

Fluid Flow Operations
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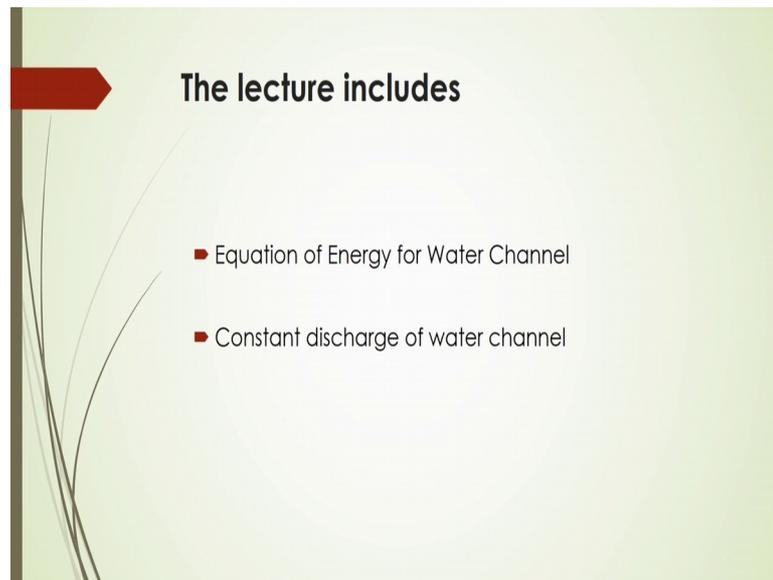
Module - 07
Flow in Water Channel-Part 2
Lecture – 19
Equation of Energy and Discharge of Water Channel

So, welcome to Massive Open Online Course on Fluid Flow Operations. In the previous lectures, we have discussed the Flow in Water Channel as a part-1, there we have discussed how the flow will be flow is flowing through the open channel with the different cross sections, round in shape and also cross sectional with the rectangular and other shapes.

So, in this lecture, we will be continuing this flow in water channel. In this case, we will discuss the equation of energy and discharge of the water channel and also what will be the critical condition for which that at a certain velocity that flow behaviour will be changed. And also at a constant discharge what should be the critical velocity at which the flow condition will be changed for a subsonic to supersonic condition or subcritical super critical condition.

So, in that case, we will again that we considering that energy equation for a calculating this optimum condition of the velocity for changing its flow behaviour from subcritical to the supercritical condition.

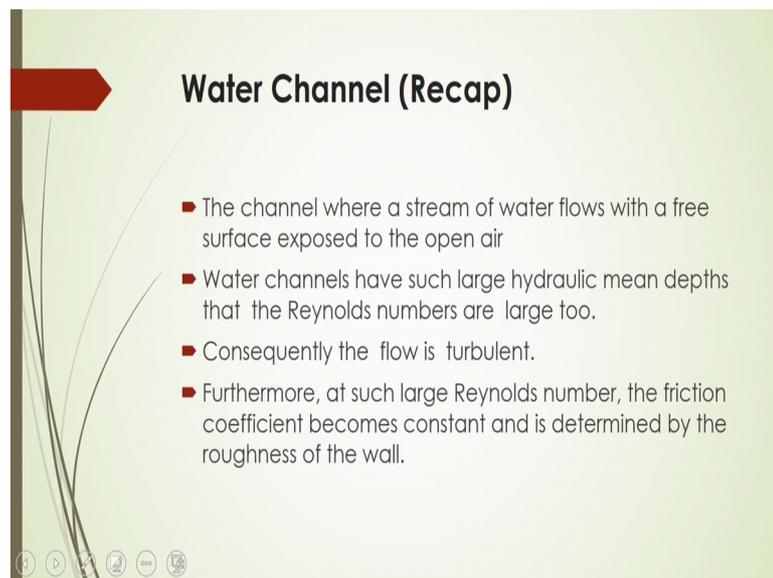
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The lecture includes

- Equation of Energy for Water Channel
- Constant discharge of water channel

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Water Channel (Recap)

- The channel where a stream of water flows with a free surface exposed to the open air
- Water channels have such large hydraulic mean depths that the Reynolds numbers are large too.
- Consequently the flow is turbulent.
- Furthermore, at such large Reynolds number, the friction coefficient becomes constant and is determined by the roughness of the wall.

Now, what we have discussed in the previous lecture that the channel where a stream of water flows with a free surface exposed to the open air, it is called the water channel. And the channels have such large hydraulic mean depths, so that the Reynolds number will be too large.

And also of course for that large Reynolds number, the flow should be turbulent in condition. And at such turbulent conditions that is at higher Reynolds number, the friction coefficient becomes constant and is determined by the roughness of the wall

there. And also we know that in the channel for different cross sectional area, the flow behaviour will be changed and also the characteristics feature of that flow will be changed in the channel.

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Equation of specific energy

- Many open channel problems can be solved using the equation of energy
- If the pressure is p at a point A in the open channel, the total head of fluid at Point A is

$$\text{Total head} = \frac{v^2}{2g} + \frac{p}{\rho g} + z + z_0$$

(Eq. 1)

Now, we will see what should be the total head, whenever fluid will be flowing through the open channel. We know that there are many channel that is opened, there will be a some problems that can be solved using the equation of energy.

We have already learned about the equation of energy like Bernoulli's equation and conservation of equation based on which we are getting three types of energy. One is operation energy, another is velocity due to the velocity, and also the gravitational energy there. So, there are three types of energy will be summing to a constant value.

And in that case if we know that particular part of that energy, we will be able to calculate other part of energy between two sections of the pipe or channel sections there. And in that case, generally this energy equation sometimes is represented by in terms of head. So, in that case, what should be the total head of this see energies; one is energy due to the velocity and energy due to the pressure, and also what will be the energy due to the gravity or you can say potential height.

Now, in that case if the pressure is p at a point A here in this figure as shown here at this particular point, if we consider that what should be the pressure? There if we considered

that pressure is p , then in this channel if it is the velocity is v the flow velocity is v and at height of this section is z .

And if it is the height from this datum level, if we consider the datum level here, then what should be the height, it will be z plus z_0 . And at this point what should be the velocity head that will be v square by $2g$. And what should be the pressure head that will be p by ρg . And what will be the elevation head that will be z and plus z_0 from the datum level, if you are considering there. So, in this case total head should be summation of this three heads here, so v square by $2g$ plus p by ρg plus z plus z_0 . So, by this equation one here, we can represent the total head, what should be that whenever the fluid will be flowing through the channel, we take gravity there.

