Mass Transfer Coefficients

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Lecture - 16
Number of ideal stages in counter current operation: graphical and algebraic methods

Welcome to the 8th lecture of module 2 on mass transfer operations. In this module we are discussing interface mass transfer and before going to the next lecture let us have small recap of our previous lectures. In the last lecture we have considered no interface mass transfer which is explained by the Lewis and Whitman Two-Film theory.

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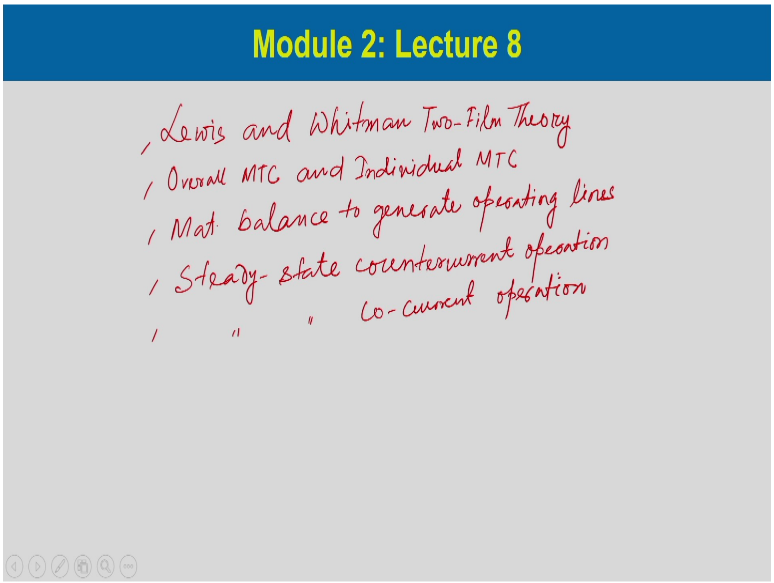

Module 2: Lecture 8

/ Lewis and Whitman Two-Film Theory
/ Overall MTC and Individual MTC
/ Mat balance to generate operating lines
/ Steady-state countercurrent operation
/ " " Co-current operation

Lewis Whitman Two-Film theory: We have discussed the relation between overall mass transfer coefficient and individual mass transfer coefficient. Then we started with general mass balance or material balance to generate operating lines indifferent stage conductors. We have considered 2 different cases 1 is steady state counter current operation and steady state co current operation. And we have seen how to draw the operating line and the equilibrium line and where the operating line and the equilibrium line lies with respect to counter current operations and co current operations.

So, in this lecture we will consider countercurrent multistage contact process and we will try to see how to calculate the number of stages using the analytical equations and 1 of them is Kremser equation.

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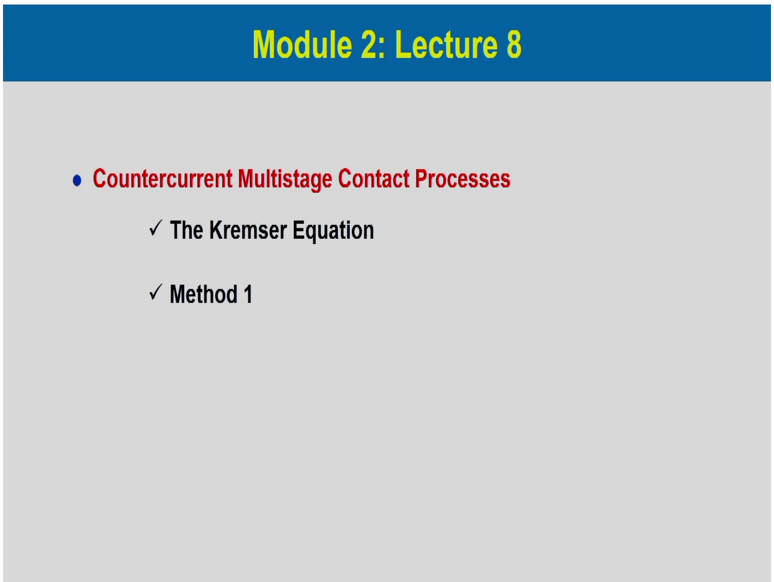

Module 2: Lecture 8

- Countercurrent Multistage Contact Processes
  - ✓ The Kremser Equation
  - ✓ Method 1

We will discuss 2 different methods how to develop the Kremser equations and to find out the number of stages in an equilibrium separation process or equilibrium conductors.

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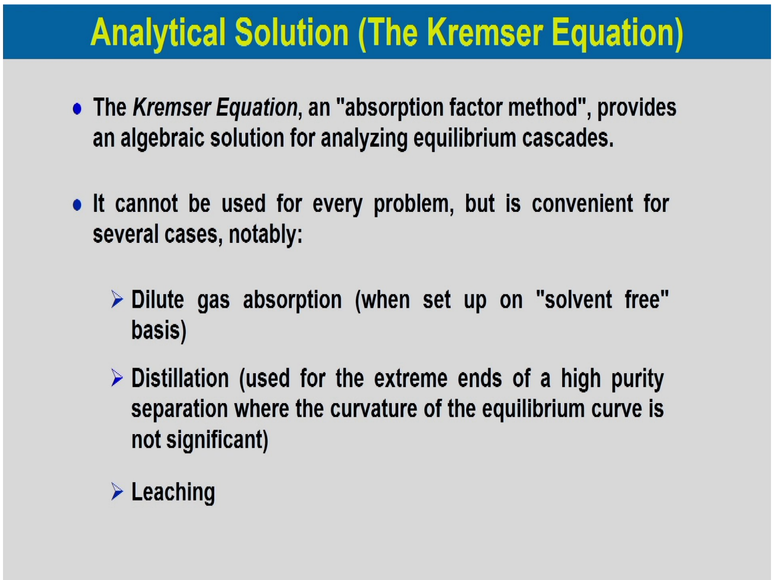

Analytical Solution (The Kremser Equation)

- The *Kremser Equation*, an "absorption factor method", provides an algebraic solution for analyzing equilibrium cascades.
- It cannot be used for every problem, but is convenient for several cases, notably:
  - ➢ Dilute gas absorption (when set up on "solvent free" basis)
  - ➢ Distillation (used for the extreme ends of a high purity separation where the curvature of the equilibrium curve is not significant)
  - ➢ Leaching

The Kremser equation is an absorption factor method, it provides an algebraic solution for analyzing equilibrium cascades. So, when we have a stage wise contact of the gas and liquids, we called it as cascade arrangement and most of the cases is very difficult to obtain the analytical solutions, but Kremser equations is 1 such method where we can apply algebraic equations to analyze the equilibriums cascade. This equation cannot be used for every problem, but it is convenient for several cases like if we consider dilute gas absorption, when we set up on solvent free basis. So, if it is dilute gas absorption, the change of molar flow rate of gas or liquid after the contacting process are very less. The solute transfer from gas phase to the liquid phase or either in other case also from the liquid phase to the gas phase are less in that case we can use this Kremser equation.

The other case is the distillation where we can use for the extreme ends of a high purity separation where the curvature of the equilibrium curve is not significant. Because at the extreme end means when we are trying to achieving the purity of the product in case of the distillation, we could see that the change of although from 99.9 percent to 99.99 percent purity we would like to go and then this concentration change is not significant. So, the solute transfer will be very less and in that case we can calculate how much number of stages required analytically using the Kremser equation.

So, in that case the equilibrium curve will also be a straight line, because there is not appreciable change in concentration. The third case is also for leaching where you have a very slow process of leach out some valuable component. The process is very slow or the solid materials are present is very low quantity. So, we can use this Kremser equation.

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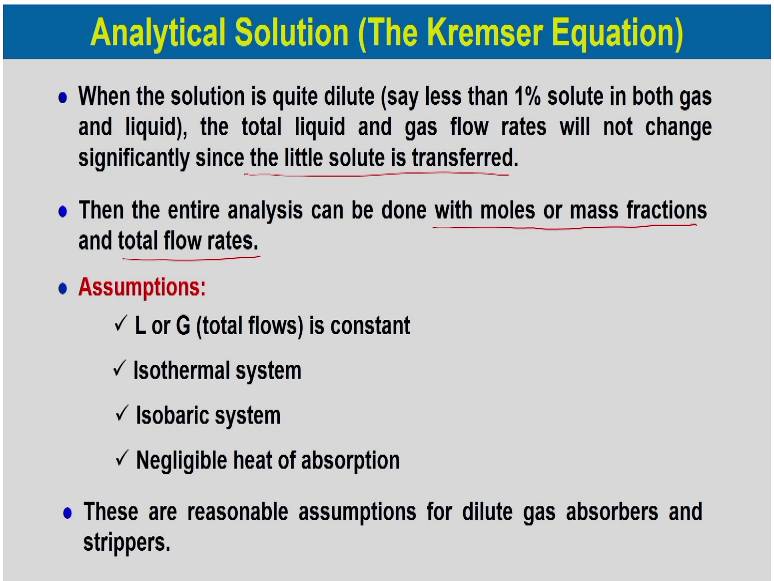

Analytical Solution (The Kremser Equation)

- When the solution is quite dilute (say less than 1% solute in both gas and liquid), the total liquid and gas flow rates will not change significantly since the little solute is transferred.
- Then the entire analysis can be done with moles or mass fractions and total flow rates.
- **Assumptions:**
  - ✓ L or G (total flows) is constant
  - ✓ Isothermal system
  - ✓ Isobaric system
  - ✓ Negligible heat of absorption
- These are reasonable assumptions for dilute gas absorbers and strippers.

When the solution is quite dilute say less than 1 percent solute in both gas and liquid, the total liquid and gas flow rates will not change significantly since the little solute is transferred. This I have already explained. So, probably less than 1 percent solute contained in a mixture the change of molar flow rate will not be appreciable. So, we can use the Kremser equation.

The entire analysis can be done with moles or mass fraction and total flow rates. So, moles are mass fractions and the total flow rate we can use we can do the entire analysis. There are some assumptions which we need to take to use this analytical method of solving the cascades equilibrium cascades. L or G that is liquid or gas total flows is constant that is the assumption.

Isothermal system; so, the system will operate at a constant or at a fixed temperature remains constant at a particular temperature throughout the operation, which is isothermal. Then isobaric system the pressure remains constant and negligible heat of absorption. So, heat of absorption should be negligible, it will behave like an ideal solution. So, these are reasonable assumption for dilute gas absorbers and strippers.

