orifice, current meters, acoustic flow meters and more other are available.

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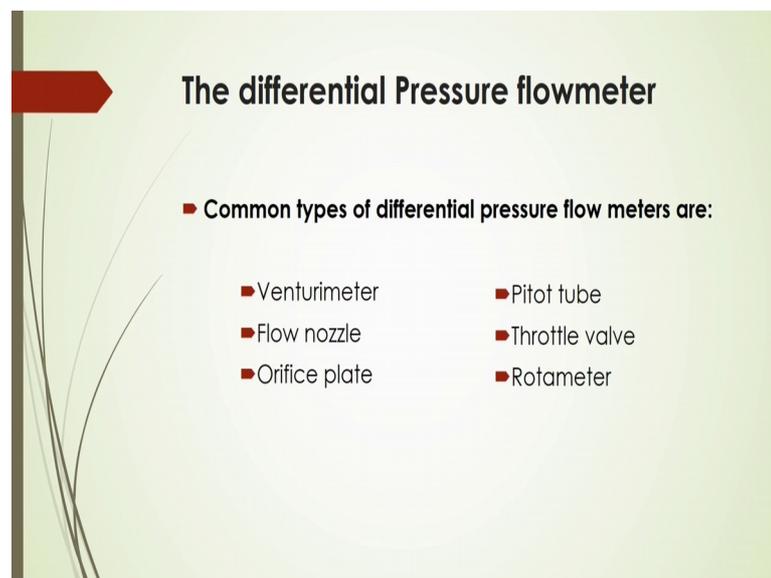
### Differential Pressure Flowmeters

- By this type of flow meters, the flow is calculated by measuring the pressure drop over an obstructions inserted in the flow.
- The differential pressure flow meter is based on the Bernoulli Equation where the pressure drop and the further measured signal is a function of the square of the flow velocity.

So, here as per this category, we will discuss the different other flow measuring devices. And differential pressure flow meters, already we have discussed in the previous lecture that by this type of flow meters. So, the flow is calculated by measuring the pressure drop over an obstruction inserted in the flow. And the differential pressure flow meter is based on the Bernoulli's equation that we have discussed how to derive those equations to calculate the velocity and if you know the velocity, then how to calculate the volumetric flow rate from this cross sectional area and the velocity.

And also other different discharge coefficients also based on the equipment or devices that already derived and how to obtain experimentally and also what would be the typical values that we have discussed.

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And based on that we have actually discussed about differential pressure flow meter like venturi meter, flow nozzle, orifice plate, pitot tube, throttle valve and rotameter. All those are based on pressure differential pressure flow meter, but here this rotameter also, it is called at the area of flow meter or drag flow meter. So, we will discuss here this flow meter also.

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### Bernoulli's Equation (uniform Flow)-Recap



For an incompressible fluid ( $\rho = \text{constant}$ ),

$$\frac{v^2}{2} + \frac{p}{\rho} + gz = \text{constant} \quad (\text{Eq. 1})$$

Unit:  $\text{m}^2/\text{s}^2$  or  $\text{J}/\text{kg}$

Multiplying each term by  $\rho$ ,

$$\frac{\rho v^2}{2} + p + \rho gz = \text{constant} \quad (\text{Eq. 2})$$

Unit:  $\text{N}/\text{m}^2$

Dividing each term by  $g$ ,

$$\frac{v^2}{2g} + \frac{p}{\rho g} + z = H = \text{constant} \quad (\text{Eq. 3})$$

Unit:  $\text{m}$

Labels for Eq. 3:

- velocity head:  $\frac{v^2}{2g}$
- pressure head:  $\frac{p}{\rho g}$
- potential head:  $z$
- total head:  $H$

And let us have an idea again that what would be the Bernoulli's equation that for an incompressible fluid, we know that for uniform flow. The density will be is equal to constant and based on which the Bernoulli's equation that we are having the energy, summation of the energy will be constant and this energy form of equation that it is represented in different form like here. This is one form equation 2 and equation 3, this is one head form, this is what is that this is an energy form and also it is energy per unit mass form. And, in that case we are getting this the Bernoulli's equation to represent the working principle of the different measuring devices.

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### Bernoulli's Equation for non-uniform flow (recap)

- In a real fluid flow across a large cross sectional area, the velocity distribution is not uniform
- The kinetic energy per unit weight of the fluid transferred across the section is greater than that calculated by using average velocity
- Hence true kinetic energy per unit weight can be expressed as

$$KE_{true} = \alpha \frac{v^2}{2g} \quad (\text{Eq. 4})$$

Then Bernoulli's Equation Takes the following form:

$$\alpha \frac{v_1^2}{2g} + \frac{p_1}{\rho g} + z_1 = \alpha \frac{v_2^2}{2g} + \frac{p_2}{\rho g} + z_2 \quad (\text{Eq. 5})$$

$\alpha$  is called kinetic energy correction factor

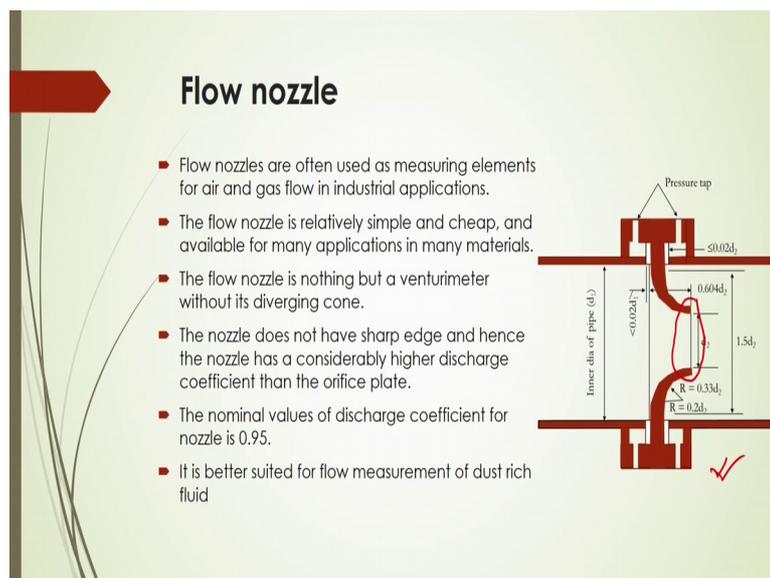
$\alpha = 1.03 \text{ to } 1.06$  for turbulent flow

$\alpha = 2$  for laminar flow

And also for the non-uniform flow in that case, we have to account the correction factor for kinetic energy change as well as in case of turbulent flow and laminar flow what will be that correction factor that already we have discussed. And this equation number 4 will give you that kinetic energy correction factor by incorporating. And, that incorporating of this kinetic energy correction factor to be represented here, in this equation number 4 to get the total kinetic energy in the actual case.

And, the Bernoulli's equation that takes this falling form after correction of this kinetic energy correction factor and it will be that correction factor will be 1.03 to 1.06 for turbulent flow and it would be 2 for laminar flow that has already been discussed in the previous class. So, you have to remember this correction factor for the laminar flow and the turbulent flow and incorporation of this correction factor in the Bernoulli's equation for non-uniform flow.