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■ If the depth of water channel is h , then

$$h = \frac{p}{\rho g} + z \quad (\text{Eq. 2})$$

■ Consequently, the total head may be described as follows:

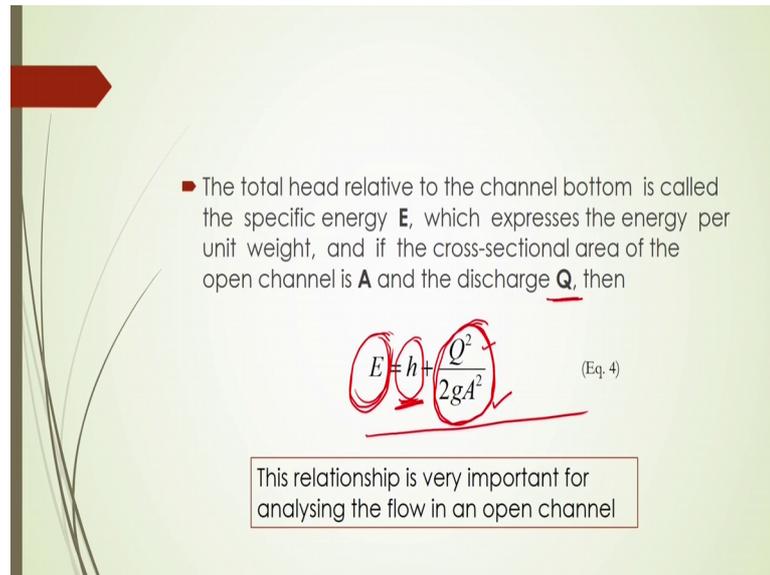
$$\text{Total head} = \frac{v^2}{2g} + h + z_0 \quad (\text{Eq. 3})$$

The diagram illustrates a channel with a datum level (horizontal) at the bottom. A vertical section of the channel is shown with a water depth h . The elevation of the channel bed at this section is z_0 . The total head is represented by the energy line, which is the sum of the velocity head $\frac{v^2}{2g}$, the pressure head $\frac{p}{\rho g}$, and the elevation head z (the height of the water surface above the datum level).

And if the depth of the water channel, if we consider that h in the channel, the water depth is suppose this one is your water depth. So, here in that case that h should be is equal to p by ρg plus z that is the summation of this 2 head here that is one is pressure head p by ρg and plus z there.

So, in this case, as a result the total head may be described as follows here. This total head should be is equal to v square by $2g$ plus h ; h is nothing but that p by ρg plus z and plus z_0 here. So, by equation-3, we can represent the total head in terms of this water head in the channel along with the velocity head and also the elevation head from the datum level there.

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- The total head relative to the channel bottom is called the specific energy **E**, which expresses the energy per unit weight, and if the cross-sectional area of the open channel is **A** and the discharge **Q**, then

$$E = h + \frac{Q^2}{2gA^2} \quad (\text{Eq. 4})$$

This relationship is very important for analysing the flow in an open channel

And total head relative to the channel bottom sometimes it will be represented by a specific energy that is E represented by E , which express the energy per unit weight. And if the cross-sectional area of the open channel if represented by A and the discharge then Q that is capital Q discharge that will be is equal to here, then total energy should be is equal to what E is equal to h plus this will be Q square by $2gA$ square.

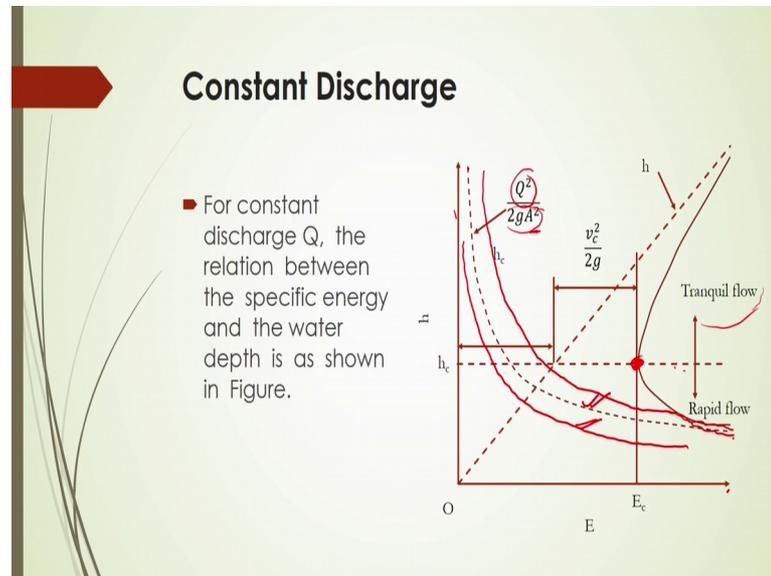
So, in this case, this total head we are considering that there will be relative to the channel bottom and this head will be represented by this specific energy E . So, this specific energy that is defined as the energy per unit weight. And if the cross-sectional area of the open channel is considered A , then we can have this energy terms here that is Q square by $2gA$ square Q is the discharge rate.

Now, this relationship is very important that is given in equation number 4, because for analysing the flow in an open channel, it is required to sometimes the calculate, what should be the energy and also what should be the discharge from the channel. So, by this equation either if you know that discharge, then what should be the energy or if you know the energy, what should be the discharge from this.

Of course, you have to know the water head through the channel there and that can be easily measured by experiment. And other parts of course Q is known to you and g and A is known to you. So, in easily you can calculate, what should be the total head relative to

the channel bottom or specific energy that you can calculate from this equation number-4.

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And if we consider that there is a constant discharge, then what should be the water head that is changed with respect to energy. Now, very interesting that these if you are changing this energy, you will see there will be a certain relationship of this head with this energy. So, for constant discharge, you will see the relation between the specific energy and the water depth as shown in the figure is like this here.

So, this one this line, this line is considered as that the profile for that water depth with the energy there. So, in that case if we change the discharge rate, then this profile will be changing accordingly there, other part will be constant here. If the cross-sectional area is fixed and of course gravitational acceleration is constants, so this parts will be constant. Only this thing that this will be changing with respect to this Q square here.

Now, if you change the if you increase the discharge, then you will see the water depth will be of course, increasing accordingly. And if Q discharge rate is decreasing, then water depth in the channel also will be decreasing there. So, we can get different type of here parabolic relationship with the water depth to the energy here.