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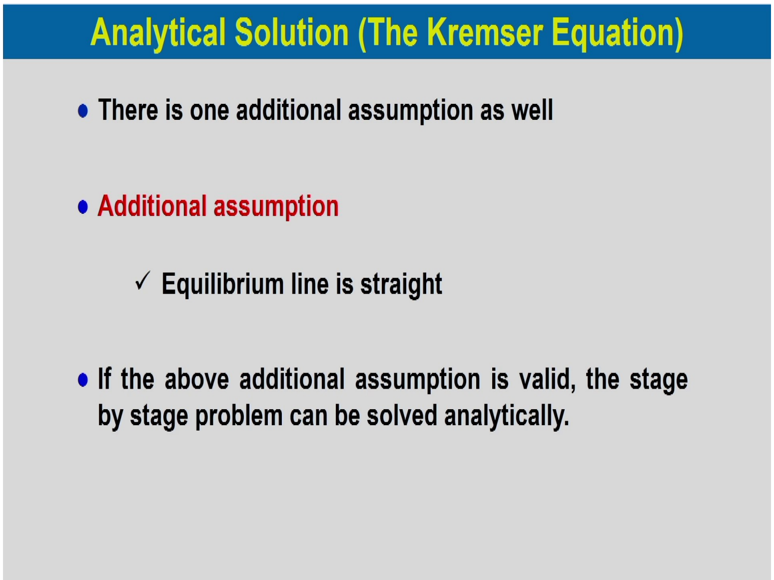

Analytical Solution (The Kremser Equation)

- There is one additional assumption as well
- Additional assumption
  - ✓ Equilibrium line is straight
- If the above additional assumption is valid, the stage by stage problem can be solved analytically.

If we considered another additional assumption, the equilibrium line is straight. So, in that case if there is additional assumption is valid, then stage by stage problem can be solved analytically with this additional assumptions. So, our equilibrium curve will be straight and also our operating line will be straight. So, we can do the stage wise calculation analytically.

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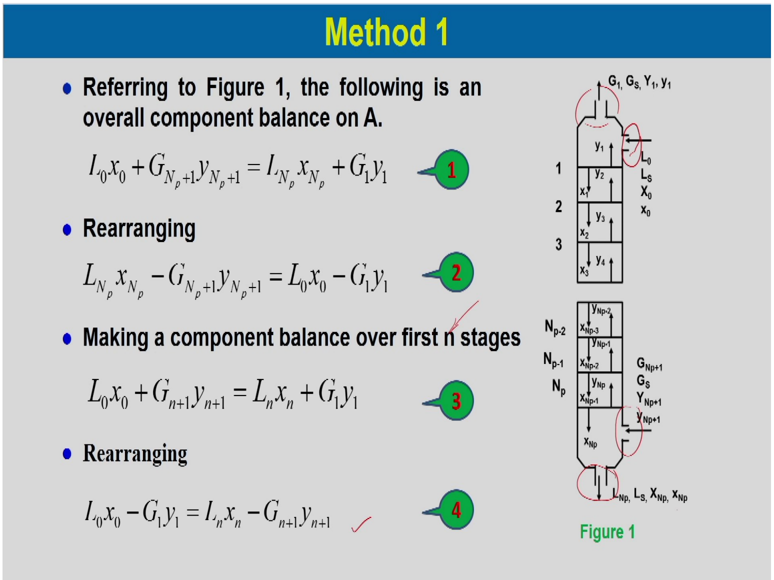

Method 1

- Referring to Figure 1, the following is an overall component balance on A.

$$L_0 x_0 + G_{N_p+1} y_{N_p+1} = L_{N_p} x_{N_p} + G_1 y_1 \quad ①$$

- Rearranging

$$L_{N_p} x_{N_p} - G_{N_p+1} y_{N_p+1} = L_0 x_0 - G_1 y_1 \quad ②$$

- Making a component balance over first n stages

$$L_0 x_0 + G_{n+1} y_{n+1} = L_n x_n + G_1 y_1 \quad ③$$

- Rearranging

$$L_0 x_0 - G_1 y_1 = L_n x_n - G_{n+1} y_{n+1} \quad ④$$

Figure 1

So, we will discuss method 1 refer to this figure you can see that in this figure no there are stages which are numbered from top to bottom. So, the top three is 1 and the next to that is 2 3 and so, on and finally, it is N p stage X capital N p. So, which is coming out the liquid from the top tree. So, liquid is entering at the top and coming out from the bottom and then the gas is entering at the bottom and going out at the top. If we do the component A balance on this, we will see that mole fractions of the components in the gas and liquid phase are numbered over here.

Gas which is coming out from stage 1 is numbered as Y 1 and the liquid which is coming out from tray 1 is X 1. Then the vapor from stage 2 which is going up each y 2 and the liquid which is coming down from stage 2 is X 2. The liquid entering at the top at a flow rate of L naught, an L s is the based on the solute free basis is the molar flow rate of the liquid and capital X naught is the mole ratio basis and small x naught is the mole fraction basis. Similarly the gas which is entering at the bottom is Y N p plus 1 it is entering to plate N p and gas flow rate is the G capital N p plus 1 and G s is based on the solute free basis.

The liquid which is coming out at the bottom having composition of X N p of a flow rate of L N p and then the gas which is going out at the top has a gas flow rate at G 1 and with a mole fractions of Y 1 or mole ratio of capital Y 1 with a gas flow rate with solute free basis G a s. Now referring to this figure, if we do the mole balance on component A we can write the overall balance component balance. So, L naught x naught plus G N p plus 1 into y N p plus 1. So, this is entering L naught x naught which is entering over here, and the gas which is entering over here is G N p plus 1 into y N p plus 1 should be equal to the coming out of that component at no bottom is L N p into x N p and then the gas which is going out at the top is G 1 y 1. So, this is equation 1.

Now if you rearrange this equation we will have L N p into x N p minus and G N p plus 1 into y N p plus 1 would be equal to L naught x naught minus G 1 y 1. So, this is equation 2. Now making a component balance over first n stages small n this n number of stages is any where inside the column from top to bottom.

So, if we do a balance on first n stages. So, we can write L naught into x naught plus G N plus 1 into y N plus 1 would be equal to L n x N plus G 1 y 1. So, this is equation 3 and if you rearrange in a similar way as we have done before. So, we will have L naught x naught minus G 1 y 1 would be equal to L n x n minus G N plus 1 into y N plus 1.

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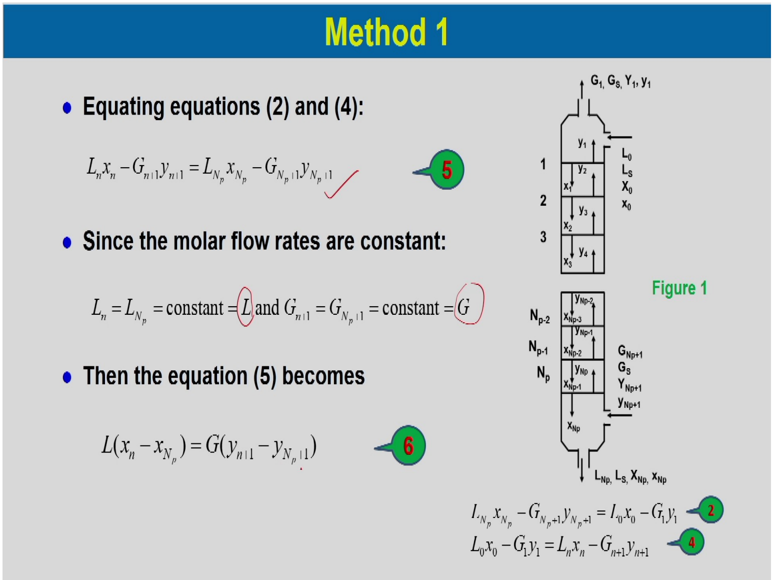

Method 1

- **Equating equations (2) and (4):**

$$L_n x_n - G_{n+1} y_{n+1} = L_{N_p} x_{N_p} - G_{N_p+1} y_{N_p+1} \checkmark \quad ⑤$$

- **Since the molar flow rates are constant:**

$$L_n = L_{N_p} = \text{constant} = L \text{ and } G_{n+1} = G_{N_p+1} = \text{constant} = G$$

- **Then the equation (5) becomes**

$$L(x_n - x_{N_p}) = G(y_{n+1} - y_{N_p+1}) \quad ⑥$$

Figure 1

$$L_{N_p} x_{N_p} - G_{N_p+1} y_{N_p+1} = L_0 x_0 - G_1 y_1 \quad ②$$

$$L_0 x_0 - G_1 y_1 = L_n x_n - G_{n+1} y_{n+1} \quad ④$$

Now, from this equation 2 and 4. So, 2 is this one and 4 is this one, from this 2 the relation if we compare if we equate them. So, L naught x naught minus G 1 y 1 we can substitute from equation 4 in equation 2 we will have L n x n minus G N plus 1, y N plus 1 would be equal to L capital N p into x capital N p minus G capital N p plus 1 into y capital N p plus 1. So, this is equation 5. Since the molar flow rates are constant so; that means, L and G are constant that is as per our assumption, because the solute transfer is negligible and hence the liquid and gas flow rates remains constant throughout the column. So, with this assumption we can write L n would be equal to L N p would be equal to constant and we consider is at L. Similarly for gas G N plus 1 plus 1 would be equal to G N p plus 1 equal to constant which is denoted as G.

So, from now onwards this liquid and gas flow rate will be denoted as L and G. So, this equation 5 would be reduced to L into x n minus x N p is equal to G into y N plus 1 minus y N p plus 1.

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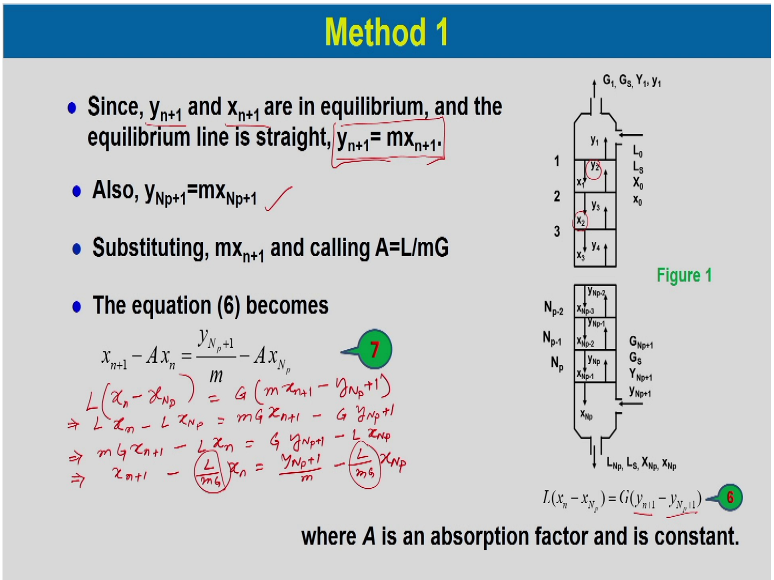

Method 1

- Since, y_{n+1} and x_{n+1} are in equilibrium, and the equilibrium line is straight, $\boxed{y_{n+1} = mx_{n+1}}$.
- Also, $y_{Np+1} = mx_{Np+1}$ ✓
- Substituting, mx_{n+1} and calling $A = L/mG$
- The equation (6) becomes

$$x_{n+1} - Ax_n = \frac{y_{N_p+1}}{m} - Ax_{N_p} \quad ⑦$$

$$L(x_n - x_{Np}) = G(m x_{n+1} - y_{Np+1})$$
$$\Rightarrow L x_m - L x_{Np} = mG x_{n+1} - G y_{Np+1}$$
$$\Rightarrow mG x_{n+1} - L x_m = G y_{Np+1} - L x_{Np}$$
$$\Rightarrow x_{m+1} - \boxed{\frac{L}{mG}} x_n = \frac{y_{Np+1}}{m} - \boxed{\frac{L}{mG}} x_{Np}$$

Figure 1

$$L(x_n - x_{N_p}) = G(y_{n+1} - y_{N_p+1}) \quad ⑥$$

where *A* is an absorption factor and is constant.

