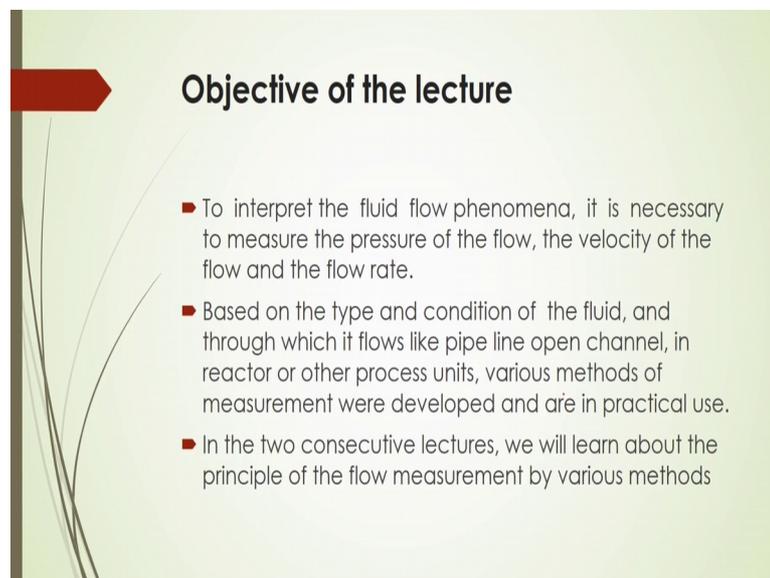


Fluid Flow Operations
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Module – 11
Lecture – 27
Measurement of Flow-Part 1

Welcome to massive open online course on Fluid Flow Operations. This is lecture number 27 under module 11 and the topics to be covered in this module, a Measurement of Flow and in this lecture has a Part 1. We will be discussing the measurement of flow.

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Objective of the lecture

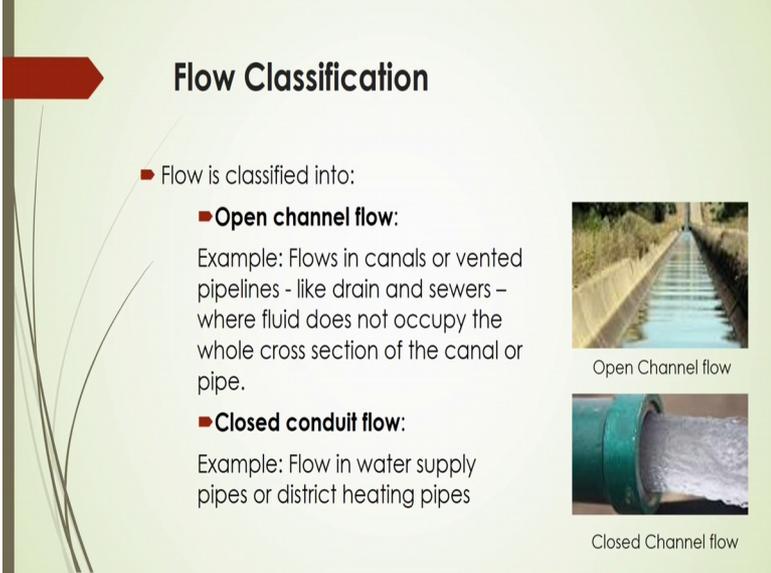
- To interpret the fluid flow phenomena, it is necessary to measure the pressure of the flow, the velocity of the flow and the flow rate.
- Based on the type and condition of the fluid, and through which it flows like pipe line open channel, in reactor or other process units, various methods of measurement were developed and are in practical use.
- In the two consecutive lectures, we will learn about the principle of the flow measurement by various methods

And the lecture includes that how the flow rate of the fluid whenever it will be flowing through the pipe or in open channel and what are the different methods and particular specific methods to be covered here. And also in the successive lecture, we will discuss more about that measurement flow.

So, here objective of the lecture is to interpret the flow phenomena and which is necessary to measure the pressure of the flow and the velocity of the fluid flow and the flow rate. And based on the type and condition of the fluid and through which it flows, like pipeline open channel in reactor or other process units various methods of measurements were developed and are in practice in use.

Now, in the two consecutive lectures, we will learn about the principles of the flow measurement by various methods and then accept these conventional methods. There are some other methods should be will be also discussed in the third lecture of this module. So, for that we have to know something what is that different type of flows there.

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The slide is titled "Flow Classification" and features a red arrow pointing right. It lists two types of flow: "Open channel flow" and "Closed conduit flow". Each type is accompanied by a photograph. The "Open Channel flow" image shows a long, straight canal with water flowing in the center. The "Closed Channel flow" image shows a close-up of a pipe with water being poured out.

Flow Classification

- Flow is classified into:
 - Open channel flow:**
Example: Flows in canals or vented pipelines - like drain and sewers – where fluid does not occupy the whole cross section of the canal or pipe.
 - Closed conduit flow:**
Example: Flow in water supply pipes or district heating pipes

Open Channel flow

Closed Channel flow

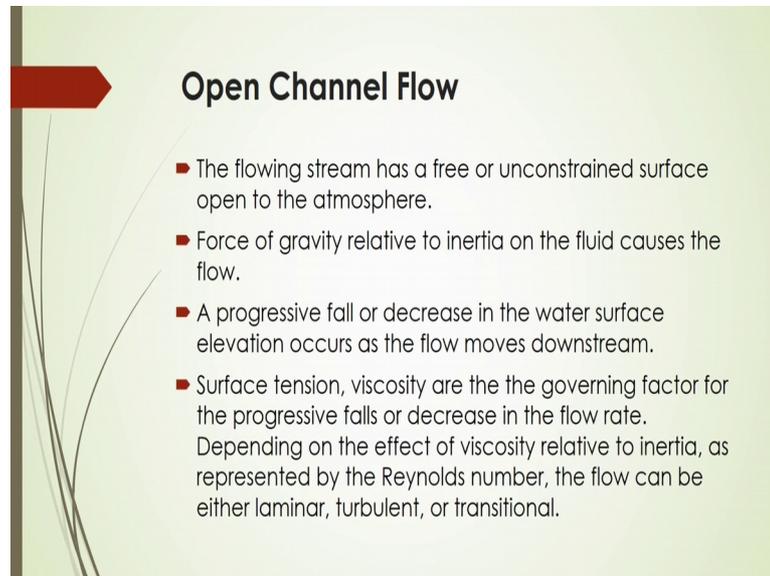
So, flow is classified into actually open channel flow and the closed conduit flow. Now the open channel flow is shown in figure here as an example that when closed in canals and vented pipelines like a drain and sewers where fluid does not occupy the whole cross section of the canal or pipe. So, in that case, this type of flow will be called or referred as a open channel flow. Already we have discuss to the principal phenomena of the open channel flow in the earlier lectures.

Here only we will discuss that how to measure that flow whenever fluid will be flowing through that channel and also there are other type of fluid is called closed conduit flow that is like flow in water supply pipes or distinct heating pipes there. So, in that case how that flow rate of the fluids based on the pressure even some other fluid properties, how it is to be measured; that will be discussed here.

Now, what is that open channel flow? In this case, the flowing stream has a free or unconstrained your surface open to the atmosphere and the force of gravity relative to the inertia that already we have discussed on the fluid that will cause the flow. And also a

progressive fall or decrease in the water surface elevation that will occurs and as a result the flow moves downstream.

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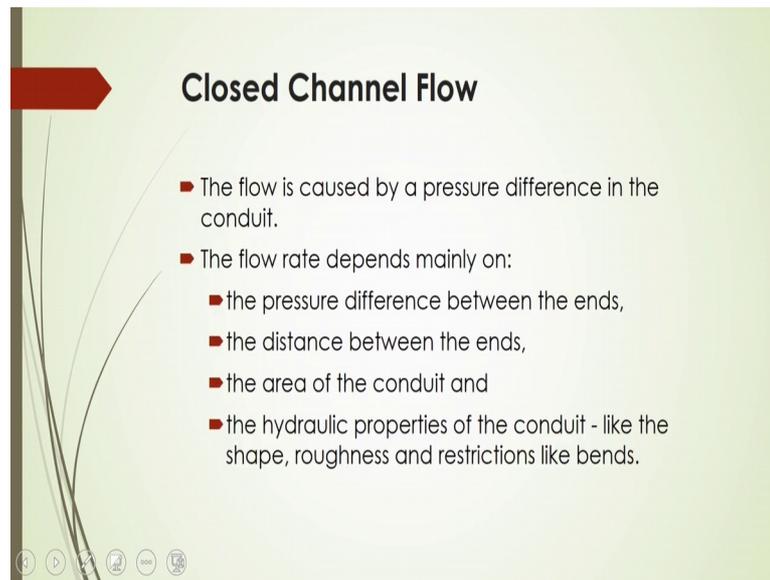
Open Channel Flow

- The flowing stream has a free or unconstrained surface open to the atmosphere.
- Force of gravity relative to inertia on the fluid causes the flow.
- A progressive fall or decrease in the water surface elevation occurs as the flow moves downstream.
- Surface tension, viscosity are the the governing factor for the progressive falls or decrease in the flow rate. Depending on the effect of viscosity relative to inertia, as represented by the Reynolds number, the flow can be either laminar, turbulent, or transitional.

And in this case, you will see that there will be surface tension viscosity of the fluid are will be the governing factor for the progressive falls or decrease in the flow rate in the channel. And depending on this fluid properties, like viscosity and relative to the by inertia represented by the Reynolds number and then flow can be either laminar turbulent or transitional.