So, from this profile you will be able to calculate. Now if energy is so high, in that case there will be a rapid flow, so that rapid flow condition, you will see there will be a

change of water depth suddenly, it will be decreasing in that case. So, if you decrease the energy, there will be a certain condition at which you will see there will be optimum value of this energy, so that is called critical energy, at which you will see what will be the corresponding critical depth of the water in the channel and for which you are getting this critical energy condition. And you will see there will be change of behaviour of the fluid flow at this condition.

So, in that case, you will see there will be a change of flow from slower velocity to the certainly jumped to the higher velocity. So, at a certain condition, where you will see this two velocity will be same. So, at this condition it is got that critical velocity. And beyond is critical velocity, you will have this profile of this water depth to the energy here.

In this case before this critical condition, you will see that will be called subcritical flow or it is called sometimes tranquil flow. And after that, you will see there will be a velocity will be so fast that, your energy will be high. And in that case, this flow should be in such condition that the behaviour, it will be like that super critical condition. So, it will be compared to that air velocity at super critical condition there.

So, we have to design that channel in such a way that, there will should not be certain jump of the flow at a certain condition, whenever the fluid is coming out from the channel there. Or if is there any suppose (Refer Time: 11:16) you are using that from the air the flow, whenever it will be coming out from that air position, there will be a sudden change of velocity and for which you can get that sudden shift of that is water velocity from this critical condition. And because of which it will be called as water jump or that is called the sudden shift of energy from this condition and because of which there will be a sudden crust of the fluid on the surface of the wall of the channel, and because of which there will be a some abrupt condition of this flow in that case.

And also if suppose there will be a corrosive fluids are flowing through the channel, then there will be a corrosion or erosion of the you can say that wall surface will be there. So, you have to avoid those condition by obtaining this critical energy condition, when what should be the water depth to be maintained in the channel. So, from this a critical condition, you will be able to find out. Now, we will see what will be that numerical value for that critical water depth in the channel there, we will be calculating subsequently here.

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Constant Discharge

- The critical point of minimum energy occurs where $dE/dh = 0$.

$$\frac{dE}{dh} = 1 - \frac{Q^2}{gA^3} \frac{dA}{dh} = 0 \quad (\text{Eq. 5})$$

Implies

$$\frac{dA}{dh} = \frac{gA^3}{Q^2} \quad (\text{Eq. 6})$$

B = gA^3 / Q^2

Now, for the critical point, there should be minimum energy should be required to get this sudden change of this flow velocity from minimum to the maximum here, so that critical point of minimum energy occurs, when if we differentiate this total energy with respect to that water height in the channel. So, it is coming, when you are considering that dE by dh will be equals to 0.

So, what will be that dE by dh , dE by dh is equal to 1 minus Q square by $g A$ cube into dA by dh , because this differentiation by is obtained from equation number 4. So, after differentiation of equation-4, we are getting this dE by dE by dh that will be is equal to 1 minus Q square by gA cube into dA by dh that should be is equal to 0 here.

So, for this optimum condition or minimum energy requirements, you have to solve this equations for dA by dh . So, this dA by dh will be is equal to gA cube by Q square. What is dA , A is what this is cross-sectional area of the channel there. So, this cross-sectional area will be of course designed in such way that should be changed to that water depth, so that the total discharge rate will be constant there.

So, based on which we are getting this dA by dh will be is equal to gA cube by Q square that is as shown in equation number-6. So, from this equation, we are having what should be the optimum condition or minimum energy requires to get this flow behaviour changing from ceasefire, lower velocity to the higher velocity there.

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Constant Discharge Critical Velocity

- When the channel width at the free surface is B , then $dA = Bdh$. So the critical area A_c , and the critical velocity u_c , become as follows.

$$A_c = \left(\frac{BQ^2}{g} \right)^{1/3} \quad (\text{Eq. 7})$$
$$v_c = \frac{Q}{A_c} = \left(\frac{gA_c}{B} \right)^{1/2} \quad (\text{Eq. 8})$$

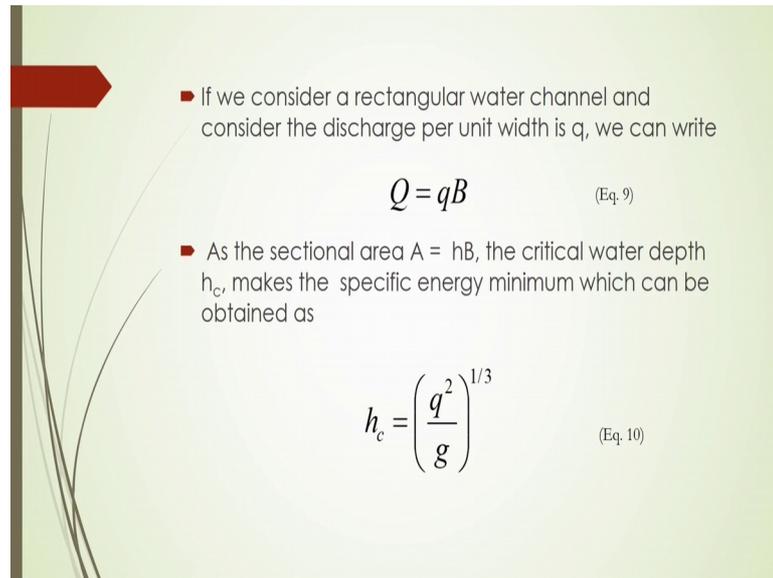
In that case, when the channel width at the free surface if we considered that if it is B , then what should be the dA for that particular point? So, dA should be is equal to B into dh . So, the critical area A_c and the critical velocity U_c that can be obtained by this equation number 7 and 8 respectively.

So, critical area will be is equal to what? BQ square by g to the power 1 by 3 here. So, this is your critical area. So, dA by dh , you have to substitute the value of dA is equal to B into dh , so it will come here. Here B will be equals to what is that gA cube by a Q square.

So, from which what should be the value of A_c here. So, A_c will be is equal to here BQ square by g to the power 1 by 3. So, again if you substitute this A_c value here in a velocity that velocity should be equals to volumetric flow rate by this critical cross-sectional area. So, this critical cross-sectional area if you substitute here, you can obtain this $g A_c$ by B to the power 1 by 2.

So, from this equation, you can calculate what should be the critical velocity and the critical cross-sectional area respectively. So, from these two equations, we can easily calculate what should be the minimum energy required for which you can get the change of velocity from this channel to its discharges point a sudden (Refer Time: 16:03).