Now, since y N plus 1 and x N plus 1 that is at any tray if you take the components of the gas and liquid which is coming out from the tray, like in case of if we consider tray 2 the wafer or the gas which is going out that is y 2 and then the liquid which is coming down from the tray that is x 2 this outgoing streams will be in equilibrium. So, y N plus 1 and x N plus 1 are in equilibrium and the equilibrium line is straight. So, you can write the equilibrium relation y N plus 1 would be is equal to m x N plus 1. So, this is equation of straight line.

Similarly, we can also write y N p plus 1 would be equal to m x N p plus 1. Now if you substitute m x N plus 1 and calling a is equal to L by m G constant A. So, this equation 6 which we have derived before is A L x n minus x N p would be equal to G y N plus 1 minus y N p plus 1 this root become x N plus 1 minus A x n is equal to y N p plus 1 by m minus A x N p.

So, basically here if we substitute this equations would be L xn minus x Np would be equal to G will substitute y N plus 1 would be m x N plus 1 minus y N p plus 1. Now if you rearrange if you just no simplify L x n minus L x N p would be equal to mG x N plus 1 minus G y Np plus 1. Now we can write m G x N plus 1 minus L x n would be equal to G y N p plus 1 minus L x Np. Now if you just divide with L m G for both the sides it would be x n plus 1 minus L by m G xn would be equal to y N p plus 1 by m minus L by m G x N p. And hence we can write x N plus 1 and this term will be A absorption factor and then this is also absorption factor. So, if we can obtain the equation 7.

So, where A is the absorption factor and is a constant.

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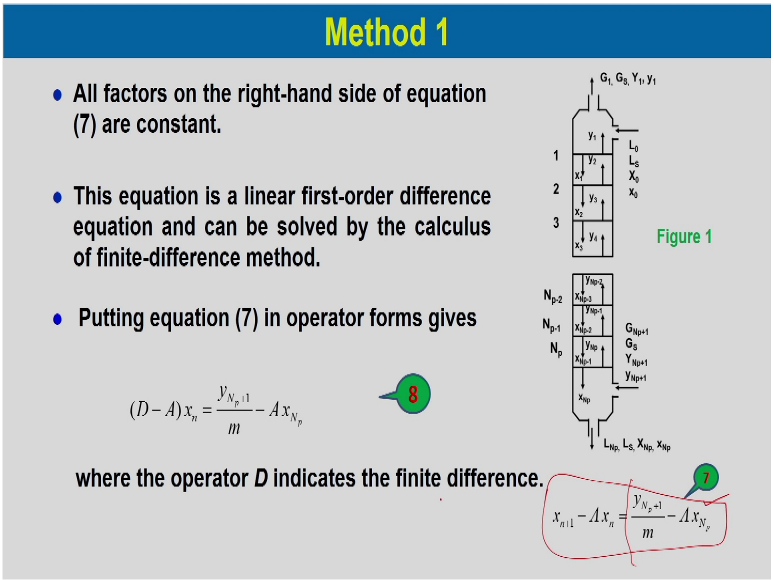

Method 1

- All factors on the right-hand side of equation (7) are constant.
- This equation is a linear first-order difference equation and can be solved by the calculus of finite-difference method.
- Putting equation (7) in operator forms gives

$$(D-A)x_n = \frac{y_{N_p+1}}{m} - A x_{N_p}$$ ⑧

where the operator *D* indicates the finite difference.

Figure 1

$$x_{n+1} - A x_n = \frac{y_{N_p+1}}{m} - A x_{N_p}$$ ⑦

Now, in this equations the right hand side all factors are constant because y N p plus 1 which is entering concentration and x N p which is exiting at the bottom. So, these are constant, m is the henrys constant for absorption. So, these are constant on the right hand side. So, this equation is a linear first order difference equation and can be solved by the calculus of finite difference method. This is a linear first order equation difference equation and we can use the finite difference method to solve this equation. Now putting equation 7 in operator form, it gives D minus A into x n would be equal to y N p plus 1 divided by m minus A x N p here the operator d indicates the finite difference.

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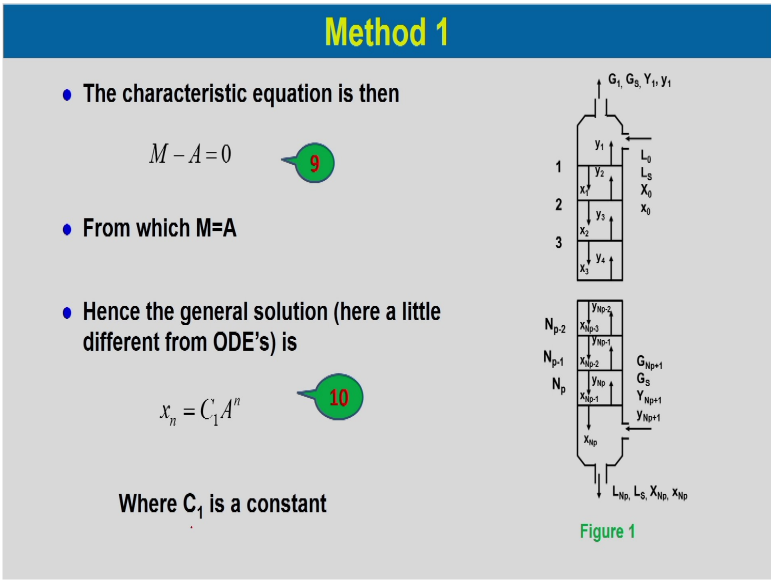

Method 1

- The characteristic equation is then

$$M - A = 0$$ ⑨

- From which M=A

- Hence the general solution (here a little different from ODE's) is

$$x_n = C_1 A^n$$ ⑩

Where C_1 is a constant

Figure 1

Now, the characteristic equation of this difference equation, we can write M minus A is equal to 0. So, this is equation 9 here we can write M is equal to A. So, the general solution it is little different than the ODEs the general solution is xn would be equal to C 1 A to the power n where C 1 is a constant.

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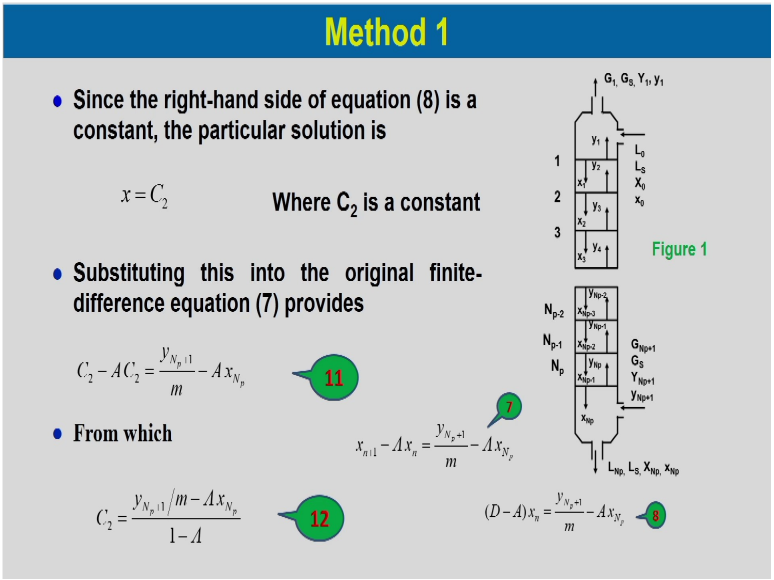

Method 1

- Since the right-hand side of equation (8) is a constant, the particular solution is

$x = C_2$ Where C_2 is a constant

- Substituting this into the original finite-difference equation (7) provides

$$C_2 - AC_2 = \frac{y_{N_p+1}}{m} - Ax_{N_p}$$ ⑪

- From which

$$x_{n+1} - Ax_n = \frac{y_{N_p+1}}{m} - Ax_{N_p}$$ ⑦

$$C_2 = \frac{y_{N_p+1}/m - Ax_{N_p}}{1 - A}$$ ⑫

$$(D - A)x_n = \frac{y_{N_p+1}}{m} - Ax_{N_p}$$ ⑧

Figure 1

Since the right hand side of the equation 8 is constant, the particular solution x should be equal to C 2. So, C 2 is another constant. So, this is equation 8 for this the right hand side of this equation is constant as we have said before and the particular solution for this x would be equal to C 2 and C 2 is constant. Now if you substitute this into the original finite difference equation. So, we will obtain this is the finite difference original equation. If we substitute this it would be C 2 minus A C 2 would be equal to y N p plus 1 divided by m minus A x N p. Now from this we can solve for C 2 C 2 would be equal to y N p plus 1 divided by m minus A x N p divided by 1 minus A. So, this is the solution for C 2. So, C 2 we know.