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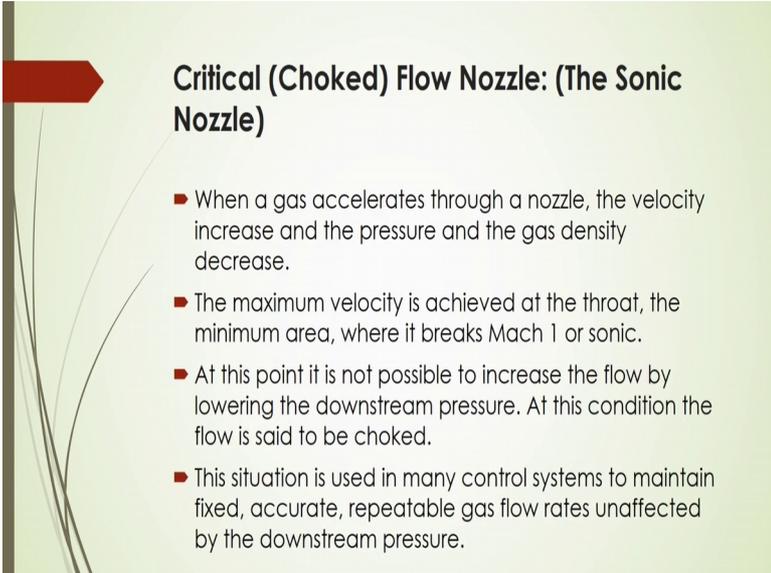


And, then we will discuss here what will be the flow nozzle and how this flow nozzle will be used to measure the flow rate there. So, in this case, again the principles of this Bernoulli's equation to be applied, but here in this case the flow nozzle is relatively simple and cheaper than other flow devices. In this case, this is working based on the venturi principle and, but in this case you have to remember that there will be no diverging cone as per venturi meter.

But the working principle will be almost the same as a venturi meter and these flow nozzles are often used as measuring elements for air or gas flow in industrial applications. And, also you will see the nominal values of the discussed coefficients is for this nozzle is 0.95 and the this nozzle a flow nozzle for this measuring of the volumetric flow rate and in this case, it will be better suited for the flow measurement of the dust rich fluids that you have to remember here. And, another point that we are in this case, the flow nozzle will not have the sharp edge and hence nozzle has a considerably higher discharge coefficient than the orifice plate.

So, here it is given the design of this orifice sorry flow nozzles here and what should be the distance of this what is that flow nozzles and also what is the diameter of that flow nozzles and what should be the at least percentage of that diameter at the main pipe of this flow nozzle systems and it is given in this figure. So, you have to follow this design to actually design a new device of this flow nozzle here.

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**Critical (Choked) Flow Nozzle: (The Sonic Nozzle)**

- When a gas accelerates through a nozzle, the velocity increase and the pressure and the gas density decrease.
- The maximum velocity is achieved at the throat, the minimum area, where it breaks Mach 1 or sonic.
- At this point it is not possible to increase the flow by lowering the downstream pressure. At this condition the flow is said to be choked.
- This situation is used in many control systems to maintain fixed, accurate, repeatable gas flow rates unaffected by the downstream pressure.

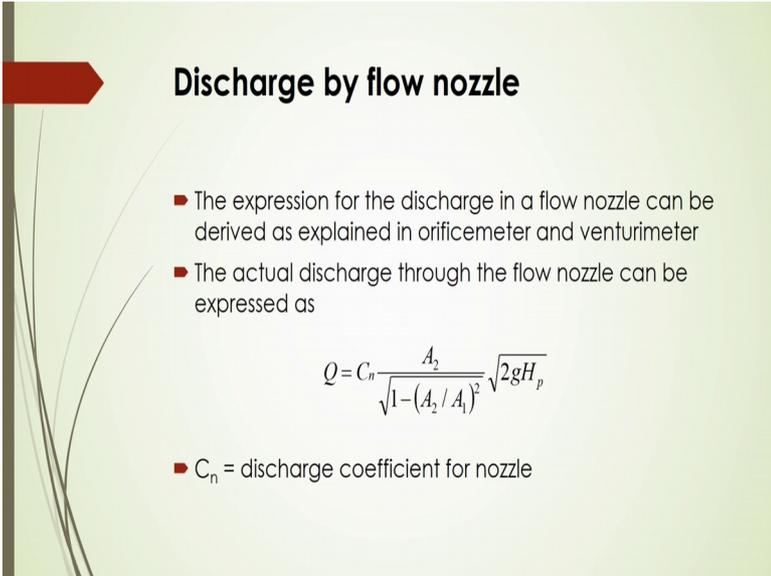
And in this case when a gas, if you are getting that accelerating through a nozzle and the velocity is increased and pressure also is increased. In that case, you will see the gas density will decrease.

So, in that case you will get some extent the maximum velocity and it will achieve at the throat and where the minimum cross sectional area will be there in the flow nozzle systems. And in that case you will see the maximum velocity you will obtain and the

maximum velocity will be such that it sometimes breaks the Mach number 1 and so, it may be higher than the Mach number 1 may be supersonic case also it will come based on the velocity. And at this point, it is not possible to increase the flow by lowering the downstream pressure as earlier what we have discussed. And at this condition the flow is say to be choked and the situation is sometimes used to control systems to maintain fixed and accurate and also repeatable gas flow rates that will be unaffected by the downstream pressure.

So, that is why this flow nozzle is sometimes it will be called as sonic nozzle where it operates the Mach number is equal to 1 or at the sound velocity they are for measuring the gas or air flow rate.

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**Discharge by flow nozzle**

- The expression for the discharge in a flow nozzle can be derived as explained in orificemeter and venturimeter
- The actual discharge through the flow nozzle can be expressed as

$$Q = C_n \frac{A_2}{\sqrt{1 - (A_2 / A_1)^2}} \sqrt{2gH_p}$$

- $C_n$  = discharge coefficient for nozzle

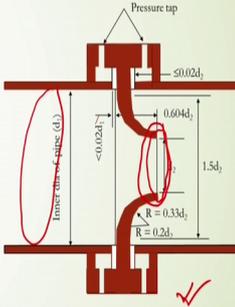
Now, how to calculate this discharge rate by this flow nozzle? And the what will be the expression for this discharge of the flow nozzle and this discharge rates can be derived as per the principles whatever given in orifice meter and venturi meter.

So, the same principle of Bernoulli's equations to be applied to get this discharge by this flow nozzle and actual discharge to the flow nozzle can be expressed by this equation here given that will be Q is equal to C n to A 2 divided by root over 1 minus A 2 by A 1 whole square and to root over 2 g H p here. So, in this case, C n is called the discharge coefficient for the nozzle and A 2 and A 1 are given respectively in this sections here this is your A 1 and A 2 is the what is that nozzle cross sectional area.

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### Flow nozzle

- Flow nozzles are often used as measuring elements for air and gas flow in industrial applications.
- The flow nozzle is relatively simple and cheap, and available for many applications in many materials.
- The flow nozzle is nothing but a venturimeter without its diverging cone.
- The nozzle does not have sharp edge and hence the nozzle has a considerably higher discharge coefficient than the orifice plate.
- The nominal values of discharge coefficient for nozzle is 0.95.
- It is better suited for flow measurement of dust rich fluid



So, if you know this nozzle cross sectional area and the pipe cross sectional area, then from the pressure difference you will be able to calculate what should be the discharge. Then for actual discharge by this flow nozzle, you have to incorporate this discharge coefficient as  $C_n$  and  $C_n$  to be incorporated or it will be taken as 0.95 in maximum cases.