So, based on which we can say that what will be the open channel flow whether this surface tension viscosity are effective they are or not relative to the inertia force. So, in that case, we will be discussing the measurement of those flow based on that energy equations which is effective based on this Reynolds number and viscosity and other critical properties of the system.

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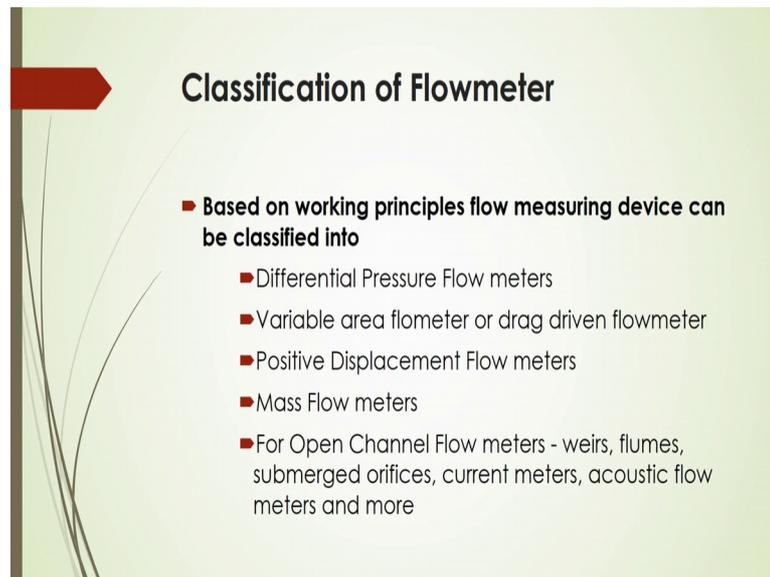
Closed Channel Flow

- The flow is caused by a pressure difference in the conduit.
- The flow rate depends mainly on:
 - the pressure difference between the ends,
 - the distance between the ends,
 - the area of the conduit and
 - the hydraulic properties of the conduit - like the shape, roughness and restrictions like bends.

Now, closed channel flow in that case, we will see the flow will be caused by a pressure difference the conduit and the flow rate depends mainly on the pressure difference between the ends, the distance between the ends, the area of the conduit and the hydraulic properties of the conduit. Like the shape, roughness and restrictions like bends here. So, these are called that closed channel flow. So, here you have to remember that what should be the pressure difference. Once you know the pressure difference, then what should be the flow rate based on which we can measure the fluid flow phenomena and what will be the rate for that in the closed channel or closed conduit.

And in this case the area of the country is very important because you are going to measure the pressure difference that what should be the resistance and what is the cross sectional area because this cross sectional area will give you the actual resistance of the fluid flow. And also hydraulic properties of the conduit like the shape roughness and restrictions like bends also will govern the fluid flow phenomena inside the closed channel.

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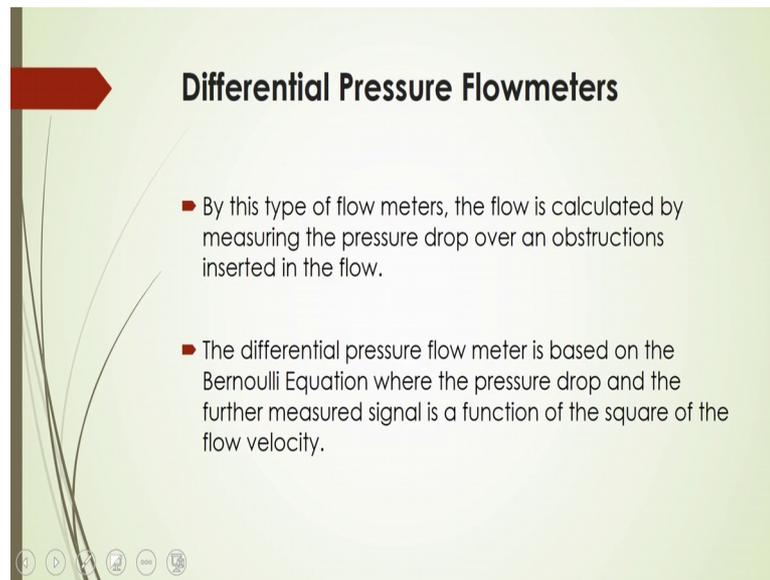
Classification of Flowmeter

- Based on working principles flow measuring device can be classified into
 - Differential Pressure Flow meters
 - Variable area flowmeter or drag driven flowmeter
 - Positive Displacement Flow meters
 - Mass Flow meters
 - For Open Channel Flow meters - weirs, flumes, submerged orifices, current meters, acoustic flow meters and more

That is already discussed in our earlier lectures what would be the different governing equations for the fluid flow in the pipe, even bend; how the resistance and then also the energy loss coefficient, how it will be there that we already we have discussed earlier lectures. And based on the working principle that we can actually, classify the flow measuring devices into different way.

Like here the flow measuring device can be classified into differential pressure flow meters, variable area flow meter or drag driven flow meter, positive displacement flow meter, mass flow meters or sometimes, it is called that we are that is being used for the flow rates for the open channel and it will be called as open channel flow meters. It is named as weirs flumes and submerged orifice even sometimes current meters, acoustic flow meters and more. There are several other equipments are available to measure the flow meter flow rates whenever it will be flowing through the pipe.

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Differential Pressure Flowmeters

- By this type of flow meters, the flow is calculated by measuring the pressure drop over an obstruction inserted in the flow.
- The differential pressure flow meter is based on the Bernoulli Equation where the pressure drop and the further measured signal is a function of the square of the flow velocity.

Now, let us consider here what is the differential pressure flow meters. In this case, you will see that differential flow meter we will calculate the flow rate based on the pressure drop over an obstruction inserted in the flow and the differential pressure flow meter based on the Bernoulli's equation where the pressure drop and the further measured signal will be a function of the square of the flow velocity.

So, it is important that since the pressure drop is a function of the flow velocity. So, once you know the pressure drop, then what should be the flow velocity and based on which what should be the flow rate that you can calculate from this pressure differences and in the pressure differential flow meters.

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The differential Pressure flowmeter

- Common types of differential pressure flow meters are:
 - Venturimeter
 - Flow nozzle
 - Weir
 - Pitot tube
 - Orifice meter
- Problems of flow under a sluice gate which is a constriction in an open channel (Weir)

And you will see there are several different types of flow meters are available based on this differential pressure flow meter. Differential pressure and which are commercially available in the market also like Venturimeter, Pitot tube, flow nozzle, orifice meter and weir. And so, in this case we will discuss one by one what will be the basic principle of that venturimeter, pitot tube, flow nozzle, orifice meter which has been used for the measurement of the flow based on this differential pressure.

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Bernoulli's Equation (uniform Flow)-Recap

For an incompressible fluid ($\rho = \text{constant}$),

$$\frac{v^2}{2} + \frac{p}{\rho} + gz = \text{constant} \quad (\text{Eq. 1})$$

Unit: m^2/s^2 or J/kg

Multiplying each term by ρ ,

$$\frac{\rho v^2}{2} + p + \rho gz = \text{constant} \quad (\text{Eq. 2})$$

Unit: N/m^2

Dividing each term by g ,

$$\frac{v^2}{2g} + \frac{p}{\rho g} + z = H = \text{constant} \quad (\text{Eq. 3})$$

Unit: m

velocity head pressure head potential head total head



Now, let us recap that principles of the Bernoulli's equation because the Bernoulli's equation is the basic form of equation to actually analyze that flow measurement devices. And a based on this, these equations are actually basic to calculate the or flow meters are working based on this principle of this Bernoulli's equation here. Now for an incompressible fluid; if we consider that in that case so, density should be constant. Then we can write here these Bernoulli's equation as this energy summation of energy will be constant here v square by 2 plus p by ρ plus $g z$ that will be equal to constant that already we have discussed in our earlier lecture.

So, this is your Bernoulli's equation and this can be represented in other way also. This will be in terms of here ρv square by 2 plus p plus $\rho g z$ that will be is equal to constant here, in terms of pressure terms and this is in terms of what is that energy terms. And also here this will be as head in terms of head here. So, this equation number 1 2 3 are the different forms of these Bernoulli's equations and this case you will see the third form here v square by 2 g as called velocity head. This is called what is that pressure head and z is called what is that potential head.

So, total head summation of this total head will be represented by this capital H and this should be constant although. So, this Bernoulli's equation should be the main governing equation to represent that principle of different flow devices which are working based on this pressure difference.

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Bernoulli's Equation for non-uniform flow

- In a real fluid flow across a large cross sectional area, the velocity distribution is not uniform
- The kinetic energy per unit weight of the fluid transferred across the section is greater than that calculated by using average velocity
- Hence true kinetic energy per unit weight can be expressed as

$$KE_{true} = \alpha \frac{v^2}{2g} \quad (\text{Eq. 4})$$

Then Bernoulli's Equation Takes the following form:

$$\alpha \frac{v_1^2}{2g} + \frac{p_1}{\rho g} + z_1 = \alpha \frac{v_2^2}{2g} + \frac{p_2}{\rho g} + z_2 \quad (\text{Eq. 5})$$

α is called kinetic energy correction factor
 $\alpha = 1.03$ to 1.06 for turbulent flow
 $\alpha = 2$ for laminar flow

Now, if we consider that if the flow in the pipe or in the conduit is the non uniform flow because in this case Bernoulli's equation, generally applied for case of uniform flow. But for non uniform flow, in a real fluid flow across a large cross sectional area; the velocity distributed to should not be uniform because there will be some obstructions. So, during the flow either by bend either by direction of the flow either by other different stiochimetry.