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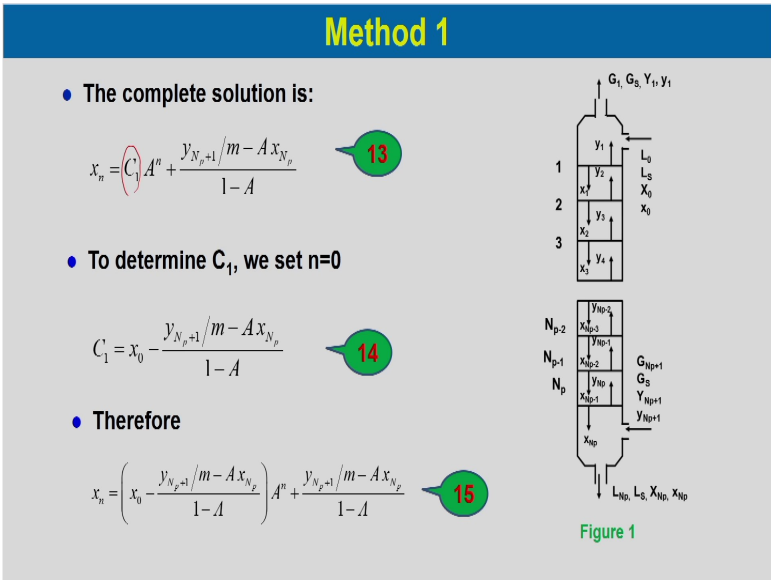

Method 1

- The complete solution is:

$$x_n = \boxed{C_1} A^n + \frac{y_{N_p+1}/m - A x_{N_p}}{1-A} \qquad ⑬$$

- To determine C_1, we set n=0

$$C_1 = x_0 - \frac{y_{N_p+1}/m - A x_{N_p}}{1-A} \qquad ⑭$$

- Therefore

$$x_n = \left(x_0 - \frac{y_{N_p+1}/m - A x_{N_p}}{1-A} \right) A^n + \frac{y_{N_p+1}/m - A x_{N_p}}{1-A} \qquad ⑮$$

Figure 1

And then the complete solution would be x n would be equal to C 1 A to the power N plus y N p plus 1 by m minus A x N p by 1 minus A. Now you have to obtain this C 1 to determine C 1 we set n is equal to 0 and then we can obtain C 1 would be equal to x not minus y N p plus 1 divided by m minus A x N p divided by 1 minus A.

So, once we get C 1 if you substitute in equation 13, we will obtain x n would be equal to x naught minus y N p plus 1 divided by m minus A x N p divided by 1 minus A into A to the power N plus y N p plus 1 by m minus A x N p divided by 1 minus A. So, this is the complete solution for this equation and then we have absorption factor A.

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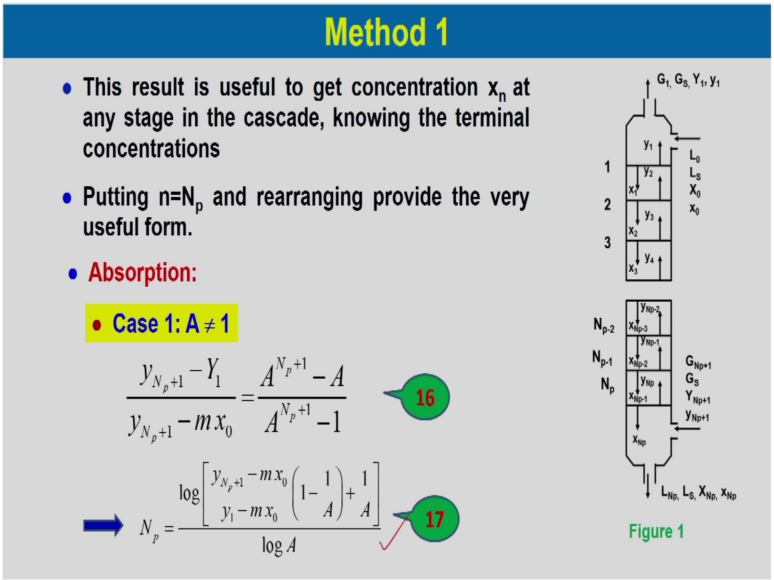

Method 1

- This result is useful to get concentration x_n at any stage in the cascade, knowing the terminal concentrations
- Putting $n=N_p$ and rearranging provide the very useful form.
- **Absorption:**
  - **Case 1: $A \neq 1$**

$$\frac{y_{N_p+1}-Y_1}{y_{N_p+1}-mx_0}=\frac{A^{N_p+1}-A}{A^{N_p+1}-1}$$ ⑯

$$\Rightarrow N_p=\frac{\log\left[\frac{y_{N_p+1}-mx_0}{y_1-mx_0}\left(1-\frac{1}{A}\right)+\frac{1}{A}\right]}{\log A}$$ ⑰

Figure 1

Now, we will consider this result is useful to get the concentration x n at any stage in the cascade knowing the terminal concentration. So, if you know the terminal concentration that is x N p which is coming out at the bottom and y N p plus 1 which is no inlet fed gas composition and x naught which is entering liquid if we know that we can calculate xn that is the composition at any tree inside the column.

Now, if we put n is equal to N p and rearrange this will provide very useful form of the equations let us put that and see what happens. Now in case of absorption we will consider few cases one is when absorption factor is not equal to 1. So, in that case we will obtain from that equation y N p plus 1 minus Y 1 divided by y N p plus 1 minus m x naught would be equal to A to the power N p plus 1 minus A divided by A to the power N p plus 1 minus 1. Now if we take the log from the both sides then and then rearrange we will get N p would be equal to log of y N p plus 1 minus m x naught divided by y minus m x naught whole into 1 minus 1 by A plus 1 by A divided by log A.

So, this is a equations we can derive when for a particular case when the absorption factor is not equal to 1 and we can calculate the number of stages required for a particular separations, knowing the terminal inlet concentration and the absorption factor.

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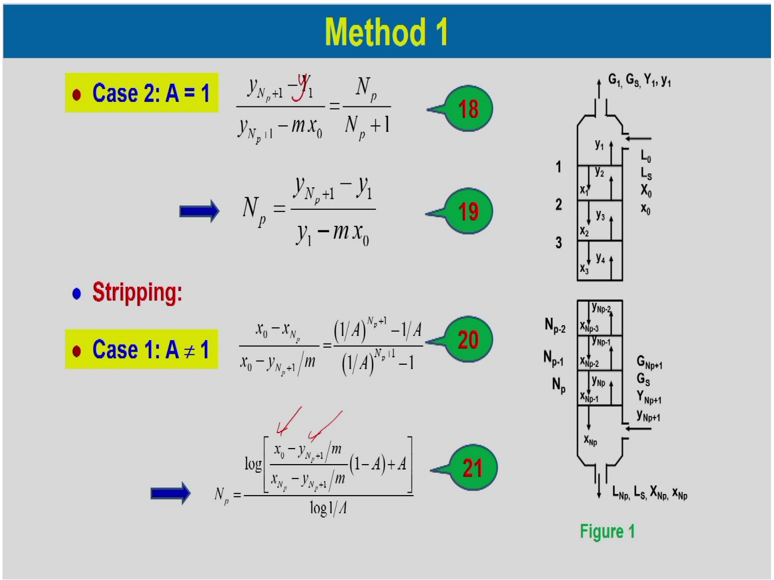

Method 1

- **Case 2: A = 1**

$$\frac{y_{N_p+1}-y_1}{y_{N_p+1}-mx_0}=\frac{N_p}{N_p+1} \quad (18)$$

$$\Rightarrow N_p=\frac{y_{N_p+1}-y_1}{y_1-mx_0} \quad (19)$$

- **Stripping:**

- **Case 1: A ≠ 1**

$$\frac{x_0-x_{N_p}}{x_0-y_{N_p+1}/m}=\frac{(1/A)^{N_p+1}-1/A}{(1/A)^{N_p+1}-1} \quad (20)$$

$$\Rightarrow N_p=\frac{\log\left[\frac{x_0-y_{N_p+1}/m}{x_{N_p}-y_{N_p+1}/m}(1-A)+A\right]}{\log 1/A} \quad (21)$$

Figure 1

So, now for another case when the absorption factor is 1 in that case we will get this equation as y N p plus 1 minus small y 1 divided by y N p plus 1 minus m x naught would be equal to N p divided by N p plus 1. If you just rearrange this equation we can get N p would be equal to y N p plus 1 minus y 1 divided by y 1 minus m x naught. So, in this case knowing the inlet concentration of gas and liquid and the exit concentration of the gas, that is the terminal concentration we can obtain the number of plates and when the absorption factor is 1.

Now, in case of stripping A naught equal to 1 in that case the equations we can derive that is x naught minus x N p divided by x naught minus y N p plus 1 divided by m which would be equal to 1 by A to the power N p plus 1 minus 1 by A whole divided by 1 by A to the power N p plus 1 minus 1. Here if we take the log then we would obtain this relation where terminal concentrations and known concentration and the knowing the values of absorption factor and all these are non factor. So, we can calculate the number of plates required for the stripping as well.

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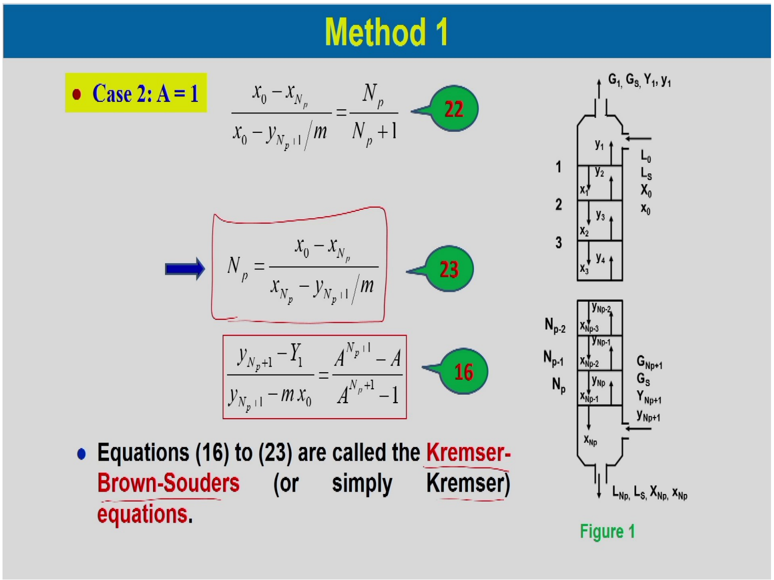

Method 1

- Case 2: A = 1

$$\frac{x_0 - x_{N_p}}{x_0 - y_{N_p+1}/m} = \frac{N_p}{N_p+1} \quad (22)$$

$$N_p = \frac{x_0 - x_{N_p}}{x_{N_p} - y_{N_p+1}/m} \quad (23)$$

$$\frac{y_{N_p+1} - Y_1}{y_{N_p+1} - m x_0} = \frac{A^{N_p+1} - A}{A^{N_p+1} - 1} \quad (16)$$

- Equations (16) to (23) are called the Kremser-Brown-Souders (or simply Kremser) equations.

Figure 1

Similarly, like in absorption in case of stripping absorption factor is 1, we will get the similar way it would be x naught minus x N p divided by x naught minus y N p plus 1 divided by m would be equal to N p divided by N p plus 1. If you rearrange to calculate the N p. So, from here we can write N p would be equal to x naught minus x N p divided by x N p minus y N p plus 1 divided by m. So, using this relations for stripping when the absorption factories 1 we can calculate the number of stages for stripper.