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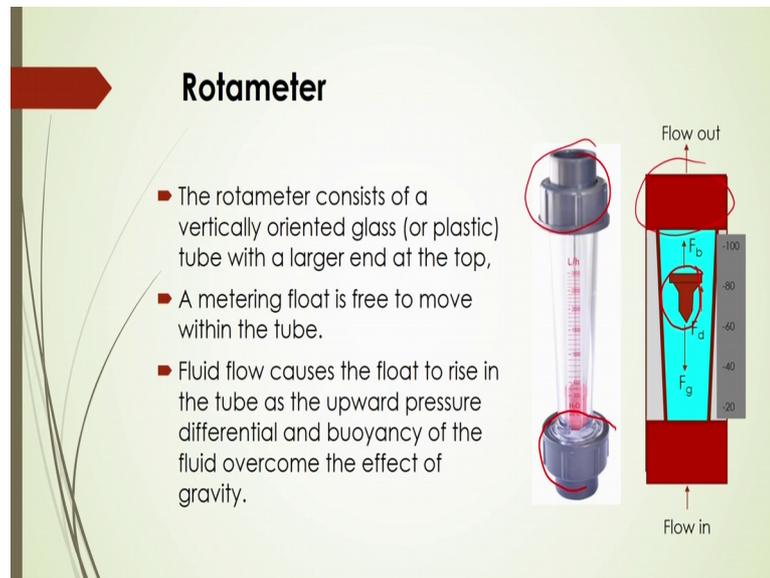
### Variable area or drag type flowmeter or gravity-type flowmeter

- Rotameters
- Turbine meters

And this variable area or drag type flow meter or gravity type flow meter also is one of the important flow meter by which you can measure the flow rate. And maximum in the

laboratory scale, you will see for the maximum use of these rotameters or turbine meters are being used to assess the flow rate or what will be the flow rate for that particular process to be measured by this rotameter.

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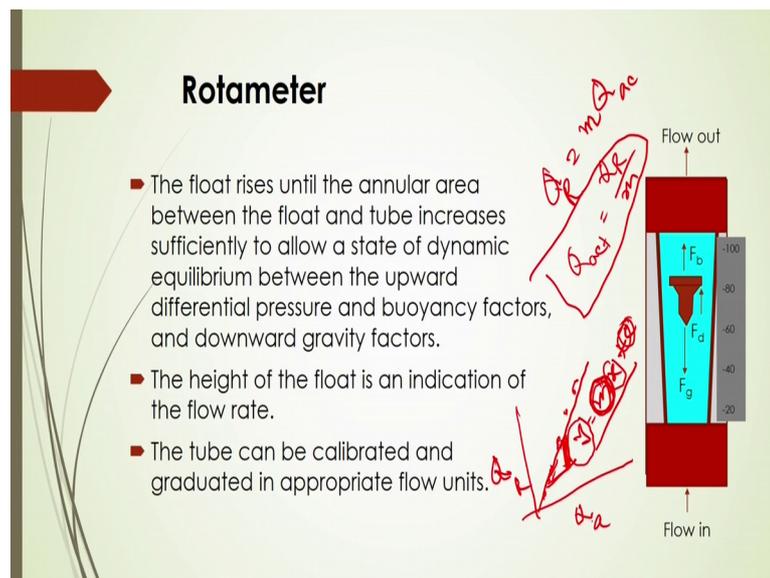
And what is that? Actually rotameter, you will see in this figure it is shown that it consists of a vertically oriented glass or plastic tube with a large end of the top here. You will see large end of the top here. In this figure here, it is given and this what is that at the bottom it will be a small cross sectional area; that means, there the column should be a taper and in this case it should be always aligned vertically, and why it will be vertically that will be discussed. And in this case, you will see a metering float is free to move within the tube.

In this case, you will see one float is seen here that will be moving freely in the vertical glass column or plastic column whatever it is, what materials will be used to make it. Generally, plastic glass or sometimes you know, the transparent glass to be used to actually design or to actually manufacture this rotameter because there will be visual observation of this float at which actually mark this float is actually state at a particular flow rate. In that case and the when this equilibrium condition at which location this float will be coming in equilibrium condition and based on which there will be a some force balance and from which you will be able to calculate what would be the flow rate. And

this scale, always this position of the float will give you the particular flow rate of the rotameter. So, we will calculate that one.

So, in this case, fluid flow causes the float to rise in the tube or that column as the upward pressure difference and also buoyancy of the fluid overcome the effect of the gravity. So, here one pressure difference will give you that its drag force should come here as well as you will see there will be some buoyancy force because this float will be floating based on the buoyancy force and also parallelly there will be a force that is gravity force. So, these three forces will be acting on this float one is drag force and other is buoyancy force and other one is gravity force. So, balance of these three forces will give you the equilibrium position of the float and from that equilibrium position, you would be able to calculate what should be the flow rate that is given by this rotameter.

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And also this float rises until the annular area between the float and the tube that will increase sufficiently to allow a state of dynamic equilibrium between the upward differential pressure and the buoyancy factors and downward gravity factors there.

So, in this case, three factors are there; one is differential pressure and another buoyancy and the downward gravity factors. This factors is nothing but the force. So, in that case, you will get height of the float in a certain equilibrium position which will indicate the flow rate and at that flow rate this float will get the equilibrium condition just by balancing this these three forces. And the tube in this case, generally being calibrated and

graduated in appropriate flow units whenever you are going to measure by this rotameter a certain flow rate.

So, before using this rotameter, you have to calibrate with a known values of flow rate and where this float is actually stands. So, in that case, you have to mark it out, then this is at this flow rate; this is stands at this mark of this what is that rotameter as whether it is the that is 20, 60 or 80 or 100 like this as per scale where it will be standing at an equilibrium position.

So, if you are changing the known flow rate of fluid and then what will happen, you will get respect; if the scale of this float where it will be in equilibrium condition. So, if you get these 2 3 5 or 5 points are there based on that known flow rate and what this rotameter is also given that the scale flow meter scale, then you can draw a like this a graph here. If you know that this is the known flow rate as  $Q$ , this is actual and this is your  $Q$  if it is  $Q$  rotameter, then this actual flow rate correspondings to the what is that rotameter given that rotameter flow rate, then you can get this type of data.

So, in that case, if you make a straight line over there, you can get a  $y$  is equal to  $m x$  plus  $c$  this type of equations you can have and after fitting this data. And then you can get that this, these  $y$  will be is equal to flow rate flow rate that is given by rotameter and  $x$  is the flow rate that is actually a known to you by just collecting the fluid with respect to time and what will be the flow rate.

And then  $c$  and  $m$  are this  $m$  are is; this  $m$  is called the slope of this equation and  $c$  is called intercept of this equation. If there is no flow rate is allowed, they are through this rotameter then of course,  $Q$  and  $Q R$  should be 0. And in that case  $c$  should be is equal to then only  $m$  slope will be coming and this slope, you can get it from this equation by two points of this what will be the slope, you can calculate. And if you once know these slope, then you can develop this equation  $Q$  rotameter that will be is equal to what is that that coefficient  $m$  into  $Q$  actual.

So, from this what will be your actual velocity, what will be your actual flow rate that you can get it  $Q$  actual,  $Q$  actual that will be is equal to  $Q$  rotameter by  $m$ . So, from these equations you can have the actual experimental flow rate whatever you are maintaining in your process. So, whenever you are going to use any device you have to calibrate it first based on your actual or known flow rate or velocity. So, then you relate with this

that is device flow rate that is the flow rate given by the device like your rotameter and other flow device also. After that make a equation like this and from those equations, you can calculate what should be the actual flow rate from the flow rate that is given by rotameter for a particular operating conditions in your process.