So, in that case you cannot say that flow will be uniform in nature. So, in that case you have to consider that energy loss during the flow and to represent this actual true kinetic energy in that flow. And based on which you have to calculate all this a flow measuring system like parameter flow meter and other things. So, in that case the kinetic energy per unit weight of the fluid that transferred across the section will be greater than that calculated by using average velocity.

Hence, the true kinetic energy per unit weight that can be expressed as that will be is equal to $v^2 / 2g$ into some multiplication factor. So, this multiplication factor, it is called kinetic energy correction factor. And if we take this kinetic energy correction factor into an account to the what is that Bernoulli's equation, then the Bernoulli's equation can be written as $\alpha \frac{v_1^2}{2g} + \frac{p_1}{\rho g} + z_1 = \alpha \frac{v_2^2}{2g} + \frac{p_2}{\rho g} + z_2$.

So, in this case alpha, generally 1.03 to 1.06 for turbulent flow and if you apply it for laminar flow, then you have to consider this alpha as 2. So, once you substitute this alpha as 2, then simply it will be as for laminar flow. In that case, you will see there will be no disturbance of the flow that is why kinetic energy correction factor will not be at all here existence based on this here. So, we can say that this Bernoulli's equation will be the main important governing equation to actually characterize that flow measuring instruments and based on which the flow rate the measuring instruments should be calculated for that pipe flow or conduit flow.

Let us consider that venturi meter. Now this is one important and mostly used measuring flow measuring device for measuring the flow rate of the fluid whenever it will be flowing through the five. Now flow rate in this case, in the pipe line is measured by narrowing a part of that you were shown in here figure.

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Venturimeter

It is a device where the flow rate in a pipe line is measured by narrowing a part of the tube

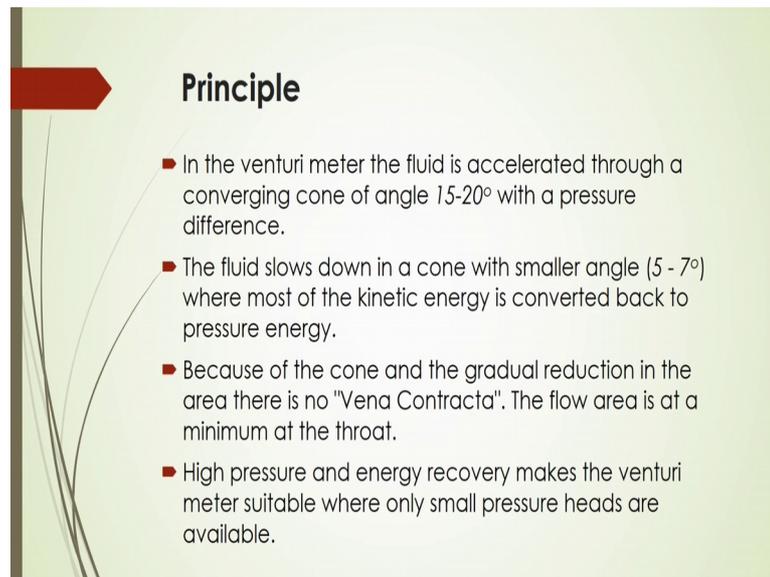
Giovanni Battista Venturi (1746-1822)
Italian physicist.

All images are taken from google image and highly acknowledged to google.com

You will see if you narrow the figure, then you will see how that pressure drop will be changing. And here a schematic diagram of this venturimeter is shown here, you will see there will be a part in this part there will be what is that inlet main section and flow through these sections will be called as main upstream flow. And then flow will be going through the converging cone and this is converging cone and then, it will flow through the throat and this section is throat. And then it will go from this throat to the diverging section here and then to the exit main stream.

And in this case, you have to measure the pressure drop between two points. Here in the main stream with this the section of this throat here. So, between these two points; if you measure the pressure drop by manometer, then you will be able to calculate what will be the flow rate there.

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Principle

- In the venturi meter the fluid is accelerated through a converging cone of angle $15-20^\circ$ with a pressure difference.
- The fluid slows down in a cone with smaller angle ($5 - 7^\circ$) where most of the kinetic energy is converted back to pressure energy.
- Because of the cone and the gradual reduction in the area there is no "Vena Contracta". The flow area is at a minimum at the throat.
- High pressure and energy recovery makes the venturi meter suitable where only small pressure heads are available.

So, let us discuss the principle about this. So, in the venturi meter the fluid is accelerated through a converging cone of the angle here. This converging angle, this generally this converging angle is considered as 15 to 20 degree with a pressure difference here.

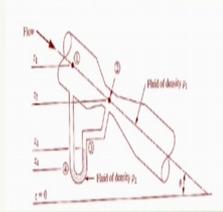
So, in this case whenever a fluid is flowing through the converging section, then velocity will be accelerated that is flow fluid will be accelerated at a certain velocity and that is because your cross section already has coming down that is because of which that you will get more velocity there. And the fluid slows down in a cone with a smaller angle of 5 to 7 degree where most of the kinetic energy converted back to the pressure energy here. And then because of this cone and the gradual reduction in the area, there is no vena contracta as like it is being used for that out of that orifice meter.

So, in that case the flow area is at minimum at the throat and we will be discussing that what is that vena contracta in the what is that in an orifice meter would be discussing. And even also we have already discussed whenever we were discussing the pressure loss in different geometry; there we have discussed the vena contracta in the earlier lecture. And also a high pressure and energy were recovery in this case makes the venturi meter suitable where only small pressure heads are available there. So, in the venturi meter, you have to remember that there would be two sections and the fluid will be just changing its energy from one section to another sections. And based on which the pressure, difference will be created and then or that you will be able to measure the flow rate.

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Principle

- It may be fitted in any position horizontal, vertical or inclined. But it is generally kept horizontal
- It should be remembered that the venturimeter must run full and it should be preceded by a straight length of pipe of not less than 5 to 10 diameter of pipe to reduce the turbulence
- Commonly, the diameter of throat is kept $\frac{1}{2}$ of the inlet pipe diameter or $\frac{1}{3}$ to $\frac{3}{4}$ th of the outlet pipe diameter



The manometry for a venturi tube or orifice should be installed below the hydraulic grade line or pipe.

And also it may be sometimes fitted in any position horizontal, vertical or inclined, but it is generally kept horizontal. You have to remember that generally, this venturi meter is being kept in horizontal direction because to get the less obstructions even there will be a turbulence see, less turbulence see accretion there. And it should be remembered that that density meter must run full and it should be preceded by a straight length of the pipe of not less than 5 to 10 diameter of the pipe to reduce the turbulence.

So, to reduce the turbulence, we have to design the venturi meter in such a way that preceded that is length of the pipe should not be less than the five to 10 meter diameter of the pipe to reduce the turbulence there. And commonly the diameter of the throat when the throat sections will see that diameter of the throat is kept half of the inlet pipe diameter or one-third to three-fourth of the outlet pipe diameter here. So, the manometry for the venturi tube or orifice should be installed below the hydraulic grade line or pipe that should be remembered here.

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From Bernoulli's equation

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 \quad (\text{Eq. 6})$$

$$v_1 = v_2 \frac{A_2}{A_1} \quad (\text{Eq. 7}) \quad z_1 = z_2$$

$$v_2 = \frac{1}{\sqrt{1 - (A_2/A_1)^2}} \sqrt{2g \left(\frac{p_1 - p_2}{\rho g} \right)} \quad \frac{v_2^2 - v_1^2}{2g} = \frac{p_1 - p_2}{\rho g}$$

$$Q = A_2 v_2 = \frac{A_2}{\sqrt{1 - (A_2/A_1)^2}} \sqrt{2gH_p} \quad \frac{p_1 - p_2}{\rho g} = H_p = H \left(\frac{\gamma_m}{\gamma_w} - 1 \right) \quad (\text{Eq. 9})$$

$$Q = C \frac{A_2}{\sqrt{1 - (A_2/A_1)^2}} \sqrt{2gH_p} \quad (\text{Eq. 10})$$

C = coeff. of discharge, from exp., Ratio of actual to theoretical, 0.91 – 0.99, depends on flow condition and losses.
H = manometric reading
 γ = specific wt.

Now, based on this Bernoulli's equation if we consider that energy balance here and this venturi meter here see here the figure, it is shown that there is a venturi meter and from that there will be sections one section 1 and section 2 section 2 is for throat. And section 1, there will a main stream and in that case, if you put that manometer tab and if we attach the manometer that manometer will give you the pressure drop and based on which you can say that in the different sections what would be the pressure drop. If we consider that pressure drop are two sections of p_1 and p_2 and also velocity accordingly the upstream and on the downstream at that throat section. If it is v_1 and v_2 respectively and also elevation is suppose z_1 and z_2 ; if it is elevation is it is horizontal, then z_1 should be equal to z_2 .

So, if we write the energy equation to be coming as equation number 6 here p_1 by ρg plus v_1 square by $2g$ plus z_1 that will be is equal to p_2 by ρg plus v_2 square by $2g$ plus here z_2 . And in this case based on the mass conservation equation, we can write here v_1 will be is equal to v_2 into A_2 by A_1 . And after substitution of this v_1 here, then you can have this v_2 will be equal to 1 by root over 1 minus A_2 by A_1 square into root over $2s$ into p_1 minus p_2 by ρg .