So, this equation 16 which we have derived here and then the series of equations for different cases we have considered special cases, when absorption factor is 1 and when it is not equal to 1 both for absorption and as well as for the stripping these equations all together are called Kremser Brown-Souders or simply Kremser equations. So, this equations are very much useful to calculate number of stages analytically in some special cases where the solute transfer is negligible or for dilute solutions.

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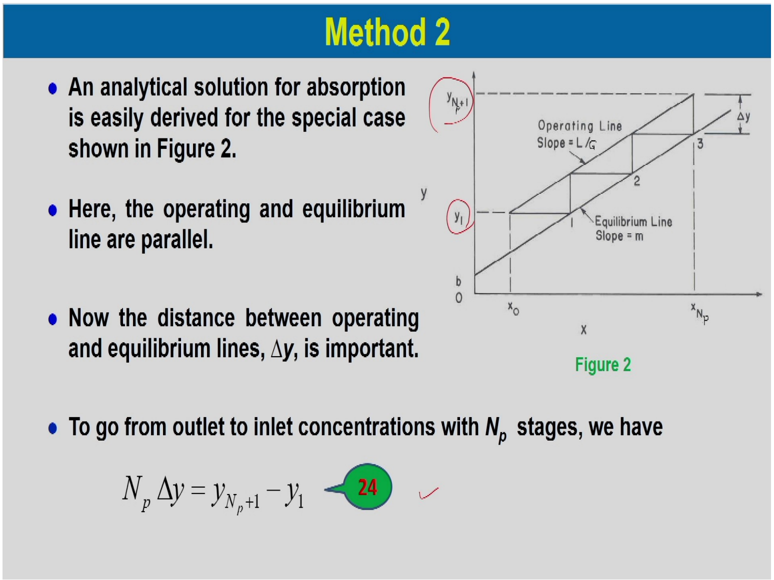

Method 2

- An analytical solution for absorption is easily derived for the special case shown in Figure 2.
- Here, the operating and equilibrium line are parallel.
- Now the distance between operating and equilibrium lines, Δy, is important.

Figure 2

- To go from outlet to inlet concentrations with N_p stages, we have

$$N_p \Delta y = y_{N_p+1} - y_1$$ ㉔

The second method which we are going to discuss here in analytical solution for absorption in which can be easily derive, we can just look into the graphs for a special case we can plot them and from there we can also obtain the similar for some special cases where we can calculate the number of stages.

Here the operating and the equilibrium line are parallel and now the distance between the operating and the equilibrium lines that is delta y that is also important. So, what is the driving force for the mass transfer over here is delta y and that is constant over here. So, since there parallel lines. So, to go from outlet to the inlet concentration and we have N p number of stages then we can write N p into delta y would be equal to y N p plus 1 minus y 1 because these are the no terminal concentration. So, this is the change of concentration from y N p plus 1 minus y 1. This is so, and it is happened throughout N p number of stages. So, N p into delta y would be equal to y N p plus 1 minus y 1. So, this is one equations we can write from this graph.

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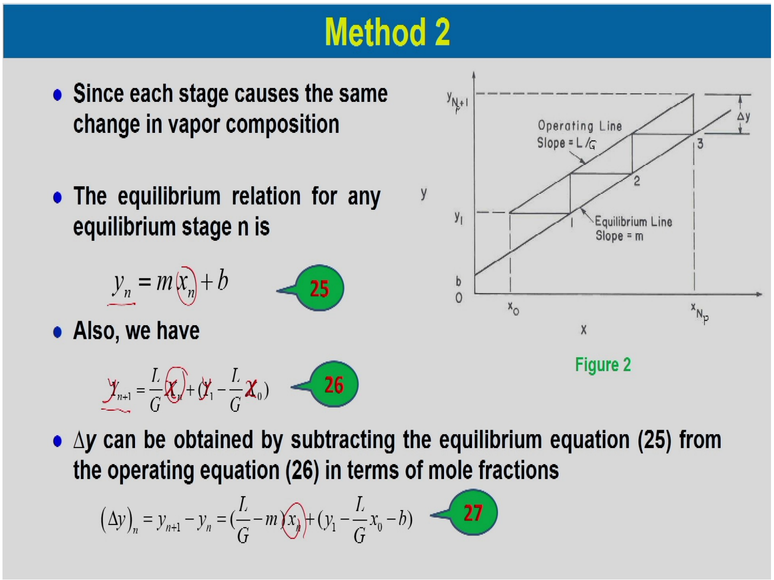

Method 2

- Since each stage causes the same change in vapor composition
- The equilibrium relation for any equilibrium stage n is

$$y_n = m x_n + b \quad (25)$$

- Also, we have

$$y_{n+1} = \frac{L}{G} x_n + \left(y_1 - \frac{L}{G} x_0\right) \quad (26)$$

- Δy can be obtained by subtracting the equilibrium equation (25) from the operating equation (26) in terms of mole fractions

$$(\Delta y)_n = y_{n+1} - y_n = \left(\frac{L}{G} - m\right) x_n + \left(y_1 - \frac{L}{G} x_0 - b\right) \quad (27)$$

Figure 2

Since each stage causes the same change in vapor composition. So, the equilibrium relation for any equilibrium stage n which can be written as yn would be equal to m x n plus b.

Now, we know the operating line equations, we have derived before this y n plus 1 would be equal to L by G X n plus Y 1 minus L by G X naught. So, delta y can be obtained by subtracting the equilibrium equation 25 from the operating line equation 26 in terms of the mole fractions. So, here it will be no mole fractions this is small y and small x. So, we can write no delta y n would be equal to y n plus 1. So, by subtracting equation 25 from equation 26. So, 26 is y N plus 1 minus y n would be equal to L by G minus m into xn. So, from this 2 this will be x n plus y 1 minus L by G x naught minus b.

So, this is the equation of delta y n for any stage n.

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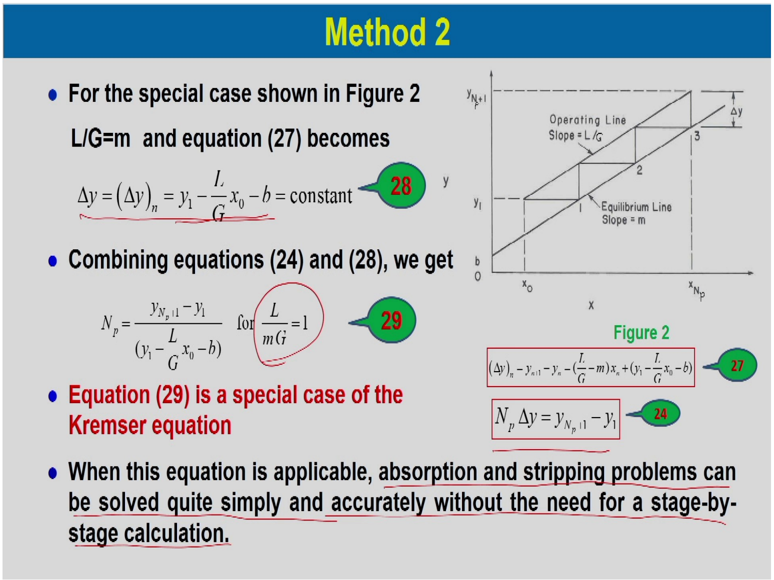

Method 2

- For the special case shown in Figure 2

  L/G=m and equation (27) becomes

  $$\Delta y = (\Delta y)_n = y_1 - \frac{L}{G}x_0 - b = \text{constant}$$ 28

- Combining equations (24) and (28), we get

  $$N_p = \frac{y_{N_p+1} - y_1}{(y_1 - \frac{L}{G}x_0 - b)} \quad \text{for } \frac{L}{mG} = 1$$ 29

- Equation (29) is a special case of the Kremser equation

- When this equation is applicable, absorption and stripping problems can be solved quite simply and accurately without the need for a stage-by-stage calculation.

Figure 2

$$(\Delta y)_n = y_{n+1} - y_n = (\frac{L}{G} - m)x_n + (y_1 - \frac{L}{G}x_0 - b)$$ 27

$$N_p \Delta y = y_{N_p+1} - y_1$$ 24

Now, for the special case shown over here L by G is equal to m that is L by mG is equal to 1. So, then equation 27 it becomes this equation this now becomes delta y would be equal to delta y n would be equal to y 1 minus L by G x naught minus b is equal to constant.

So, combining this equation 24 and 28 we get this is 24 and then 28 this equation and this equation if we combine both these 2 equations 24 and 28 we can write Np would be equal to N p plus 1 minus y 1 divided by y 1 minus L by G x not minus b and this is true or it is valid only for L by mG is equal to 1.

So, this equation 29 is a special case of the Kremser equation as we have derived before. So, this is another equations we can derived from the graph. When this equations is applicable the absorption and stripping problems can be solved simply and accurately without need of stage by stage calculation. So, from here we can easily calculate the number of stages required using analytical equations.

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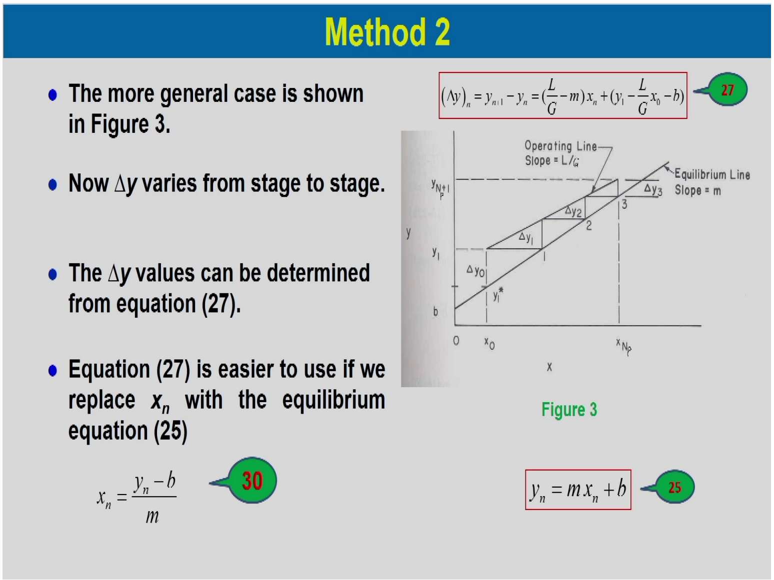

Method 2

- The more general case is shown in Figure 3.
- Now Δy varies from stage to stage.
- The Δy values can be determined from equation (27).
- Equation (27) is easier to use if we replace x_n with the equilibrium equation (25)

$$\boxed{(\Delta y)_n = y_{n+1} - y_n = (\frac{L}{G} - m)x_n + (y_1 - \frac{L}{G}x_0 - b)}$$ 27

$$x_n = \frac{y_n - b}{m}$$ 30

$$\boxed{y_n = mx_n + b}$$ 25

Figure 3

Now, let us consider the more general case where delta y the driving force is not constant throughout. So, it is varying, but the equilibrium curve and the operating line curve they are straight line. So, delta y varies from stage to stage. The delta y values can be determined from equation 27. Equation 27 is easier to use no this is equation 27 and from there we can calculate stage to stage no change of concentration from their equation twenty seven is easier to use if we replace X n with the equilibrium relation or equilibrium equation. So, I over here is y n is equal to m xn plus b. So, if you substitute xn then here xn would be from this equation we can write y n minus b by m.