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▶ A force balance on the metering float yields

$$F_d + F_b = F_g \quad (\text{Eq. 1})$$

$$F_d + \rho_f V_{float} g = \rho_{float} V_{float} g \quad (\text{Eq. 2})$$

$$F_d = C_d A_{float} \frac{\rho_f U_m^2}{2} \quad (\text{Eq. 3})$$

$\rho_f$  = density of the fluid for gas it is p/RT  
 $\rho_{float}$  = density of the float  
 $V_{float}$  = total volume of float  
 $C_d$  = drag coefficient  
 $A_{float}$  = frontal area of the float  
 $U_m$  = mean flow velocity in the annular space between float and the tube  
 $F_d$  = Drag force,  $F_b$  = Buoyancy force,  $F_g$  = Gravity force

And then we can say that what will be the force balance on the metering float that yields in this rotameter. Now we know that there are three forces are acting over this floats to get its equilibrium condition. After balancing this equations then you can have certain scale at which it will maintain a certain flow rate. And in that case what will be the force balance, then you will see that the drag force will be acting upward and buoyancy force also will be acting upward direction.

So, drag force and buoyancy force will be balancing the drag force which will be acting downward direction. So, these two forces of drag force and buoyancy force will be balanced this gravity force here. And what is that drag force that? Drag force will be calculated by this already, we have discussed in the earlier lecture that how to calculate the drag. So, for that you have to have the drag coefficient also what would be the projectional area of the float and also what would be the density of the fluid and also what would be the mean velocity of the fluid. Once you know this from this equation number 3, you can calculate what should be the drag force there.

And what would be the buoyancy force that? Buoyancy force, you can calculate from the volume of the float and also what will be the displacement volume by the float that will be accounted here. And if you multiply by the density of the fluid, then you will get the mass of the fluid that is displaced by this float. And then what would be the mass of the fluid here? You can calculate  $F$  into  $b$  float into  $g$  and this is called buoyancy force as per archimedes principle. And what will the gravity force that is  $m$  into  $g$ , then  $m$  into  $g$  what is that  $m$   $g$  this is what that  $\rho_{float}$  into  $v_{float}$  into  $g$ . The what will be the volume of fluid that is displaced by the what is the float that you can calculate from this you can invert what would be the weight of the float that you can calculate volume of the float into density of the float, then you can calculate mass of the float and then gravitational acceleration.

So, from this you can calculate what would be the gravitational force that is or the what is that float and also what would be the buoyancy force that is as per archimedes principle that you can calculate by this and what would be the drag force. And after substitution of all these forces, here drag force and buoyancy force and gravity force and after simplification, you can get the velocity of the fluid from this equation number 3. Because this equation number 3 will give you the terms velocity. So, after balancing and the simplification, you can get the mean velocity of the fluid.

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Combining Eqs. 2 and 3 we can get the volumetric flowrate as

$$Q = Au_m = A \left[ \frac{1}{C_d} \frac{2gV_{float}}{A_{float}} \left( \frac{\rho_{float}}{\rho_f} - 1 \right) \right]^{1/2} \quad (\text{Eq. 4})$$

Where  $A$  is the annular area.

The flowrate can also be determined from the continuity equation as follows:

$$Q = kA\sqrt{gH_p} \quad (\text{Eq. 5})$$

$$H_p = \frac{\Delta p}{\rho_f g} \quad (\text{Eq. 6})$$

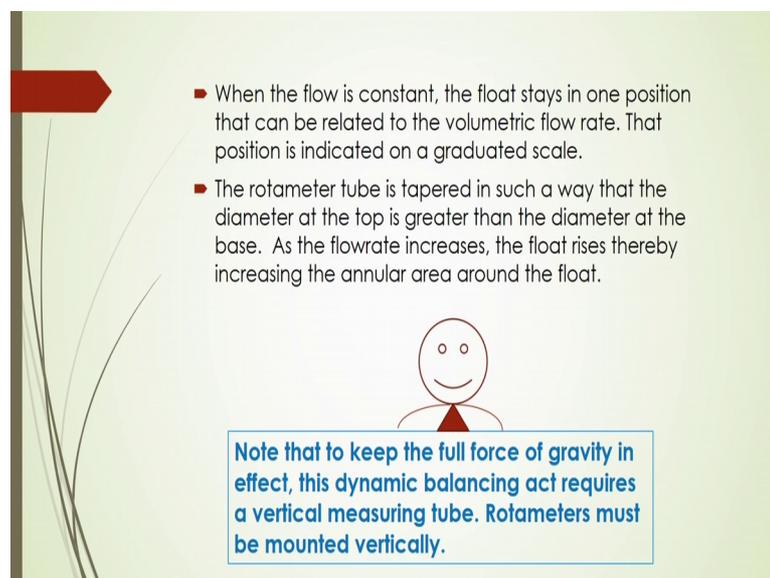
$Q$	= the volumetric flowrate ( $\text{m}^3/\text{s}$ )
$k$	= a constant
$A$	= the annular area available for flow ( $\text{m}^2$ )
$g$	= the acceleration due to gravity ( $\text{m}/\text{s}^2$ )
$H_p$	= the pressure/head loss across the rotameter (m)

And once you know the mean velocity of the fluid, then you can calculate what would be the discharge or volumetric flow rate of the fluid that is given by this rotameter.

So, from this  $Q$  will be equal to after substitution of  $u_m$  and  $A$  cross sectional area, then you can get is one  $a$  into  $1$  by  $C_d$  into  $2gV_{float}$  by a float into  $\rho_{float}$  by  $\rho_F$  minus  $1$  to the power  $1$  by half. So, it can be obtained after simplification where  $A$  is called the annular area that is very important here. This annular area means through which that fluid will be flowing. And the flow rate can also be determined from the continuity equation that you can directly obtain by this equation number 5 here as  $Q$  is equal to  $kA$  into root over  $gH_p$  and  $H_p$  will be is equal to  $\Delta p$  by  $\rho F g$ .

So, after substitution of this pressure difference and you can calculate what would be the flow rate. In this case, you have to know the constant  $k$  that  $k$  constant should be it is called rotameter constant. So, that will be obtained based on the experimental observation.

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- When the flow is constant, the float stays in one position that can be related to the volumetric flow rate. That position is indicated on a graduated scale.
- The rotameter tube is tapered in such a way that the diameter at the top is greater than the diameter at the base. As the flowrate increases, the float rises thereby increasing the annular area around the float.

**Note that to keep the full force of gravity in effect, this dynamic balancing act requires a vertical measuring tube. Rotameters must be mounted vertically.**

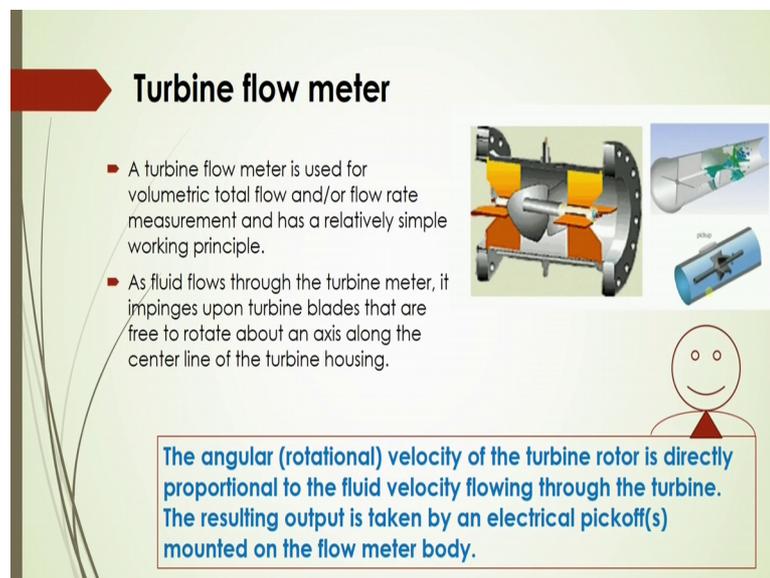
And also you have to remember that the, you have to keep the rotameter in full scale. So, that to keep the full force of the gravity in the effect and in that case, the dynamic balancing act requires a vertical measuring tube.