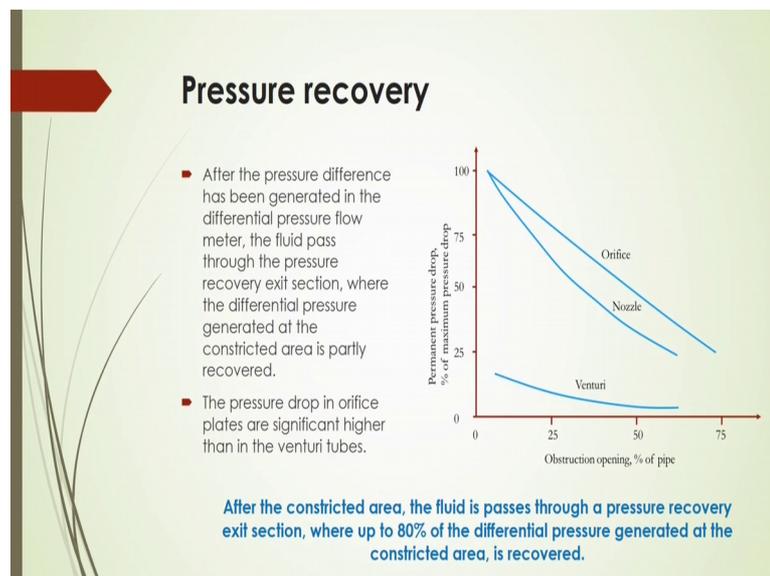
So, this equation is representing the velocity at section r_2 that is in the throat section and based on this velocity, we can calculate what should be the flow rate there. So, flow rate should be is equal to velocity into cross sectional area. So, velocity is v_2 . So, cross

sectional area is A_2 . So, v_2 into A_2 is giving you the flow rate there. So, v_2 what is that v_2 ? v_2 value is nothing, but here we have substitute this v_2 here as this is like this v_1 by this A_1 for the from equation number here. These 8, then we can calculate what should be the volumetric flow rate here.

So, it will come here this in this way. After simplification; now in this case if we substitute that z_1 is equal to z_2 , then we can have this v_2^2 square minus v_1^2 square by $2g$ that p_1 minus p_2 by ρg . So, based on which we can write here these and finally, then we can have this Q will be is equal to C into A_2 by root over p_1 minus A_1^2 by A_1 whole square into root over $2g$ into H_p .

So, H_p is what? H_p is here defined as this pressure energy here p_1 minus p_2 by ρg and it will be represented by this H into γ_m by γ_w minus 1. What is that γ_m ? γ_m is the specific weight of the manometric fluid and γ_w is the what is that what are specific weight and C is here this is coefficient discharge from experiment and ratio of actual to theoretically to would be 0.91 to 0.99 depends on flow condition and losses and H manometric reading here. So, based on which you can calculate what will be the volumetric flow rate there.

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Now, in this case very interesting that after the pressure difference has been generated in the differential pressure flow meter that the fluid pass through the pressure recovery in the exit section where the differential pressure generated at the constricted area is partly

recovered there. So, as shown in here figure, you will see there will be some pressure recovery during this flow through the venturi meter or orifice meter or to nozzle. So, in that case the pressure drop in orifice plate are significantly higher than in the venturi tube. So, you have to remember this that venturi tube always will be the pressure drop in the venturi it will be lower than the orifice plate and even you will see in the through nozzle.

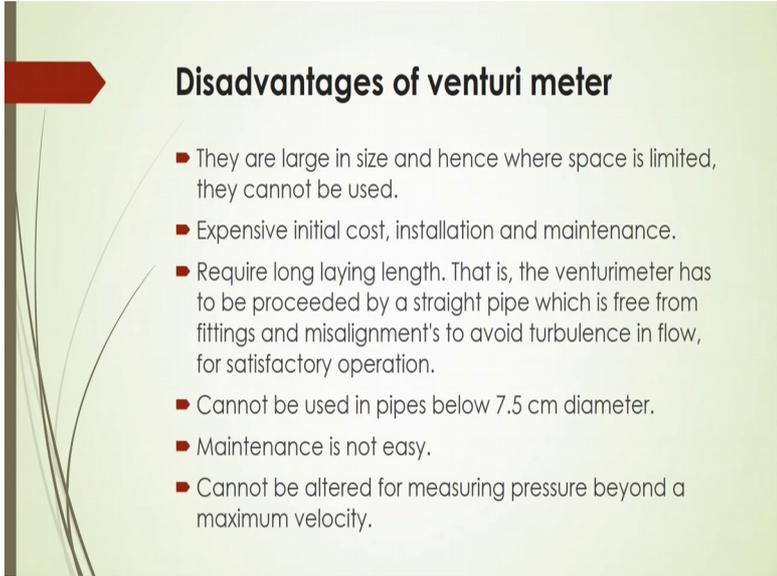
So, after the constricted area, the fluid is passes through a pressure recovery exit section where up to 80 percent of the differential pressure generated at the constricted area will be recovered there. So, how the pressure is recovered in the what is that orifice meter that you can have from this figure.

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Now, what would be the advantages of the venturi meter are to estimate the flow rate there. In this case, you will see less chances of getting clogged with sediments and also coefficients of discharge is high and its behavior can be predicted perfectly and it can be installed vertically, horizontally even an inclined. They are more accurate and can be used for wide range of flows also around 90 percent of the pressure drop can be recovered in this venturi meter.

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Disadvantages of venturi meter

- They are large in size and hence where space is limited, they cannot be used.
- Expensive initial cost, installation and maintenance.
- Require long laying length. That is, the venturimeter has to be preceded by a straight pipe which is free from fittings and misalignments to avoid turbulence in flow, for satisfactory operation.
- Cannot be used in pipes below 7.5 cm diameter.
- Maintenance is not easy.
- Cannot be altered for measuring pressure beyond a maximum velocity.

But it has some disadvantage also and they are large in size and hence, where you can say space is limited they cannot be used for measuring this flow rate. And also it is sometimes expensive initial cost installation and maintenance and it would require long laying length and that is why you can say that venturi has to be preceded by a straight 5 which is free from fittings and misalignments to avoid turbulence in flow for the satisfactory operation. And this venturi meters cannot be used in pipes below 7.5 centimeter diameter because high pressure even some turbulence should be more there.

So, because of which you can see, it cannot be used there and also sometimes it will make some vibrations. So, that will actually give you some error in your measurement and also this type of venturimeter since, there is a convergent divergent even throat section. So, it is sometimes maintenance of this flow even these what is that cleaning of the venturi meter sometimes it is not easy. And also you cannot actually use these two alter for measuring pressure beyond the maximum velocity there. Let us do an example to calculate the flow rate based on this venturi meter.

Now, calculate the flow of water flowing through a 40 into 15 centimeter horizontal a venturi meter. When the differential gauge are connected to the inlet end of the vein meter and its throat shows 25 centimeter mercury. In this case, assume that coefficient of discharge should be 0.98.

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Example: Calculate the flow of water flowing through a 40x15 cm horizontal venturimeter, when the differential gauge connected to the inlet end of the meter and its throat shows 25 cm Hg. Assume the coefficient of discharge is 0.98

Solution

$A_1 = 0.1256 \text{ m}^2$; $A_2 = 0.01765 \text{ m}^2$

$H_p = 0.25(13600/1000-1) = 3.15 \text{ m}$ of flowing fluid i.e. water here

Therefore the flow of water as per Eq. 10 we can get

$$Q = C \frac{A_2}{\sqrt{1 - (A_2/A_1)^2}} \sqrt{2gH_p} = 0.137 \text{ m}^3/\text{s}$$

So, in this case, what should be the flow rate? So, first of all you have to calculate what should be the cross sectional area at section 1 as A_1 and what should be the cross sectional area at section 2 A_2 . Then after that what should be the h_p value based on this formula given in your slides here in equation number 9 and then after that we are getting this around 3.15 meter of flowing fluid of water here based on this example. And then after that what should be the flow rate of water as per equation number 10, we can calculate and we can get this 0.137 meter cube per second.

So, here important to that this though you can actually, then have what should be the flow rate based on this energy equation and based on this what is that venturi principle and how to calculate that flow rate you can get based on these examples here.

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Pitot tube



Henry de Pitot
(1692-1771)
France. A civil
Engineer,

- In Early eighteenth century, Pitot, procured a simple measuring device of flow rate. It was made by bending of a glass tube by 90° of its lower end.
- He measured the flow velocity by measuring the increased height of the water level due to pressure.
- He did his first experiment in the River Seine carrying a glass tube with a bent end.