So, if you substitute this equation in equation 27 we would obtain this equation.

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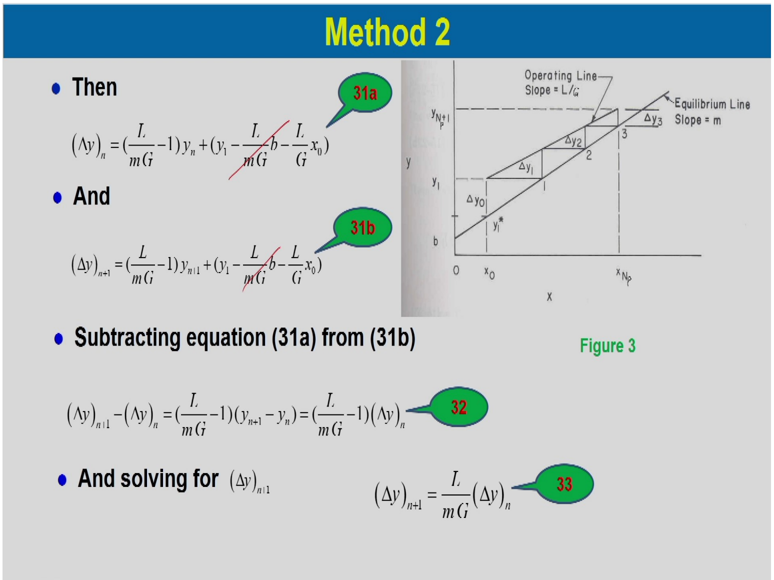

Method 2

- Then

$$(\Delta y)_n = \left(\frac{L}{mG} - 1\right) y_n + \left(y_1 - \cancel{\frac{L}{mG} b} - \frac{L}{G} x_0\right)$$

31a

- And

$$(\Delta y)_{n+1} = \left(\frac{L}{mG} - 1\right) y_{n+1} + \left(y_1 - \cancel{\frac{L}{mG} b} - \frac{L}{G} x_0\right)$$

31b

- Subtracting equation (31a) from (31b)

$$(\Delta y)_{n+1} - (\Delta y)_n = \left(\frac{L}{mG} - 1\right)(y_{n+1} - y_n) = \left(\frac{L}{mG} - 1\right)(\Delta y)_n$$

32

- And solving for $(\Delta y)_{n+1}$

$$(\Delta y)_{n+1} = \frac{L}{mG} (\Delta y)_n$$

33

Figure 3

Delta yn would be equal to L by m G minus 1 into yn plus y 1 minus L by mG into b minus L by G into x naught and also we can write for any other stage in plus 1 we can write delta y N plus 1 would be equal to L by mG minus 1 y N plus 1 plus y 1 minus L mG into b minus L by G x naught. Now from this equation 31 a and 31 b we can also write if we just subtract 31 a from 31 b. So, this 2 will cancel out and we will be having delta y n plus 1 minus delta y n would be equal to L by mG minus 1 into y n plus 1 minus y n we should be equal to L by mG minus 1 into delta yn. So, this is equation 32.

And now if you solve from this equation for delta y n plus 1 we will have L by mG into delta y n.

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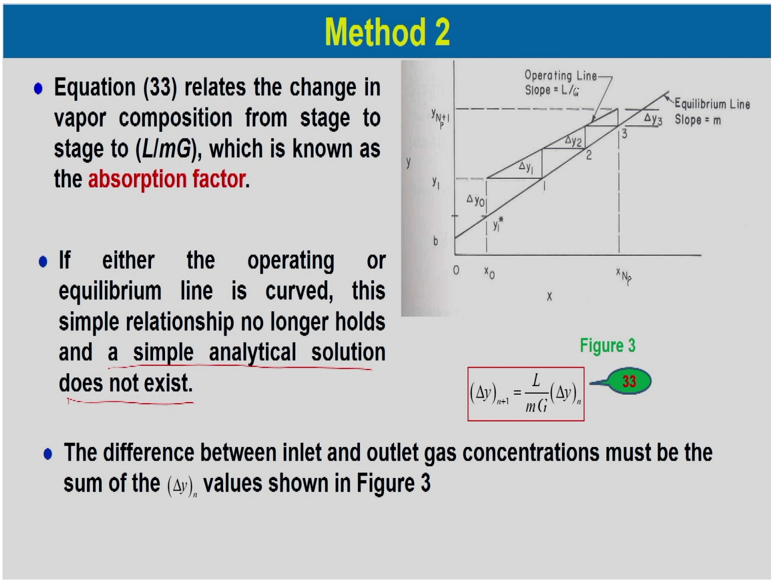

Method 2

- Equation (33) relates the change in vapor composition from stage to stage to (*L*/*mG*), which is known as the absorption factor.
- If either the operating or equilibrium line is curved, this simple relationship no longer holds and a simple analytical solution does not exist.

Figure 3

$$(\Delta y)_{n+1} = \frac{L}{mG}(\Delta y)_n \qquad (33)$$

- The difference between inlet and outlet gas concentrations must be the sum of the $(\Delta y)_n$ values shown in Figure 3

Now, this equation 33 relates the change in vapor composition from stage to stage to L by m G which is known as the absorption factor and it varies. So, delta y N plus 1 would be equal to L by mG delta y n. If either the operating or the equilibrium line is called this simple relationship no longer holds and a simple analytical solutions does not exist. The difference between the inlet and outlet gas concentrations must be the sum of delta y n values shown in this figure.

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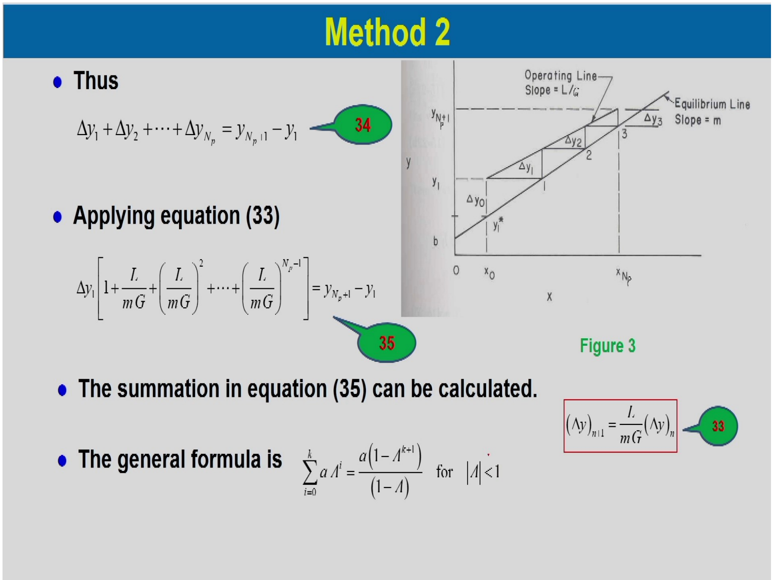

Method 2

- Thus

$$\Delta y_1 + \Delta y_2 + \cdots + \Delta y_{N_p} = y_{N_p+1} - y_1 \qquad (34)$$

- Applying equation (33)

$$\Delta y_1 \left[1 + \frac{L}{mG} + \left(\frac{L}{mG} \right)^2 + \cdots + \left(\frac{L}{mG} \right)^{N_p - 1} \right] = y_{N_p+1} - y_1 \qquad (35)$$

Figure 3

- The summation in equation (35) can be calculated.

$$(\Delta y)_{n+1} = \frac{L}{mG}(\Delta y)_n \qquad (33)$$

- The general formula is $\sum_{i=0}^{k} aA^i = \frac{a(1-A^{k+1})}{(1-A)}$ for $|A| < 1$

So, we can write delta y 1 plus delta y 2 and up to delta y N p would be equal to y N p plus 1 minus y 1. So, this would be some would be constant. So, with this difference inlet to outlet concentration.

Now, applying equation 33, we can write delta y 1 into 1 plus L by mG plus L by mG square and so, on it will be plus L by mG to the power N p minus 1 would be equal to y N p plus 1 minus y 1. Now summation in equation 35 can be calculated and the general formula we can use is summation over i is equal to 0 to k a to the power I would be equal to a into 1 minus A to the power k plus 1 divided by 1 minus A for no mode of A less than 1.

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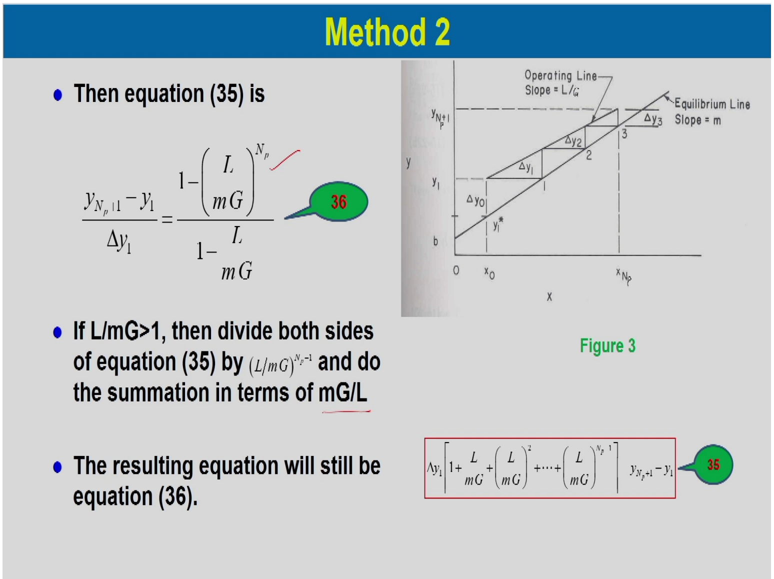

Method 2

- Then equation (35) is

$$\frac{y_{N_p+1}-y_1}{\Delta y_1}=\frac{1-\left(\frac{L}{mG}\right)^{N_p}}{1-\frac{L}{mG}}$$

36

- If L/mG>1, then divide both sides of equation (35) by $(L/mG)^{N_p-1}$ and do the summation in terms of mG/L
- The resulting equation will still be equation (36).

$$\boxed{\Delta y_1\left[1+\frac{L}{mG}+\left(\frac{L}{mG}\right)^2+\cdots+\left(\frac{L}{mG}\right)^{N_p\ 1}\right]\ y_{N_p+1}-y_1}$$

35

Figure 3

So, then from this equation 35 we can write y N p plus 1 minus y 1 divided by delta y 1 would be equal to 1 minus L by mG to the power N p divided by 1 minus L by mG. If the L by mG is greater than 1 then divide both sides of equation 35 by L by mG to the power N p minus 1 and do the summation in terms of mG by L. So, if we do so, the resulting equations will still be equation 36 the same equations we will obtain.