So, in that case rotameters of course, must be mounted vertically there. So, because you are getting the equilibrium position of the rotameter and you can get it only in the

vertical directions because the all the dynamic forces that will balance at that particular position of this float. So, when the flow is constant the float stays in open in one position that can be related to the volumetric flow rate and that position will be indicated on a graduated scale in the rotameter. And the rotameter tube is tapered in such a way that the diameter at the top is greater than the diameter at the base and as the flow rate increases the float rises thereby increasing the annular area around the float.

So, that is why through the annular area, you will see there will be flow of fluid and based on that annular area and the mean velocity of the fluid, you can get the flow rate based on this equation number 4.

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**Turbine flow meter**

- A turbine flow meter is used for volumetric total flow and/or flow rate measurement and has a relatively simple working principle.
- As fluid flows through the turbine meter, it impinges upon turbine blades that are free to rotate about an axis along the center line of the turbine housing.

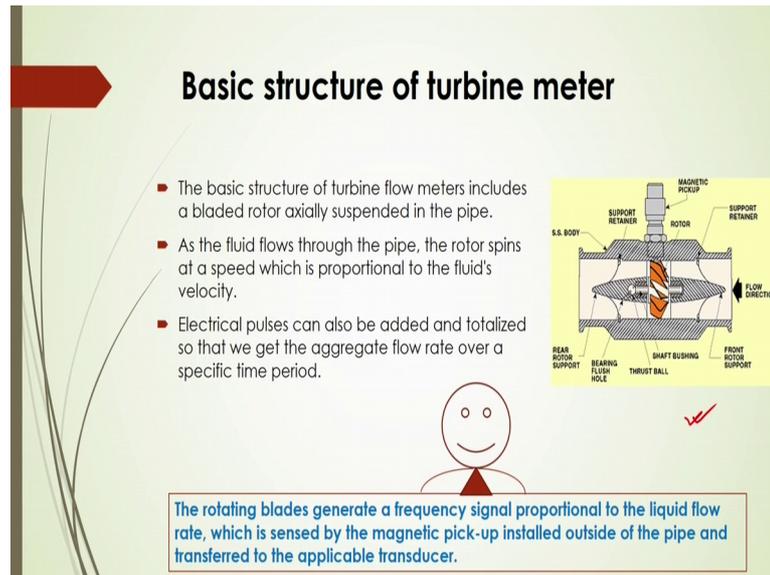
The angular (rotational) velocity of the turbine rotor is directly proportional to the fluid velocity flowing through the turbine. The resulting output is taken by an electrical pickoff(s) mounted on the flow meter body.

Now, another important device that is being used to calculate the flow rate of the fluid, that is passed through the pipe. So, in that case, it will be actually based on the what is that is called sometimes called turbine meter and there will be some turbine blades and based on the rotating of the turbine blades, the flow rate is actually calculated.

So, a turbine flow meter is used for volumetric total flow and or flow rate measurement and has a relatively simple working principle here. As fluid flows through the turbine meter, it impinges upon the turbine blades that are free to rotate about an axis along the centerline of the turbine housing. So, in this case, you have to remember that this angular rotating velocity of the turbine rotor is directly proportional to the fluid velocity that is flowing through the turbine and the resulting output of this rotational velocity that will

be taken by an electrical pickups that is mounted on the flow meter body. And that will give you the fluid velocity and this is proportional to the angular velocity and from which you will be able to calculate what would be the flow rate; because that fluid velocity will give you the flow rate just by multiplying the cross sectional area of the pipe.

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And also what will be the basic structure of the turbine meter that you have to know here in the figure, it is shown that some components of this turbine meter for its basic structure. And in this case it will include say, bladed rotor that will be actually suspended in the pipe and as the fluid flow through the pipe the rotor spins at a speed which is proportional to the fluids velocity. And also there should be an electrical pulse and that electrical pulse can also be added and totalized it. So, that we get the aggregate flow rate over a specific time period.

So, in that case, you have to remember what would be those basic structure of this turbine meter and where that electrical pulse will be actually added and also how this aggregate flow rate over the specific time period can be calculated. There will be certain principles and we will discuss this principle here.

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- The radius of the rotor (the radius at the roots of the blades) is  $a$
- The radius of the turbine (radius measured at the outer edges of the blades) is  $R$ ,
- The width of blades is  $c$ ,
- The distance between blades is  $S$ .
- The incoming flow with velocity  $V$  causes the turbine to rotate at angular velocity  $\omega$ .
- If there is no velocity loss, the ideal angular velocity  $\omega_i$  can be related to the flow velocity  $V$  by a simple trigonometric formula

$$\bar{r} \omega_i = V \tan \beta \Rightarrow \frac{\omega_i}{V} = \frac{\tan \beta}{\bar{r}} \quad (7)$$

$$\bar{r} = \sqrt{\frac{R^2 + a^2}{2}} \quad (8)$$

Let us consider the radius of the rotor as shown in figure here and the radius at the roots of the blades, it is called and it is denoted by  $a$ . And the radius of the turbine radius measured at the outer edge of the blades it is denoted by  $R$  and the width of the blade; if it is considered  $c$ , then what will be the distance between the blades? It is  $S$  and the incoming flow with the velocity that will cause the turbine to rotate at an angular velocity that is denoted by  $\omega$ . And if there is no velocity loss, then the ideal angular velocity  $\omega_i$  it is denoted that can be related to the fluid velocity  $V$  by a simple trigonometric formula here.

So, this is  $\bar{r} \omega_i$  that will be is equal to  $V \tan \beta$  and that implies the  $\bar{r}$  would  $\omega_i$  by  $V$  that will be is equal to  $\tan \beta$  by  $\bar{r}$ . And  $\bar{r}$  is nothing but the root of  $R^2 + a^2$  by  $2$ ; that means, here what will be the radius of the turbine and what will be the radius of the rotor that you have to know. Once you know that radius of the turbine and rotor and square of this turbine and rotor and then, add it up and get it averaged and then take the square root, then you will get the average  $R$  value.

Once you know that average  $R$  value, then what should be the ideal angular velocity. Based on the velocity that is incoming flow velocity and then, it will be represented by this  $\omega_i$  by  $V$  that will be  $\tan \beta$  by  $\bar{r}$ . Now, this  $\beta$  is what is that you will see the rotor is inclined at an angle  $\beta$  with the horizontal or flow direction there.