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- If we consider a rectangular water channel and consider the discharge per unit width is q , we can write
$$Q = qB \quad (\text{Eq. 9})$$
- As the sectional area $A = hB$, the critical water depth h_c , makes the specific energy minimum which can be obtained as
$$h_c = \left(\frac{q^2}{g} \right)^{1/3} \quad (\text{Eq. 10})$$

And then if we consider a rectangular water channel and consider the discharge per unit width is Q . If we consider that there is a discharge per unit width, and it is denoted by q , then you can write here Q should be is equal to small q into B as shown in equation number-9.

Now, as the sectional area, we know that A will be is equal to hB , then the critical water depth h_c , we can calculate which makes the specific energy minimum and that can be represented by this equation number 10 here. So, h_c should be is equal to q square by g to the power 1 by 3. So, this is your critical water depth in the channel for which you can get the minimum specific energy and also for which you can say that critical minimum specific energy will be required to change your velocity from this a lower to the higher there.

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The specific energy (total head) in the critical situation

- At the critical water depth h_c ,

$$E_c = \frac{q^2}{2gh_c^2} + h_c \quad (\text{Eq. 11})$$

From previous equation

$$q^2 = gh_c^3 \quad (\text{Eq. 12})$$

We get

$$E_c = \frac{h_c}{2} + h_c = 1.5h_c \quad (\text{Eq. 13})$$

The specific energy in the critical situation E_{c_c} is 1.5 times the critical water depth h_c .

And the specific energy or total head in the critical situation, we can say that at this critical water depth h_c , then critical energy can be calculated by this equation number-11. If you substitute this h_c and q value there, then you can easily calculate the critical specific energy.

And from the previous equation that is we can have this what is that q^2 will be equals to g into h^3 . Here we have already given that what is the q value, here from this equation number 10. Here we can say that very simple if we just take the cube of this equation number 10 on both sides, then we can have this h^3 will be equals to q^2 by g from which you can calculate that q^2 will be equals to g into h^3 .

And after substitution of this q value here in equation number 11, then we can have this E_c will be is equal to h_c by 2 plus h_c , then it will be equals to 1.5 into h_c . So, very interesting point is that we are having that the specific energy in the critical situation E_c will be 1.5 times of the critical water depth h_c .

So, also if we know this E_c at this critical situation, you will be able to calculate, what should be the specific energy there for a per unit width. And also the discharge per unit also width that to be calculated as Q , in that case what should be the that Q and Q that is critical discharge there, you can calculate. And also what should be the critical height also, you can calculate.

Now, this relationship very interesting that either one you have to calculate. If you know that a critical energy, then you can easily calculate, what will be the critical depth of the water in the channel? Once you know that a critical depth of the channel, then you will be able to calculate, what will be the critical area of that? Once you know that critical area and critical depth, then easily you can calculate what should be the discharge rate that critical discharge rate at that critical situation of this energy.

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Example: What is the maximum discharge for 2 m specific energy in an open channel with a rectangular section 5 m wide?

Solution:
 Maximum discharge at critical specific energy $E_c = 1.5 h_c$
 Which implies, $h_c = E_c / 1.5 = 2 / 1.5 = 1.33$ m

$q^2 = gh_c^3$ From (Eq. 12)

$q = (gh_c^3)^{1/2} = 4.82$ m²/s

$Q = 24.11$ m³/s as $Q = qB$ From (Eq. 9)

And if we do an example for this, if you are asked to find out the maximum discharge for 2 meter specific energy in an open channel with a rectangular section of 5 meter wide, then you have to use those equations like equation 12 and equation 9 to calculate all those things.

Now, maximum discharge that critical specific energy E_c that will be is equal to 1.5 into E_c that we have got by this equation number 13 here. So, from this equation number 13, if you calculate this h_c as E_c by 1.5, then you can get this critical height as per this problem, it will be found 1.33 meter.

And again if you use this equation number 12 to calculate the discharge rate per unit width as q , then q^2 is equal to gh_c^3 from this equation, you can calculate this q will be is equal to then, after substitution of this h_c value, you can get 4.82 meter square per second.

And then after substitution of this q small q in equation number 9, then you can calculate this discharge that will be is equal to 24.11 meter cube per second. In this case, this capital Q will be is equal to small q in to B , this B is known to you that is 5 meter width wide. So, what will be the discharge per unit width, it is calculated from this equation number 12 at this critical height of the water channel based on this critical specific energy.

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Critical velocity

- The corresponding critical velocity v_c , becomes,

$$v_c = \frac{q}{h_c} = \sqrt{gh_c} \quad (\text{Eq. 14})$$

- In the critical condition, the flow velocity coincides with the travelling velocity of a wave in a water channel of small depth, what is called long wave.

Now, you have to calculate what should be the critical velocity at this minimum energy required for this optimum condition of the channel there. So, the corresponding critical a velocity u_c or v_c you can say here, it will become as that is v_c will be is equal to q by h_c . q is nothing but that discharge per unit width. So, discharge per unit width divided by h_c , this will give you the corresponding critical velocity there. So, this it will come as root over g into h_c , you have to substitute this here q you know that it is a $g h^3$ and then by divided by h , then simply you can have this root over g into h_c .

In the critical condition, then what should be that flow velocity what should be the flow velocity that will be related to the travelling velocity of the wave. Whenever in the channel there will be fluid will be flowing through certain wave and that wave velocity that is called travelling velocity of a wave in a water channel. And that can be obtained, if there is a certain depth of smaller from smaller beyond it is a critical condition.

So, in that case you will see after a critical condition, beyond this energy that is minimum energy, you can have this flow velocity in such a way that, the velocity will be just considered as a travelling velocity, and the fluid will be moving as a wave. So, in that case wave velocity will be considered as the travelling velocity there.

So, in the critical condition this flow velocity will of course will be a same as that travelling velocity of the wave in a water channel. And there will be certain depth that is of course, it will be smaller than this regular velocity and the flow channel. And then at this critical condition, the travelling velocity of the wave in the water channel for that particular depth will be called as long wave travelling velocity there.

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Example: Find the critical water depth and the critical velocity when $12 \text{ m}^3/\text{s}$ of water is flowing in an open channel with a rectangular section 4 m wide.