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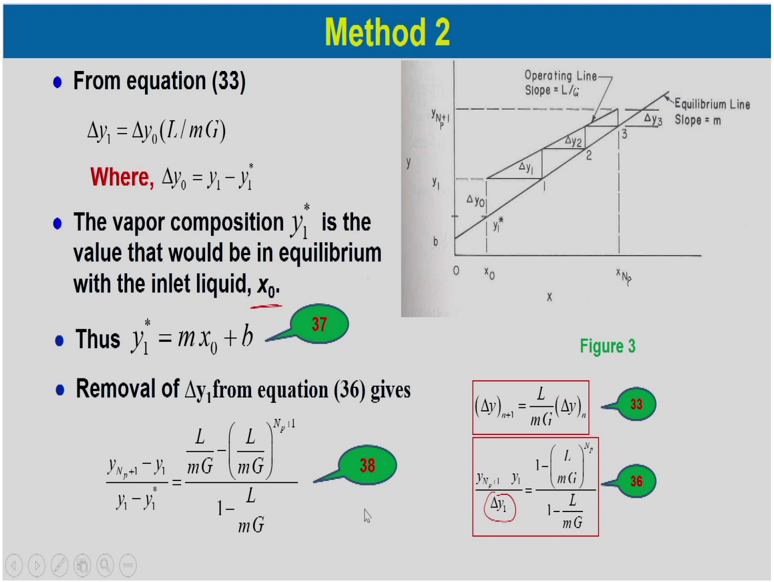

Method 2

- From equation (33)

$$\Delta y_1 = \Delta y_0 (L/mG)$$

Where, $\Delta y_0 = y_1 - y_1^*$

- The vapor composition y_1^* is the value that would be in equilibrium with the inlet liquid, x_0.
- Thus $y_1^* = m x_0 + b$ ㊲
- Removal of Δy_1 from equation (36) gives

$$\frac{y_{N_p+1} - y_1}{y_1 - y_1^*} = \frac{\frac{L}{mG} - \left(\frac{L}{mG}\right)^{N_p+1}}{1 - \frac{L}{mG}} \quad (38)$$

Figure 3

$$(\Delta y)_{n+1} = \frac{L}{mG}(\Delta y)_n \quad (33)$$

$$\frac{y_{N_p+1} - y_1}{\Delta y_1} = \frac{1 - \left(\frac{L}{mG}\right)^{N_p}}{1 - \frac{L}{mG}} \quad (36)$$

Now, from equation 33 we can write delta y 1 would be equal to delta y naught into L by m G. And here delta y not would be equal to y 1 minus y 1 star it is y 1 the vapor composition y 1 star is the value that would be in equilibrium with the inlet liquid x naught. So, these are equilibrium compositions. So, we can write y 1 star would be equal to m x naught plus b. Removal of delta y 1 from equation 36 know which will give this is equation 36 and we would like to remove delta y 1 over here which is given who were here. So, we can write y N p plus 1 minus y 1 divided by y 1 minus y 1 star, which is given over here would be equal to L by m G minus L by mG to the power N p plus 1 divided by 1 minus L by mG. So, this is equation 38.

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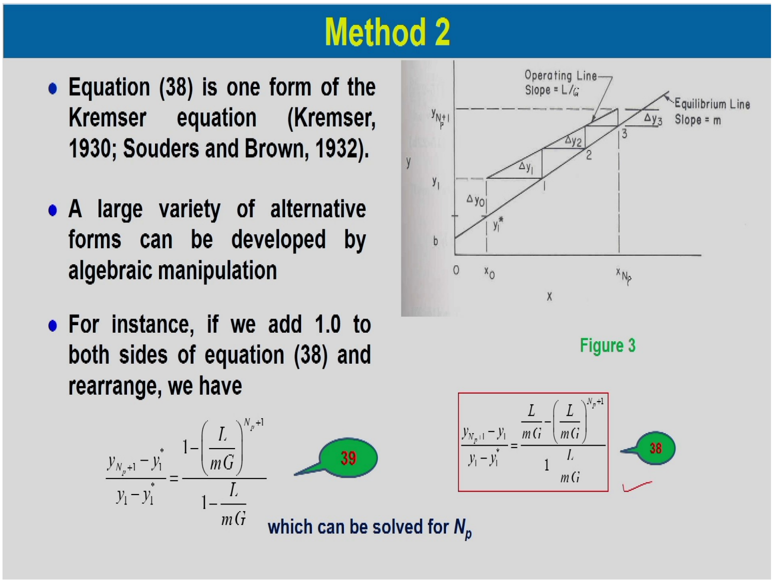

Method 2

- Equation (38) is one form of the Kremser equation (Kremser, 1930; Souders and Brown, 1932).
- A large variety of alternative forms can be developed by algebraic manipulation
- For instance, if we add 1.0 to both sides of equation (38) and rearrange, we have

Figure 3

$$\boxed{\frac{y_{N_p+1}-y_1}{y_1-y_1^*}=\frac{\frac{L}{mG}-\left(\frac{L}{mG}\right)^{N_p+1}}{1-\frac{L}{mG}}} \quad (38)$$

$$\frac{y_{N_p+1}-y_1^*}{y_1-y_1^*}=\frac{1-\left(\frac{L}{mG}\right)^{N_p+1}}{1-\frac{L}{mG}} \quad (39)$$

which can be solved for N_p

And then the equation 38 is one form of the Kremser equation and it is also called Kremser Brown Souder equation. this thirty equation 38 which no similar equations we have developed with different manipulations we can develop different and algebraic equations. For instance if we add one to both sides of the equation 38 and then we rearrange we will get no this equations; y N p plus 1 minus y 1 star divided by y 1 minus y 1 star would be equal to 1 minus L by mG to the power N p plus 1 divided by 1 minus L by mG.

So, many forms of these equations after mathematical manipulations can be obtained and then we can which can be solved for N p.

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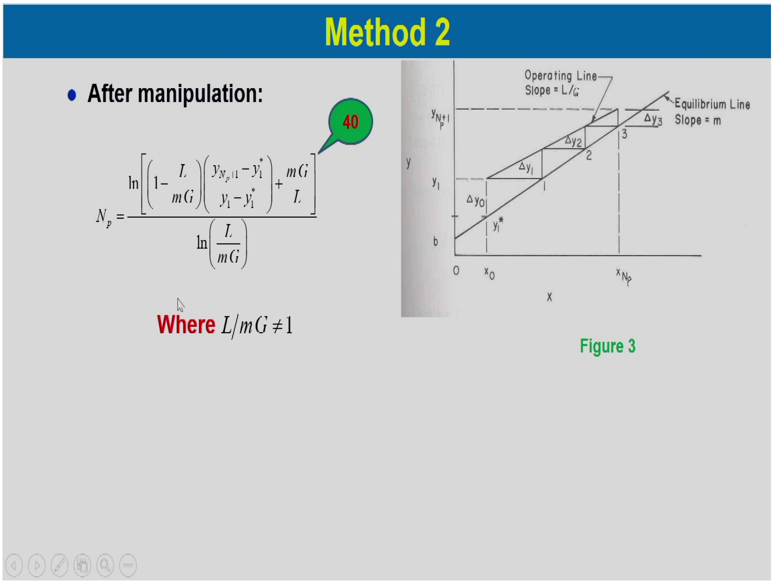

Method 2

- **After manipulation:**

$$N_p = \frac{\ln\left[\left(1-\frac{L}{mG}\right)\left(\frac{y_{N_p+1}-y_1^*}{y_1-y_1^*}\right)+\frac{mG}{L}\right]}{\ln\left(\frac{L}{mG}\right)}$$

40

Where $L/mG \neq 1$

Figure 3

If we do the manipulations the N p equation should be N p would be equal to L n, 1 minus L by mG whole into y N p plus 1 minus y 1 star divided by y 1 minus y 1 star plus mG by L divided by L n L by mG. Now L by mG which is absorption factor is not equal to 1 in this case.

So, this is variable absorption factor where L by m G is not equal to 1. So, in this case we can obtain this equation.

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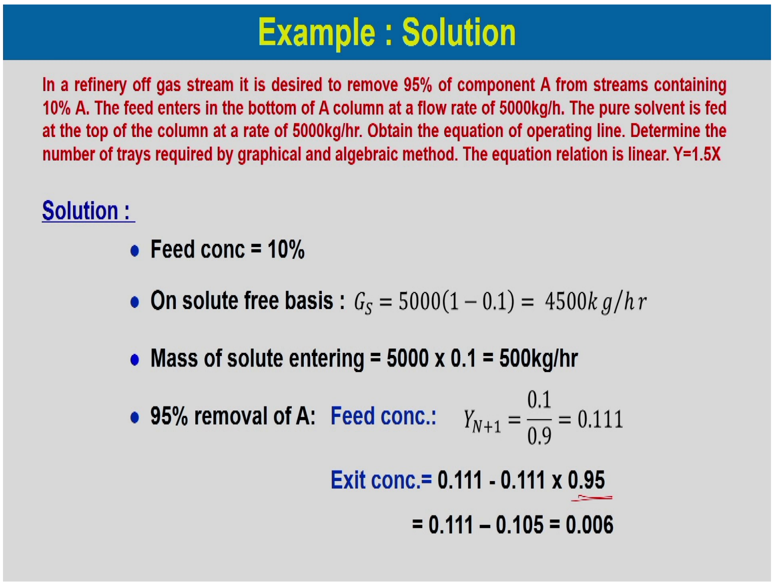

Example : Solution

In a refinery off gas stream it is desired to remove 95% of component A from streams containing 10% A. The feed enters in the bottom of A column at a flow rate of 5000kg/h. The pure solvent is fed at the top of the column at a rate of 5000kg/hr. Obtain the equation of operating line. Determine the number of trays required by graphical and algebraic method. The equation relation is linear. Y=1.5X

Solution :

- Feed conc = 10%
- On solute free basis : $G_S = 5000(1-0.1) = 4500kg/hr$
- Mass of solute entering = 5000 x 0.1 = 500kg/hr
- 95% removal of A: Feed conc.: $Y_{N+1} = \frac{0.1}{0.9} = 0.111$

Exit conc.= 0.111 - 0.111 x 0.95

= 0.111 – 0.105 = 0.006

Now, let us take an example how to calculate the number of stages analytically using the Kremser equation. In a refinery off gas stream it is desired to remove 95 percent of component A, from streams containing 10 percent A. The feed enters in the bottom of A of the column at a flow rate of 5000 kg per hour the pure solvent is fed at the top of the column at a rate of 5000 kg per hour. Now obtain the equation of operating line then determine the number of trays required by graphical and algebraic method the equation relation is linear that is Y is equal to 1.5 X.