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■ The flow applies a torque  $T$  to the turbine to make it rotate. The torque is

$$T = \int_a^R \frac{\rho AV 2\pi(r\omega_i - r\omega)}{\pi(R^2 - a^2)} dr = \rho AV \frac{R^2 + a^2}{2} (\omega_i - \omega)$$
$$= \rho AV (\bar{r})^2 (\omega_i - \omega) \quad (9)$$

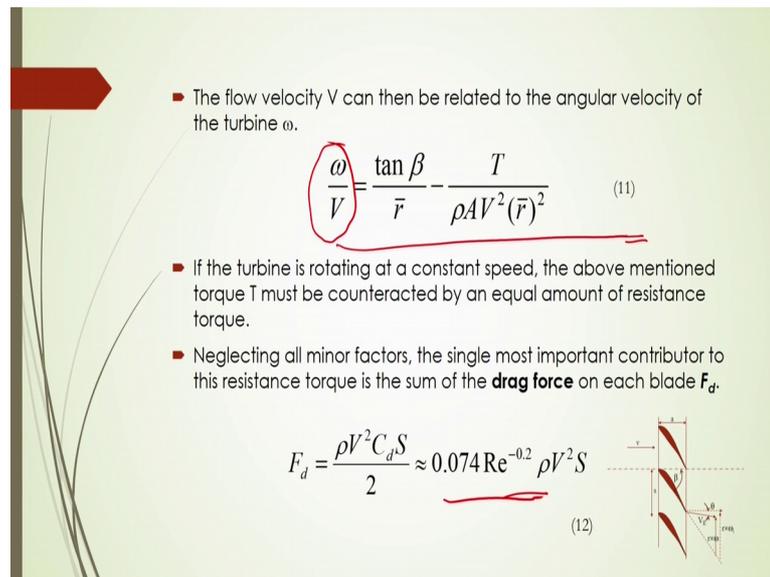
■ Or

$$\omega = \omega_i - \frac{T}{\rho AV (\bar{r})^2} \quad (10)$$

And the flow applies a torque whenever it will be rotating at a certain angular velocity. Then this torque to the turbine to make it rotate and in that case, what should be the torque how to calculate this torque of course, will be depending on that rotational velocity that is called what is that torque and this torque will be measured by this equation here. That would be finally is coming rho AV into r bar square into omega i minus omega after simplification and here so, this equation number 9 will be used to calculate this torque.

Once you know the torque, then what should be the angular velocity that is given by the what is that turbine meter that will be is equal to what is that ideal angular velocity minus this torque by what is that rho AV by r bar square. So, from this equation, you can get what should be the angular velocity of this turbine meter.

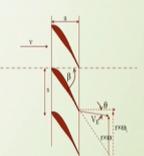
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- The flow velocity  $V$  can then be related to the angular velocity of the turbine  $\omega$ .

$$\frac{\omega}{V} = \frac{\tan \beta}{\bar{r}} = \frac{T}{\rho A V^2 (\bar{r})^2} \quad (11)$$

- If the turbine is rotating at a constant speed, the above mentioned torque  $T$  must be counteracted by an equal amount of resistance torque.
- Neglecting all minor factors, the single most important contributor to this resistance torque is the sum of the **drag force** on each blade  $F_d$ .

$$F_d = \frac{\rho V^2 C_d S}{2} \approx 0.074 \text{Re}^{-0.2} \rho V^2 S \quad (12)$$


And once you know that angular velocity from the equation number 11, you can calculate what will be the velocity of the fluid. The flow velocity can be then obtained from this equation number 11. Once you know the angle beta  $\bar{r}$  even  $T$  that is torque and also density of the fluid cross sectional area of the pipe. So, from which you can calculate this ratio of this angular velocity to the flow velocity.

If the turbine is rotating at a constant speed, the above mentioned torque here that is in equation number 11; the torque must be counteracted by an equal amount of resistance torque. Now neglecting all minor factors, the single most important contributed to the resistance torque will be the sum of drag force on each blade that is denoted by  $F_d$ .

Now, this  $F_d$  should be is equal to  $\rho V^2 C_d S$  by 2.  $S$  is what is that? That the distance between these two plates here and once you know this then what should be the value for drag force? Even after simplification of this  $C_d$ , you can substitute here  $0.074 \text{Re}^{-0.2}$  to the power of minus 0.2 and then into  $\rho V^2 S$ . So, from this you can calculate this drag force. So, once you know this drag force you can say that the torque whichever it is given by this rotating of this fluid, then equal amount of resistance of the torque to be developed by this drag force.

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■ The torque  $T$  then becomes

$$T = n \left( \frac{R+a}{2} \right) F_d \sin \beta = 0.037 \text{Re}^{-0.2} n(R+a) \rho V^2 S \sin \beta \quad (13)$$

where  $n$  is the number of blades. Using this expression, the  $\omega$  to  $V$  ratio can be written as:

$$\frac{\omega}{V} = \frac{\tan \beta}{\bar{r}} - \frac{0.037 \text{Re}^{-0.2} n(R+a) S \sin \beta}{A(\bar{r})^2} \quad (14)$$

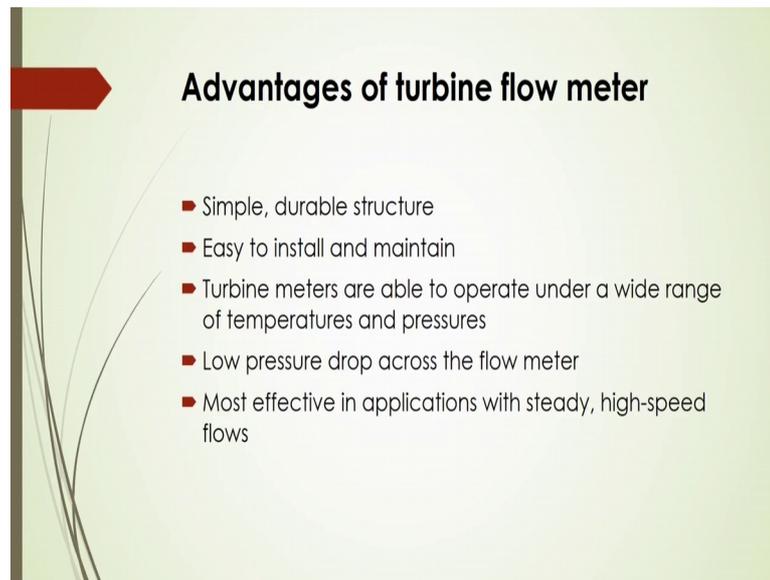
■ The volume flow rate  $Q$  can then be expressed in terms of the angular velocity of the turbine

$$Q = VA = \frac{\omega(\bar{r})^2 A^2}{\bar{r} A \tan \beta - 0.037 \text{Re}^{-0.2} n(R+a) S \sin \beta} \quad (15)$$

So, you have to equate this drag force to the torque here and after that you can get the torque as here,  $T$  is equal to  $n$  into  $R$  plus  $a$  by  $2$  into  $F_d$  into sine beta. So, after substitution of this drag force here, you can get this equation number 13 as a simplified form here where  $n$  is the number of blades which is imposed for this turbine meter to measure this flow rate. And using this expression the angular velocity  $\omega$  to  $V$  ratio can, then be written as after substitution all this parameter here you can get this simplified form of equation number 14 to get this ratio of angular velocity to the flow velocity.

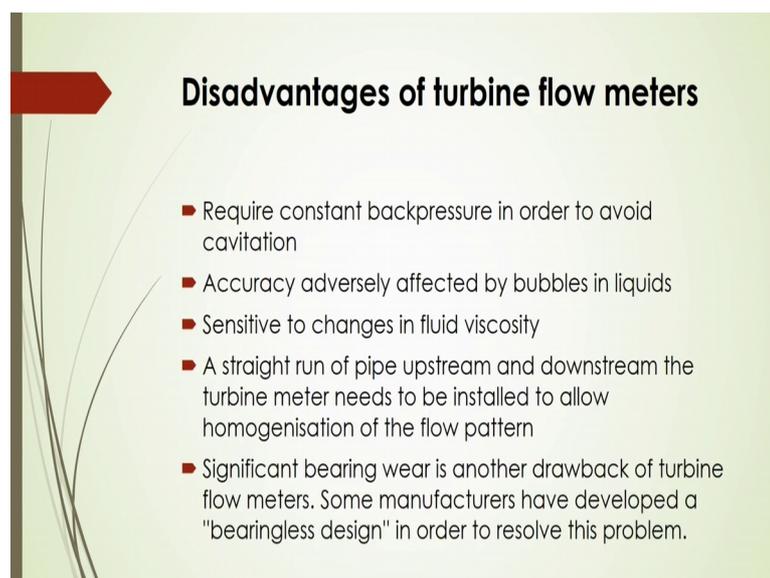
And once you know this equation 14 for this velocity of the fluid, then you can calculate the volumetric flow rate by this equation here;  $Q$  will be is equal to  $V$  into  $A$  where  $V$  should be calculated based on this equation number 14. And after substitution of this  $p$  here, then you can have this final form of equation to calculate the volumetric flow rate here as given in equation number 15. So, this equation number 15 to be used to calculate the volume flow rate based on the angular velocity of the turbine.