"If tube faces into the flow, water in the tube goes up. From this height, the flow velocity can be estimated"



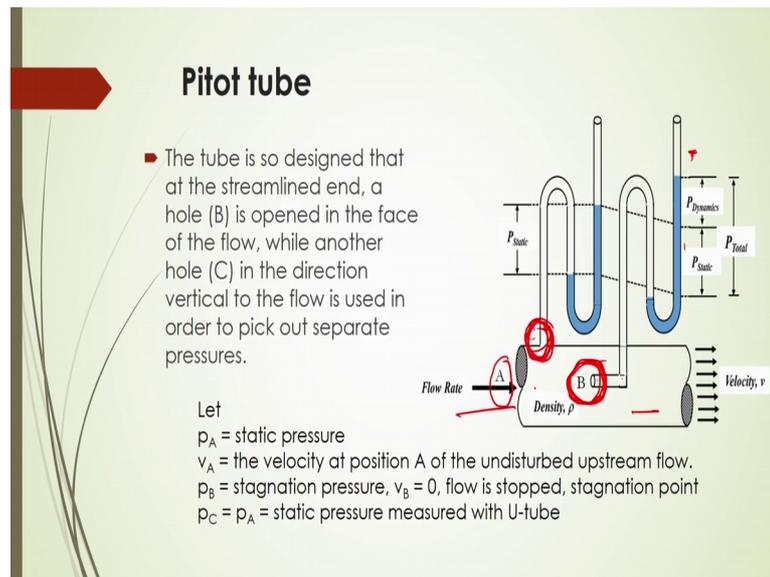
Seine River

Now, another important flow measuring device, it is called Pitot tube. Actually this pitot tube, it is developed by Henry de Pitot. He is a French civil engineer and he did his first experiment just for testing whenever he was actually spending the time in near about that river seine. And it is in early 18 century and he procured a simple measuring device of flow rate in the at that time and it was made by bending of a glass tube by 90 degree of its lower end as shown in here figure. See this case, he measured the flow velocity by measuring the increased height of the water level due to pressure.

Whenever he placed this glass tube one end is bent at 90 degree and he placed this there is hole of this glass tube here in the direction that is this or the flow in which direction the fluids were opposite to that facing that the flow and he installed or he just placed that tube inserts in the river. He observed that there will be rise of liquid in the what is that at last you.

This rise of liquid in the glass tube because of the pressure and from that concept, t he first actually discovered this pitot tube and he did his first experiment in this river seine carrying a glass tube with a bent tube here. And based on this rise of how much rise of actually liquid in the tube and that is actually pressure difference and based on which he has actually estimated the flow rate of the water in the river.

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And in this case the tube is to be designed in such way that at the streamlined end a hole B as shown in figure here. Let us see here this, p how it is actually installed here this is one pipe and this is flow rate at which this a fluid will be flowing and here this is your pitot tube and this face of this pitot tube has B hole is actually faced to the fluid flow and then how this pressure is giving during this flow and that will be measured by this what is that manometer. And also parallelly, you have to measure what is that free stream pressure here in the flow. And based on these two pressure difference, he has actually estimated the a flow rate of the fluid.

And this tube so that is why design in such a way that the streamline end A whole B we are assume here figure is opened in the face of the flow. While another hole C in the direction vertical to the flow is used to pick out separate pressure. Here this is C and another point here this is actually the, but this normal to the flow is placed and here this parallel to the flow. So, this you will see this in this case at C, the pressure will be measured and here and this you will see there will be a pressure at this point p . It is it would be called as stagnation pressure and p at and that is here C or at section A, the pressure will be called as a static pressure.

So, what will be that difference in static pressure and what is the stagnation pressure is there. And here at section B, the stagnation pressure will be there because at this section B velocity will be 0 because flow will be stopped at that location at that point and it is

called stagnation point and in that case based on this stagnation pressure and that pressure difference of this free stream that is measured in this section C by placing that manometer particle to the flow. And based on which you will be able to calculate what would be the velocity there.

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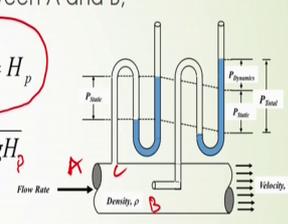
Pitot tube

- Apply Bernoulli's equation between A and B,

$$\frac{p_A}{\rho g} + \frac{v_A^2}{2g} = \frac{p_B}{\rho g}$$

$$v_A = \sqrt{2 \left(\frac{p_B - p_C}{\rho} \right)}$$

$$\frac{p_B - p_C}{\rho g} = H_p$$

$$v_A = \sqrt{2gH_p}$$


with an actual Pitot tube, since some loss occurs due to its shape and the fluid viscosity, the equation is modified as :

$$v_A = C_v \sqrt{2gH}$$

C_v is called the coefficient of velocity, Pitot tube coefficient

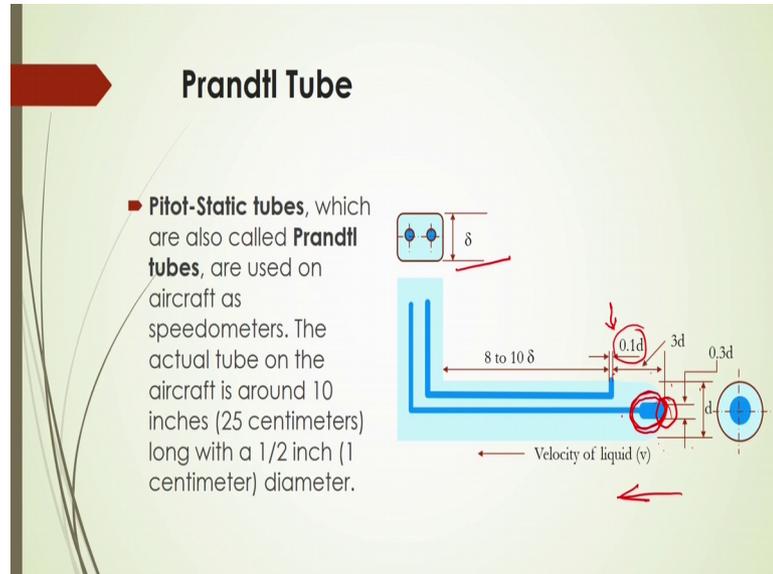
Now, the applying that Bernoulli's equations again between these sections A and B, we can have that p_A minus p_A by ρg plus v square by $2z$ at the sections A. That would be is equal to what that section 2 at section 2; that means, at B sections, you can say here it is B and here C here A. So, between A and B; if we consider, then p_A by ρg plus v A square by $2g$ that will be is equal to p_B by ρg plus here what is that B; $B A$ square by $2g$ since it is B B is equal to 0 at that stagnation point, then we can say that simply here p_A by ρg plus $v A$ square by $2g$ that will be is equal to p_B by ρg .

So, that will come here v_A will be is equal to what root over 2 into p_B minus p_C by ρg after simplification from this equation. Now since p_B minus p_C by ρg is represented as H_p , then v_A should be is equal to what this is $2g$ into H_p here; root over $2g$ into H_p . So, this actual pitot tube since some loss occurs due to it is a shape and the fluid viscosity, the equation is modified as v_A is equal to C_v into root over $2gH$ here.

Now, this C_v is called the coefficient of velocity pitot tube coefficient. It is sometimes it is called pitot tube coefficient also. So, C_v is called the coefficient of velocity or pitot tube coefficient. So, based on these principles, we can then calculate what should be the

velocity at point A; that means, here what should be the velocity that is measured by the pitot tube.

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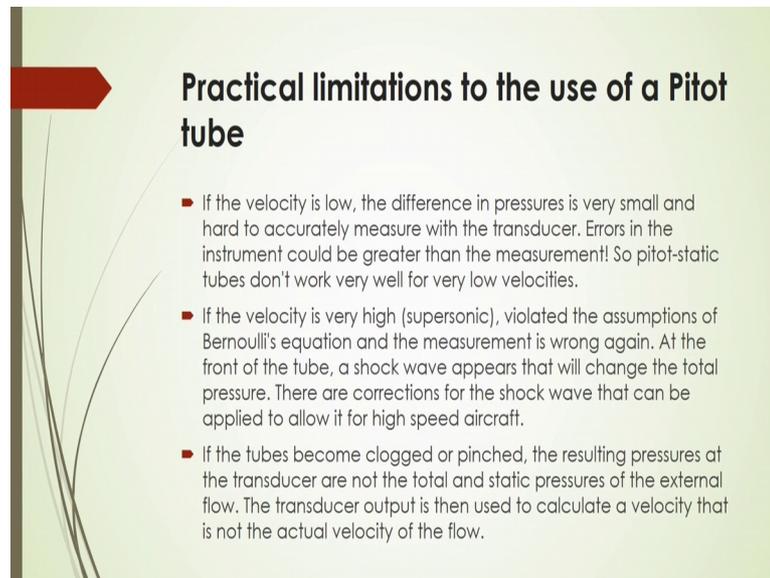
Now, another important point that earlier stage the another Prandtl tube actually pitot tube is a design based on the concept of that Prandtl tube. That is why the Prandtl pitot tube, it is sometimes called as Prandtl tubes. And in this case, this Prandtl tubes are pitot static tubes you can say are used on aircrafts as speedometers. The actual tube on the aircraft is around 10 inches that is 25 centimeters long with a half inch or 1 centimeter diameter here.

So, in that case how this actually Prandtl tube designed there at that age? So, here this is this, you will see this is the tube and this is one what is that through the channel the fluid will be coming out and then here this the channel width or diameter should be 0.1 into diameter. And this diameter is this diameter of the tube is this and here this pipe should be here. What is the channel through which this fluid will be flowing? So, in that case from this stagnation point and whenever fluid will be giving the pressure here and this diameter should be designed that this should be is equal to 0.3 of d and these two these distance would be $3d$ and these two these distance will be 8 to or 10 d .