■ **Solution:**
Critical water depth can be obtained from

$$h_c = \left(\frac{q^2}{g} \right)^{1/3} \quad \text{From Eq. 12}$$

$$Q = qB \quad \text{from Eq. 9}$$

$$v_c = \frac{q}{h_c} = \sqrt{gh_c} \quad \text{from Eq. 14}$$

$Q = 12 \text{ m}^3/\text{s}; B = 4 \text{ m};$ So, $h_c = 0.305 \text{ m}$
 $v_c = (9.81 * 0.305)^{1/2} = 1.73 \text{ m/s}$

And in that case, what should be the critical velocity, if you have the critical water depth, and the critical velocity there. And when if you know that the water is flowing in an open channel with a flow rate of critical flow rate as per metre cube per second, then you have to find out the critical water depth for the channel of rectangular in shape with 4 meter wide.

So, in that of critical water depth can be obtained from again the equation number 12 and from equation 9 and also from equation number 14 here. From equation number 12, we have to calculate this h_c ; h_c means critical water depth that is related with this discharge per unit width q . And this and then discharge is Q will be is equal to small q into B and v_c is given to you here.

Now, in this case first of all you have to calculate what will be the small q , because you know this capital Q here and also B here. So, very simple you can calculate what should be the q and a small q . And after getting this small q , you can have this h_c just by calculating this v_c here, because this v_c can be calculated from that is critical velocity, it is given here root over $g h_c$. And then otherwise, you can calculate it from your what is that mass conservation of equation there and in this way.

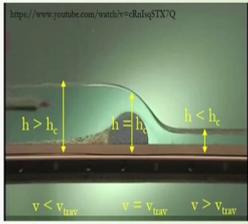
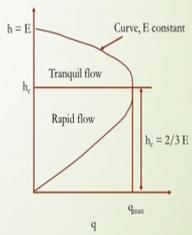
So, we can calculate here first what should be the h_c is given that Q capital Q is given to 12 metre cube per second and small b is given to you and also h_c from which you can calculate this q by h_c is equal to root over $g h_c$ and it will be is equal to what is that from which you can calculate h_c .

And what is that v_c , then v_c will be is equal to what simply root over $g h_c$. And from this then we can have the critical, then water velocity at critical water depth for this critical flow rate of the water for the 4 meter wide of rectangular channel.

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Subcritical and supercritical flow

- If the flow depth is deeper or shallower than h_c , the flow behaviour is different.
- **Subcritical flow:** When the water is deeper than h_c , the velocity is smaller than the travelling velocity of the long wave. This type of flow is called tranquil (or subcritical) flow.
- **Supercritical flow:** When the water depth is shallower than h_c , the velocity is larger than the travelling velocity of the long wave. This type of flow is called rapid (or supercritical) flow

Now, let us consider that flow conditions in the channel, before and after the critical condition. Now, before the critical conditions, the flow behaviour will be represented by this supercritical and beyond this it will be supercritical flow. Now, when and how that it will be defined you just see this video here, you will see how this whenever liquid is flowing through the channel, you will see there will be an object that is some obstruction by a what is that circular stone and over which that flow is flowing.

And before this point, you will see there will be a depth of water channel will be there, it will be represented by h and at that condition of critical condition where from this stone, you will see the flow will be suddenly a drop to its higher velocity there, where this h that water depth will be smaller than other parts. So, in this case if they are h is greater than h_c , it will be called as subcritical and when h is equal to h_c that will be your what is that critical condition. If h is less than h_c , it will be called as a super critical condition.

Now, this is very interesting some this phenomena is sometimes, it is called that hydraulic jump. We will come to that point hydraulic jump. So, from this figure, we are observing that whenever fluid is flowing through the channel at this condition here, at this condition the water depth will be higher and the flow velocity will be is less than this critical velocity. And after this your critical condition, you will see the water depth will be lower, whereas the velocity will be greater than the critical velocity.

And at this critical condition, both the height will be considered the critical height and velocity should be the travelling velocity of the wavelength there. So, if the flow depth is a deeper or you can say that shallower than h_c , the flow behaviour will be different, whenever fluid will be flowing through the channel. Now, when the water us deeper than h_c ? The velocity is a smaller than the travelling velocity of the long wave and this type of flow will be called as the tranquil or subcritical flow.

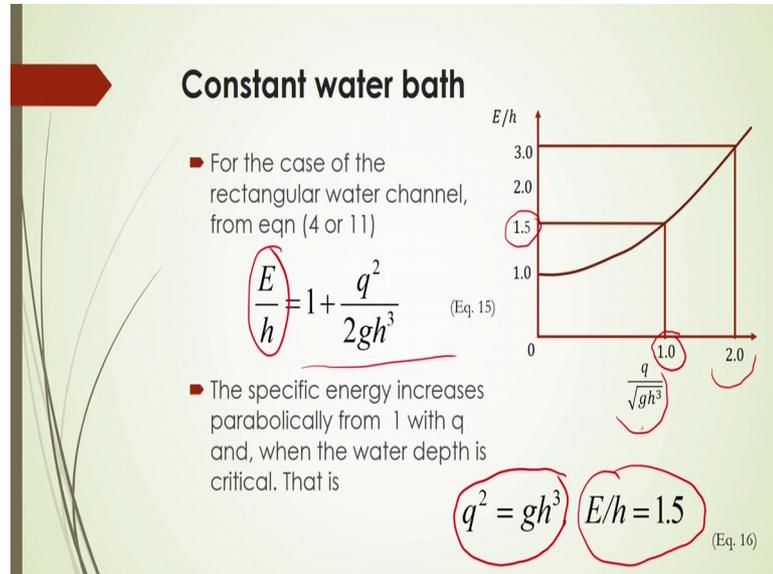
And if the water depth is a shallower or smaller than the h_c , then the velocity is larger than the travelling velocity of the long wave that is produced after this critical condition. And this type of flow is called the rapid or you can say supercritical flow. Now, as shown in figure here, the profile of this water depth to this discharge rate per unit width is given this profile if you show that at this height at the maximum height, you will see there will be a that is here this minimum flow rate.

And in that case the energy should be higher and also in that case the liquid height will be lower that case. And also you will see at this critical condition this where q_{max} will be there. In that case, there will be a condition from which you can get the separation of these supercritical and subcritical flow or tranquil or rapid flow there.

So, this relationship will give you at exactly what location that you can get this critical flow condition of the channel flow. And very interesting that we have already obtained that that h_c should be is equal to $2/3 E$ or E should be is equal to 1.5 of critical, what

is that water depth there in the channel. So, up to this is your q maximum will be obtained at this, what is that critical what are water depth there.