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And what are the advantages of this turbine flow meter? Generally you will see, it will be very simple, durable structure, easy to install and maintain and also turbine meters are able to operate under a wide range of temperatures and pressures, low pressure drop across the flow meter and most effective in applications with steady and high speed flows.

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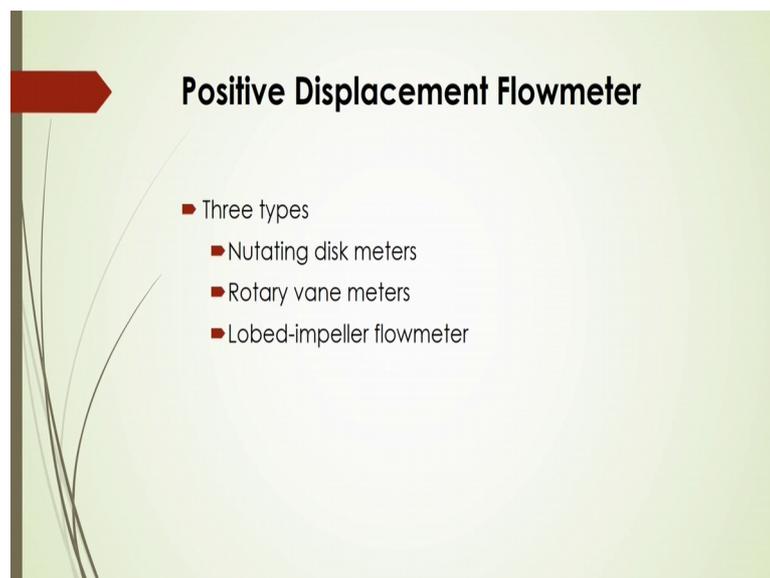


Some disadvantages also are seen. A sometimes it requires constant back pressure in order to avoid the cavitation and the accuracy sometimes affected by bubbles in liquids

also it is sensitive to changes in fluid viscosity. And also the significant bearing, we are using another drawback of turbine flow meters and sometimes a straight run of pipe upstream and downstream. The turbine meter needs to be installed to allow homogenous and of the flow pattern.

And also you will see some manufacturers, they have developed a bearingless what is that design in order to resolve this problem of we are in the bearing. So, these are the some disadvantages of this turbine meters and generally, it is being used for what is that large scale operation of this fluid flow phenomena and measuring the pipe flow an industrial that is to supply it. Suppose liquid in mass scale in that case, you have to measure this what is the flow rate by this turbine flow meter.

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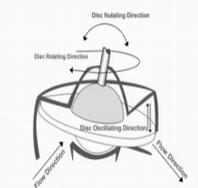
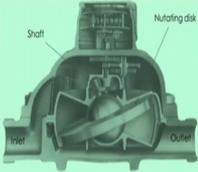


Another important flow meter, it is called positive displacement flow meter. There are three types of positive displacement flow meters are there; nutating disk meters, rotating vane meters and lobed impeller flow meters are the important three types of this positive displacement flow meter.

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## Nutating Disk Flowmeter

- Get their name from the idea of nutation, which means nodding or rocking.
- Comprise basically of round disk mounted on a concentric sphere
- A spindle in the centre of the sphere is tracked through the gear arrangement at the top
- Tracking the movement of the spindle, the flowmeter determines the number of times the chamber traps and empties fluid.
- This can be used to determine flow rate



And here the nutating disk flow meters picture of this kind, it is given. In this case, this name is actually obtained from the idea of nutation which means that nodding or rocking and it generally comprise a round disk which is mounted on a concentric spear as shown in figure here. And also a spindle in the center of the sphere is tracked through the gear arrangement at the top and to track the movement of the spindle and you have to arrange this spindle in the center of the sphere. And after tracking the movement of the spindle, the flow rate is determined based on the number of times the chamber traps and empties these fluids and this can be used to determine the flow rate.

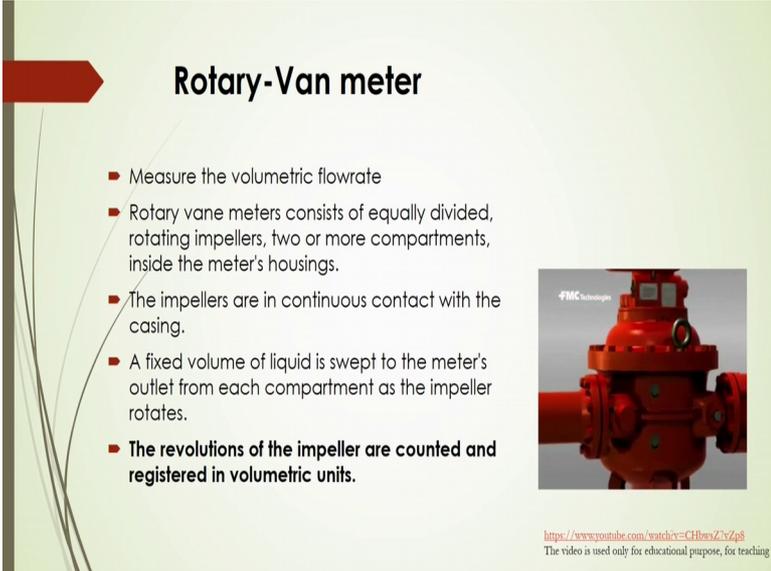
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## Advantage & Disadvantage

Advantage	Disadvantage
■ May be constructed from a variety of materials.	■ Accuracy is adversely affected by viscosities below the meter's designated threshold.
■ High accuracy and repeatability.	

And here also, it has some advantage and disadvantages like it may be useful because of that constructed from the variety of materials and also it may be advantageous or you can say selectively based on the high accuracy and the repeatability. And sometimes accuracies adversely affected by the viscosities of the fluid and below the meters and that is designated threshold. So, these are the advantages and maximum cases we are getting these flow meters as giving the accurate flow rate.

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### Rotary-Van meter

- Measure the volumetric flowrate
- Rotary vane meters consists of equally divided, rotating impellers, two or more compartments, inside the meter's housings.
- The impellers are in continuous contact with the casing.
- A fixed volume of liquid is swept to the meter's outlet from each compartment as the impeller rotates.
- **The revolutions of the impeller are counted and registered in volumetric units.**



<https://www.youtube.com/watch?v=CH8wvZVZq8>  
The video is used only for educational purpose, for teaching

And that is why these are being used in industry and also another type of nutating disk flow meters sorry it is called rotating van meter and these van meter is also advantageous. In this case, you will see the some impeller is being actually used to design this rotary diameter and this rotary van meter will give you the volumetric flow rate.