So, velocity of the fluid, it will be flowing through in this direction and through this portion and also through this. So, in this case the pressure will be obtained that is perpendicular to the fluid and here, this is parallel to the fluid. This is your stagnation

pressure and this is will be your what is the free stream pressure and based on this the pressure difference will be able to calculate, what should be the flow rate or velocity of the fluid.

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Practical limitations to the use of a Pitot tube

- If the velocity is low, the difference in pressures is very small and hard to accurately measure with the transducer. Errors in the instrument could be greater than the measurement! So pitot-static tubes don't work very well for very low velocities.
- If the velocity is very high (supersonic), violated the assumptions of Bernoulli's equation and the measurement is wrong again. At the front of the tube, a shock wave appears that will change the total pressure. There are corrections for the shock wave that can be applied to allow it for high speed aircraft.
- If the tubes become clogged or pinched, the resulting pressures at the transducer are not the total and static pressures of the external flow. The transducer output is then used to calculate a velocity that is not the actual velocity of the flow.

Now, there will be a some practical limitations to the use of a pitot tube. If the velocity is low, the difference in pressures is very small and hard to accurately measure with the transducer. And in this case, errors in the instrument could be greater than the measurement and so, pitot static tubes do not work very well for very low velocities. And also you will see that if the velocity is very high little supersonic; that means, velocity is greater than sound velocity.

Then violated the assumptions of the Bernoulli's equations and in that case, measurement should be wrong and at the front of the tube then a shock wave will be generated and that will change the total pressure. And there are corrections for the shock wave that can be applied to allow it for high speed aircraft and that is why you have to consider that consider the correction factor whenever you are going to use these for high velocity. And if the tubes become clogged or piece to the resulting pressures at the transducer; if you are using for measuring pressure drop are not the total and the static pressures of the external flow. And the transducer output is then used to calculate the velocity that is not the actual velocity of the flow.

So, in that case, you have to remember; if you are you going to use the transducer, then you will not be able to calculate the accurate velocity of the flow.

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Example: A submarine moves horizontally in sea and has its axis 15 m below the surface of the water. A pitot tube properly placed just in front of the submarine and along its axis, is connected to the two limbs of the U-tube manometer containing mercury. The difference of the mercury level is found to be 17 cm. What is the speed of the submarine. The density of the sea water is 1.026 with respect to normal water.

■ Solution

$$\frac{p_1 - p_2}{\rho g} = H_p = H \left(\frac{\gamma_m}{\gamma_w} \right) = 0.17 \left(\frac{13.6}{1} \right) = 2.081$$

$$v_A = \sqrt{2gH_p} = 6.391 \text{ m/s} = 23 \text{ km/h.}$$

Let us do an examples with the calculation that is the with the theory of this pitot tube. How it can be calculated the velocity? A submarine moves horizontally in sea and has its axis 15 meter below the surface of the water. Now, a pitot tube properly placed just in front of the submarine and along its axis and is connected to the two limbs of the U tube manometer and containing mercury.

And the difference of the mercury label is found to be 17 centimeter, then what should be the speed of the submarine? The density of the sea water to be considered as 1.206 within respect to the normal water and in that case if we apply this pitot tube theory here, then it will be here $p_1 - p_2$ by ρg that should be is equal to H_p . Then what would be the H_p value? Based on this problem you can calculate here. It is given it is coming 2.081, then v_A will be is equal to root over $2g$ into H_p , then should be simply after substitution of this H_p ; then you can get here it should be 23 kilometer per hour.

So, based on this problem very simple to solve and what should be the velocity, then you can calculate once you know the pressure difference.

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Flow through Orifice meter

- the case where water is discharging from a small hole. Such a hole is called an orifice (dia = d_o).
- the spouting flow contracts to have its smallest section 2 a small distance from the hole.
- the flow lines are almost parallel so that the pressures are uniform from the periphery to the centre of the flow.
- This part of the flow is called the vena contracta (dia = d_{vc})
- It is a cheaper device as compared to venturimeter.
- This also work on the same principle as that of venturimeter.

The diameter of orifice is generally 0.5 times the diameter of the pipe (D), although it may vary from 0.4 to 0.8 times the pipe diameter.

Now, what should be the flow through orifice meter ? Now in this case, you will see the case where water is discharging from a small tube. In that case, the whole of this orifice meter is important and the hole is called an orifice. This here in this case very interesting that that fluid will be passing through a small hole that is called orifice. And then there will be spouting of the flow and that is spouting flow contracts to have its a smallest section 2 as shown in figure here the section 1 here and section 2 here; the section 2 here.

In this case, you will see there will be if you are considering this section 2 here and then in between you will see you are using that obstruction as a hole here. It is called orifice hole and then, there will be a then spouting flow and which will contracts to have its a smallest section 2 and as a small distance from the hole. And the flow lies are almost parallel to that the pressure and it will be uniform from the periphery to the center of the flow. And also you will see this part of the flow is called that vena contracta.

It is shown in figure here which portion is vena contracta. This is here; this is your vena contracta. This portion and this case this vena contracta sometimes it will be lengthy and that is suit a certain distance. And in that case, the diameter of the orifice is generally 0.5 times the diameter of the pipe. It is considered although it may vary from 0.42 0.8 times the pipe diameter and it is a cheaper device as compared to the venturimeter. And, these also work on the same principle as that of the venturimeter meter.

So, very interesting that here also you have to allow the fluid through a hole. Whenever it will be coming through a pipe so, you can say that make a flow in such way that would be a some spouting flow because of that using of that a small hole through which this fluid will be passing. And that contrast to have it is a smallest section a small distance from the hole will be considered as a vena contracta.

And it is at the diameter of the vena contracta is represented d_{VC} here. And sometimes if you are not considering that vena contracta with a certain distance; that means, here small distance from its the hole, then in that case the section 2 will be almost equals to that vena contracta there. And also for that you will see, there will be some energy loss whenever you are just giving this obstruction of hole during this flow.

So, this energy loss will give you the pressure, energy loss that pressure energy to be actually calculated and based on which that you we will be able to calculate this flow rate. Now, assuming that the fluid particle at section 1 has flowed down to section 2 here and then if we apply the Bernoulli's theorem, then we can say that v_1^2 square by $2g$ plus p_1 by ρg plus z_1 .

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Flow through Orifice meter

- Assume that fluid particle at section 1 has flowed down to section 2. Then, from Bernoulli's theorem

$$\frac{v_1^2}{2g} + \frac{p_1}{\rho g} + z_1 = \frac{v_2^2}{2g} + \frac{p_2}{\rho g} + z_2$$
- As pipe is horizontal $z_1 = z_2$

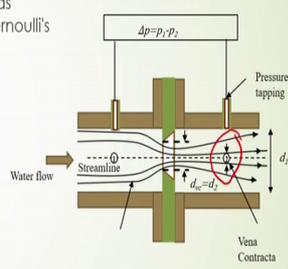
$$\frac{p_1 - p_2}{\rho g} = \frac{v_2^2 - v_1^2}{2g}$$

$$\Rightarrow v_2 = \sqrt{2gh + v_1^2}$$

Where

$$H_p = \frac{p_1 - p_2}{\rho g}$$

= difference of pressure heads at sections 1 and 2.



That would be is equal to v_2^2 square by $2g$ plus p_2 by ρg plus z_2 . The two sections, we are applying that energy here and the summation of the energy at these two sections will be constants. So, as pipe is horizontal in this case so, we are getting this z_1 will be is equal to z_2 here. And as pipe since horizontal, then we can cancelled out this z_1 and z_2 .

2 and finally, we are getting the equation as a simple one $p_1 - p_2 = \rho g z$ that will be equal to $v_2^2 - v_1^2 = 2g z$. And then v_2 , what should be the v_2 at the sections 2? Then you can have this by this equation $v_2 = \sqrt{2g z + v_1^2}$.

Now, this case H_p will be equal to then $p_1 - p_2 = \rho g z$. This will be your difference of pressure heads at section 1 and 2.

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Let A_0 is the area of the orifice.

Coefficient of contraction $C_c = \frac{A_2}{A_0}$ $A_0 > A_2$

From the continuity equation at sections 1 and 2, we obtain

$A_1 v_1 = A_2 v_2$ $v_1 = \frac{C_c A_0 v_2}{A_1}$

Hence $v_2 = \sqrt{2gH_p + \frac{C_c^2 A_0^2 v_2^2}{A_1^2}}$

$v_2 = \frac{\sqrt{2gH_p}}{\sqrt{1 - \frac{A_0^2}{A_1^2} C_c^2}}$

Note
The expression is for ideal condition and is known as theoretical discharge.

Now after substitution of this pressure head here in this equation, then you can get what should be the v_2 there and based on which you will be able to calculate the flow rate. Now let us do that if A_0 is the area of the orifice to be considered, then the ratio of A_2 by A_0 that is section 2. What is the sectional area at this section 2? If we divide it by this orifice cross sectional area, in this case A_0 should be is greater than A_2 , then you can say that you will get the coefficient of contraction.

And from the continuity equation at sections 1 and 2 we obtain then $A_1 v_1$ that will be is equal to $A_2 v_2$ and then v_1 should be is equal to what? v_1 should be is equal to $C_c A_0 v_2$ by A_1 and in this case, C_c will be is equal to A_2 by A_0 . Hence v_2 will be is equal to $\sqrt{2gH_p + \frac{C_c^2 A_0^2 v_2^2}{A_1^2}}$. Then finally, it is coming as v_2 will be is equal to $\frac{\sqrt{2gH_p}}{\sqrt{1 - \frac{A_0^2}{A_1^2} C_c^2}}$. So, in this case, the expression is for ideal condition and is known as theoretical discharge.