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Now, if we consider the case, where the rectangular water channel from equation number 4 and 11, we can express this energy per unit that is depth of water channel, then it will be represented by the equation number 15 as E by h will be is equal to 1 plus q square by 2 g h cube. Now, this specific energy it will increase the parabolically as shown in the figure here from 1 with q small q and you will see when this water depth is a critical at that conditions. So, at critical condition if we plot this E by h versus q by root over g h cube, we are getting this parabolic shape of the nature of flow characteristics.

So, in this case if this q square will be is equal to g h cube; g h cube, then you can have this E by h will be is equal to 1.5. So, here this is you E by h, this is 1.5 and where this q square will be is equal to g h cube, so from which you can calculate.

Similarly, if you change this q value, then if it is coming as q will be equals to 2 into root over g h cube, then you can have this value of E by h will be equals to double this critical water depth. So, if you increase the double of this q, then immediately you can calculate, what should be the critical water depth in the channel from this relationship.

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Example: In an open channel with a rectangular section 3 m wide, 15 m³/s of water is flowing at 1.2 m depth. Is the flow rapid or tranquil and what is the specific energy?

Solution

$$h_c = \left(\frac{q^2}{g} \right)^{1/3} \quad \text{From Eq. 12}$$
$$Q = qB \quad \text{from Eq. 9}$$
$$\frac{E}{h} = 1 + \frac{q^2}{2gh^3} \quad \text{from Eq. 15}$$

$Q = 15 \text{ m}^3/\text{s}; B = 3 \text{ m}; q = 15/3 = 5 \text{ m}^2/\text{s}$
 $h_c = (5^2/9.81)^{1/3} = 0.85 \text{ m}$

Since the water (1.2 m depth) is deeper than h_c , the velocity is smaller than the travelling velocity of the long wave. This type of flow is called **tranquil (or subcritical) flow**.

$E = 2.083 \text{ m}$ By Eq. 15

Now, if we consider an open channel with rectangular cross sections of 3 meter wide, and 15 meter cube per second of water is flowing at 1.2 meter depth of water. Now, we have to say that is the flow rapid or tranquil and what is the specific energy is there.

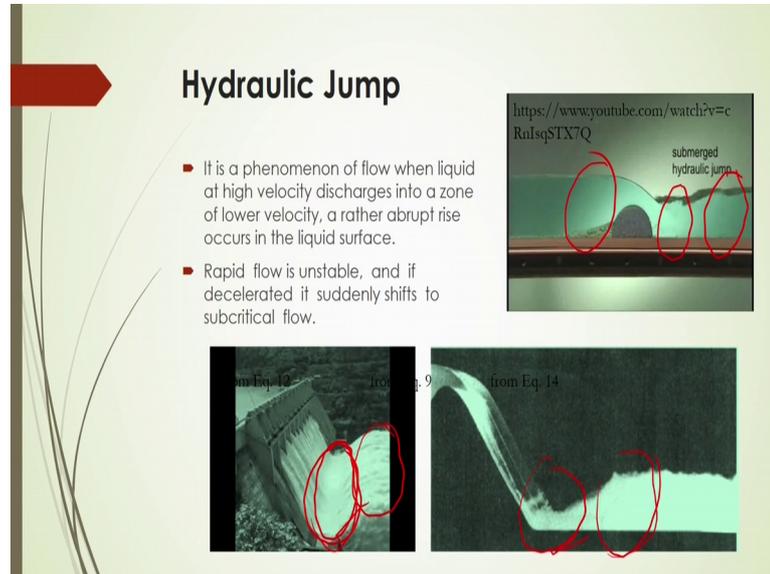
And we know that from equation number 12 that we have discussed that h_c should be equal to q^2 by g that should be to the power 1 by 3 and from this equation you can calculate what should be the h_c . For this you need the value of a discharge per unit width, because unit width is given to you from which you can calculate here Q , then Q is equal to capital Q by B , this will be your discharge per unit width.

So, this Q will be as 5 metre per second here. And from equation number 15, then you can calculate this E by h . If you know this h_c value, h_c value you can calculate from equation number 12, after substitution of value of small q there. And if you substitute again this q value small q and h_c from this equation number 12, then you can have h_c will be equal to 0.85 and also respective E will be equal to 2.083.

Now, in this case water depth is 1.2 and it is deeper than h_c . We are having because h_c we have calculated from this equation number 12 as 0.85, whereas water depth is given as per this problem, it is 1.2 meter. So, this 1.2 meter of course it is greater than this critical depth. So, we can say that the flow, the velocity is smaller than the travelling velocity of the long wave. And then we can say that as per definition that the flow will be

tranquil or you can say that it will be sub critical flow. So, based on this example, we will be able to calculate whether the velocity will be sub-critical or super-critical.

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Now, let us have an discussion of hydraulic jump here. Already you have seen this video here, what is that whenever fluid will be flowing through the channel and if it is coming out from a certain flow obstruction, then you will see there will be a regular flow obstruction maybe air or some other that is surface over which this flow will be overflow. And in that case the flow whenever it will be flowing over the surface and there will be certain jump of this velocity or you can say fluid mass into the channel of this remaining part. Then this phenomena, you will observed as shown this video.

So, it is a phenomena you can say that flow, when the liquid at high velocity that will discharges into a zone of lower velocity and they there are other abrupt rise occurs in the liquid surface there.

And we will see and here in this here it is the phenomena, where this is your higher velocity high velocity, where in this region it will be lower velocity. Whereas here in this video, it will be your lower velocity and here in this con composition it will be your higher velocity, again it will gain and here lower velocity there.

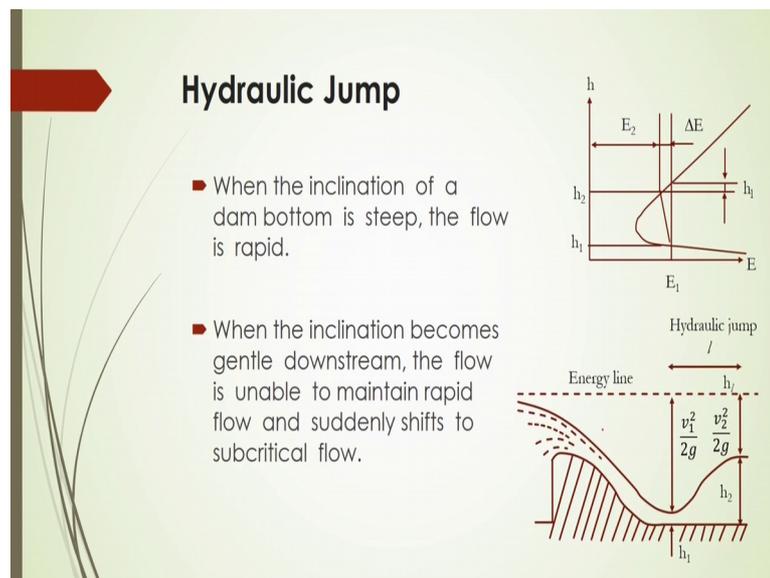
So, in this case there will be a creation of some phenomena that it will be called as that hydraulic jump is there. So, we will see in this video in this in this picture also, there will

be sudden jump of this liquid. And it falls and hits at the surface, and after that the fluid is getting its velocity that will be lower than this here.