So, linear equation now let us see the solution for this, the feed concentration is given 10 percent of a component A. Now on solute free basis if we calculate G s would be equal to 5000 into 1 minus 0.1. So, it is 4500 kg per hour. So, that is the G s mass of solute entering is 5000 into 0.1 it is 500 kg per hour and 95 percent removal of A is required. Feed concentration molar ratio unit we can calculate y N plus 1 would be equal to 0.1 by 0.9 which is equal to 0.111. Now exit concentration is no 0.111 is the inlet minus 0.111 into 0.95 because we have to remove 95 percent. So, this is the exit concentration would come around 0.006.

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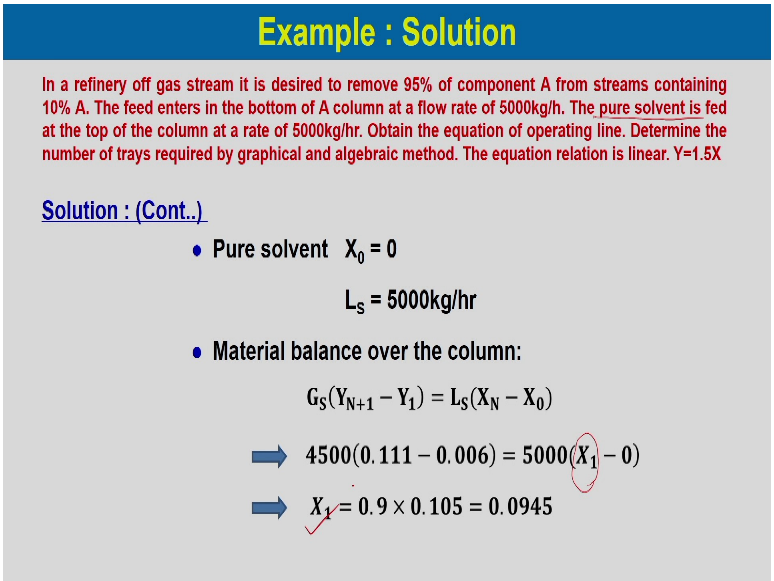

Example : Solution

In a refinery off gas stream it is desired to remove 95% of component A from streams containing 10% A. The feed enters in the bottom of A column at a flow rate of 5000kg/h. The pure solvent is fed at the top of the column at a rate of 5000kg/hr. Obtain the equation of operating line. Determine the number of trays required by graphical and algebraic method. The equation relation is linear. Y=1.5X

Solution : (Cont..)

- Pure solvent $X_0 = 0$

  $L_S = 5000\text{kg/hr}$

- Material balance over the column:

$$G_S(Y_{N+1} - Y_1) = L_S(X_N - X_0)$$

$$\Rightarrow 4500(0.111 - 0.006) = 5000(X_1 - 0)$$

$$\Rightarrow X_1 = 0.9 \times 0.105 = 0.0945$$

So, now it is given that the solvent which enters is a pure solvent over here which is fed solvent is pure solvent and so, X naught would be 0 and L s would be 5000 kg per hour which is given. So, because since it is solute free basis the L s is 5000 kg per hour which is given and material balance over the column if you do Gs into Y N plus 1 minus Y 1 would be equal to L s X N minus X naught. So, if we substitute we can calculate the exit mole ratio capital X 1 would be equal 0.9 into 0.105 which is equal to 0.0945. So, x 1 now is known to us.

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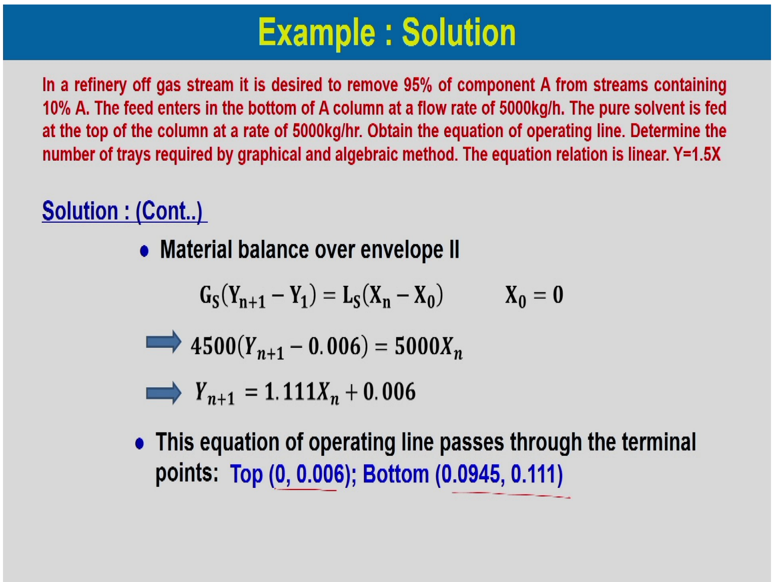

Example : Solution

In a refinery off gas stream it is desired to remove 95% of component A from streams containing 10% A. The feed enters in the bottom of A column at a flow rate of 5000kg/h. The pure solvent is fed at the top of the column at a rate of 5000kg/hr. Obtain the equation of operating line. Determine the number of trays required by graphical and algebraic method. The equation relation is linear. Y=1.5X

Solution : (Cont..)

- Material balance over envelope II

$$G_S(Y_{n+1} - Y_1) = L_S(X_n - X_0) \qquad X_0 = 0$$

$$\Rightarrow 4500(Y_{n+1} - 0.006) = 5000X_n$$

$$\Rightarrow Y_{n+1} = 1.111X_n + 0.006$$

- This equation of operating line passes through the terminal points: Top (0, 0.006); Bottom (0.0945, 0.111)

Now, if you do the material balance then we can get G s Y n plus 1 minus Y 1 is equal to L s X n minus X naught. So, Xn and Xn plus 1 at any tray no inside the column and your x naught is 0.

So, we can calculate no or we can get a relations between Y n plus 1 and x n. So, it will be y n plus 1 would be equal to 1.111 Xn plus 0.006. This is the equation of operating line which passes through the terminal points. So, top is 0 and 0.006 this is the point at top and the bottom is at 0.0945 and 0.111 this is at the bottom the terminal points at the bottom at the top.

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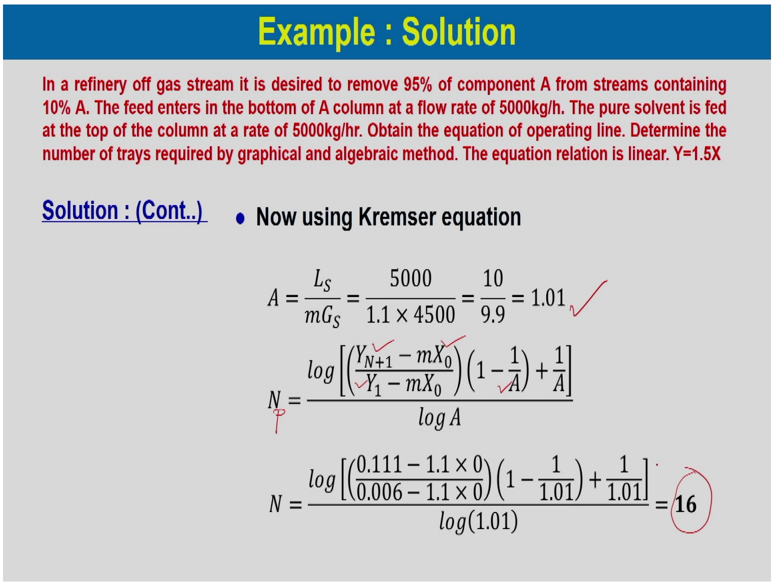

Example : Solution

In a refinery off gas stream it is desired to remove 95% of component A from streams containing 10% A. The feed enters in the bottom of A column at a flow rate of 5000kg/h. The pure solvent is fed at the top of the column at a rate of 5000kg/hr. Obtain the equation of operating line. Determine the number of trays required by graphical and algebraic method. The equation relation is linear. Y=1.5X

Solution : (Cont..)

- Now using Kremser equation

$$A = \frac{L_S}{mG_S} = \frac{5000}{1.1 \times 4500} = \frac{10}{9.9} = 1.01 \checkmark$$

$$\underset{p}{N} = \frac{log\left[\left(\frac{Y_{N+1} - mX_0}{Y_1 - mX_0}\right)\left(1 - \frac{1}{A}\right) + \frac{1}{A}\right]}{log\,A}$$

$$N = \frac{log\left[\left(\frac{0.111 - 1.1 \times 0}{0.006 - 1.1 \times 0}\right)\left(1 - \frac{1}{1.01}\right) + \frac{1}{1.01}\right]}{log(1.01)} = \boxed{16}$$

Now, if we use the Kremser equation over here, the absorption factor we can calculate L s by m Gs which is 5000 divided by 1.1 into 4500 which is 10 by 9.9 which is equal to 1. 0 1 0 1. So, absorption factor is known to us now if we this is np the number of no stages would be equal to log y N p plus 1 minus m X naught divided by Y 1 minus m X naught into 1 minus 1 by a plus 1 by a now if you substitutes the values which are this is given this is 0 and y 1 we have calculated 0.006 absorption factor we have calculated now we substitute over here we will obtain 16 number of stages required for this separation.

So, using the Kremser equations we can simply calculate the number of stages required for a particular problem, but we have to be very cautious about the concentration of the salute which is transferred and how far it is valid to use the Kremser equations. If the solute transfer is appreciable we should not use the Kremser equation, but this gives only the raw estimate if you use the equation in case of change of concentration for change of molar flow rate of the gas and liquid. with this we conclude for this modules module 2, where we have considered the fundamentals of inter phase mass transfer and then we have developed the equilibrium stages we will continue our discussion on mass transfer in the next class.