And in this case, it generally consists of equally divided rotating impellers two or more compartments inside the meters housing and also the impellers are in continuous contact with the casing. And a fixed volume of liquid is generally swept to the meters, outlet from each compartment as the impeller which will be rotating. And then the rotating speed, it is called revolutions of the impeller. It is generally are being counted and based on that counting of these revolutions the what will be the amount of fluid is passing out that will be actually registered as a volumetric units.

So, based on the revolution of the impeller that this volumetric amount of fluid is how much volumetric amount of fluid is passing that can be calculated based on this rotary van meter.

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## Rotary-Van meter

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<https://www.youtube.com/watch?v=CH8evsZvZp8>  
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So, here see are this rotary vane meter is acting how it is acting in this video. Here see these are the structure of this rotary vane meters, some impellers are placed here. In that case, how it will be rotating and based on that rotation revolutionary rotation, you can get how much fluid is actually displaced by this rotation. And also and number of this revolutions, that will give you the amount of fluid flow by this rotary vane meter.

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## Advantage and Disadvantage

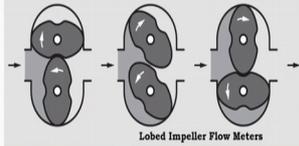
- **Advantage**
  - Sweeping action of vanes prevents buildup of sediment and keeps compartment clean.
  - Wide choice of construction materials.
  - High accuracy and repeatability.
  - Low pressure drop over entire range of flow measuring capabilities.
- **Disadvantage**
  - Relatively complex design increases cost.

And, it will also have some advantage and disadvantage in this case and some advantages like a sweeping action of vane prevents buildup of sediment and keeps the compartment clean and wide choice of construction materials, high accuracy and repeatability, low pressure drop over the entire range of flow measuring capabilities. But, in this case relatively complex design that will increase more cost and it is not economic for some extent.

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## Lobed-impeller meter

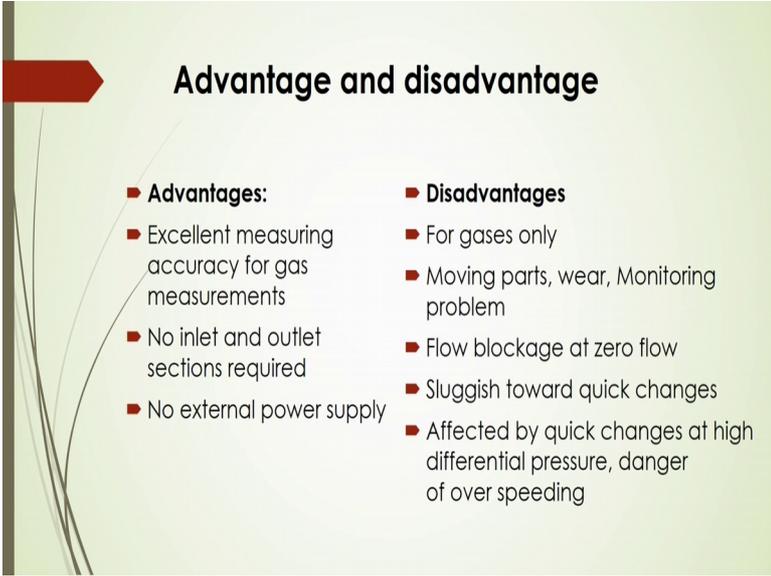
- Two lobed impellers (rotors) are mounted on parallel shafts and are synchronized to keep them correctly positioned in relation to each other
- Incoming fluid is trapped between the rotors and passed to the outlet as a result of the rotation
- **The number of revolutions of the rotors is used to calculate the volumetric flow rate**
- Impellers and case are carefully machined so that accurate fit is maintained



Lobed Impeller Flow Meters

And some other devices like here, another important this nutating type flow device it is called lobed impeller meter. In this case two lobed impellers or it is called rotors are mounted on a parallel shafts as shown in figure and are synchronized to keep them a correctly position in relation to each other. In this case, again the number of revolutions of the rotors is used to calculate the volumetric flow rate. And in this case, impellers and case are carefully may machined. So, that the accurate fit is maintained here and incoming fluid is trapped between the rotors and passed to the outlet as a result of the rotation. So, that rotation will give you the volumetric flow rate.

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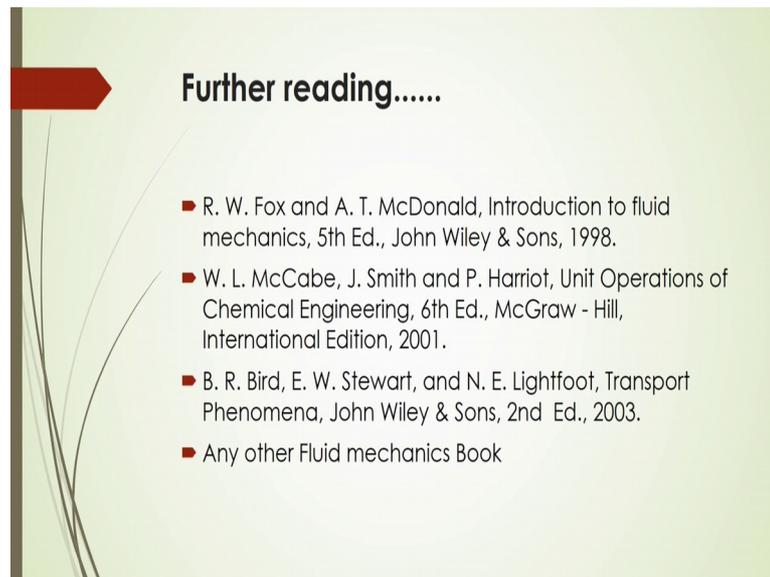


**Advantage and disadvantage**

- **Advantages:**
  - Excellent measuring accuracy for gas measurements
  - No inlet and outlet sections required
  - No external power supply
- **Disadvantages**
  - For gases only
  - Moving parts, wear, Monitoring problem
  - Flow blockage at zero flow
  - Sluggish toward quick changes
  - Affected by quick changes at high differential pressure, danger of over speeding

And it has advantage because it will give you the excellent measuring accuracy for gas measurements. No inlet and outlet sections will be required; no external power supply also required and also you can say that there will be a what is that accuracy of the measurement. But some disadvantages here, it is generally being used for only gases and there is a moving parts we are monitoring problem is there. Flow blockage at zero flow, sluggish towards quick changes and affected by quick changes at high differential pressure and it is sometimes danger of over speeding there.

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**Further reading.....**

- R. W. Fox and A. T. McDonald, Introduction to fluid mechanics, 5th Ed., John Wiley & Sons, 1998.
- W. L. McCabe, J. Smith and P. Harriot, Unit Operations of Chemical Engineering, 6th Ed., McGraw - Hill, International Edition, 2001.
- B. R. Bird, E. W. Stewart, and N. E. Lightfoot, Transport Phenomena, John Wiley & Sons, 2nd Ed., 2003.
- Any other Fluid mechanics Book

So, in this lecture we have discussed somewhat is that area flow meter, drag flow meter and what will be the nutating type disk flow meter and also what is that positive displacement flow meter. So, some devices which are being used and in industrial scale to measure the flow rate and in the previous lecture also we have discussed some flow devices. I would suggest you to go further for basic principles of the flow device and also design. And based on these so, what is that textbooks and also other books also you can follow for further design and further basic fundamentals. Because more details of these fundamentals are not scope of this here course.

So, I will suggest to read further this textbook and next class or next lecture, we will provide more information some other different types of flow devices which are being used in industry as well as in laboratory scale. So, thank you for this lecture today.

Thank you.