And then rapid you are observing that there will be a certain rapid flow, and it will be very unstable. And if it is decelerated and then it suddenly shifts to it is of critical flow. So, there are two flow phenomenon here; one is lower velocity, another is higher velocity, and there will be some critical velocity there. So, the velocity lower than critical velocity, it will be called as subcritical flow and if it is greater than that, critical velocity it will be supercritical flow.

So, the hydraulic jump it is a phenomenon of the flow, when the liquid at high velocity discharges or low velocity discharges into a zone of lower velocity or the higher velocity that is two ways, it can happen as shown in figure here. So, a rather abrupt rise occurs in the liquid surface is there, so this phenomena will be called as hydraulic jump.

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Now, this hydraulic jump of course will be coming as that based on that critical condition here. So, when the inclination of a dam bottom is the suppose deep or steep, you can say the flow will be very rapid. So, in that case it will be a supercritical flow. And when the inclination becomes a gentle downstream for a dam, the flow is unable to maintain rapid flow. And it suddenly shifts to sub critical flow there.

So, it is important to say that there you will see whenever the fluid is coming from a higher position, where water depth with the lower water depth they are, you will see there will be a sudden change of energy. And that energy will converted to the what is that may be elevation energy or a static energy to be converted to the velocity energy is there. So that energy sometimes, it will damage the bottom surface of the channel, and because of which you have to get an idea, when that that optimum condition or how to avoid or how to design the channel surface. In such a way that at that critical high energy of that velocity convert that is energy conversion from this potential to the velocity head or velocity energy and by which you can of course design the channel there.

So, here as shown in figure very interesting that the designing of that dam or channel that you have to make the inclination of that dam bottom, whether it should be steep or not. So, if it is very steep, then the flow will be repeat. And if it is very gentle downstream, then the flow is unable to maintain rapid flow and subsequently or suddenly, you can say shifts to the subcritical flow, so there is a jump. So, you have to maintain that critical condition, where that this change is of happened. And that critical a water depth are to be maintained to get this optimum condition of channel design.

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William Froude (1810-79)
Born in England

Wave velocity during hydraulic jump

- The travelling velocity of the long wave of the small depth of h of the water channel is

$$V_{t, wave} = \sqrt{gh}$$

- The ratio of flow velocity to the travelling velocity of the wave is called Froude number.
- The Froude number of the subcritical flow is less than one

$Fr = 1$

$V_{cr} = \sqrt{gh_c}$

And what should be that wave velocity during the hydraulic jump? The travelling velocity of that long wave, after this critical condition, it will be of the small depth of h of the water channel. And then it will be calculated as here root over g into h . So, this

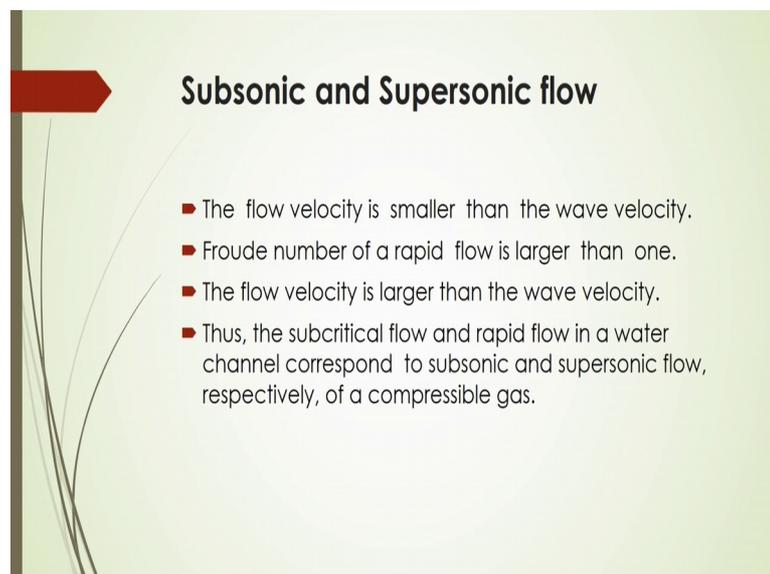
will be represented as travelling velocity of the long wave. And the ratio of this flow velocity to the travelling velocity, it would be represented by a dimensionless number, it will be called as Froude number.

So, Froude number very interesting that here long wave velocity, what should be that whether it should be higher than the long wave velocity travelling velocity higher than the flow velocity or not based on this Froude number. If Froude number is equal to 1, then you can say that subcritical flow will be less than 1 that case of course the critical condition, you can obtain that means here Froude number, if it is suppose travelling a velocity is less than is suppose Froude number is equal to if Froude number is equal to 1, then of course V travelling will be is equal to V flow velocity is there. So, both will be same that is at critical condition.

If it is less than 1; if it is less than 1, then of course this travelling velocity should be what is that less than flow velocity, so it will be coming at this sub critical condition. So, sub critical condition, when again if it is greater than 1, of course it will be that flow velocity will be or travelling velocity will be greater than this flow velocity. So, in that case you can have this, so supercritical flow.

Now, of course that supercritical flow will not be the actually they are the desirable condition, so Froude number should be always less than 1 in that condition, so that you can get the subcritical flow condition, whenever you are designing the dam there.

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Subsonic and Supersonic flow

- The flow velocity is smaller than the wave velocity.
- Froude number of a rapid flow is larger than one.
- The flow velocity is larger than the wave velocity.
- Thus, the subcritical flow and rapid flow in a water channel correspond to subsonic and supersonic flow, respectively, of a compressible gas.