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Thus discharge $Q = A_2 v_2 = v_2 A_0 C_c = \frac{A_0 C_c \sqrt{2gH_p}}{\sqrt{1 - \frac{A_0^2}{A_1^2} C_c^2}}$

If C_d is the co-efficient of discharge for orifice meter, which is defined as

$$C_d = \frac{C_c}{\sqrt{1 - \frac{A_0^2}{A_1^2} C_c^2}}$$

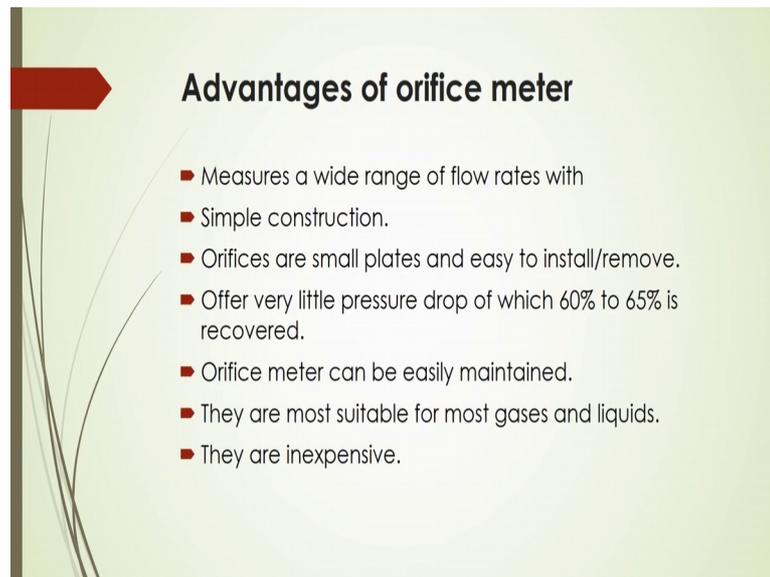
Therefore discharge $Q = C_d \frac{A_0 A_1 \sqrt{2gH_p}}{\sqrt{A_1^2 - A_0^2}}$

Remember
The coefficient of discharge of the orifice meter is much smaller than that of a venturimeter.

So, you will see that what will be the theoretical research for this, you can calculate from this equation here as v_2 as given. And thus discharge Q should be is equal to what? A_2 into v_2 , what would be the cross section? Already at this section A_2 and what will be the velocity that is calculated by this equation and then after substitution of A_2 here, we can get that v_2 into A_0 into C_c . And after substitution of this is what is that A_0 and also you can say that A_2 value, then we can get here no after substitution of this v_2 value. Here, then we can get this equation here.

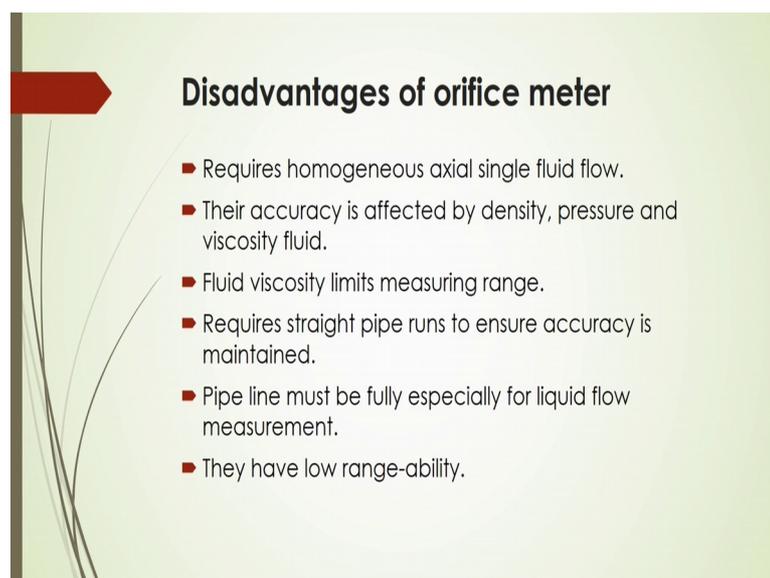
So, if C_d is considered as the coefficient of discharge for orifice meter and defining as by this equation that will be C_c by root over one minus A_0 square by A_1 square into C_c square, then the discharge should be represented as C_d into $A_0 A_1$ to root over $2gH_p$ by A_1 square minus A_0 square. And in this case, you have to remember that the coefficient of discharge of the orifice meter is much smaller than that of a venturi meter.

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What are the advantages of the orifice meter? These orifice meter measures a wide range of flow rates with simple construction. Orifices are small plates and easy to install or remove. Offer very little pressure of which 60 percent to 65 percent is recovered and orifice meter can be easily maintained. They are most suitable for most gaseous and liquids and they are inexpensive.

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Now, some disadvantage also can be seen for measuring the fluid flow rate by orifice meter. Whenever you are going to use this orifice meter, it will sometimes require that

homogeneous axial a single fluid flow and it is not actually applicable for that homogenous axial single liquid flow. And their accuracy is affected by density pressure and viscosity and also fluid viscosity limits the measuring range. In this case, requires a straight 5 runs to ensure accuracy is maintained and also pipeline must be fully especially for liquid flow measurement and also they have low range ability to measure the fluid flow rate.

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Example: From an experiment of water flow through an orificemeter, the exhibit pressure drop during the flow was found to be 15 kPa. The pipe and orifice diameters are 30 and 15 cm respectively. What was the discharge or flowrate of the water? Coefficient of contraction is 0.625

- Solution
- $A_1 = (\pi/4) \cdot 0.3 \cdot 0.3 = 0.071$ ✓
- $A_0 = (\pi/4) \cdot 0.15 \cdot 0.15 = 0.018$ ✓
- Coefficient of contraction is $C_c = 0.625$ ✓

$$C_d = \frac{C_c}{\sqrt{1 - (A_0^2 / A_1^2) C_c^2}} = 0.632$$

$$Q = C_d \frac{A_0 A_1 \sqrt{2gH_p}}{\sqrt{A_1^2 - A_0^2}} = 0.063 \text{ m}^3/\text{s} = 63 \text{ L/s}$$

Let us do an example based on this principle of this orifice meter. Now from an experiment of water flow through an orifice meter, the exhibit a pressure drop during the flow was found to be 15 kilo Pascal. And the pipe and orifice diameters are 30 and 15 centimeter respectively, then what was the discharge of discharge of flow rate of the water.

Now, coefficient of contraction is given as 0.625. Now in this case, you have to first calculate what should be the A_1 , that is cross section at this section 1. What will be the cross section at this A_0 that is in bena contracta, you can say. And then coefficient of contraction is C_c that will be here. And then what should be the C_d ? C_d will be is equal to as per equation. It is C_c by root over 1 minus say 0 square by A_1 square into C_c square that will be is equal to 0.632. And finally, after substitution of this C_c value and C_d value and you can get this discharge rate by this equation here and it will be coming finally, 0.063 meter cube per second or 63 litre per second. So, these examples also you

have to practice that how to actually calculate the discharge flow rate based on the principle of orifice meter.

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Weir

- In the case where a water channel is stemmed by a board or a wall, over which the water flows, such a board or wall is called a weir.
- A weir is used to adjust the flow rate.

Assume:
 dz = a minute depth at a given depth z from the water level.
 b = the width of the water channel
 Bdz = a minute area as an orifice.

From Bernoulli's equation $v = \sqrt{2gz}$

$$dQ = Cbdz\sqrt{2gz} \quad Q = Cb\sqrt{2g} \int_0^H \sqrt{z} dz = \frac{2}{3} Cb\sqrt{2g} H^{3/2}$$

By measuring H , the discharge Q can be computed

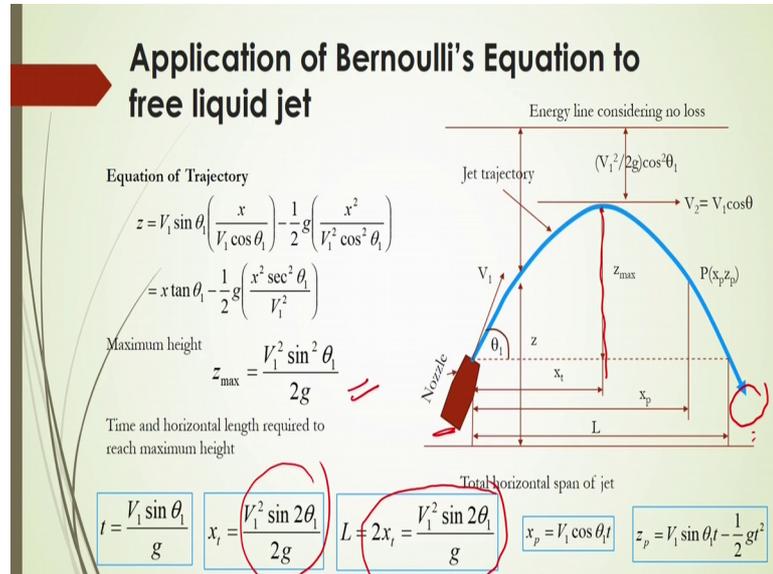
Another one is we are we are also we are being used we are using for measuring the flow rate in the water channel. In the case of where water channel is streamed by a board or what is that wall or over which the water flows such a board or wall that will be called as weir here. And it is shown in here figure this and a weir is used to adjust the flow rate here assume dz a minute depth of a given depth z from the water level as shown in figure here. In this case, this z dz to be considered and B the width of the water channel and then B into dz , that will be a minute area as an orifice.

Then from the Bernoulli's equation, we can have this equation from the energy balance, then v will be is equal to root over $2gz$. And then what should be the discharge? Then discharge should be is equal to after differentiate this dQ and then it will come as Cs b into dz into root over $2gz$, then it will be here Q will be is equal to C b into root over $2g$ into integration of what is that 0 to H for the whole height of this water column. Here then you can get finally, as $\frac{2}{3}$ into C b into root over $2g$ H to the power $3/2$.

So, by measuring the H , the discharge q can be computed. Now this H will be calculated, what would be the H here? Based on this here, we are that is liquid height, then you will be able to calculate what should be the discharge flow rate. So, only thing is that you

have to calculate this mass of water depth there. So, from which you will be able to calculate what will be the discharge rate.

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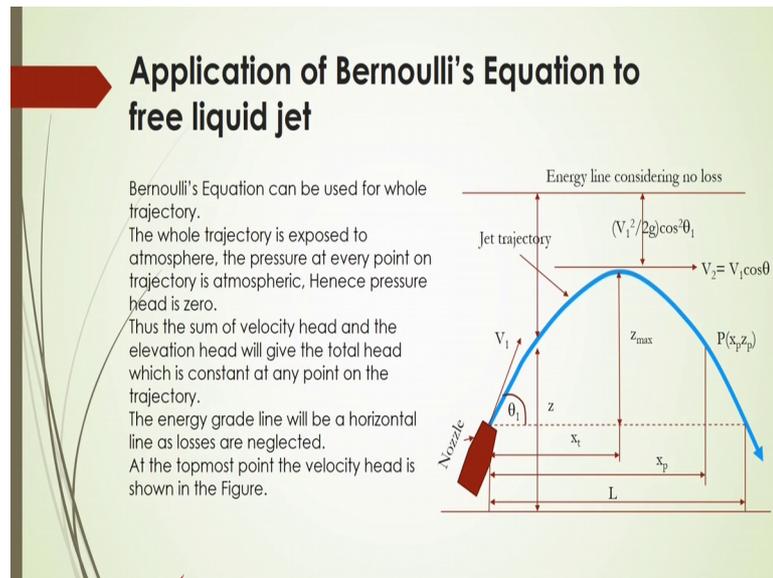
And again the liquid jet to calculate that liquid jet free liquid jet how to use the Bernoulli's equation there, you can also obtain from the slides that how to use this. So, whenever and nozzle is actually exposed to the atmosphere, there at a certain velocity. And whenever it will be heating in a ground at a certain velocity, what should be the velocity and what will be the time to require to reach at this location here from this beginning.

And then equation of trajectory can be used for this jet velocity and then maximum height of that free liquid jet, how to calculate this jet maximum? This will be calculated based on this equation and the time and horizontal length required to reach the maximum height, it will be like this. And here and what should be the length of that at time t, how to horizontal length that he travelled that will be calculated by this equation. And what will be the total length he can travel; that will be calculated based on this equation and what will be the different component of this x p and what is that z p, that also can be calculated based on this equation.

So, go to these slides to what is the how this Bernoulli's equation can be applied to calculate what is that jet trajectory equation and also energy line there where the based on this Bernoulli's equation and what will be the different velocity components during

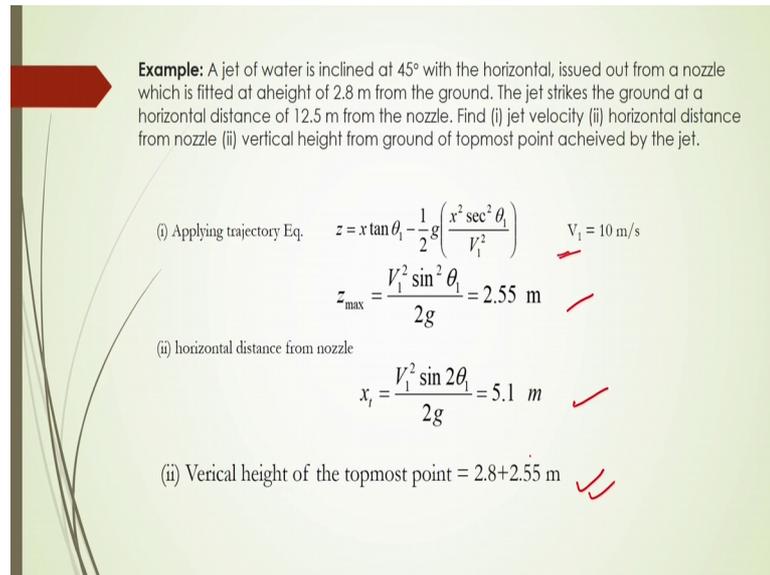
this flow of this jet. And based on which you will be able to calculate what should be the what is that flow rate at each location of this gear fluid at that particular location based on the velocity. And also whatever the diameter of the jet is coming based on which you can have the flow rate there.

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So, application of the Bernoulli's equation to the free liquid jet here again, this can be used to whole trajectory. The whole trajectory is exposed to atmosphere the pressure at every point on trajectory is atmospheric. Hence pressure head will be 0, does the sum of the velocity head and the elevation head will be will give the total head which is constant at any point on the trajectory. The energy grade line will be a horizontal line as losses are neglected at the topmost point the velocity head is shown in here figure in this case.

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Example: A jet of water is inclined at 45° with the horizontal, issued out from a nozzle which is fitted at a height of 2.8 m from the ground. The jet strikes the ground at a horizontal distance of 12.5 m from the nozzle. Find (i) jet velocity (ii) horizontal distance from nozzle (iii) vertical height from ground of topmost point achieved by the jet.

(i) Applying trajectory Eq. $z = x \tan \theta_1 - \frac{1}{2} g \left(\frac{x^2 \sec^2 \theta_1}{V_1^2} \right)$ $V_1 = 10 \text{ m/s}$

$z_{\text{max}} = \frac{V_1^2 \sin^2 \theta_1}{2g} = 2.55 \text{ m}$

(ii) horizontal distance from nozzle

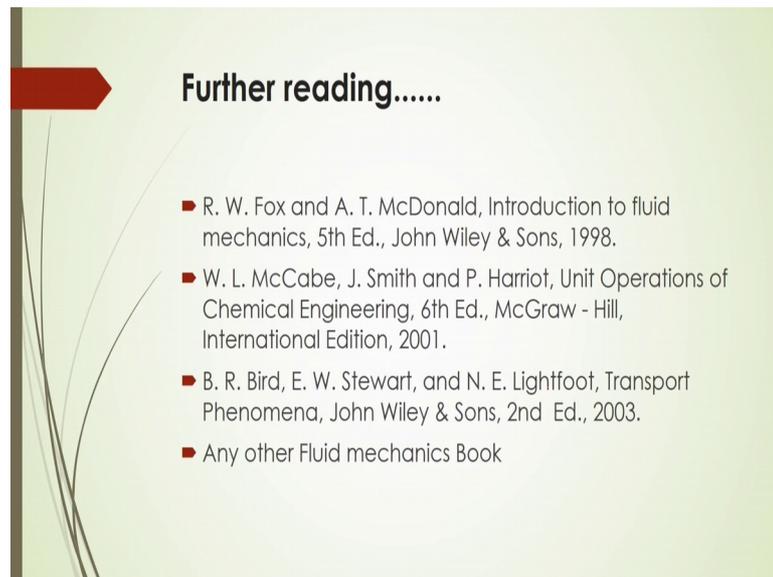
$x_1 = \frac{V_1^2 \sin 2\theta_1}{2g} = 5.1 \text{ m}$

(iii) Vertical height of the topmost point = $2.8 + 2.55 \text{ m}$

So, based on these principles if we do an examples that a jet of water is inclined at 45 degree with the horizontal issued out from a nozzle which is fitted at a height of 2.8 meter from the ground. The jet strikes the ground at a horizontal distance of 12.5 meter from the nozzle, then what should be the jet velocity horizontal distance from the nozzle? Vertical height from ground of topmost point achieved by the jet, then you can calculate based on this equation here.

So, it will be z and then z max, then what should be the x t and vertical height of the topmost point should be like this. So, based on these examples you will be able to calculate the trajectory point and also height there for the liquid jet.

(Refer Slide Time: 54:05)



Further reading.....

- R. W. Fox and A. T. McDonald, Introduction to fluid mechanics, 5th Ed., John Wiley & Sons, 1998.
- W. L. McCabe, J. Smith and P. Harriot, Unit Operations of Chemical Engineering, 6th Ed., McGraw - Hill, International Edition, 2001.
- B. R. Bird, E. W. Stewart, and N. E. Lightfoot, Transport Phenomena, John Wiley & Sons, 2nd Ed., 2003.
- Any other Fluid mechanics Book

So, I would suggest to read further about this specific principles of the measurement and also measuring devices like orifice meter, venturi meter and also pitot tube how they are working based on the Bernoulli's equations. And for the calculation of the flow rate, you can design the venturi meter orifice meter and pitot tube based on the principles.

So, thank you for this lecture. Next class, we will next lecture we will discuss something more about the measuring instruments to actually measure the fluid flow rate.

Thank you.