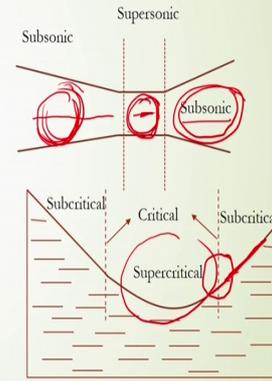
Now, if the flow velocity is smaller than the wave velocity, then it will be called as subsonic or supersonic also. So, the Froude number of the rapid flow is larger than one, for the subsonic or supersonic flow. In that case, the flow velocity is larger than the wave velocity, thus the supercritical flow and the rapid flow in a water channel correspond to subsonic or supersonic flow, respectively of a compressible gas.

So, here this subcritical will be represented by this subsonic. And rapid flow or supercritical, it will be as the supersonic flow there. And in that case, you will see a there will be a certain analogue between the hydraulic jump and the shockwave.

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Analogy between the hydraulic jump and the shock wave.

- For the flow of gas in a convergent-divergent nozzle supersonic flow which has gone through the nozzle stays supersonic if the back pressure is low.
- If the back pressure is high, the flow suddenly shifts to the subsonic flow with a shock wave.



For the flow of gas in a convergent-divergent nozzle the supersonic flow, which has gone through the nozzle stays. If you are considering one nozzle, then you will see that in this figure here flow is coming from this position to this position and then this position.

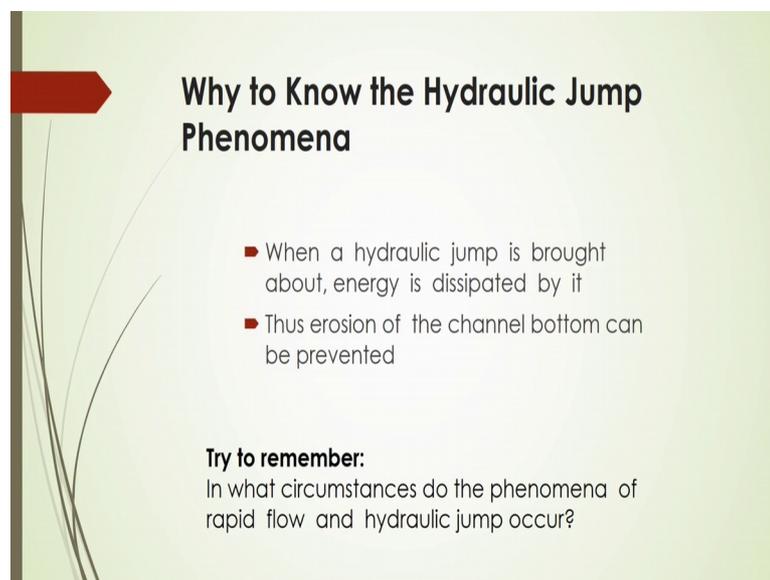
So, in this case here velocity will be higher, whereas in this section velocity will be lower and in this position velocity will be higher, whereas in this position again it will be lower. So, it will be what is that the subsonic, it will be supersonic and it will be what is that subsonic again. So, it is analogous to this convergent-divergent nozzle flow.

So, if the back pressure is high in this case, then the flow suddenly shifts to the subsonic flow with a shock wave there. So, it depends on that how much pressure is creating over

there, so the flow suddenly shifts to the subsonic flow, if it is here. In this case, from this supercritical flow again suddenly, after jumping it will do that subsonic flow, then at this condition, it will be considered as a shock wave is there.

So, you can have this hydraulic jump as per that convergent-divergent nozzle flow there. So, in that case only thing is that, whether it should be sub critical or subsonic or supercritical or supersonic or water this critical or sonic. So, this analogue will be maintained or you can consider in that case of this channel.

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Why to Know the Hydraulic Jump Phenomena

- When a hydraulic jump is brought about, energy is dissipated by it
- Thus erosion of the channel bottom can be prevented

Try to remember:
In what circumstances do the phenomena of rapid flow and hydraulic jump occur?

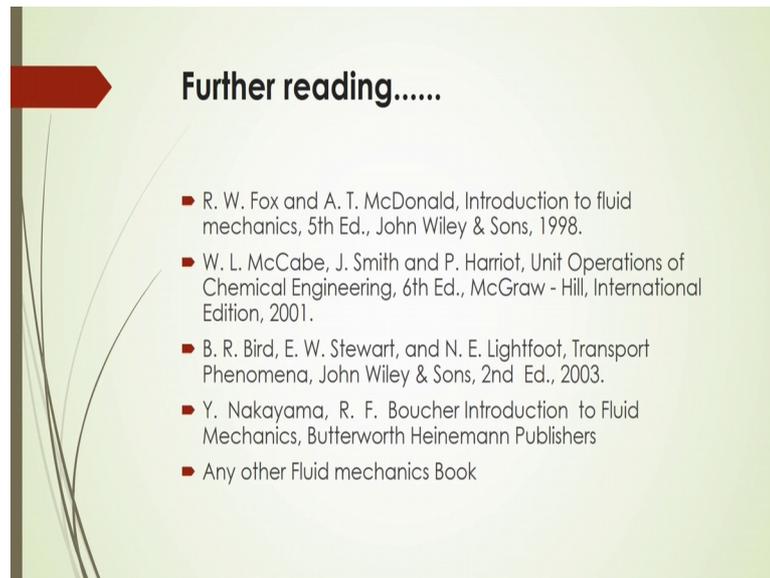
Now, why to know this hydraulic jump phenomenon? That interesting that you have to always remember that when a hydraulic jump is brought about, energy is dissipated by it. Thus the erosion of the channel bottom can be prevented based on this idea of this hydraulic jump phenomena.

In that case, when this critical flow will be happen and at which water depth this critical flow will be happen. And when these subsonic, when this supersonic or sub sonic sub critical or super critical will happen that you have to calculate based on this formula that here discussed.

So, I think you have learned something about this what is that hydraulic jump and this supercritical subcritical flows. And what is that analogue and also how to calculate the water depth critical water depth water depth and also what should be the critical cross-

sectional area for the design what will be the energy required, so that you can I have the idea to prevent the erosion by a sudden, what is that thrust of the water by rapid flow in the water channel, whenever it will be flowing is coming out from this lower velocity to the higher velocity zone.

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So, further reading, I can suggest this again this books, for better understanding of course. And this is the basic things that we have discussed here. I think from this basic knowledge, you can calculate or you can have a certain interpretation and also the knowledge by which you can go further for a design of this flow device based on this fluid flow.

So, thank you for this lecture.